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Research on Normal Pythagorean Neutrosophic Set Choquet Integral Operator and Its Application

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ABSTRACT

We first propose the normal Pythagorean neutrosophic set (NPNS) in this paper, which synthesizes the distribution of the incompleteness, indeterminacy, and inconsistency of the Pythagorean neutrosophic set (PNS) and normal fuzzy number. We also define some properties of NPNS. For solving the decision-making problem of the non-strictly independent and interacting attributes, two kinds of NPNS Choquet integral operators are proposed. First, the NPNS Choquet integral average (NPNSCIA) operator and the NPNS Choquet integral geometric (NPNSCIG) operator are proposed. Then, their calculating formulas are derived, their properties are discussed, and an approach for solving the interacting multi-attribute decision making based on the NPNS is studied. Finally, the two kinds of operators are applied to validate the stability of the new method.

KEYWORDS

Normal pythagorean neutrosophic set (NPNS); NPNS choquet integral average (NPNSCIA) operator; NPNS choquet integral geometric (NPNSCIG) operator; multi-attribute decision making (MADM)

1 Introduction

As an important branch of modern decision theory, MADM is widely used in many fields such as economy, management, military, and engineering. One of the core problems of MADM is how to give the attribute values under each attribute. Since Zadeh [1] proposed the concept of fuzzy set (FS), FS has become a hot research topic. Different scholars apply various methods and new theories to fuzzy decision making, improve and optimize the existing problems in fuzzy decision making, and make its development more perfect and reasonable. To popularize the FS, in 1986, Atanassov [2] introduced the concept of intuitionistic fuzzy set (IFS) and studied it. IFS can express membership degree and non-membership degree at the same time, and their sum is less than or equal to 1. Compared with traditional fuzzy sets, IFS is more suitable for describing fuzziness and uncertainty in practical problems. However, in the process of dealing with decision making, the sum of membership degree and non-membership degree may be greater than 1. So

Yager [3] proposed the Pythagorean fuzzy set (PFS), which allows the sum of membership degree and non-membership degree to be greater than 1, but its sum of squares is not more than 1. PFS is an extension of IFS. Therefore, the processing of fuzzy and uncertain information has a stronger performance. On the basis of Yager's research, different scholars applied various methods and new theories to Pythagorean fuzzy decision making. For example, Peng et al. [4] put forward the Pythagoras soft set by integrating Pythagoras with the soft set. Liang and Peng et al. [5,6] studied interval-valued Pythagorean fuzzy sets. Liu et al. [7] proposed the Pythagorean hesitation fuzzy set by combining the Pythagorean and hesitation fuzzy set. Fan et al. [8] generalized the membership degree and non-membership degree of the Pythagorean fuzzy number and proposed the triangular Pythagorean fuzzy set. But IFS can only deal with incomplete information but not with uncertain and inconsistent information. Therefore, Smarandache [9,10] proposed the neutrosophic set (NS) theory based on IFS. It is a generalization of FS and IFS by adding independent uncertainty on the basis of IFS. In the NS theory, decision-makers can use the degree of truth, indeterminacy, and falsity to describe the evaluation of objective things. Since it was proposed, it has attracted extensive attention and research. Wang et al. [11] proposed a concept of single-valued neutrosophic sets (SVNS). Liu et al. [12] proposed a weighted aggregation operator and decision making method based on interval value neutrosophic sets (IVNS). Fan et al. [13–15] proposed decision making methods based on SVNS, linguistic neutrosophic multisets(LNM), and refined-SVNS. Liu et al. [16,17] proposed the normal neutrosophic set (NNS). Ye [18] proposed Correlation Coefficients of NNS. Jansi et al. [19] in 2019 proposed Pythagorean neutrosophic set (PNS) as an extension of Ajay et al. applied PNS in fuzzy graphs [20].

Ideally, the constructed decision indicator system should have the conditions of completeness, representativeness, and independence. However, in many practical cases, these attributes are usually not independent but correlated. In order to solve the MADM problem of attribute correlation, Marichal [21] in 2000 generalized the fuzzy measure defined by Sugeno [22] and proposed the Choquet integral [23]. Since Choquet integral was put forward, it has been hotly discussed by many scholars. Xu [24] applied Choquet integral to multi-attribute decision making of intuitional fuzzy sets and proposed intuitional fuzzy correlated average operator, intuitional fuzzy correlated geometric operator, etc. Tan et al. [25] proposed intuitionistic fuzzy Choquet integral average operator and intuitionistic fuzzy Choquet integral geometric operator. Qu et al. [26] applied the Choquet integral to the interval-hesitation fuzzy multi-attribute decision making and proposed the interval-hesitation fuzzy Choquet integral operator. Peng et al. [27] applied Choquet integral to The Pythagorean fuzzy decision making environment and proposed the Pythagorean fuzzy Choquet integral average operator and geometric operator. Dong et al. [28] proposed Generalized Choquet Integral Operator of Triangular Atanassov's Intuitionistic Fuzzy Numbers. Wan et al. [29] proposed Generalized Shapley Choquet integral operator based method for interactive interval-valued hesitant fuzzy uncertain linguistic. When solving MADM problems, Choquet integral operator can effectively deal with the redundant part of attribute compatibility, and solve the problem of attribute correlation by balancing the influence degree between attributes. In view of the Choquet integral operator can be used to consider the relationship between information, this paper proposes the normal Pythagorean neutrosophic set (NPNS) and generalize Choquet integral operator to the NPNS environment. NPNS synthesizes the distribution of the incompleteness, indeterminacy, and inconsistency of PNS and the normal neutrosophic number, which is more reasonable than PNS and NNS on expressing the decision making information. NPNS Choquet integral operator not only considers the importance between attributes but also reflects the relation between attributes. Then the properties of this operator are discussed, and an algorithm for solving the MADM problem is proposed based on this operator.

We organize the paper as follows. Section 2 describes basic concepts. Section 3 defines two Choquet integral operators of NPNS. Section 4 establishes the DM model based on NPNSCIA or NPNSCIG Operator. Section 5 provides an example. Section 6 gives conclusions.

2 Some Basic Concepts

2.1 Pythagorean Fuzzy set (PFS)

Definition 1 [3]. Set Y as an object set, a PFS is expressed as $\Gamma = \{y, (u_\Gamma(y), v_\Gamma(y))y \in Y\}$, $u\Gamma$: Y $\in$ [0, 1] denotes membership *and* $v\Gamma$: Y $\in$ [0, 1] denotes non-membership, and $0 \leq (u_\Gamma(y))^2 + (v_\Gamma(y))^2 \leq 1$ for each $y \in Y$.

2.2 Normal Fuzzy Number (NFN)

Definition 2 [30]. Set $\Gamma(y) = e^{-\left(\frac{y-\alpha}{\varepsilon}\right)^2}$ $(\varepsilon > 0)$ as the membership-function of Γ, and then $\Gamma = (\alpha, \varepsilon)$ is an NFN and all NFNs are denoted as N.

2.3 Neutrosophic Set (NS)

Definition 3 [10,11]. Set Y as an object set, $\bar{\Gamma} = \{y, \langle J_{\bar{\Gamma}}(y), K_{\bar{\Gamma}}(y), L_\Gamma\rangle(y) | y \in Y\}$, then $\bar{\Gamma}$ is called neutrosophic set (NS), $J_{\bar{\Gamma}}(y)$ express truth-membership , $K_{\bar{\Gamma}}(y)$ express indeterminacy-membership and $L_{\bar{\Gamma}}(y)$ express falsity-membership. $\forall y \in Y$, $J_{\bar{\Gamma}}(y), K_{\bar{\Gamma}}(y), L_{\bar{\Gamma}}(y) \in [0, 1]$ and $3 \geq J_{\bar{\Gamma}}(y) + K_{\bar{\Gamma}}(y) + L_{\bar{\Gamma}}(y) \geq 0$.

2.4 Pythagorean Neutrosophic Set (PNS)

Definition 4 [19]. Set Y as an object set, $\bar{\Gamma} = \{y, \langle J_{\bar{\Gamma}}(y), K_{\bar{\Gamma}}(y), L_\Gamma(y)\rangle | y \in Y\}$, then $\bar{\Gamma}$ is called Pythagorean neutrosophic set (PNS), $J_{\bar{\Gamma}}(y)$ express truth-membership, $K_{\bar{\Gamma}}(y)$ express indeterminacy-membership and $L_{\bar{\Gamma}}(y)$ express falsity-membership. Here $J_{\bar{\Gamma}}(y)$ and $L_{\bar{\Gamma}}(y)$ are dependent and $K_{\bar{\Gamma}}(y)$ is independent. $\forall y \in Y, J_{\bar{\Gamma}}(y), K_{\bar{\Gamma}}(y), L_{\bar{\Gamma}}(y) \in [0, 1]$ and $1 \geq J_{\bar{\Gamma}}(y) + L_{\bar{\Gamma}}(y) \geq 0$, $1 \geq (J_{\bar{\Gamma}}(y))^2 + (L_{\bar{\Gamma}}(y))^2 \geq 0$, $1 \geq K_{\bar{\Gamma}}(y) \geq 0$.

2.5 Normal Pythagorean Neutrosophic Set (NPNS)

Definition 5. Set Y is an object set, $(\alpha, \varepsilon) \in N$, then $\hat{\Gamma} = \{\langle y, (\alpha, \varepsilon), (J_{\hat{\Gamma}}(y), K_{\hat{\Gamma}}(y), L_{\hat{\Gamma}}(y))\rangle | y \in Y\}$ is defined as the normal Pythagorean neutrosophic set (NPNS) in *Y*, $J_{\hat{\Gamma}}(y)$ express the truth-membership, $K_{\hat{\Gamma}}(y)$ express indeterminacy-membership and $L_{\hat{\Gamma}}(y)$ express falsity-membership. Here $J_{\hat{\Gamma}}(y)$ *and* $L_{\hat{\Gamma}}(y)$ are dependent and $K_{\hat{\Gamma}}(y)$ is independent. $\forall y \in Y$, , $J_{\bar{\Gamma}}(y), K_{\bar{\Gamma}}(y), L_{\bar{\Gamma}}(y) \in [0, 1]$ *and* $1 \geq J_{\bar{\Gamma}}(y) + L_{\bar{\Gamma}}(y) \geq 0$, $1 \geq (J_{\bar{\Gamma}}(y))^2 + (L_{\bar{\Gamma}}(y))^2 \geq 0$, $1 \geq K_{\bar{\Gamma}}(y) \geq 0$, $J_{\hat{\Gamma}}(y) = (J_{\hat{\Gamma}})^2 e^{-\left(\frac{y-\alpha}{\varepsilon}\right)^2}$, $K_{\hat{\Gamma}}(y) = 1 - (1 - (K_{\hat{\Gamma}})^2) e^{-\left(\frac{y-\alpha}{\varepsilon}\right)^2}$ *and* $L_{\hat{\Gamma}}(y) = 1 - (1 - (L_{\hat{\Gamma}})^2) e^{-\left(\frac{y-\alpha}{\varepsilon}\right)^2}$.

For convenience, a normal Pythagorean neutrosophic element (NPNE) is denoted as $\hat{\epsilon} = \langle (\alpha, \varepsilon), (J, K, L)\rangle$.

Definition 6. Set $\hat{\epsilon}_1 = \langle (\alpha_1, \varepsilon_1), (J_1, K_1, L_1)\rangle$ and $\hat{\epsilon}_2 = \langle (\alpha_2, \varepsilon_2), (J_2, K_2, L_2)\rangle$ as two NPNEs, then we define the NPNEs' operations:

i. $$\hat{\epsilon}_1 \oplus \hat{\epsilon}_2 = \left\langle \begin{array}{c} (\alpha_1 + \alpha_2, \varepsilon_1 + \varepsilon_2), \\ ((J_1)^2 + (J_2)^2 - (J_1)^2 (J_2)^2, (K_1)^2 (K_2)^2, (L_1)^2 (L_2)^2) \end{array} \right\rangle \tag{1}$$

ii. $$\hat{\epsilon}_1 \otimes \hat{\epsilon}_2 = \left\langle \begin{array}{c} \left(\alpha_1\alpha_2, \alpha_1\alpha_2\sqrt{\frac{\varepsilon_1{}^2}{\alpha_1{}^2} + \frac{\varepsilon_2{}^2}{\alpha_2{}^2}}\right), \\ (\,(J_1)^2(J_2)^2, (K_1)^2 + (K_2)^2 - (K_1)^2(K_2)^2, (L_1)^2 + (L_2)^2 - (L_1)^2(L_2)^2 \end{array} \right\rangle \quad (2)$$

iii. $$\chi\hat{\epsilon}_1 = \left\langle (\chi\alpha_1, \chi\varepsilon_1), \left(1 - \left(1 - (J_1)^2\right)^\chi, \left((K_1)^2\right)^\chi, \left((L_1)^2\right)^\chi\right) \right\rangle \chi > 0 \quad (3)$$

iv. $$\hat{\epsilon}_1{}^\chi = \left\langle \left(\alpha_1{}^\chi, \chi^{1/2}\alpha_1{}^{\chi-1}\varepsilon_1\right), \left(\left((J_1)^2\right)^\chi, 1 - \left(1 - (K_1)^2\right)^\chi, 1 - \left(1 - (L_1)^2\right)^\chi\right) \chi > 0 \right\rangle \quad (4)$$

Definition 7. Set $\hat{\epsilon} = \langle(\alpha, \varepsilon), (J, K, L)\rangle$ as a NPNE, and then its score functions are

$$\Phi_1(\hat{\epsilon}) = \alpha(2 + J^2 - K^2 - L^2) \quad (5)$$

$$\Phi_2(\hat{\epsilon}) = \varepsilon(2 + J^2 - K^2 - L^2) \quad (6)$$

and its accuracy functions are

$$\Psi_1(\hat{\epsilon}) = \alpha(2 + J^2 - K^2 + L^2) \quad (7)$$

$$\Psi_2(\hat{\epsilon}) = \varepsilon(2 + L^2 - M^2 + N^2) \quad (8)$$

Definition 8. Set $\hat{\epsilon}_1 = \langle(\alpha_1, \varepsilon_1), (J_1, K_1, L_1)\rangle$ *and* $\hat{\epsilon}_2 = \langle(\alpha_2, \varepsilon_2), (J_2, K_2, L_2)\rangle$ as two NPNEs, then we have

if $\Phi_1(\hat{\epsilon}_1) > \Phi_1(\hat{\epsilon}_2)$ *then* $\hat{\epsilon}_1 > \hat{\epsilon}_2$;
if $\Phi_1(\hat{\epsilon}_1) = \Phi_1(\hat{\epsilon}_2)$ *then*
if $\Psi_1(\hat{\epsilon}_1) > \Psi_1(\hat{\epsilon}_2)$ *then* $\hat{\epsilon}_1 > \hat{\epsilon}_2$;
if $\Psi_1(\hat{\epsilon}_1) = \Psi_1(\hat{\epsilon}_2)$ *then*
if $\Phi_2(\hat{\epsilon}_1) < \Phi_2(\hat{\epsilon}_2)$ *then* $\hat{\epsilon}_1 > \hat{\epsilon}_2$;
if $\Phi_2(\hat{\epsilon}_1) = \Phi_2(\hat{\epsilon}_2)$ *then*
if $\Psi_2(\hat{\epsilon}_1) < \Psi_2(\hat{\epsilon}_2)$ *then* $\hat{\epsilon}_1 > \hat{\epsilon}_2$;
if $\Psi_2(\hat{\epsilon}_1) = \Psi_2(\hat{\epsilon}_2)$ *then* $\hat{\epsilon}_1 = \hat{\epsilon}_2$.

2.6 Fuzzy Measure (FM)

Definition 9 [22]. Set $D(Y)$ as the power set to $Y = \{y_1, y_2, \ldots, y_n\}$, $\eta(y_i)$ expresses the weight of y_i, and then $\eta : D(Y) \in [0, 1]$ is called the *FM* of Y while satisfying the following conditions:

i. $\eta(\emptyset) = 0, \eta(Y) = 1$;
ii. $\forall\Gamma, \Upsilon \in D(Y)$, *if* $\Gamma \subseteq \Upsilon$ *then* $\eta(\Gamma) \leq \eta(\Upsilon)$;
iii. $\eta(\Gamma \cup \Upsilon) = \eta(\Gamma) + \eta(\Upsilon) + \chi\eta(\Gamma)\eta(\Upsilon), \forall\Gamma, \Upsilon \in D(Y)$, *and* $\chi \in (-1, \infty)$.

If $\chi = 0$ then $\eta(\Gamma \cup \Upsilon) = \eta(\Gamma) + \eta(\Upsilon)$, which indicates that the attribute sets Γ and Υ are independent of each other; if $-1 < \chi < 0$ then $\eta(\Gamma \cup \Upsilon) < \eta(\Gamma) + \eta(\Upsilon)$, which indicates that there is information redundancy between attribute sets Γ and Υ; if $\chi > 0$ then $\eta(\Gamma \cup \Upsilon) > \eta(\Gamma) + \eta(\Upsilon)$, which indicates that there is information complementarity between attribute sets Γ and Υ.

If $\forall y_\rho \in Y, \rho, \varrho = 1, 2, \ldots, n, \rho \neq \varrho, y_\rho \cap y_\varrho = \emptyset$, then $\bigcup_{\rho=1}^{n} y_\rho = Y$ and the fuzzy measure

$$\vartheta \, satisfies: \eta(Y) = \eta\left(\bigcup_{\rho=1}^{n} y_\rho\right) = \begin{cases} \frac{1}{\vartheta}\left(\prod_{\rho=1}^{n}(1+\vartheta\eta(y_\rho)) - 1\right), \vartheta \neq 0, \\ \sum_{\rho=1}^{n} \eta(y_\rho), \ \vartheta = 0. \end{cases} \tag{9}$$

For $\eta(Y) = 1$, according to Eq. (9), when $\vartheta \neq 0$, η can be confirmed:

$$\vartheta + 1 = \prod_{\rho=1}^{n}(1+\vartheta\eta(y_\rho)) \tag{10}$$

2.7 *Choquet Integral (CI)*

Definition 10 [21]. Set $Y = \{y_1, y_2, \ldots, y_n\}$ and η as an FM on Y and ξ as a nonnegative real value function. Then we define the Choquet integral of function ξ about η:

$$C_\eta(\xi) = \sum_{\rho=1}^{n} \xi_{\gamma(\rho)}(\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})), \tag{11}$$

In which $(\gamma(1), \gamma(2), \ldots, \gamma(n))$ express an arrangement and can make $\xi_{\gamma(1)} \geq \xi_{\gamma(2)} \geq \ldots \geq \xi_{\gamma(n)}$ and $Z_{\gamma(\varrho)} = \{y_{\gamma(\varsigma)} | \varsigma \leq \varrho\} (\varrho \geq 1)$, $Z_{\gamma(0)} = \emptyset$, $y_{\gamma(\varsigma)}$ expresses the corresponding weight to $\xi_{\gamma(\varsigma)}$.

3 Two Choquet Integral Operators of NPNS

In this section, we define two normal Pythagorean neutrosophic set based Choquet integral operators, one is the normal Pythagorean neutrosophic set Choquet integral averaging (NPNSCIA) operator, and the other is the normal Pythagorean neutrosophic set Choquet integral geometric (NPNSCIG) operator. Meantime, we discuss some properties of them.

3.1 *The NPNSCIA Operator*

Definition 11. Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = \langle(\alpha_\rho, \varepsilon_\rho), \ (J_\rho, K_\rho, L_\rho)\rangle \ (\rho = 1, 2, \ldots, n)$, while

$$NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) = \oplus_{\rho=1}^{n}(\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}))\hat{\epsilon}_{\gamma(\rho)} \tag{12}$$

NPNSCIA is called the NPNS Choquet integral averaging operator, in which $(\gamma(1), \gamma(2), \ldots, \gamma(n))$ express an arrangement and can make $\hat{\epsilon}_{\gamma(1)} \geq \hat{\epsilon}_{\gamma(2)} \geq \ldots \geq \hat{\epsilon}_{\gamma(n)}$ and $Z_{\gamma(\varrho)} = \{y_{\gamma(\varsigma)} | \varsigma \leq \varrho\} (\varrho \geq 1), Z_{\gamma(0)} = \emptyset \ and \ y_{\gamma(\varsigma)}$ is the corresponding weight of $\hat{\epsilon}_{\gamma(\varsigma)}$.

According to some relevant operation rules of NPNE, we can get the form of the NPNSCIA operator shown in Theorem 1.

Theorem 1. Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = \langle(\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho)\rangle (\rho = 1, 2, \ldots, n)$, after using the NPNSCIA operator, the collective value obtained is also an NPNE and is denoted by

$$NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) = \left\langle \begin{pmatrix} \sum_{\rho=1}^{n} (\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}))\alpha_{\gamma(\rho)}, \\ \sum_{\rho=1}^{n} ((\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})))\varepsilon_{\gamma(\rho)} \end{pmatrix}, \begin{pmatrix} 1 - \prod_{\rho=1}^{n} (1 - (J_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})}, \\ \prod_{\rho=1}^{n} ((K_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})}, \\ \prod_{\rho=1}^{n} ((L_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})} \end{pmatrix} \right\rangle \tag{13}$$

In which $(\gamma(1), \gamma(2), \ldots, \gamma(n))$ express an arrangement and can make $\hat{\epsilon}_{\gamma(1)} \geq \hat{\epsilon}_{\gamma(2)} \geq \ldots \geq \hat{\epsilon}_{\gamma(n)}$ and $Z_{\gamma(\varrho)} = \{y_{\gamma(\varsigma)} | \varsigma \leq \varrho\} (\varrho \geq 1), Z_{\gamma(0)} = \emptyset$ *and* $y_{\gamma(\varsigma)}$ is the corresponding weight of $\hat{\epsilon}_{\gamma(\varsigma)}$.

Now, we proof Eq. (13).

Proof:

When n = 1, we can easily get Eq. (13).

When n = 2, we get:

$$NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2) = (\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)}))\hat{\epsilon}_{\gamma(1)} \oplus (\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)}))\hat{\epsilon}_{\gamma(2)} =$$
$$\begin{pmatrix} (\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)}))\alpha_{\gamma(1)}, \\ (\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)}))\varepsilon_{\gamma(1)} \end{pmatrix}, \quad \begin{pmatrix} (\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)}))\alpha_{\gamma(2)}, \\ (\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)}))\varepsilon_{\gamma(2)} \end{pmatrix},$$
$$\left\langle \begin{pmatrix} 1 - (1 - (J_{\gamma(1)})^2)^{\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)})}, \\ ((K_{\gamma(1)})^2)^{\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)})}, \\ ((L_{\gamma(1)})^2)^{\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)})} \end{pmatrix} \right\rangle \oplus \left\langle \begin{pmatrix} 1 - (1 - (J_{\gamma(2)})^2)^{\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)})}, \\ ((K_{\gamma(2)})^2)^{\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)})}, \\ ((L_{\gamma(2)})^2)^{\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)})} \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} (\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)}))\alpha_{\gamma(1)} + (\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)}))\alpha_{\gamma(2)}, \\ (\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)}))\varepsilon_{\gamma(1)} + (\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)}))\varepsilon_{\gamma(2)} \end{pmatrix}$$

$$= \left\langle \begin{pmatrix} 1 - (1 - (J_{\gamma(1)})^2)^{\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)})} + 1 - (1 - (J_{\gamma(2)})^2)^{\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)})} \\ -(1 - (1 - (J_{\gamma(1)})^2)^{\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)})})(1 - (1 - (J_{\gamma(2)})^2)^{\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)})}), \\ ((K_{\gamma(1)})^2)^{\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)})}((K_{\gamma(2)})^2)^{\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)})}, \\ ((L_{\gamma(1)})^2)^{\eta(Z_{\gamma(1)}) - \eta(Z_{\gamma(0)})}((L_{\gamma(2)})^2)^{\eta(Z_{\gamma(2)}) - \eta(Z_{\gamma(1)})} \end{pmatrix} \right\rangle$$

$$\left(\begin{array}{l}\sum_{\rho=1}^{2}(\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)}))\alpha_{\gamma(\rho)},\\ \sum_{\rho=1}^{2}((\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})))\varepsilon_{\gamma(\rho)}\end{array}\right),$$

$$=\left\langle\left(\begin{array}{c}1-\prod_{\rho=1}^{2}(1-(J_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})},\\ \prod_{\rho=1}^{2}((K_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})},\\ \prod_{\rho=1}^{2}((L_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})}\end{array}\right)\right\rangle$$

Making a hypothesis, when $n=\varsigma$ the Eq. (13) is established:

$$NPNSCIA(\hat{\epsilon}_1,\hat{\epsilon}_2,\ldots,\hat{\epsilon}_\varsigma)=\left\langle\begin{array}{c}\left(\begin{array}{l}\sum_{\rho=1}^{\varsigma}(\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)}))\alpha_{\gamma(\rho)},\\ \sum_{\rho=1}^{\varsigma}((\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})))\varepsilon_{\gamma(\rho)}\end{array}\right),\\ \left(\begin{array}{c}1-\prod_{\rho=1}^{\varsigma}(1-(J_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})},\\ \prod_{\rho=1}^{\varsigma}((K_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})},\\ \prod_{\rho=1}^{\varsigma}((L_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})}\end{array}\right)\end{array}\right\rangle$$

Then $n=\varsigma+1$,

$$NPNSCIA(\hat{\epsilon}_1,\hat{\epsilon}_2,\ldots,\hat{\epsilon}_\varsigma,\hat{\epsilon}_{\varsigma+1})=\left\langle\begin{array}{c}\left(\begin{array}{l}\sum_{\rho=1}^{\varsigma}(\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)}))\alpha_{\gamma(\rho)},\\ \sum_{\rho=1}^{\varsigma}((\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})))\varepsilon_{\gamma(\rho)}\end{array}\right),\\ \left(\begin{array}{c}1-\prod_{\rho=1}^{\varsigma}(1-(J_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})},\\ \prod_{\rho=1}^{\varsigma}((K_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})},\\ \prod_{\rho=1}^{\varsigma}((L_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})}\end{array}\right)\end{array}\right\rangle$$

$$\oplus\left\langle\begin{array}{c}\left(\begin{array}{l}(\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)}))\alpha_{\gamma(\varsigma+1)},\\ (\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)}))\varepsilon_{\gamma(\varsigma+1)}\end{array}\right),\\ \left(\begin{array}{c}1-(1-(J_{\gamma(\varsigma+1)})^2)^{\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)})},\\ ((K_{\gamma(\varsigma+1)})^2)^{\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)})},\\ ((L_{\gamma(\varsigma+1)})^2)^{\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)})}\end{array}\right)\end{array}\right\rangle$$

$$\left(\begin{array}{l}\sum_{\rho=1}^{\varsigma}(\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)}))\alpha_{\gamma(\rho)}+(\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)}))\alpha_{\gamma(\varsigma+1)},\\ \sum_{\rho=1}^{\varsigma}((\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})))\varepsilon_{\gamma(\rho)}+(\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)}))\varepsilon_{\gamma(\varsigma+1)}\end{array}\right),$$

$$=\left\langle\left(\begin{array}{l}\left(1-\prod_{\rho=1}^{\varsigma}(1-(J_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})}\right)+(1-(1-(J_{\gamma(\varsigma+1)})^2)^{\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)})})-\\ \left(1-\prod_{\rho=1}^{\varsigma}(1-(J_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})}\right)(1-(1-(J_{\gamma(\varsigma+1)})^2)^{\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)})}),\\ \left(\prod_{\rho=1}^{\varsigma}((K_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})}\right)((K_{\gamma(\varsigma+1)})^2)^{\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)})},\\ \left(\prod_{\rho=1}^{\varsigma}((L_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})}\right)((L_{\gamma(\varsigma+1)})^2)^{\eta(Z_{\gamma(\varsigma+1)})-\eta(Z_{\gamma(\varsigma)})}\end{array}\right)\right\rangle$$

$$\left(\begin{array}{l}\sum_{\rho=1}^{\varsigma+1}(\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)}))\alpha_{\gamma(\rho)},\\ \sum_{\rho=1}^{\varsigma+1}((\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})))\varepsilon_{\gamma(\rho)}\end{array}\right),$$

$$=\left\langle\left(\begin{array}{l}1-\prod_{\rho=1}^{\varsigma+1}(1-(J_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})},\\ \prod_{\rho=1}^{\varsigma+1}((K_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})},\\ \prod_{\rho=1}^{\varsigma+1}((L_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)})-\eta(Z_{\gamma(\rho-1)})}\end{array}\right)\right\rangle$$

This proves Theorem 1.

Theorem 2. Set η as a fuzzy measure on $Y=\{y_1,y_2,\ldots,y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho=\langle(\alpha_\rho,\varepsilon_\rho),(J_\rho,K_\rho,L_\rho)\rangle(\rho=1,2,\ldots,n)$, if $\hat{\epsilon}_\rho=\hat{\epsilon}=(\alpha,\varepsilon),(J,K,L)$, then

$$NPNSCIA(\hat{\epsilon}_1,\hat{\epsilon}_2,\ldots,\hat{\epsilon}_n)=NPNSCIA(\hat{\epsilon},\hat{\epsilon}\ldots,\hat{\epsilon})=\hat{\epsilon} \tag{14}$$

Proof:

Since $\hat{\epsilon}_\rho = \epsilon$ according to Definition 11, we can get:

$$NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) = NPNSCIA(\hat{\epsilon}, \hat{\epsilon} \ldots, \hat{\epsilon}) = \left\langle \left(\begin{pmatrix} \sum_{\rho=1}^{n} (\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}))\alpha, \\ \sum_{\rho=1}^{n} ((\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})))\varepsilon \end{pmatrix}, \begin{pmatrix} 1 - \prod_{\rho=1}^{n} (1 - J^2)^{(\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}))}, \\ \prod_{\rho=1}^{n} (K^2)^{(\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}))}, \\ \prod_{\rho=1}^{n} (L^2)^{(\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}))} \end{pmatrix} \right) \right\rangle$$

for $\sum_{\rho=1}^{n} (\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})) = 1$, according to the definition of the fuzzy measure, then $NPNSCIA$
$(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) = \langle (\alpha, \varepsilon), (J, K, L) \rangle = \hat{\epsilon}$.

Theorem 3. Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = \langle (\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho) \rangle (\rho = 1, 2, \ldots, n)$, if $\hat{\epsilon}_\rho{}'$ *is a replacement of* $\hat{\epsilon}_\rho$ then

$$NPNSCIA(\hat{\epsilon}_1{}', \hat{\epsilon}_2{}', \ldots, \hat{\epsilon}_n{}') = NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \tag{15}$$

It is easy to prove it according to Definition 11, here we omit.

Theorem 4. Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = \langle (\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho) \rangle$ *and* $\hat{\epsilon}_\rho{}^* = \langle (\alpha_\rho{}^*, \varepsilon_\rho{}^*), (J_\rho{}^*, K_\rho{}^*, L_\rho{}^*) \rangle$ $(\rho = 1, 2, \ldots, n)$ as two collections of NPNE on Y, and their ranking orders are $\hat{\epsilon}_{\gamma(1)} \geq \hat{\epsilon}_{\gamma(2)} \geq \ldots \geq \hat{\epsilon}_{\gamma(n)}$ *and* $\hat{\epsilon}_{\gamma(1)}{}^* \geq \hat{\epsilon}_{\gamma(2)}{}^* \geq \ldots \geq \hat{\epsilon}_{\gamma(n)}{}^*$, if $\hat{\epsilon}_{\gamma(\rho)} \leq \hat{\epsilon}_{\gamma(\rho)}{}^*$ for all ρ, which means $\alpha_{\gamma(\rho)} \leq \alpha_{\gamma(\rho)}{}^*, \varepsilon_{\gamma(\rho)} \geq \varepsilon_{\gamma(\rho)}{}^*, (J_{\gamma(\rho)})^2 \leq ((J_{\gamma(\rho)})^2)^*, (K_{\gamma(\rho)})^2 \geq ((K_{\gamma(\rho)})^2)^*$ *and* $(L_{\gamma(\rho)})^2 \geq ((L_{\gamma(\rho)})^2)^*$, then

$$NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \leq NPNSCIA(\hat{\epsilon}_1{}^*, \hat{\epsilon}_2{}^*, \ldots, \hat{\epsilon}_n{}^*) \tag{16}$$

Proof:

Since $\alpha_{\gamma(\rho)} \leq \alpha_{\gamma(\rho)}{}^*, \varepsilon_{\gamma(\rho)} \geq \varepsilon_{\gamma(\rho)}{}^*$ for all ρ and $\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}) \geq 0$ (can be got in Definition 10), then

$$\sum_{\rho=1}^{n} (\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}))\alpha_{\gamma(\rho)} \leq \sum_{\rho=1}^{n} (\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}))\alpha_{\gamma(\rho)}{}^*$$

$$\textit{and} \sum_{\rho=1}^{n} ((\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})))\varepsilon_{\gamma(\rho)} \geq \sum_{\rho=1}^{n} ((\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})))\varepsilon_{\gamma(\rho)}{}^*$$

Since $(J_{\gamma(\rho)})^2 \le ((J_{\gamma(\rho)})^2)^*, (K_{\gamma(\rho)})^2 \ge ((K_{\gamma(\rho)})^2)^*$ *and* $(L_{\gamma(\rho)})^2 \ge ((L_{\gamma(\rho)})^2)^*$ for all ρ then

$$1 - \prod_{\rho=1}^{n} (1 - (J_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})} \le 1 - \prod_{\rho=1}^{n} (1 - ((J_{\gamma(\rho)})^2)^*)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})}$$

$$\prod_{\rho=1}^{n} ((K_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})} \ge \prod_{\rho=1}^{n} (((K_{\gamma(\rho)})^2)^*)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})}$$

$$\prod_{\rho=1}^{n} ((L_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})} \ge \prod_{\rho=1}^{n} (((L_{\gamma(\rho)})^2)^*)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})}$$

Then, with Eq. (5), $\Phi_1(NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n)) \le \Phi_1(NPNSCIA(\hat{\epsilon}_1{}^*, \hat{\epsilon}_2{}^*, \ldots, \hat{\epsilon}_n{}^*))$, so we can get $NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \le NPNSCIA(\hat{\epsilon}_1{}^*, \hat{\epsilon}_2{}^*, \ldots, \hat{\epsilon}_n{}^*)$.

Theorem 4 has been proved.

Theorem 5. (Boundedness). Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = \langle(\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho)\rangle (\rho = 1, 2, \ldots, n)$, let $\hat{\epsilon}^- = \langle(min(\alpha_\rho), max(\varepsilon_\rho)), (\min(J_\rho{}^2), max(K_\rho{}^2), max(L_\rho{}^2))\rangle$ and $\hat{\epsilon}^+ = \langle(max(\alpha_\rho), min(\varepsilon_\rho)), (\max(J_\rho{}^2), min(K_\rho{}^2), min(L_\rho{}^2))\rangle$, then

$$\hat{\epsilon}^- \le NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \le \hat{\epsilon}^+ \tag{17}$$

Proof:

To Theorem 2,

$$\hat{\epsilon}^- = NPNSCIA(\hat{\epsilon}^-, \hat{\epsilon}^- \ldots, \hat{\epsilon}^-),\ \hat{\epsilon}^+ = NPNSCIA(\hat{\epsilon}^+, \hat{\epsilon}^+ \ldots, \hat{\epsilon}^+)$$

To Theorems 3–4,

$$NPNSCIA(\hat{\epsilon}^-, \hat{\epsilon}^- \ldots, \hat{\epsilon}^-) \le NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \le NPNSCIA(\hat{\epsilon}^+, \hat{\epsilon}^+ \ldots, \hat{\epsilon}^+)$$

Then $\hat{\epsilon}^- \le NPNSCIA(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \le \hat{\epsilon}^+$.

3.2 NPNSCIG Operator

Definition 12. Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = \langle(\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho)\rangle$ ($\rho = 1, 2, \ldots, n$), while

$$NPNSCIG(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) = \oplus_{\rho=1}^{n} \hat{\epsilon}_{\gamma(\rho)}{}^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}} \tag{18}$$

NPNSCIG is called the NPNS Choquet integral geometric operator, in which $(\gamma(1), \gamma(2), \ldots, \gamma(n))$ express an arrangement and can make $\hat{\epsilon}_{\gamma(1)} \ge \hat{\epsilon}_{\gamma(2)} \ge \ldots \ge \hat{\epsilon}_{\gamma(n)}$ and $Z_{\gamma(\varrho)} = \{y_{\gamma(\varsigma)} | \varsigma \le \varrho\} (\varrho \ge 1), Z_{\gamma(0)} = \emptyset$ *and* $y_{\gamma(\varsigma)}$ is the corresponding weight of $\hat{\epsilon}_{\gamma(\varsigma)}$.

According to some relevant operation rules of NPNSs, we can get the form of the NPNSCIG operator shown in Theorem 6.

Theorem 6. Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = (\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho)(\rho = 1, 2, \ldots, n)$, after using the NPNSCIG operator, the collective value obtained is also an NPNE, denoted by

$$NPNSCIG(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) = \left\langle \begin{pmatrix} \begin{pmatrix} \prod_{\rho=1}^{n} \alpha_{\gamma(\rho)}{}^{(\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})}, \\ \prod_{\rho=1}^{n} \alpha_{\gamma(\rho)}{}^{(\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})} \sqrt{\sum_{\rho=1}^{n} (\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)}) \frac{\varepsilon_{\gamma(\rho)}{}^2}{\alpha_{\gamma(\rho)}{}^2}} \end{pmatrix}, \\ \begin{pmatrix} \prod_{\rho=1}^{n} ((J_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})}, \\ 1 - \prod_{\rho=1}^{n} (1 - (K_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})}, \\ 1 - \prod_{\rho=1}^{n} (1 - (L_{\gamma(\rho)})^2)^{\eta(Z_{\gamma(\rho)}) - \eta(Z_{\gamma(\rho-1)})} \end{pmatrix} \end{pmatrix} \right\rangle \tag{19}$$

Theorem 7. Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = \langle(\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho)\rangle(\rho = 1, 2, \ldots, n)$, if $\hat{\epsilon}_\rho = \hat{\epsilon} = (\alpha, \varepsilon), (J, K, L)$, then

$$NPNSCIG(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) = NPNSCIG(\hat{\epsilon}, \hat{\epsilon} \ldots, \hat{\epsilon}) = \hat{\epsilon} \tag{20}$$

Theorem 8. Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = \langle(\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho)\rangle(\rho = 1, 2, \ldots, n)$, if $\hat{\epsilon}_\rho{}'$ is a replacement of $\hat{\epsilon}_\rho$ then

$$NPNSCIG(\hat{\epsilon}_1{}', \hat{\epsilon}_2{}', \ldots, \hat{\epsilon}_n{}') = NPNSCIG(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \tag{21}$$

Theorem 9. Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, $\hat{\epsilon}_\rho = \langle(\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho)\rangle$ and $\hat{\epsilon}_\rho{}^* = \langle(\alpha_\rho{}^*, \varepsilon_\rho{}^*), (J_\rho{}^*, K_\rho{}^*, L_\rho{}^*)\rangle(\rho = 1, 2, \ldots, n)$ as two collections of NPNE on Y, and their ranking orders are $\hat{\epsilon}_{\gamma(1)} \geq \hat{\epsilon}_{\gamma(2)} \geq \ldots \geq \hat{\epsilon}_{\gamma(n)}$ and $\hat{\epsilon}_{\gamma(1)}{}^* \geq \hat{\epsilon}_{\gamma(2)}{}^* \geq \ldots \geq \hat{\epsilon}_{\gamma(n)}{}^*$, if $\hat{\epsilon}_{\gamma(\rho)} \leq \hat{\epsilon}_{\gamma(\rho)}{}^*$ for all ρ, that is $\alpha_{\gamma(\rho)} \leq \alpha_{\gamma(\rho)}{}^*, \varepsilon_{\gamma(\rho)} \geq \varepsilon_{\gamma(\rho)}{}^*, (J_{\gamma(\rho)})^2 \leq ((J_{\gamma(\rho)})^2)^*, (K_{\gamma(\rho)})^2 \geq ((K_{\gamma(\rho)})^2)^*\ and\ (L_{\gamma(\rho)})^2 \geq ((L_{\gamma(\rho)})^2)^*$, then

$$NPNSCIG(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \leq NPNSCIG(\hat{\epsilon}_1{}^*, \hat{\epsilon}_2{}^*, \ldots, \hat{\epsilon}_n{}^*) \tag{22}$$

Theorem 10. (Boundedness). Set η as a fuzzy measure on $Y = \{y_1, y_2, \ldots, y_n\}$, a collection of NPNE $\hat{\epsilon}_\rho = \langle(\alpha_\rho, \varepsilon_\rho), (J_\rho, K_\rho, L_\rho)\rangle(\rho = 1, 2, \ldots, n)$, let $\hat{\epsilon}^- = \langle(min(\alpha_\rho), max(\varepsilon_\rho)), (\min(J_\rho{}^2)\ max(K_\rho{}^2), max(L_\rho{}^2))\rangle$ and $\hat{\epsilon}^+ = \langle(max(\alpha_\rho), min(\varepsilon_\rho)), (\max(J_\rho{}^2), min(K_\rho{}^2), min(L_\rho{}^2))\rangle$ then

$$\hat{\epsilon}^- \leq NPNSCIG(\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_n) \leq \hat{\epsilon}^+ \tag{23}$$

It is easy to prove Theorems 6–10.

4 Decision Making Methods Based on NPNSCIA or NPNSCIG Operator

The multi-attribute decision problem with decision information of NPNS is described as follows: There are m schemes $H = \{h_1, h_2, \ldots, h_m\}$ and n attributes $T = \{t_1, t_2, \ldots, t_n\}$. The decision maker measures the schemes h_i by attribute t_j and gets the value $\hat{\epsilon}_{ij}$ of attributes, in which $h_{ij} = \langle(\alpha_{ij}, \varepsilon_{ij}), (J_{ij}, K_{ij}, L_{ij})\rangle$ is an NPNE. Then, we can get the NPNE decision matrix C=$(h_{ij})_{mn}$.

Considering the correlation between attributes, by using the NPNS Choquet integral operator, a MADM method is presented in the NPNS environment.

Step 1: Build up $C=(h_{ij})_{mn}$;

Step 2: Calculate the fuzzy measure of attribute sets;

Step 3: Calculate by using the NPNSCIA or NPNSCIG operator;

Step 4: Calculate each scheme's score value;

Step 5: Choose the best scheme.

5 An Illustrative Example

The company plans to choose one of the four suppliers $H=\{h_1,h_2,h_3,h_4\}$ with the strongest comprehensive ability as its long-term supplier, h_1 expresses the supplier A, h_2 expresses the supplier B, h_3 expresses the supplier C and h_4 expresses the supplier D. When making decisions, the company considers four attributes $T=\{t_1,t_2,t_3,t_4\}$ of suppliers: quality, production capacity, after-sales service and management ability, and their fuzzy measures are $\eta_1=0.3$, $\eta_2=0.3$, $\eta_3=0.3$ *and* $\eta_4=0.2$.

Step 1: In Table 1, the decision makers give the evaluation values described by NPNE.

Table 1: Matrix C

	t_1	t_2	t_3	t_4
h_1	$(3, 0.4), \left(\sqrt{0.7}, \sqrt{0.1}, \sqrt{0.3}\right)$	$(7, 0.6), \left(\sqrt{0.3}, \sqrt{0.2}, \sqrt{0.4}\right)$	$(5, 0.4), \left(\sqrt{0.3}, \sqrt{0.2}, \sqrt{0.6}\right)$	$(6, 0.5), \left(\sqrt{0.7}, \sqrt{0.3}, \sqrt{0.3}\right)$
h_2	$(4, 0.2), \left(\sqrt{0.5}, \sqrt{0.2}, \sqrt{0.2}\right)$	$(8, 0.4), \left(\sqrt{0.2}, \sqrt{0.1}, \sqrt{0.7}\right)$	$(6, 0.7), \left(\sqrt{0.5}, \sqrt{0.2}, \sqrt{0.5}\right)$	$(7, 0.6), \left(\sqrt{0.8}, \sqrt{0.1}, \sqrt{0.2}\right)$
h_3	$(3.5, 0.3), \left(\sqrt{0.4}, \sqrt{0.2}, \sqrt{0.4}\right)$	$(6, 0.2), \left(\sqrt{0.3}, \sqrt{0.1}, \sqrt{0.7}\right)$	$(5.5, 0.6), \left(\sqrt{0.3}, \sqrt{0.2}, \sqrt{0.6}\right)$	$(4, 0.4), \left(\sqrt{0.7}, \sqrt{0.2}, \sqrt{0.3}\right)$
h_4	$(5, 0.5), \left(\sqrt{0.2}, \sqrt{0.1}, \sqrt{0.4}\right)$	$(7, 0.5), \left(\sqrt{0.4}, \sqrt{0.3}, \sqrt{0.2}\right)$	$(4.5, 0.5), \left(\sqrt{0.8}, \sqrt{0.1}, \sqrt{0.2}\right)$	$(6, 0.5), \left(\sqrt{0.2}, \sqrt{0.3}, \sqrt{0.6}\right)$

Step 2: Using Eq. (10), we calculate $\vartheta=-0.2317$;

Step 3: Using Eq. (9) and ϑ, we get the fuzzy measure:

$\eta(x_1,x_2)=\eta(x_1,x_3)=\eta(x_2,x_3)=0.5791$;

$\eta(x_1,x_4)=\eta(x_2,x_4)=\eta(x_3,x_4)=0.4861$;

$\eta(x_1,x_2,x_3)=0.8389$; $\eta(x_1,x_2,x_4)=\eta(x_1,x_3,x_4)=\eta(x_2,x_3,x_4)=0.7523$;

$\eta(x_1,x_2,x_3,x_4)=1$.

Step 4: Using the NPNSCIA operator, we calculate each supplier's comprehensive value.

$h_1=\langle(5.1185,0.4719),(0.5363,0.1752,0.3892)\rangle$;

$h_2=\langle(6.1232,0.4626),(0.5323,0.1466,0.3600)\rangle$;

$h_3=\langle(4.7609,0.3677),(0.4346,0.1663,0.4960)\rangle$;

$h_4=\langle(5.5694,0.5000),(0.5129,0.1622,0.2939)\rangle$.

Step 5: Each supplier's score value can be gotten by using Eqs. (5), (6);

$\Phi_1(h_1) = 10.7778; \Phi_1(h_2) = 13.0562; \Phi_1(h_3) = 9.1181; \Phi_1(h_4) = 11.9763.$

$\Phi_2(h_1) = 0.9937; \Phi_2(h_2) = 0.9864; \Phi_2(h_3) = 0.7042; \Phi_2(h_4) = 1.0752.$

Step 6: According to the value $\Phi_1(h_i)$ and the Definition 8, we rank each supplier $h_2 \succ h_4 \succ h_1 \succ h_3$ and choose the excellent supplier h_2 .

While using NPNSCIG operator:

Step 1'–3': Just as Steps 1–3;

Step 4': Using the NPNSCIG operator, we calculate each supplier's comprehensive value.

$h_1 = \langle (4.8535, 0.4894), (0.4529, 0.1916, 0.4202) \rangle;$

$h_2 = \langle (5.9026, 0.4630), (0.4276, 0.1562, 0.4615) \rangle;$

$h_3 = \langle (4.6323, 0.3960), (0.3829, 0.1745, 0.5438) \rangle;$

$h_4 = \langle (5.5115, 0.5143), (0.3391, 0.1967, 0.3437) \rangle.$

Step 5': Each supplier's score value can be gotten by using Eqs. (5), (6).

$\Phi_1(h_1) = 9.6674; \Phi_1(h_2) = 11.4833; \Phi_1(h_3) = 8.4328; \Phi_1(h_4) = 10.7924$

$\Phi_2(h_1) = 0.9748; \Phi_2(h_2) = 0.9007; \Phi_2(h_3) = 0.7209; \Phi_2(h_4) = 1.0071.$

Step 6': According to the value $\Phi_1(h_i)$ and Definition 8, we rank each supplier $h_2 \succ h_4 \succ h_1 \succ h_3$ and choose the excellent supplier h_2.

Compared with the literatures [16–19], NPNS Choquet integral operator can not only model the weight of attributes and attribute-set in MADM problem, measure the correlation, complementary correlation, and preference correlation between attributes, but also can fully consider the importance between attributes, so as to make the decision results more objective.

6 Conclusions

In this paper, NPNS and Choquet integral operators are combined to define the NPNSCIA operator and NPNSCIG operator, which can consider the incidence relation between indicators. It is proved that they have power equality, displacement invariance, ordered monotonicity, and boundedness. A nonlinear programming model is established and the FM of indicators and indicator sets is solved objectively. In the NPNS environment, by using the defined operators and the established model, the problem of related MADM with attribute weight information unknown or partially attribute weight information unknown is solved effectively. Finally, the case proves that this method is easy and reasonable. This study extended the Choquet integral to NPNS, making the Choquet integral better applied and developed in the related MADM problems.

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References

1. Zedeh, L. A. (1965). Fuzzy sets. *Information and Control, 8(3),* 338–353. DOI 10.1016/S0019-9958(65)90241-X.
2. Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20(1),* 87–96. DOI 10.1016/S0165-0114(86)80034-3.
3. Yager, R. R. (2013). Pythagorean fuzzy subsets. *IFSA World Congress and NAFIPS Annual Meeting, 2013,* 57–61. DOI 10.1109/IFSA-NAFIPS.2013.6608375.
4. Peng, X. D., Yang, Y., Song, J. P., Jiang, Y. (2015). Pythagorean fuzzy soft set and its application. *Computer Engineering, 41(7),* 224–229. DOI 10.3969/j.issn.1000-3428.2015.07.043.
5. Liang, W., Zhang, X., Liu, M. (2015). The maximizing deviation method based on interval-valued pythagorean fuzzy weighted aggregation operator for multiple criteria group decision analysis. *Discrete Dynamics in Nature and Society, 2015,* 1–15. DOI 10.1155/2015/746572.
6. Peng, X., Yang, Y. (2016). Fundamental properties of interval-valued pythagorean fuzzy aggregation operators. *International Journal of Intelligent Systems, 31(5),* 444–487. DOI 10.1002/int.21790.
7. Liu, W. F., He, X.(2016). Pythagoras hesitation fuzzy set. *Fuzzy Systems and Mathematics, 30(4),* 107–115.
8. Fan, J. P., Yan, Y., Wu, M. Q. (2019). Multi-criteria decision making method based on triangle pythagorean fuzzy set. *Control and Decision Making, 34(8),* 1601–1608.
9. Smarandache, F. (1998). *Neutrosophy: Neutrosophic probability, set, and logic*. Rehoboth, DE, USA: American Research Press.
10. Smarandache, F. (1999). A unifying field in logics. In: *Neutrosophy: Neutrosophic probability, set and logic*. Rehoboth: American Research Press.
11. Wang, H., Smarandache, F., Zhang, Y. Q., Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure, 4,* 410–413.
12. Liu, P. D., Tang, G. L. (2016). Some power generalized aggregation operators based on the interval neutrosophic numbers and their application to decision making. *Journal of Intelligent & Fuzzy Systems, 30,* 2517–2528. DOI 10.3233/IFS-151782.
13. Fan, C. X. (2019). Correlation coefficients of refined-single valued neutrosophic sets and their applications in multiple attribute decision-making. *Journal of Advanced Computational Intelligence and Intelligent Informatics, 23(3),* 421–426. DOI 10.20965/jaciii.2019.p0421.
14. Fan, C. X., Hu, K. L., Feng, S., Ye, J., Fan, E. (2019). Heronian mean operators of linguistic neutrosophic multisets and their multiple attribute decision-making methods. *International Journal of Distributed Sensor Networks, 15(4),*155014771984305. DOI 10.1177/1550147719843059.
15. Fan, C. X., Fan, E., Chen, J. H., Ye, J., Zhou, K. (2020). Single-value neutrosophic cosine measure for evaluation of port logistics competitiveness. *Journal of Intelligent & Fuzzy Systems, 39(3),* 4667–4675. DOI 10.3233/JIFS-200598.
16. Liu, P. D., Teng, F. (2018). Multiple attribute decision-making method based on normal neutrosophic generalized weighted power averaging operator. *International Journal of Machine Learning and Cybernetics, 9(2),* 281–293. DOI 10.1007/s13042-015-0385-y.
17. Liu, P. D., Li, H. G. (2017). Multiple attribute decision-making method based on some normal neutrosophic Bonferroni mean operators. *Neural Computing and Applications, 28,* 179–194. DOI 10.1007/s00521-015-2048-z.
18. Ye, J. (2017). Multiple attribute decision-making method using correlation coefficients of normal neutrosophic sets. *Symmetry, 9,* 80. DOI 10.3390/sym9060080.

19. Jansi, R., Mohana, K., Smarandache, F. (2019). Correlation measure for pythagorean neutrosophic sets with t and f as dependent neutrosophic components. *Neutrosophic Sets and Systems, 30,* 202–212.
20. Ajay, D., Chellamani, P. (2020). Pythagorean neutrosophic fuzzy graphs. *International Journal of Neutrosophic Science, 11(2),* 108–114. DOI 10.5281/zenodo.4172124.
21. Marichal, J. L. (2001). An axiomatic approach of the discrete choquet integral as a tool to aggregate interacting criteria. *IEEE Transactions on Fuzzy Systems, 8(6),* 800–807. DOI 10.1109/91.890347.
22. Sugeno, M. (1974). *Theory of fuzzy integral and its application (Doctorial Dissertation)*. Tokyo Institute of Technology.
23. Choquet, G. (1954). Theory of capacities. *Ann Institut Fourier, 5,* 131–295. DOI 10.5802/aif.53.
24. Xu, Z. S. (2010). Choquet integrals of weighted intuitionistic fuzzy information. *Information Sciences, 180,* 726–736. DOI 10.1016/j.ins.2009.11.011.
25. Tan, C. Q., Chen, X. H. (2010). Intuitionistic fuzzy choquet integral operator for multi-criteria decision making. *Expert Systems with Applications, 37(1),* 149–157. DOI 10.1016/j.eswa.2009.05.005.
26. Qu, G., Qu, W., Li, C. (2018). Some new interval-valued dual hesitant ant fuzzy choquet integral aggregation opeators and their applications. *Journal of Intelligent & Fuzzy Systems, 34,* 245–266. DOI 10.3233/JIFS-171162.
27. Peng, X., Yang, Y. (2016). Pythagorean fuzzy choquet integral based MMBAC method for multiple attribute group decision making. *International Journal of Intelligent Systems, 31(10),* 989–1020. DOI 10.1002/int.21814.
28. Dong, J. Y., Lin, L. L., Wan, S. P., Feng, W. (2016). Generalized choquet integral operator of triangular atanassov's intuitionistic fuzzy numbers and application to multi-attribute group decision making, international journal of uncertainty. *Fuzziness and Knowledge-Based Systems, 24(5),* 647–683. DOI 10.1142/S0218488516500306.
29. Wan, S. P., Xiao, J. (2020). Generalized shapley choquet integral operator based method for interactive interval-valued hesitant fuzzy uncertain linguistic multi-criteria group decision making. *IEEE Access, 8,* 202194–202215. DOI 10.1109/Access.6287639.
30. Wang, J. Q., Li, K. J., Zhang, H. Y., Chen, X. H. (2013). A score function based on relative entropy and its application in intuitionistic normal fuzzy multiple criteria decision making. *Journal of Intelligent & Fuzzy Systems, 25,* 567–576. DOI 10.3233/IFS-120662.