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ARTICLE

Fixed-Time Adaptive Time-Varying Matrix Projective Synchronization of Time-Delayed Chaotic Systems with Different Dimensions

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ABSTRACT

This paper deals with the fixed-time adaptive time-varying matrix projective synchronization (ATVMPS) of different dimensional chaotic systems (DDCSs) with time delays and unknown parameters. Firstly, to estimate the unknown parameters, adaptive parameter updated laws are designed. Secondly, to realize the fixed-time ATVMPS of the time-delayed DDCSs, an adaptive delay-unrelated controller is designed, where time delays of chaotic systems are known or unknown. Thirdly, some simple fixed-time ATVMPS criteria are deduced, and the rigorous proof is provided by employing the inequality technique and Lyapunov theory. Furthermore, the settling time of fixed-time synchronization (Fix-TS) is obtained, which depends only on controller parameters and system parameters and is independent of the system's initial states. Finally, simulation examples are presented to validate the theoretical analysis.

KEYWORDS

Time-varying matrix projective synchronization (TVMPS); fixed-time control; unknown parameters; different dimensions; time-delayed chaotic systems (TDCSs)

1 Introduction

Since Pecora et al. [1] introduced the concept of chaotic synchronization, chaotic synchronization, as a significant dynamical behavior of chaotic systems, has gained much attention in varieties of fields such as chemical reactions, image processing, secure communications, etc. So far, various forms of synchronization in chaotic systems have been observed and developed, including complete synchronization [2,3], lag synchronization [4,5], generalized synchronization [6,7], function projective synchronization [8] and matrix projective synchronization [9,10].

It is well noticed that most of synchronization problems are studied between two identical dimensional systems. However, in practice, the synchronization often occurs in different dimensional systems, especially systems in biological or social science. For instance, the synchronization behavior between circulatory system and respiratory system [11] is a typical application. Thus, it is essential and significant to study synchronization topic in DDCSs. Up to now, in regard to synchronization of DDCSs, numerous remarkable results have been reported in [12–16]. For example, the matrix projective synchronization was studied in [17,18], where the scaling matrices can be time-varying or time-invariant. In addition, Cai et al. [14–16] studied finite-time generalized synchronization of DDCSs.

It is universally acknowledged that time delays are frequently inevitable for a number of nonlinear dynamics systems such as physical and biological systems, which can lead to oscillations or instability in systems. So, it is difficult to reach the synchronization of TDCSs. Hence, the synchronization of TDCSs has become a hot issue and lots of related literature have been published [17–19]. Nevertheless, as far as the authors know, there exist few works for the synchronization of DDCSs with time delays up to now. Moreover, it is observed that all above aforementioned results of synchronization of TDCSs mainly consider that the parameters in master-slave systems are known with certainty. However, in practical applications, due to the influence of environment noises and limitations of equipment, it is often impossible to precisely acquire the system's parameters in advance. These parametric uncertainties may cause performance degradation or instability of real systems. Thus, it can be reasonable to investigate the chaotic systems' synchronization with unknown parameters via adaptive control strategies. Recently, some studies considered the synchronization of identical dimensional chaotic systems only with uncertainty parameters or only with time delays [15,20,21]. It means that previous studies did not deal with both the time delays and the parametric uncertainties simultaneously for identical dimensional chaotic systems, especially for DDCSs.

In practical applications, convergence rate is significantly important to reflect the synchronization effectiveness, that is to say, for the purpose of meeting specific requirements, it is preferable to realize synchronization in a limited time. Considering this point, various types of finite-time control strategies have been developed [22–26]. But, the settling time of finite-time synchronization (Fin-TS) relies on systems' initial states which are required to be known in advance. In fact, in real-word systems, the initial conditions are usually nonadjustable or even unavailable sometimes which means the settling time is imprecise. To break through these limitations, the fixed-time stabilization of nonlinear dynamic systems was proposed by [27]. From then, various literature [28–31] have studied the Fix-TS. In contrast to Fin-TS, the Fix-TS contributes to a more precise estimation of the settling time. In other words, the settling time can be estimated by some constants which are simply relevant to the parameters design in control strategies, regardless of initial states. For the sake of realizing the Fix-TS, some control methods have been put forward, for example, sliding model control [32,33], impulsive control [34], pining control [35,36], adaptive control [37], Kalman filter [38,39] and, etc. Among them, adaptive control has the characteristics of fast and real-time response, which can adjust the controller parameters to realize synchronization of systems. But, as far as authors know, these results are most for chaotic systems with the same dimensions and there are few results for the Fin-TS or Fix-TS of DDCSs. In [40], the authors first studied the generalized Fin-TS of DDCS. Then the Fin-TS of DDCS with uncertain parameters was studied in [41]. Recently, in [42], the authors investigated global Fix-TS of DDCSs. However, up to now, the fixed-time strategies are still not present in the research of adaptive TVMPS of time-delayed DDCSs.

Inspired by the aforementioned concerns, the fixed-time ATVMPS of DDCSs with time delays and unknown parameters is addressed in this paper. The main contributions can be found as following: Firstly, compared with the synchronization problem of TDCSs with known and certainty parameters [17–19], this paper designs the adaptive parameter updated laws to identify the unknown parameters. Secondly, on the basis of the time delays of chaotic systems are unknown, an adaptive delay-unrelated controller is designed to realize fixed-time ATVMPS of time-delayed DDCSs. Thirdly, some simple fixed-time ATVMPS criteria are deduced, and the rigorous proof is given by utilizing inequality technique and Lyapunov theory. Fourthly, compared with finte-time synchronization [40,41], the settling time for fixed-time ATVMPS is obtained, which is regardless of initial states but only depends on controller parameters and system parameters. Finally, simulations demonstrate the effectiveness of main results.

This paper is structured as following: The model description and preliminaries are briefly presented in Section 2. Based on the unknown delays, a novel adaptive delay-unrelated controller is designed in Section 3. Section 4 shows illustrative examples. Section 5 gives some conclusions and future extension of this paper.

Notations: $\mathbb{R}^r$ represents the real space with dimension r. $\mathbb{R}^+$ and $\mathbb{R}^{r\times l}$ denote a space composed of all nonnegative real numbers and $r\times l$ real matrices, respectively. Let $\|x\| = (x^T x)^{1/2}$, for $x \in \mathbb{R}^r$. Let A^T stand for matrix transpose of A. Let I denote the identity matrix.

2 Problem Formulation and Preliminaries

Consider the time-delayed DDCSs (1) and (2) with unknown parameters. The master system is described by:

$$\dot{x}(t) = G(x(t)) + g_1(x(t))\varphi_1 + g_2(x(t-\tau_1))\varphi_2, \tag{1}$$

in which $x(t) \in \mathbb{R}^r$ denotes the state vector; $G : \mathbb{R}^r \to \mathbb{R}^r$, $g_1 : \mathbb{R}^r \to \mathbb{R}^{r\times m_1}$, $g_2 : \mathbb{R}^r \to \mathbb{R}^{r\times n_1}$ are all continuous functions; $\varphi_1 \in \mathbb{R}^{m_1}$ and $\varphi_2 \in \mathbb{R}^{n_1}$ are unknown parameters; $\tau_1 > 0$ is the delay of the master system. Similarly the slave system is given by:

$$\dot{y}(t) = H(y(t)) + h_1(y(t))\psi_1 + h_2(y(t-\tau_2))\psi_2 + u(t), \tag{2}$$

in which $y(t) \in \mathbb{R}^l$ denotes the state vector; $H : \mathbb{R}^l \to \mathbb{R}^l$, $h_1 : \mathbb{R}^l \to \mathbb{R}^{l\times m_2}$, $h_2 : \mathbb{R}^l \to \mathbb{R}^{l\times n_2}$ are continuous functions; $\psi_1 \in \mathbb{R}^{m_2}$ and $\psi_2 \in \mathbb{R}^{n_2}$ are unknown parameters; $\tau_2 > 0$ is the delay of slave system; $u(t) \in \mathbb{R}^l$ denotes control input designed to achieve Fix-TPS of systems (1) and (2).

Remark 1. (1) and (2) are the same dimensions when $r = l$. The time delays $\tau_i (i = 1, 2)$ are positive constants which can exist in the well-known chaotic and hyperchaotic systems, including Chen system, Lorenz system, Lü system, hyperchaotic Lozenz system, etc. Then, the synchronization problem for the delayed chaotic systems also has been studied [43–45]. However, up to now, there is no paper to investigate the adaptive TVMP synchronization of TDCSs with different dimensions within a fixed time.

Assumption 2.1. In this paper, we assume the dimensions r and l of master and slave systems satisfy that $l \le r$.

To consider fixed-time ATVMPS of time-delayed DDCSs, some crucial lemmas and definitions are described as follows:

Definition 2.1. [13] Suppose there is a time-varying full row rank scaling function matrix $M(t) = (m_{ij}(t)) \in \mathbb{R}^{l\times r}$, then, the synchronization error between (1) and (2) is defined as $\varepsilon(t) = y(t) - M(t)x(t)$. If there is a function $T : U\backslash\{0\} \to (0, +\infty)$ in which $U \subset \mathbb{R}^l$ is an open neighborhood of the origin satisfying $\varepsilon_0 = y(0) - M(0)x(0) \in U$ and

$$\lim_{t\to T(\varepsilon_0)} \|\varepsilon(t)\| = \lim_{t\to T(\varepsilon_0)} \|y(t) - M(t)x(t)\| = 0, \tag{3}$$

$$\|\varepsilon(t)\| \equiv 0, t > T(\varepsilon_0), \tag{4}$$

the finite-time TVMPS for the (1) and (2) can be realized. If $U = \mathbb{R}^l$, then the global finite-time TVMPS for the systems (1) and (2) can be realized.

Remark 2. In Definition 2.1, it is not allowed that the element in each row of the time-varying matrix $M(t)$ cannot be equal to zero

$$\varepsilon_i(t) = y_i - (0 \quad 0 \quad \cdots \quad 0)\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_r \end{pmatrix} = y_i,$$

then it is meaningless for chaotic system's synchronization. Additionally, the time-varying matrix $M(t)$ is a full row rank which ensures the existence of solutions of (3).

Definition 2.2. [27] The master system (1) reaches fixed-time TVMPS with slave system (2), if their error system $\varepsilon(t) = y(t) - M(t)x(t)$ is globally finite-time stable, and there is a positive constant $T_{\max}$ so that the settling time $T(\varepsilon_0)$ satisfies $T(\varepsilon_0) \leqslant T_{\max}$ for any initial values.

Definition 2.3. Definite the vector functions $\phi_1(x)$ and $\phi_2(x)$ as:

$$\phi_1(x) = \begin{cases} \|x\|^{-1}x, & if x \neq 0 \\ 0, & if x = 0 \end{cases}, \quad \phi_2(x) = \begin{cases} \|x\|^{-2}x, & if x \neq 0 \\ 0, & if x = 0 \end{cases}.$$

Lemma 2.1. [46] Let $\chi_1, \chi_2, \cdots, \chi_N \geq 0$. Then,

$$\sum_{i=1}^{N} \chi_i^p \geqslant \left(\sum_{i=1}^{N} \chi_i\right)^p, \quad \text{when} \quad 0 < p \leqslant 1; \quad \text{and} \quad \sum_{i=1}^{N} \chi_i^p \geqslant N^{1-p}\left(\sum_{i=1}^{N} \chi_i\right)^p, \quad \text{when} \quad p > 1.$$

Lemma 2.2. [27] Given the differential equation:

$$\dot{x}(t) = f(x,t), \quad x(0) = x_0, \tag{5}$$

where $f(t, x(t)) : \mathbb{R}^+ \times \mathbb{R}^N \to \mathbb{R}^N$ is continuous. Let $V(t)$ be a positive definite and continuous radially unbounded function. If the following inequality for any solution $x(t)$ of (5) holds,

$$\dot{V}(x(t)) \leq -\alpha V^m(x(t)) - \beta V^n(x(t)),$$

with constants $\alpha, \beta > 0, 0 < m < 1$ and $n > 1$, then Eq. (5) is fixed-time stable, and the settling time holds

$$T(x) \leq T_{max} := \frac{1}{\alpha(1-m)} + \frac{1}{\beta(n-1)}.$$

3 Main Results

This part addresses the fixed-time TVMPS for time-delayed DDCSs (1) and (2) by designing the adaptive controller. Then, an adaptive controller will be designed for slave system (2) based on the unknown time delays. Furthermore, the rigorous proof will also be given. To get the theoretical results, we suppose that the following conditions are satisfied:

Assumption 3.1. The unknown parameter vectors are norm bounded, i.e., $\|\varphi_1\| \le \theta_{\varphi_1}$, $\|\varphi_2\| \le \theta_{\varphi_2}$, $\|\psi_1\| \le \theta_{\psi_1}$, $\|\psi_2\| \le \theta_{\psi_2}$, where $\theta_{\varphi_1}, \theta_{\varphi_2}, \theta_{\psi_1}, \theta_{\psi_2}$ are positive constants.

In this section, the synchronization problem of systems (1) and (2) with unknown time delays τ_1 and τ_2 will be studied, where the proposed adaptive controller is independent of delays. In this case, some assumptions shall be needed.

Assumption 3.2. Assume the matrix function $M(t)$ is piecewise continuous, differentiable, and norm bounded for any t, i.e., $\|M(t)\| \le \rho$, for any t, where ρ is a positive constant.

Assumption 3.3. The nonlinear continuous functions $g_i(\cdot)$ and $h_i(\cdot)(i=1,2)$ in systems (1) and (2) satisfy $g_i(0) = f_i(0) = 0$ and Lipschitz conditions. That is to say, there exist constants $L_{g_i}, L_{h_i} > 0$ to ensure that following conditions hold:

$$\|g_i(u_1) - g_i(v_1)\| \le L_{g_i}\|u_1 - v_1\|, \quad \|h_i(u_2) - h_i(v_2)\| \le L_{h_i}\|u_2 - v_2\|,$$

where $u_1, v_1 \in \mathbb{R}^r, u_1 \ne v_1,\ u_2, v_2 \in \mathbb{R}^l, u_2 \ne v_2$.

Assumption 3.4. The state vector of master system (1) satisfies $\|x(t)\| \le \delta$ for any $t > 0$, where δ is a positive constant.

Assumption 3.5. In this part, we suppose that the unknown delay τ_2 is bounded, i.e., $0 \le \tau_2 \le \bar{\tau}, \bar{\tau}$ is known as a positive constant.

Remark 3. As is widely acknowledged, chaotic system has bounds, and its trajectory is always confined to a certain region, that is, the domain of chaotic attraction. No matter how unstable the interior of the chaotic system is, its orbit will not go out of the domain of chaotic attraction. So, the Assumption 3.4 is reasonable.

To realize fixed-time ATVMPS of chaotic systems (1) and (2) with unknown delays, the adaptive delay-unrelated controller is presented as follows:

$$\begin{aligned} u(t) = & -H(y(t)) - h_1(y(t))\hat{\psi}_1 - h_2(y(t))\hat{\psi}_2 + \dot{M}(t)x(t) + M(t)G(x(t)) \\ & + M(t)g_1(x(t))\hat{\varphi}_1 + M(t)g_2(x(t))\hat{\varphi}_2 - \sigma_1\phi_1(\varepsilon(t)) - \sigma_2\varepsilon(t) \\ & - k_1\beta_1(t) - \lambda_1\Theta_1\phi_2(\varepsilon(t)) - \eta_1\left\{\int_{t-\bar{\tau}}^{t} \varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_1}{2}} \phi_2(\varepsilon(t)) \\ & - k_2\beta_2(t) - \lambda_2\Theta_2\phi_2(\varepsilon(t)) - \eta_2\left\{\int_{t-\bar{\tau}}^{t} \varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_2}{2}} \phi_2(\varepsilon(t)), \end{aligned} \tag{6}$$

where $\beta_1(t) = [sign(\varepsilon_1(t))|\varepsilon_1(t)|^{\mu_1}, \ldots, sign(\varepsilon_l(t))|\varepsilon_l(t)|^{\mu_1}]^T$; $\beta_2(t) = [sign(\varepsilon_1(t))|\varepsilon_1(t)|^{\mu_2}, \ldots, sign(\varepsilon_l(t))|\varepsilon_l(t)|^{\mu_2}]^T$; $\sigma_1 = \delta L_{h_2}\theta_{\psi_2} + \rho\delta L_{h_2}\theta_{\psi_2} + 2\rho\delta L_{g_2}\theta_{\varphi_2}$; $\sigma_2 = 2L_{h_2}\theta_{\psi_2}$; $Q = L_{h_2}\theta_{\psi_2}I$; $\Theta_1 = (\|\hat{\varphi}_1\| + \theta_{\varphi_1})^{\mu_1+1} + (\|\hat{\varphi}_2\| + \theta_{\varphi_2})^{\mu_1+1} + (\|\hat{\psi}_1\| + \theta_{\psi_1})^{\mu_1+1} + (\|\hat{\psi}_2\| + \theta_{\psi_2})^{\mu_1+1}$; $\Theta_2 = (\|\hat{\varphi}_1\| + \theta_{\varphi_1})^{\mu_2+1} + (\|\hat{\varphi}_2\| + \theta_{\varphi_2})^{\mu_2+1} + (\|\hat{\psi}_1\| + \theta_{\psi_1})^{\mu_2+1} + (\|\hat{\psi}_2\| + \theta_{\psi_2})^{\mu_2+1}$; $\hat{\varphi}_1, \hat{\varphi}_2, \hat{\psi}_1$ and $\hat{\psi}_2$ denote the estimations of unknown parameters $\varphi_1, \varphi_2, \psi_1$ and ψ_2, respectively; $k_1, k_2, m_1, \lambda_1, \lambda_2, \eta_1$, and η_2 are arbitrary positive constants; μ_1, μ_2 are constants satisfying $0 < \mu_1 < 1, \mu_2 > 1$.

To address the unknown parameters, the following updated laws are designed as:

$$\dot{\hat{\varphi}}_1 = -(M(t)g_1(x(t)))^T\varepsilon(t), \tag{7}$$

$$\dot{\hat{\varphi}}_2 = -(M(t)g_2(x(t)))^T\varepsilon(t), \tag{8}$$

$$\dot{\hat{\psi}}_1 = h_1^T(y(t))\varepsilon(t), \tag{9}$$

$$\dot{\hat{\psi}}_2 = h_2^T(y(t))\varepsilon(t). \tag{10}$$

Hence, the error system satisfies

$$\begin{aligned}\dot{\varepsilon}(t) =& \dot{y}(t) - \dot{M}(t)x(t) - M(t)\dot{x}(t)\\ =& -h_1(y(t))\tilde{\psi}_1 - h_2(y(t))\tilde{\psi}_2 + M(t)g_1(x(t))\tilde{\varphi}_1 + M(t)g_2(x(t))\tilde{\varphi}_2\\ &+ (h_2(y(t-\tau_2)) - h_2(y(t)))\psi_2 + M(t)(g_2(x(t)) - g_2(x(t-\tau_1)))\varphi_2\\ &- \sigma_1\phi_1(\varepsilon(t)) - k_1\beta_1(t) - \lambda_1\Theta_1\phi_2(\varepsilon(t)) - \eta_1\left\{\int_{t-\bar{\tau}}^{t}\varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_1}{2}}\phi_2(\varepsilon(t))\\ &- \sigma_2\varepsilon(t) - k_2\beta_2(t) - \lambda_2\Theta_2\phi_2(\varepsilon(t)) - \eta_2\left\{\int_{t-\bar{\tau}}^{t}\varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_2}{2}}\phi_2(\varepsilon(t)),\end{aligned} \tag{11}$$

where $\tilde{\varphi}_1 = \hat{\varphi}_1 - \varphi_1, \tilde{\varphi}_2 = \hat{\varphi}_2 - \varphi_2, \tilde{\psi}_1 = \hat{\psi}_1 - \psi_1,\ \tilde{\psi}_2 = \hat{\psi}_2 - \psi_2$.

Remark 4. The controller (6) and updated laws (7)–(10) do not contain delays τ_1 and τ_2. It implies that the controller (6) is independent of delays of systems (1) and (2), but is only related with $\bar{\tau}$.

Theorem 3.6. Supposing Assumptions 2.1, 3.1 and 3.2–3.5 hold, the master-slave systems (1)–(2) can reach fixed-time adaptive TVMP synchronization using the proposed controller (6) and parameters updated laws (7)–(10). Moreover, the settling time is bounded

$$T_2 \le \frac{1}{\alpha_1(1-\mu_1)} + \frac{1}{\alpha_2(\mu_2-1)}, \tag{12}$$

where $\alpha_1 = \min\{k_1, \lambda_1, \eta_1\}$, $\alpha_2 = 3^{\frac{1-\mu_2}{2}}\min\left\{k_2 l^{\frac{1-\mu_2}{2}}, \lambda_2 4^{\frac{1-\mu_2}{2}}, \eta_2\right\}$, l is the dimension of the slave system, μ_1 and μ_2 are positive constants and satisfy Lemma 2.2.

Proof: Let the Lyapunov-Krasovskii function:

$$V(t) = \varepsilon(t)^T\varepsilon(t) + \tilde{\varphi}_1{}^T\tilde{\varphi}_1 + \tilde{\varphi}_2{}^T\tilde{\varphi}_2 + \tilde{\psi}_1{}^T\tilde{\psi}_1 + \tilde{\psi}_2{}^T\tilde{\psi}_2 + \int_{t-\tau_2}^{t}\varepsilon^T(s)Q\varepsilon(s)ds. \tag{13}$$

The derivative of $V(t)$ along (7)–(11) obeys with

$$\begin{aligned}\dot{V}(t) =& 2\varepsilon^T(t)\dot{\varepsilon}(t) + 2\tilde{\varphi}_1{}^T\dot{\hat{\varphi}}_1 + 2\tilde{\varphi}_2{}^T\dot{\hat{\varphi}}_2 + 2\tilde{\psi}_1{}^T\dot{\hat{\psi}}_1 + 2\tilde{\psi}_2{}^T\dot{\hat{\psi}}_2\\ &+ \varepsilon^T(t)Q\varepsilon(t) - \varepsilon^T(t-\tau_2)Q\varepsilon(t-\tau_2)\\ &\le 2\varepsilon(t)^T M(t)[g_2(x(t)) - g_2(x(t-\tau_1))]\varphi_2 + 2\varepsilon(t)^T[h_2(y(t-\tau_2)) - h_2(y(t))]\psi_2\end{aligned}$$

$$
\begin{aligned}
&-2\sigma_1\|\varepsilon(t)\|-2\sigma_2\varepsilon^T(t)\varepsilon(t)+\varepsilon^T(t)Q\varepsilon(t)-\varepsilon^T(t-\tau_2)Q\varepsilon(t-\tau_2)\\
&-2k_1\sum_{i=1}^{l}\varepsilon_i(t)sign(\varepsilon_i(t))|\varepsilon_i(t)|^{\mu_1}-\eta_1\left\{\int_{t-\tau_2}^{t}\varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_1}{2}}\frac{1}{\|\varepsilon(t)\|^2}\sum_{i=1}^{l}\varepsilon_i^2(t)\\
&-2k_2\sum_{i=1}^{l}\varepsilon_i(t)sign(\varepsilon_i(t))|\varepsilon_i(t)|^{\mu_2}-\eta_2\left\{\int_{t-\tau_2}^{t}\varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_2}{2}}\frac{1}{\|\varepsilon(t)\|^2}\sum_{i=1}^{l}\varepsilon_i^2(t)\\
&-2\lambda_1\Theta_1\frac{1}{\|\varepsilon(t)\|^2}\sum_{i=1}^{l}\varepsilon_i^2(t)-2\lambda_2\Theta_2\frac{1}{\|\varepsilon(t)\|^2}\sum_{i=1}^{l}\varepsilon_i^2(t).
\end{aligned}
\tag{14}
$$

Based on Assumptions 3.1–3.4 as well as the fact $\frac{1}{\|\varepsilon(t)\|^2}\sum_{i=1}^{l}\varepsilon_i^2(t)=1$, we have

$$
\begin{aligned}
&2\varepsilon(t)^T M(t)[g_2(x(t))-g_2(x(t-\tau_1))]\varphi_2\\
&\quad\le 2\|\varepsilon(t)\|\ \|M(t)\|\ \|g_2(x(t))-g_2(x(t-\tau_1))\|\|\varphi_2\|\\
&\quad\le 2\|\varepsilon(t)\|\rho\theta_{\varphi_2}L_{g_2}\|x(t)-x(t-\tau_1)\|\\
&\quad\le 4\rho\delta L_{g_2}\theta_{\varphi_2}\|\varepsilon(t)\|,
\end{aligned}
\tag{15}
$$

and

$$
\begin{aligned}
&2\varepsilon(t)^T[h_2(y(t-\tau_2))-h_2(y(t))]\psi_2\\
&\quad\le 2\|\varepsilon(t)\|\ \|h_2(y(t-\tau_2))-h_2(y(t))\|\|\psi_2\|\\
&\quad\le 2\theta_{\psi_2}L_{h_2}\|\varepsilon(t)\|\ \|y(t-\tau_2)-y(t)\|\\
&\quad= 2\theta_{\psi_2}L_{h_2}\|\varepsilon(t)\|\|\varepsilon(t-\tau_2)+M(t)x(t-\tau_2)-\varepsilon(t)-M(t)x(t)\|\\
&\quad\le 2\theta_{\psi_2}L_{h_2}\|\varepsilon(t)\|\|\varepsilon(t-\tau_2)\|+2\theta_{\psi_2}L_{h_2}\|\varepsilon(t)\|^2+2\rho\delta L_{h_2}\theta_{\psi_2}\|\varepsilon(t)\|+2\delta L_{h_2}\theta_{\psi_2}\|\varepsilon(t)\|\\
&\quad\le 3L_{h_2}\theta_{\psi_2}\|\varepsilon(t)\|^2+L_{h_2}\theta_{\psi_2}\|\varepsilon(t-\tau_2)\|^2+2\rho\delta L_{h_2}\theta_{\psi_2}\|\varepsilon(t)\|+2\delta L_{h_2}\theta_{\psi_2}\|\varepsilon(t)\|.
\end{aligned}
\tag{16}
$$

Hence, (14) is rewritten as

$$
\begin{aligned}
\dot V(t)\le&(4\rho\delta L_{g_2}\theta_{\varphi_2}+2\delta L_{h_2}\theta_{\psi_2}+2\rho\delta L_{h_2}\theta_{\psi_2}-2\sigma_1)\|\varepsilon(t)\|+(3L_{h_2}\theta_{\psi_2}-2\sigma_2)\|\varepsilon(t)\|^2\\
&+L_{h_2}\theta_{\psi_2}\|\varepsilon(t-\tau_2)\|^2-2k_1\left(\varepsilon^T(t)\varepsilon(t)\right)^{\frac{1+\mu_1}{2}}-2k_2 l^{\frac{1-\mu_2}{2}}\left(\varepsilon^T(t)\varepsilon(t)\right)^{\frac{1+\mu_2}{2}}\\
&-2\eta_1\left\{\int_{t-\tau_2}^{t}\varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_1}{2}}-2\eta_2\left\{\int_{t-\tau_2}^{t}\varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_2}{2}}\\
&-\lambda_1\Theta_1-\lambda_2\Theta_2+\varepsilon^T(t)Q\varepsilon(t)-\varepsilon^T(t-\tau_2)Q\varepsilon(t-\tau_2).
\end{aligned}
\tag{17}
$$

Substituting σ_1, σ_2 and Q into (17), there is

$$
\begin{aligned}
\dot{V}(t) \leq & -2k_1\left(\varepsilon^T(t)\varepsilon(t)\right)^{\frac{1+\mu_1}{2}} - 2\eta_1\left\{\int_{t-\tau_2}^{t}\varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_1}{2}} \\
& -2k_2 l^{\frac{1-\mu_2}{2}}\left(\varepsilon^T(t)\varepsilon(t)\right)^{\frac{1+\mu_2}{2}} - 2\eta_2\left\{\int_{t-\tau_2}^{t}\varepsilon^T(s)Q\varepsilon(s)ds\right\}^{\frac{1+\mu_2}{2}} \\
& -2\lambda_1(\tilde{\varphi}_1{}^T\tilde{\varphi}_1 + \tilde{\varphi}_2{}^T\tilde{\varphi}_2 + \tilde{\psi}_1{}^T\tilde{\psi}_1 + \tilde{\psi}_2{}^T\tilde{\psi}_2)^{\frac{1+\mu_1}{2}} \\
& -2\lambda_2 4^{\frac{1-\mu_2}{2}}(\tilde{\varphi}_1{}^T\tilde{\varphi}_1 + \tilde{\varphi}_2{}^T\tilde{\varphi}_2 + \tilde{\psi}_1{}^T\tilde{\psi}_1 + \tilde{\psi}_2{}^T\tilde{\psi}_2)^{\frac{1+\mu_1}{2}} \\
\leq & -2\alpha_1 V_2^{\frac{1+\mu_1}{2}}(t) - 2\alpha_2 V_2^{\frac{1+\mu_2}{2}}(t),
\end{aligned} \tag{18}
$$

where $\alpha_1 = \min\{k_1, \lambda_1, \eta_1\}$, $\alpha_2 = 3^{\frac{1-\mu_2}{2}}\min\left\{k_2 l^{\frac{1-\mu_2}{2}}, \lambda_2 4^{\frac{1-\mu_2}{2}}, \eta_2\right\}$.

From Lemma 2.2, it follows $V(t) \equiv 0$ for $t \geq T_{\max} = \frac{1}{\alpha_1(1-\mu_1)} + \frac{1}{\alpha_2(\mu_2-1)}$. That is to say, the global fixed-time stability of (11) can be guaranteed, then the fixed-time ATVMPS of systems (1) and (2) is obtained with the aid of controller (6). Also, the settling time is estimated by (12). Consequently, we finalize the derivation.

Remark 5. If we change the time-varying matrix $M(t) = (m_{ij}(t)) \in \mathbb{R}^{l\times r}$ to a time-invariant matrix which also satisfies all conditions of this paper. Then, all above results are still valid.

4 Simulation Results

To verify the correctness of Theorem 3.6, we here respectively select a four-dimensional delayed hyperchaotic Lü system [47] and a three-dimensional delayed Lorenz system [48] as the master and slave systems.

The delayed hyperchaotic Lü system can be described as:

$$
\begin{cases}
\dot{x}_1(t) = -a_1x_1(t) + a_1x_2(t), \\
\dot{x}_2(t) = b_1x_2(t) - x_1(t)x_3(t) + h_1x_4(t-\tau_1), \\
\dot{x}_3(t) = x_1(t)x_2(t) - c_1x_3(t), \\
\dot{x}_4(t) = -d_1x_1(t) - e_1x_2(t),
\end{cases} \tag{19}
$$

and the delayed Lorenz system is given

$$
\begin{cases}
\dot{y}_1(t) = a_2(y_2(t) - y_1(t)) + h_2y_1(t-\tau_2) + u_1(t), \\
\dot{y}_2(t) = b_2y_1(t) + c_2y_2(t) - y_1(t)y_3(t) + u_2(t), \\
\dot{y}_3(t) = y_1(t)^2 - d_2y_3(t) + u_3(t).
\end{cases} \tag{20}
$$

In this simulation, the parameters of master-slave systems are selected as $a_1 = 35$, $b_1 = 20$, $c_1 = 3$, $d_1 = 2$, $e_1 = 2$, $h_1 = 1$, $a_2 = 20$, $b_2 = 14$, $c_2 = 10.6$, $d_2 = 2.8$, $h_2 = 3$ and the values of delays are chosen as $\tau_1 = 1, \tau_2 = 0.001$. In this case, the systems (19) and (20) show the hyperchaotic and chaotic behaviors. Let $\varphi_1 = [a_1, b_1, c_1, d_1, e_1]^T$, $\varphi_2 = h_1$, $\psi_1 = [a_2, b_2, c_2, d_2]^T$, $\psi_2 = h_2$. After some calculations, the norm bounds of parameters can be obtained as $\|\varphi_1\| \le \theta_{\varphi_1} = 42$, $\|\varphi_2\| \le \theta_{\varphi_2} = 1.5$, $\|\psi_1\| \le \theta_{\psi_1} = 27$, $\|\psi_2\| \le \theta_{\psi_2} = 4.5$. Suppose the scaling function matrix is chosen as following:

$$M(t) = \begin{bmatrix} 0.2e^{-t}\sqrt{5e^t} & 0.1 & 0 & e^{-t}\sqrt{7e^t} \\ 0.01 & e^{-t}\sqrt{5e^t} & 0.01e^{-t}\sqrt{3e^t} & 0 \\ -2e^{-t}\sqrt{e^t - 0.5} & -e^{-t}\sqrt{5e^t - 1} & e^{-2t}\sqrt{2e^t} & -0.002 \end{bmatrix}.$$

Let the initial states of (19) and (20) as $x(0) = (-2, 3, 0.4, 0.2)^T$, $y(0) = (20, 20, 20)^T$. The initial values of adaptive laws (7)–(10) of unknown parameters are chosen as $\hat{a}_1(0) = \hat{b}_1(0) = \hat{c}_1(0) = \hat{d}_1(0) = \hat{e}_1(0) = 0, \hat{h}_1(0) = 2.4, \hat{a}_2(0) = 1.2, \hat{b}_2(0) = 1.2, \hat{c}_2(0) = 1, \hat{d}_2(0) = 1, \hat{e}_2(0) = -2, \hat{h}_2(0) = -3$. Required parameters are selected as $\bar{\tau} = 0.002$, $L_{h_2} = 1, L_{g_2} = 0.9$, $\rho = 12, \delta = 5$, $k_1 = 0.0001$, $k_2 = 0.0001$, $\lambda_1 = 0.0001$, $\lambda_2 = 10.6$, $\eta_1 = 0.001$, $\eta_2 = 0.001$, $\mu_1 = 1.15$, $\mu_2 = 0.25$. Fig. 1 shows that the synchronization errors of systems (19) and (20) can tend to zero in a fixed time which validates Theorem 3.6. Figs. 2–5 show the evolutions of systems parameters, from which the system parameters also tend to fixed constants in a fixed time.

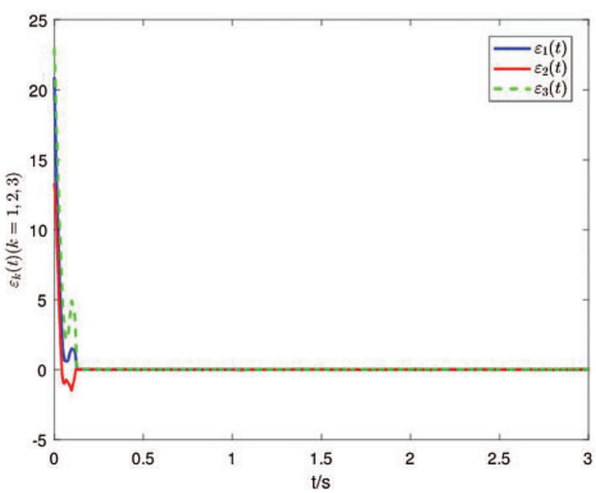

Figure 1: The trajectories of the errors $\varepsilon_k(t)(k = 1, 2, 3)$

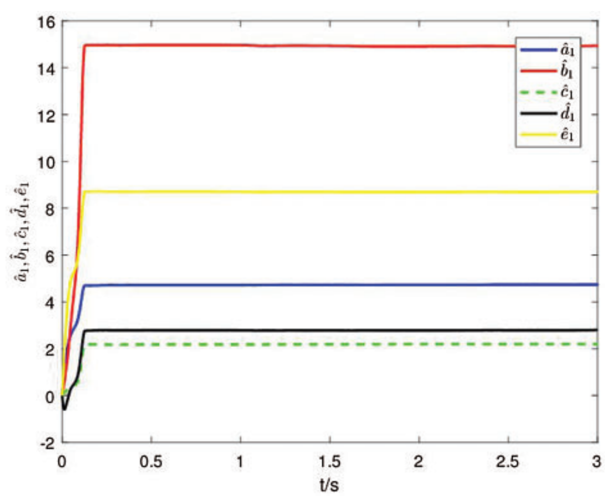

Figure 2: The evolutions of parameters $\hat{a}_1, \hat{b}_1, \hat{c}_1, \hat{d}_1, \hat{e}_1$

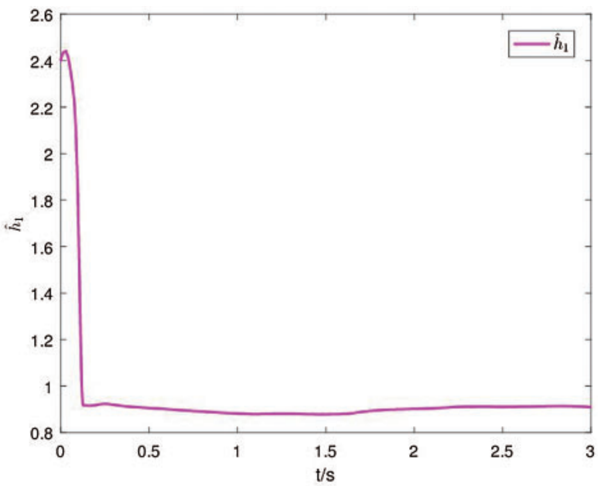

Figure 3: The evolutions of parameter $\hat{h}_1$

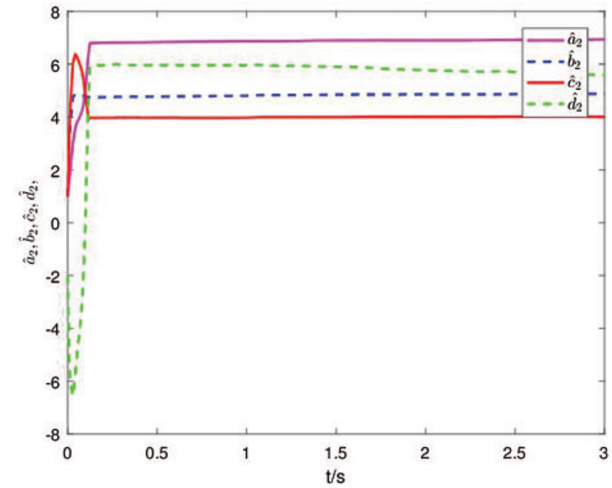

Figure 4: The evolutions of parameters $\hat{a}_2, \hat{b}_2, \hat{c}_2, \hat{d}_2$

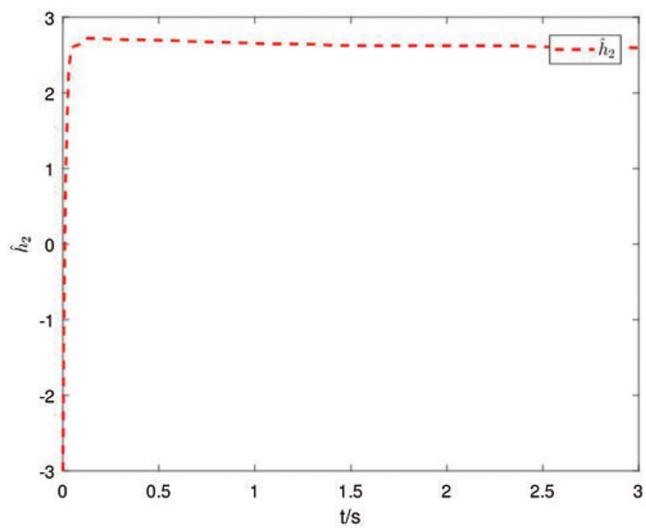

Figure 5: The evolutions of parameter $\hat{h}_2$

5 Conclusions

In this work, the issue with regard to the fixed-time ATVMPS of time-delayed DDCSs with unknown parameters has been investigated. To realize fixed-time ATVMPS between two nonidentical chaotic systems, one the basis of the unknown delays, an adaptive controller and parameters' updated laws have been designed. Then, from the perspective of practice, the estimation of settling time in Fix-TS can be determined in advance, which is distinct from Fin-TS. In future work, we will consider the fixed-time ATVMPS of time-delayed DDCSs under pinning control. In addition, we will also consider Fix-TS of neural networks and complex networks with different dimensions as well as lag synchronization of the time-delayed DDCSs.

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