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ARTICLE

Dynamic Fatigue Reliability Analysis of Transmission Gear Considering Failure Dependence

Yonghua Li[1,*], Xiaoning Bai[2], Shanshan Shi[1] and Shenxuan Wang[1]

[1]College of Locomotive and Rolling Stock Engineering, Dalian Jiaotong University, Dalian, 116028, China

[2]School of Mechanical Engineering, Dalian Jiaotong University, Dalian, 116028, China

[*]Corresponding Author: Yonghua Li. Email: yonghuali@163.com

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ABSTRACT

Multiple failure modes and strength degradation are usually inherent in the gear transmission system, which brings new challenges for conducting fatigue reliability analysis and design. This paper proposes a novel dynamic fatigue reliability analysis method for failure dependence and strength degradation based on the combination of the Copula function and Gamma process. Firstly, the dynamic simulation model of the gear transmission system is established to obtain the dynamic stress-time history. The Gamma process is then used to describe the strength degradation to establish the dynamic stress-strength interference model. The marginal distribution functions of tooth contact fatigue and dedendum bending fatigue are calculated respectively based on the dynamic interference model. Finally, the joint distribution of the two failure modes can be obtained by the t-Copula function to characterize the failure dependence, and so the dynamic fatigue reliability considering failure dependence can be estimated. The effectiveness of the proposed method is illustrated with examples. The results reveal the temporal law of reliability and the effects of failure dependence on dynamic fatigue reliability.

KEYWORDS

Gear transmission system; failure dependence; dynamic fatigue reliability; the Gamma process; the Copula function

1 Introduction

The gear transmission system has the advantages of compact structure, high carrying capacity, and high efficiency, and it is one of the most widely used transmission modes in the modern industry. Although various fields have different requirements for using the gear transmission system, the most fundamental requirement is high stability and high reliability [1]. There are many potential failure modes in a gear transmission system due to uncertain factors such as load cases, geometric parameters, and material properties [2–5]. Furthermore, the above uncertainties have homology, and there will be different degrees of the dependence relationship between failure modes [6]. When one failure occurs, the process of another failure mode will be affected. In the design process of the gear transmission system, whether to consider the failure dependence has a significant impact on the reliability. Meanwhile, the structural strength will gradually degenerate with increased service time, which aggravates gear failure probability growth, so the reliability shows a downward trend. Therefore, it is necessary to introduce failure dependence and dynamic reliability modeling in the whole design process to provide a more accurate reliability evaluation for gear transmission system design.

Many scholars have studied reliability considering failure dependence and obtained outstanding achievements. Many traditional methods are used to analyze the failure correlation of mechanical components, most of these methods are based on correlation coefficients. Ditlevsen [7,8] given the narrow reliability bounds theory through strict reasoning and proof, is widely accepted by many scholars. This theory can solve mechanical structure reliability with multiple failure modes, which is undoubtedly a milestone. Nevertheless, this method depends on the correlation coefficient, and it is difficult to achieve accurate calculations with the increase of failure modes number. Besides, the calculation result of this method is only an interval, which cannot give accurate reliability. Based on Ditlevsen's theory, Low et al. [9] simplified the calculation of fault probability boundary by programming, and effectively expanded the application value of narrow reliability bounds theory. Wang et al. [10] introduced the dependence function to quantify the failure dependence among the components, and the method is applied to the parallel system. The research results focused on the influence of the dependence relationship on structural reliability. Although the above research effectively solves the problem of reliability solutions under the coexistence of multiple failure modes, the description of dependency based on correlation coefficient has the following limitations. First, the disadvantage of the linear correlation coefficient is that it may lose the possibility of the theoretical solution, because it cannot determine the joint probability density function. Second, the difficulty of solving the linear correlation coefficient with the increase of failure modes is insurmountable. Most importantly, the linear correlation coefficient can only reveal the first-order linear relationship, but it cannot describe the more complex and higher-order actual relationship between failure modes [11,12]. In recent years, the applicability of the Copula function in solving the problem of correlation is favored by many scholars and has achieved good results. Tang et al. [13] studied the differences of two-dimensional variable correlation models constructed by different Copula functions and analyzed their influence on reliability analysis results. Pan et al. [14] established the dependence relationship between two output performance characteristics of a structural system based on the Copula function. Eryilmaz et al. [15] developed a Copula-based reliability modeling method that uses the multivariate Copula. Because the Copula function only uses one parameter to describe the dependence relationship between variables, it is not suitable for describing a high-dimensional situation. Therefore, some scholars transform multivariate dependence into binary dependence, making the Copula function more practical [16–18]. Shen et al. [19] evaluated the structural reliability under various failure modes by using Rosenblatt transform and Monte Carlo simulation method. This research provides a new way to solve the multivariate reliability problem.

In this paper, the Copula function is used to characterize the dependence relationship of failure modes in solving the fatigue reliability of the gear transmission system. The Gamma process is introduced to describe the strength degradation, and combined with the dynamic stress-time history to establish the dynamic fatigue reliability model. The research results reveal the influence of failure dependence and strength degradation on gear transmission systems reliability, which has an excellent guiding significance for the design and maintenance of gears.

This paper has four parts. The Copula function theory is introduced in Section 2, and the reliability model of the gear transmission system considering failure dependence is constructed. The strength degradation and dynamic stress-strength interference model are detailed in Section 3. The Gamma process is used to reveal the law of strength degradation. Section 4 proposes an engineering practice example to validate the proposed approach. It is shown to be more in line with the engineering practice than the reliability calculation results without failure dependence.

2 Reliability Model of the Gear Transmission System

The coexistence of multiple failure modes is an essential feature of the mechanical structure. There is a different degree of dependence relationship that exists between various failure modes. For example, there is a positive relationship between fatigue crack growth and fatigue pitting of gear teeth. The failure dependence will seriously affect the structure safety function and make the reliability analysis and modeling more complex. According to the mechanical principle and gear transmission characteristics, the reliability model under different failure modes is established. The reliability of the gear transmission system considering the failure dependence is researched.

2.1 Reliability Model Based on Dedendum Bending Fatigue

It is easy for the dedendum to produce fatigue crack in the transmission process, resulting in the bending fatigue fracture. The dedendum bending stress of gears can be calculated as [20]

$$\sigma_F = Y_{Fa} Y_{Sa} Y_e Y_\beta \frac{F_t}{bm_n} K_A K_V K_{F\beta} K_{F\alpha} \tag{1}$$

where F_t is the rated tangential tooth force at the transverse pitch, b is the active face width, m_n is the normal module, Y_{Fa} is the tooth form factor, Y_{Sa} is the bending stress concentration coefficient, Y_e is the contact ratio factor, Y_β is the helix angle coefficient, K_A is the work condition coefficient, K_V is the dynamic load coefficient, $K_{F\beta}$ is the longitudinal load distribution coefficient, $K_{F\alpha}$ is the load distribution coefficient.

Dedendum bending fatigue strength of gears can be calculated as

$$\sigma_{FS} = \sigma_{F\lim} Y_{ST} Y_{NT} Y_{\delta relT} Y_{RrelT} Y_X \tag{2}$$

where $\sigma_{F\lim}$ is the experimental gear bending fatigue strength, Y_{ST} is the experimental gear tooth stress concentration coefficient, Y_{NT} is the life coefficient, $Y_{\delta relT}$ is the relative sensitive coefficient, Y_{RrelT} is the relative surface condition coefficient, Y_X is the size coefficient.

According to the stress-strength interference theory, the performance function of dedendum bending fatigue can be expressed as

$$g_1 (X_1) = \sigma_{FS} - \sigma_F \tag{3}$$

If the value of the performance function is negative, the structure is a failure. On the contrary, if the value is positive, the structure is safe. The border between the negative and positive domains is called the limit state ($g_1 (X_1) = 0$). X_1 contains all the random variables in Eqs. (1) and (2).

2.2 Reliability Model Based on Tooth Contact Fatigue

The tooth contact fatigue is the common cause of gear failure and is affected by design geometry, material, manufacturing, and other variables. The contact stress of gears can be calculated as [20]

$$\sigma_H = Z_H Z_E Z_e Z_\beta \sqrt{\frac{F_t}{d_1 b}\frac{u \pm 1}{u} K_A K_V K_{H\alpha} K_{H\beta}} \tag{4}$$

where d_1 is the pinion pitch diameter, u is the gear ratio, $K_{H\alpha}$ is the longitudinal load distribution coefficient, $K_{H\beta}$ is the transverse load distribution coefficient, Z_H is the nodal field coefficient, Z_E is the elastic coefficient, Z_e is the contact ratio coefficient, Z_β is the spiral angle coefficient.

Allowable contact fatigue strength of gears can be calculated as

$$\sigma_{HS} = \sigma_{H\lim} Z_{NT} Z_L Z_V Z_R Z_W Z_X \tag{5}$$

where $\sigma_{H\lim}$ is the experimental flank contact fatigue strength, Z_{NT} is the life coefficient, Z_L is the lubricant coefficient, Z_V is the velocity coefficient, Z_R is the tooth fineness coefficient, Z_W is the hardening coefficient, Z_X is the size coefficient.

The performance function of tooth contact fatigue can be expressed as

$$g_2(X_2) = \sigma_{HS} - \sigma_H \tag{6}$$

X_2 contains all the random variables in Eqs. (4) and (5). It can be seen that X_1 and X_2 have the same random variables, so there is a certain degree of the dependence relationship between the two failure modes. Therefore, it is necessary to establish a joint distribution among failure modes to describe their correlation accurately.

2.3 Copula Function

Sklar's theorem: Any multivariate joint distribution can be written in terms of univariate marginal distribution functions and a Copula function, which describes the dependence structure between the two variables. In other words, the Copula function connects the joint distribution of multivariate random variables with their respective marginal distribution. The Copula function has the following properties [16,21,22].

(1) $C(u_1, u_2, \cdots, u_n)$ is the n-dimensional distribution function defined on $[0,1]^n$, $C: I^n = [0,1]^n \to [0,1]$;

(2) For vector $u = (u_1, u_2, \cdots, u_n)$, if there is any vector $u_i = 0, i = 1, 2, \cdots, n$, $C_n(u) = 0$;

(3) When $u_{i,i\neq k,i=1,2,\cdots,n} = 1$, $\forall 0 \leq u_k \leq 1$, $C_n(u) = C_n(1, \cdots, 1, u_k, 1, \cdots, 1) = u_k$;

(4) $C(u_1, u_2, \cdots, u_n)$ is monotonically increasing for any of its variables.

Supposing that $x_1, x_2, \cdots, x_n$ are random variables, their marginal distributions are $F_1(X_1), F_2(X_2), \cdots, F_n(X_n)$, respectively, and their joint distribution is $H(x_1, x_2, \cdots, x_n)$. Then, there is a Copula function $C(\cdot)$ that connects the marginal distribution and the joint distribution.

$$H(x_1, x_2, \cdots, x_n) = C(F_1(X_1), F_2(X_2), \cdots, F_n(X_n)) \tag{7}$$

According to the inverse transformation of cumulative distribution function for marginal distribution $x_i = F^{-1}(u_i), i = 1, 2, \cdots, n$, the Copula function of the n-dimensional random variables can be formulated as

$$C(u_1, u_2, \cdots, u_n) = H\left[F^{-1}(u_1), F^{-1}(u_2), \cdots, F^{-1}(u_n)\right] \tag{8}$$

When the random variables are two-dimensional, the Copula function is $C(u_1, u_2) = H\left[F^{-1}(u_1), F^{-1}(u_2)\right]$. Combined with the above two failure modes, the reliability of gear transmission system considering failure dependence can be expressed as

$$\begin{aligned} R = R_C &= 1 - P(g_1 \le 0) - P(g_2 \le 0) + P(g_1 \le 0, g_2 \le 0) \\ &= 1 - F_{g_1}(g_1) - F_{g_2}(g_2) + C\left(F_{g_1}(0), F_{g_2}(0)\right) \\ &= C(1,1) - C\left(F_{g_1}(0), 1\right) - C\left(1, F_{g_2}(0)\right) + C\left(F_{g_1}(0), F_{g_2}(0)\right) \end{aligned} \tag{9}$$

3 Dynamic Stress-Strength Interference Model

3.1 Dynamic Stress-Strength Interference Theory

The normal operation of a mechanical structure depends on the relationship between strength and stress. In the design service period, if the stress at any moment is greater than the structural strength, the structure will fail; when the stress is less than the strength, the structure will appear cumulative fatigue damage, reducing the structure strength until it fails [23–27]. The traditional structural reliability analysis based on the static stress-strength interference model does not consider the influence of time-variant strength on reliability. Due to the impact of random factors such as material oxidation, load fluctuation, and environmental corrosion, the structural strength will gradually decrease in engineering practice, showing a deterioration trend called strength degradation [28–30]. The dynamic stress-strength interference model is constructed, as shown in Fig. 1.

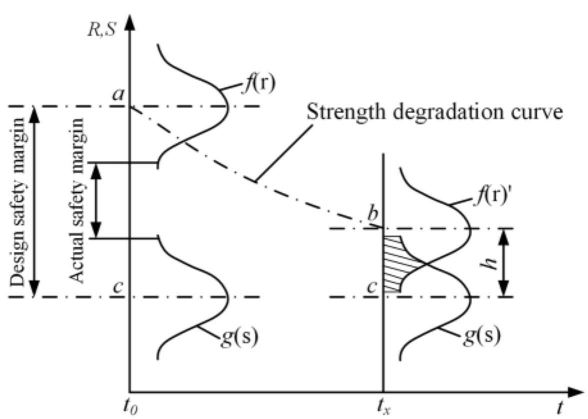


Figure 1: Dynamic stress-strength interference theory

As time goes on, the mean value of structural strength and its probability density distribution function $f(r)$ decreases gradually, while the failure probability of the structure increases. When

$t = 0$, there is a certain safety margin between strength and stress, and the structure will not fail. At t_x moment, the $f(r)$ decreases to $f(r)'$ and intersects with the probability density function of structural stress $g(s)$, leading to structural failure. The structural function considering the strength degradation can be expressed as

$$Z(t) = g[R(t), S(t)] = R(t) - S(t) \tag{10}$$

where $Z(t)$ is the reliability over time, $R(t)$ is the random process of structural strength, $S(t)$ is the random process of structural stress, t is the service time.

The structural reliability considering the strength degradation during the service period T is

$$P_S(T) = P\{Z(t) > 0, t \in [0, T]\} = P\{R(t) > S(t), t \in [0, T]\} \tag{11}$$

3.2 Gamma Process

Strength degradation is an external characterization of structural performance degradation under the action of various stochastic factors, which will lead to the reduction of structural safety and reliability. From the macroscopic point of view, strength degradation is a continuous process with randomness and irreversibility, which is caused by the characteristics of the structure as well as stress effects. The effects caused by strength degradation should be fully considered when performing structural dynamic reliability analysis. Stochastic process theory is usually used to describe the general law of strength degradation. Since the Gamma process [31,32] is an independent and non-negative incremental process, it is considered a natural choice to describe the degradation process. The Gamma process can describe both small degradation fluctuations and drastic steps, and is well characterized for the degradation mechanism of various structures. The probability density function Q describing the structural strength degradation using the Gamma process can be formulated as

$$Ga(q|\, v, u) = \frac{u^v}{\Gamma(v)} q^{v-1} \exp(-uq)\, I_{(0,\infty)}(q) \tag{12}$$

where $\Gamma(v) = \int_{t=0}^{\infty} t^{v-1} e^{-t} dt$ is the Gamma function, $I_A(q)$ is an indicative function, if $q \in A$, then $I_A(q) = 1$, if $q \notin A$, then $I_A(q) = 0$, $u > 0$ is the dimension parameter, $v(t) > 0$ is the shape parameter.

The Gamma process has the following characteristics.

(1) The probability of $Q(0) = 0$ is 1;

(2) When $\zeta > t \geq 0$, $Q(\zeta) - Q(t) \sim Ga(v(\zeta) - v(t), u)$;

(3) $Q(t)$ has the independent increment.

The mathematical expectation $E(Q(t))$ and variance $Var(Q(t))$ of the Gamma process are

$$\begin{cases} E(Q(t)) = \dfrac{v(t)}{u} \\ Var(Q(t)) = \dfrac{v(t)}{u^2} \end{cases} \tag{13}$$

Although there are some fluctuations in the strength degradation process, it is generally a stationary random process during engineering experience. Therefore, the shape parameter is a linear function $v(t) = at$, where a is a constant. So, Eqs. (12) and (13) can be written as

$$Ga(q|\,at, u) = \frac{u^{at}}{\Gamma(at)} q^{at-1} \exp(-uq)\, I_{(0,\infty)}(q) \tag{14}$$

$$\begin{cases} E(Q(t)) = \dfrac{at}{u} \\ Var(Q(t)) = \dfrac{at}{u^2} \end{cases} \tag{15}$$

Considering the essential characteristics of structural strength degradation, the $P-S-N$ curve is used to determine the values of a and u. The steps are as follows:

(1) The $S-N$ curve is transformed into the $S-t$ curve. The α and β are two constants.

$$N = f(t), S = \sqrt[\alpha]{\beta/f(t)} \tag{16}$$

(2) Let $S_{Pi}(t_j) - S_{Pi}(t_{j+1}) = \Delta\hat{Q}_{ij}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$ denote the strength degradation of the $S-t$ curve with survival rate P_i in the time $[t_j, t_{j+1}]$. Then the estimated values of the mean value and variance of the strength degradation are as follows.

$$\begin{cases} \hat{\mu}_j = \dfrac{1}{m}\displaystyle\sum_{i=1}^{m} \Delta\hat{Q}_{ij} \\ \hat{\sigma}_j^2 = \dfrac{1}{m-1}\displaystyle\sum_{i=1}^{m}\left(\Delta\hat{Q}_{ij} - \dfrac{1}{m}\displaystyle\sum_{i=1}^{m}\Delta\hat{Q}_{ij}\right)^2 \end{cases} \tag{17}$$

(3) The estimated values of characteristic parameters a and u of the Gamma process are calculated. Finally, the final parameters are estimated by averaging a and u obtained from all-time intervals.

$$\begin{cases} \hat{a} = \dfrac{1}{n}\displaystyle\sum_{j=1}^{n} \dfrac{u_j^2\left(\hat{\mu}_j + \hat{\sigma}_j^2\right)}{(t_{j+1} - t_j)(u_j + 1)} \\ \hat{u} = \frac{1}{n}\displaystyle\sum_{j=1}^{n} \dfrac{\hat{\mu}_j}{\hat{\sigma}_j^2} \end{cases} \tag{18}$$

Combined with the above introduction, the flow chart of this paper is shown in Fig. 2.

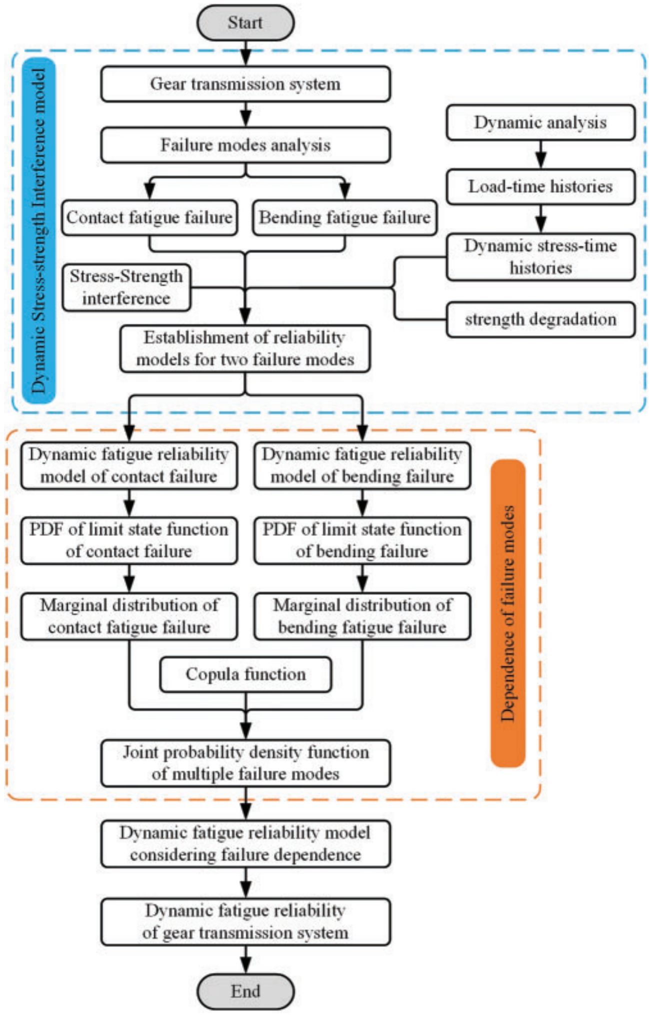


Figure 2: Flow chart of the thesis

4 Dynamic Fatigue Reliability Analysis Considering Failure Dependence

4.1 Dynamic Fatigue Reliability Model

Taking a certain type of EMU gear transmission system as the research object, the dynamic model is constructed according to actual geometric parameters and constraints. According to the 615 kW input power and 4,200 r/min driving gear speed, constant traction torque is applied to the model. The geometric parameters of the gear pair are shown in Table 1. The values of each random variable in X_1 and X_2 are obtained by referring to the standards and manuals, as shown in Table 2.

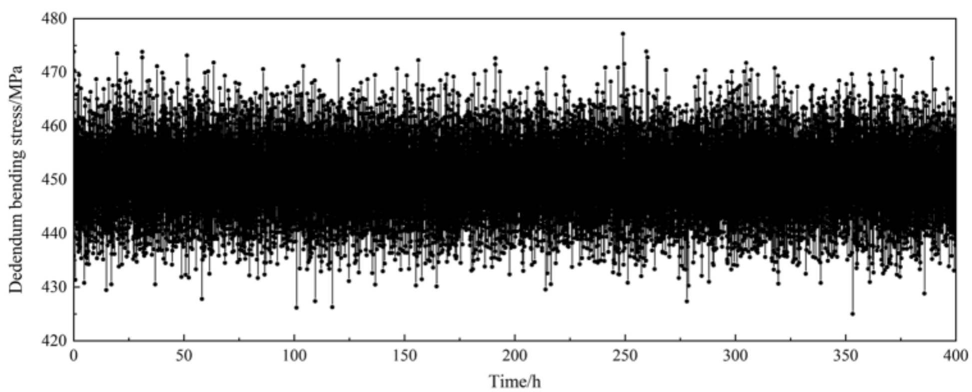
Table 1: Geometric parameters of the gear pair

Gear	Number of teeth	Tooth width/mm	Pressure angle/(°)	Normal module/mm	Helix angle/(°)
Pinion	35	70	20	6	17.5
Wheel	85	68			

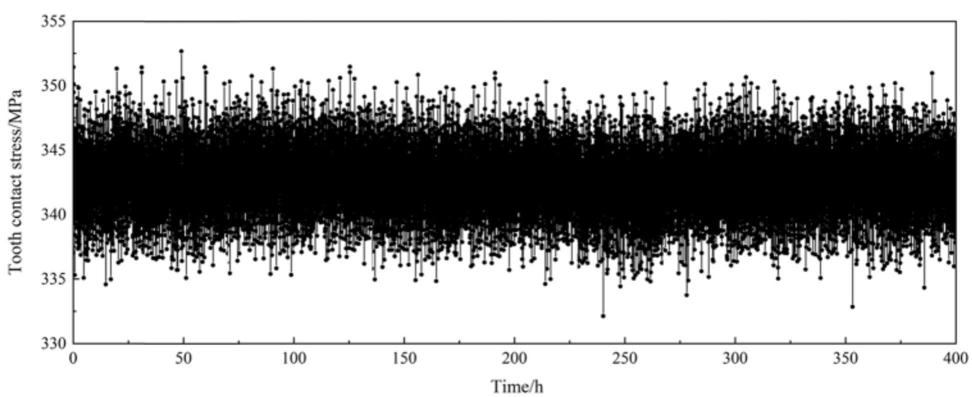
Table 2: Information of X_1 and X_2

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
Y_{ST}	2.000	Y_{NT}	0.980	$Y_{\delta relT}$	0.600	Y_{RrelT}	1.120
Y_X	1.000	Y_{Fa}	1.249	Y_{Sa}	1.152	Y_e	1.100
Y_β	0.107	K_A	1.100	K_V	1.060	$K_{F\beta}$	1.200
K_{Fa}	1.060	F_t	73180.370	$\sigma_{F\lim}$	405.000	Z_H	0.876
Z_β	1.000	Z_e	0.690	Z_E	146.000	$K_{H\alpha}$	0.336
$K_{H\beta}$	0.532	$\sigma_{H\lim}$	510.000	Z_{NT}	1.000	Z_L	0.920
Z_V	1.026	Z_R	1.014	Z_W	1.000	Z_X	1.000

The load-time history of the gear transmission system is obtained by dynamic analysis. The stress-time history is obtained by substituting the load-time history into Eqs. (1) and (4), as shown in Fig. 3. It can be seen that the fluctuation of dedendum bending stress and tooth contact stress fluctuate randomly with time, respectively. Therefore, when the strength degenerates to a certain extent, the gear transmission system may fail, even if the average strength is not less than the stress.



(a)



(b)

Figure 3: Stress-time history of the gear pair (a) Dedendum bending stress (b) Tooth contact stress

The strength degradation is described by Gamma random process. The $P-S-N$ curve of the gear material can be obtained by querying the corresponding mechanical material manual, as shown in Fig. 4, and the specific parameters are shown in Table 3. Since the $P-S-N$ curves of gear and base materials are similar, this paper directly uses the $P-S-N$ curves of the base material to characterize the gear performance.

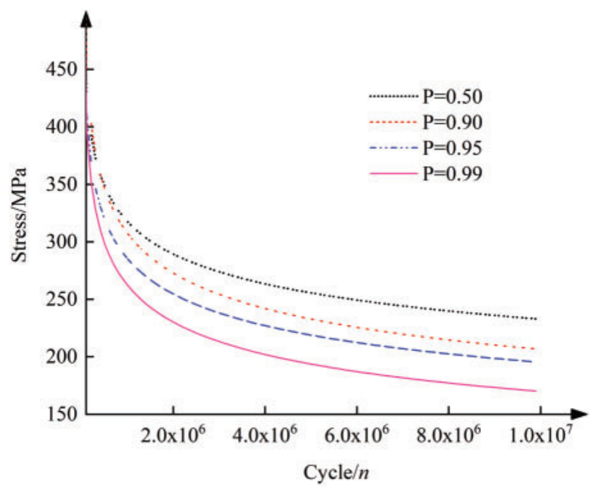


Figure 4: The *P–S–N* curve of the gear material

Table 3: Parameters estimation of the $P-S-N$ curve

Parameters	Survival rate P			
	0.50	0.90	0.95	0.99
α	24.49	20.88	20.38	18.83
β	−7.39	−6.06	−5.30	−4.77
$S-N$ curve expression: $S^{\beta} \times N = \alpha$				

Define the unit of operating time of the gearing system as h, the $N = f(t) = 4200 \times 60t = 2.52 \times 10^5 t$. Substituting the data in Table 3 into Eq. (16) to obtain the S–t curve data, as shown in Eq. (19).

$$\begin{cases} P = 0.50, & S = \sqrt[-7.39]{10^{24.49}/2.52 \times 10^5 t} \\ P = 0.90, & S = \sqrt[-6.06]{10^{20.88}/2.52 \times 10^5 t} \\ P = 0.95, & S = \sqrt[-5.30]{10^{20.38}/2.52 \times 10^5 t} \\ P = 0.99, & S = \sqrt[-4.77]{10^{18.83}/2.52 \times 10^5 t} \end{cases} \quad (19)$$

The value of strength degradation $\hat{D}_{ij}$ in any time interval can be calculated from the function of the S–t curve. The parameters u_j and a_j of the Gamma process are obtained by substituting the $\hat{D}_{ij}$ in different time intervals into Eqs. (17) and (18), as shown in Table 4. Then, $u = 29.6223$, $a = 19.2490$.

Table 4: Values of u_j and a_j corresponding to time interval $[t_j, t_{j+1}]$

j	2	3	4	5	6	7	8	9
u_j	2.4791	6.0127	10.6402	16.2586	22.7800	30.1248	38.2194	46.9945
a_j	10.0644	12.9571	15.3054	17.3441	19.1609	20.8006	22.2906	23.6495

Through the above analysis, the strength degradation of the gear pair can be described as a Gamma process with a shape parameter of $19.2490t$ and a dimension parameter of 29.6223. By substituting the dynamic stress-time history and Gamma process into Eq. (3) and Eq. (6), the dynamic fatigue reliability model of the gear is constructed. The performance function values of tooth contact and dedendum bending are obtained by numerical calculation.

4.2 Characterization of Failure Dependence

Nonparametric kernel distribution estimation is used to select the optimal Copula function. The core idea of this method is to use nonparametric kernel distribution to estimate the marginal distribution of $F(g_1)$ and $F(g_2)$, and preliminarily screen out the Copula functions that can describe the correlation of failure modes. At the same time, the relevant parameters of each Copula function are calculated. Finally, the optimal Copula function is selected by comparing the Square Euclidean distance between each Copula function and the empirical Copula function.

The marginal distribution functions of $F(g_1)$ and $F(g_2)$ are determined by the kernel distribution estimation method, and the accuracy of the marginal distribution functions is verified by comparing with the empirical distribution function. The results are shown in Fig. 5. It can be seen that the kernel distribution estimation and the empirical distribution coincide, so the marginal distribution functions obtained by the kernel distribution estimation can be used to construct the Copula function.

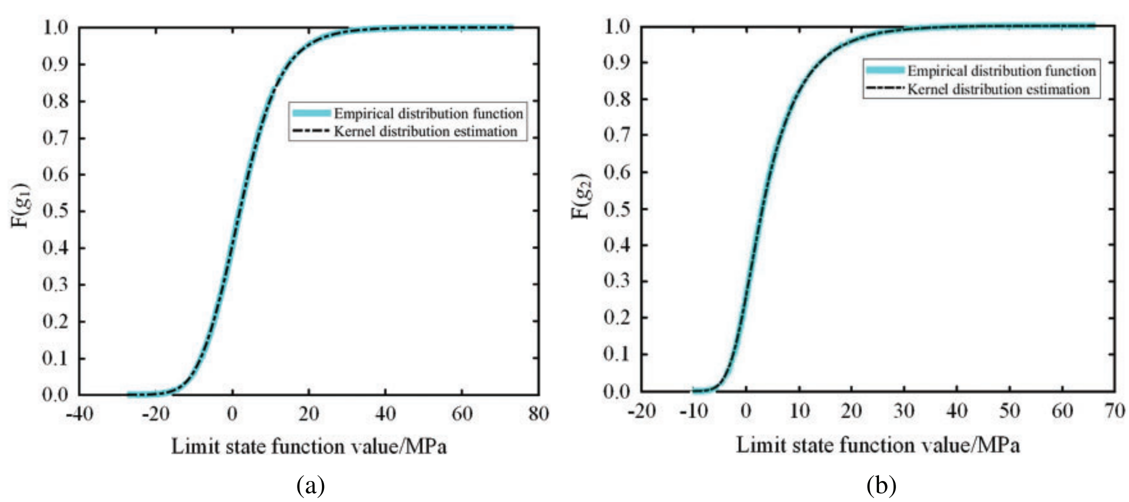


Figure 5: Marginal distribution of limit state functions (a) Dedendum bending (b) Tooth contact

The bivariate frequency histogram (shown in Fig. 6) is drawn as the joint density function for the two failure modes to choose the appropriate Copula function. Fig. 6 shows the frequency histogram has tail symmetry, so the Gauss Copula function and t-Copula function are preliminarily selected to fit the dependence relationship of two failure modes. The expressions of the Gauss Copula function and t-Copula function are as follows:

$$C_{Ga}(g_1, g_2, R) = \int_{-\infty}^{\Phi-1(g1)} \int_{-\infty}^{\Phi-1(g2)} \frac{1}{2\pi\sqrt{1-R^2}} \exp\left(\frac{2RX_1X_2 - X_1^2 - X_2^2}{2(1-R^2)}\right) dX_1 dX_2 \tag{20}$$

$$C_t(g_1, g_2; R, m) = T_{R,m}\left(t_m^{-1}(g_1), t_m^{-1}(g_2)\right) \tag{21}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of the standard normal distribution function. R is an n-dimensional coefficient matrix. $T_{R,m}(\cdot)$ is the n dimension distribution function with the coefficient matrix R, and the degree of freedom m. t_m^{-1} is the inverse function of the one-dimensional distribution function.

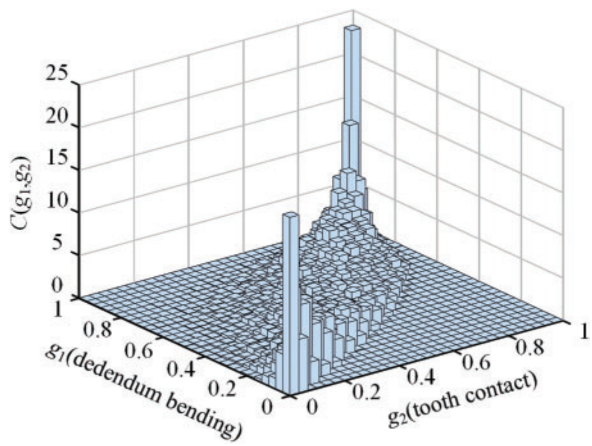


Figure 6: Bivariate frequency histogram

The parameters estimation results for the Gauss Copula and t-Copula functions are calculated, respectively. The coefficient matrix R of Gauss Copula is as follows:

$$R_{Ga} = \begin{bmatrix} 1.0000 & 0.9269 \\ 0.9269 & 1.0000 \end{bmatrix} \tag{22}$$

The coefficient matrix R and degree of freedom m of t-Copula are as follows:

$$R_t = \begin{bmatrix} 1.0000 & 0.9276 \\ 0.9276 & 1.0000 \end{bmatrix}, \quad m = 7.6936 \tag{23}$$

It is substituting the R and m into Eqs. (20) and (21) to calculate the density function and distribution function of the two Copula functions. The Square Euclid distance between the alternative Copula function and the empirical Copula function was then calculated through Eqs. (24) and (25). Eq. (24) is the expression of the empirical Copula function. Eq. (25) is the Square Euclid distance of the Gauss Copula function and t-Copula function, reflecting the fitting degree of the Copula function to the failure dependence. The smaller the distance, the better the fitting effect. The results are shown in Table 5.

$$\hat{C}(u,v) = \frac{1}{n}\sum_{i=1}^{n} I_{[F_n(x_i)\le u]} I_{[G_n(y_i)\le v]}, u, v \in [0,1] \tag{24}$$

$$d_{gau}^2 = \sum_{i=1}^{n} \left|\hat{C}_n(u_i, v_i) - C_{gau}(u_i, v_i)\right|^2, d_t^2 = \sum_{i=1}^{n} \left|\hat{C}_n(u_i, v_i) - C_t(u_i, v_i)\right|^2 \tag{25}$$

where $I_{[\cdot]}$ is a characteristic function, when $F_n(x_i) \le u$, the $I_{[F_n(x_i)\le u]} = 1$, otherwise, $I_{[F_n(x_i)\le u]} = 0$.

According to the results in Table 5, the Square Euclid distance between the t-Copula function and the empirical Copula function was the smallest. It indicates that the t-Copula function is more appropriate to describe the dependence relationship. The density function and distribution function of the t-Copula is shown in Fig. 7. It can be seen that the t-Copula function has good tail correlation characteristics, which can well fit the dependence relationship between the tooth contact fatigue and dedendum bending fatigue.

Table 5: The square Euclid distance of two Copula functions

Copula function	Gauss Copula	t-Copula
Value	0.4586	0.4342

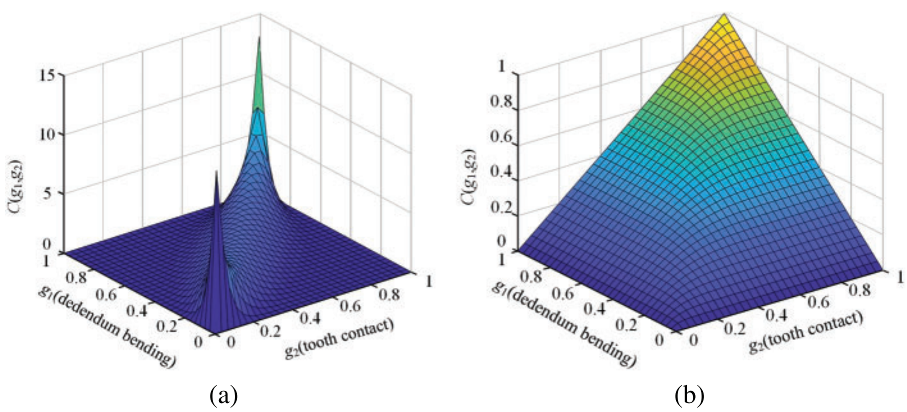


Figure 7: The t-Copula function diagram (a) Density function (b) Distribution function diagram

4.3 Dynamic Fatigue Reliability Analysis Results

Substituting the above-calculated parameters into Eq. (9), the dynamic fatigue reliability of gear transmission system considering failure dependence is obtained, as shown in Fig. 8.

It can be seen from Fig. 8 that the reliability gradually decreases due to the strength degradation with the increase of service time. The dynamic fatigue reliability of the gear transmission system has an inevitable fluctuation. The main reason for the reliability fluctuation is that both the dynamic contact stress and dedendum bending stress has randomness. Because of the dependence relationship between the two failure modes, the reliability is lower than that of the single failure mode, and the fluctuation is more obvious. The results show that the failure dependence greatly influences the reliability of the gear transmission system. In this paper, only infant mortality (assume 400 hours) is analyzed, and the reliability is following the description of the bathtub curve. The study provides a theoretical basis for considering the correlation in the design stage. Compared with the deterministic reliability analysis, the result is more suitable for engineering practice.

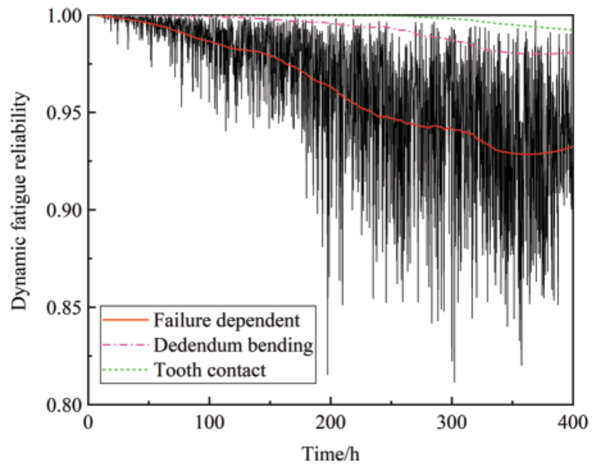


Figure 8: Dynamic fatigue reliability of the gear transmission system

5 Conclusions

In this paper, a novel dynamic fatigue reliability analysis method for failure dependence and strength degradation is proposed, which provides an effective approach for dynamic fatigue reliability analysis of gear transmission systems in the design stage. We first constructed each failure mode's function based on stress-strength interference and gear transmission theory, then used the Copula function to characterize the dependence relationship between failure modes. Simultaneously, we introduce the Gamma process to describe structural strength degradation to construct a dynamic fatigue reliability model.

A practical engineering example illustrated the dynamic fatigue reliability considering failure dependence is more in line with the engineering practice, reflecting the influence of failure dependence and strength degradation on reliability. The results show that the gear strength degradation follows the Gamma process with a shape parameter of $19.2490t$ and a dimension parameter of 29.6223. The reliability of the gear transmission system considering failure dependence is about 0.94 after 400 h, which is lower than that under a single failure mode. It shows that the dependence relationship has a significant influence on structural reliability. The safety of the design can be guaranteed by considering the failure dependence and strength degradation.

This study opens avenues for more accurate calculation of failure probability and determination of design parameters in the design stage. Moreover, the research results can provide essential data for making maintenance cycles and plans.

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