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Failure Mode and Effects Analysis Based on Z-Numbers and the Graded Mean Integration Representation

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ABSTRACT

Failure mode and effects analysis (FMEA) is a widely used safety assessment method in many fields. Z-number was previously applied in FMEA since it can take both possibility and reliability of information into consideration. However, the use of fuzzy weighted mean to integrate Z-valuations may have some drawbacks and is not suitable for some situations. In this paper, an improved method is proposed based on Z-numbers and the graded mean integration representation (GMIR) to deal with the uncertain information in FMEA. First, Z-numbers are constructed based on the evaluations of risk factors O , S , D for each failure mode by different experts. Second, weights of the three risk factors and experts are determined. Third, the integration representations of Z-numbers are obtained by a new method based on the GMIR method. Finally, risk priorities of the failure modes are derived considering the weights of experts and risk factors. Two examples and a case study are given to show the use of the proposed method and comparison with other methods. The results show that the proposed method is more reasonable, universal and simple in calculation.

KEYWORDS

Safety assessment; FMEA; risk priority number; Z-number; the graded mean integration representation

1 Introduction

Safety assessment is an important task for large complex systems, such as nuclear power plants [1], aerospace systems [2], metro tunnels [3,4], bridges [5], and gas pipelines [6,7]. It includes structure reliability analysis [8,9], component reliability analysis [10,11] and even human reliability analysis [12], and can be conducted by using various methods, both qualitative [13,14] and quantitative [15–17]. Failure mode and effects analysis (FMEA) that originated in 1955 [18] is a widely used safety assessment method in many industry fields such as aircraft manufacturing, military systems, and engineering designs [19–22]. In these fields, there are multiple failure modes with different associated probabilities and consequences. FMEA is a method which aims to aggregate information for risk management decisions. It not only helps people to pick out failure modes by considering the causes and consequences, but also helps people to pick out the key point in designing which requires improvement and special control.



Risk priority number (RPN) which is used to prioritize failure modes in FMEA has been conducted a lot of considerable research by investigators over the years [23–28]. The traditional RPN method has been widely used to solve the problem of risk evaluation and maintenance strategy design under limited time and resources in a quantity of industries. However, the use of the traditional RPN in FMEA exposes some drawbacks.

Uncertainty is an important problem in failure analysis in traditional RPN. In order to deal with uncertain information more effectively, fuzzy set theory was first introduced to represent the uncertainty in risk evaluations. It relies on the membership functions and reasons by the if-then rules and is developing constantly [29–31]. Wang et al. [27] used fuzzy set theory to make the uncertainty quantitative in failure analysis in RPN. Failure modes' risk factors with their relative weights can be evaluated through the method. However, in situations where the systems are lack of experimental data, the fuzzy weighted geometric mean method (FWGM) proposed by Wang et al. [27] cannot be used.

Dempster-Shafer (D-S) evidence theory is another effective method to deal with uncertainty [32,33]. Yang et al. [34] applied D-S evidence theory in handling uncertain evaluations of the risk factors in RPN. In their method, a modified D-S evidence theory is employed to gather different opinions with inaccuracy and uncertainty. However, in the situation that experts evaluate risk factors with different and accurate values, there will be highly conflict evidence in the basic belief assignments (BBAs) and Dempster's combination rule can not fuse it. In order to avoid conflicting evidence in combination in FMEA, the boundaries of the focal elements are made fuzzy and a new BBA is constructed through the method to handle conflicting evidence [35]. However, in Su et al.'s method [35], the generalization is not complete and the basic assumption of D-S evidence theory that the basic elements in the frame of discernment are mutually exclusive is unrealistic. Thus, D numbers which can be seen as the generalization of D-S evidence theory is also investigated in the RPN evaluation in FMEA [36–38]. It can represent both mutually related uncertain information and incomplete situations.

Information in reality not only has uncertainty limits on values, but also has partial reliability. Zadeh [39] proposed Z-number to describe information more fully in reality. According to the definition of Z-number, a Z-number can be expressed as $Z = (A, B)$. Z-number contains two parts: the first part A represents the uncertainty of the valuation; the second part B measures the reliability of part A . Z-numbers has attracted many researchers' attention and has been developing quickly [40–42]. To represent the reliability in uncertain valuations, Jiang et al. [43] utilized Z-number [39] in FMEA. The method of Jiang et al.'s [43] is practical and useful since it takes the reliability part (experts' confident part) into consideration and does not need to define a great number of if-then rules. However, the fuzzy weighted mean process for integrating Z-valuations in their method may get wrong results when the reliability part B is equal to zero or a specialized trapezoidal fuzzy set with the same four numbers (which means the reliability part is a crisp number).

In this paper, an improved method based on the Z-numbers and the graded mean integration representation method is proposed. The subsequent parts of the paper are structured as follows. In Section 2, basic concepts are briefly reviewed. In Section 3, a new method based on Z-number and the graded mean integration presentation is proposed. In Section 4, two examples are given to point out the advantages of the proposed method. In Section 5, a case study is used to illustrate the application of the proposed method and compare results with other methods. Section 6 provides concluding remarks.

2 Preliminaries

2.1 Risk Priority Number (RPN)

RPN is the product of three risk factors, i.e., the occurrence (O), severity (S) and detection (D) of a failure. It is always expressed as Eq. (1):

$$RPN = S \times O \times D \tag{1}$$

where S is the severity of a failure effect, O is the probability of occurrence of a failure mode and D is the probability of a failure being detected. Each risk factor is evaluated using the 10 ratings from 1 to 10. Take risk factor Occurrence (O) as an example (see Table 1): if the probability of occurrence of the failure is nearly impossible (i.e., the possible failure rate $\leq 1/1,500,000$), then the rating is evaluated as “1”; if the probability of occurrence is extremely high (failure almost inevitable, the possible failure rate $\geq 1/2$), then the rating is evaluated as “10” [34]. The failure modes whose RPNs are higher are considered riskier and will be ranking in higher positions for revision.

Table 1: Suggested criteria of rating for occurrence of a failure in FMEA [34]

Rating	Probability of occurrence	Possible failure rate
10	Extremely high: failure almost inevitable	$\geq 1/2$
9	Very high	$1/3$
8	Repeated failures	$1/8$
7	High	$1/20$
6	Moderately high	$1/80$
5	Moderate	$1/400$
4	Relatively low	$1/2000$
3	Low	$1/15,000$
2	Remote	$1/150,000$
1	Nearly impossible	$\leq 1/1,500,000$

2.2 Fuzzy Numbers

Information that is uncertain and indistinct can be represented by fuzzy numbers [44].

Definition 2.2.1 A discrete and finite fuzzy set A which membership function is $\mu_A(\chi) \rightarrow [0, 1]$ in the universe of discourse X is defined as Eq. (2) [45,46]:

$$A = \{(\chi, \mu_A(\chi)) | \chi \in X\} \tag{2}$$

Definition 2.2.2 Fuzzy sets can be expressed with intervals, which are named α -level sets. Assume A is a fuzzy set on the universe of discourse X , the α -level sets of A are defined as Eq. (3) [44]:

$$A_\alpha = \{\chi \in X | \mu_A(\chi) \geq \alpha\} \\ = [\min \{\chi \in X | \mu_A(\chi) \geq \alpha\}, \max \{\chi \in X | \mu_A(\chi) \geq \alpha\}] \tag{3}$$

The defuzzified centroid of A_α is determined as Eq. (4):

$$\begin{aligned} \bar{A}_\alpha &= \{\chi \in X \mid \mu_A(\chi) \geq \alpha\} \\ &= \frac{[\min \{\chi \in X \mid \mu_A(\chi) \geq \alpha\}, \max \{\chi \in X \mid \mu_A(\chi) \geq \alpha\}]}{2} \end{aligned} \quad (4)$$

Definition 2.2.3 A discrete triangular fuzzy number A and a discrete trapezoidal fuzzy number A with membership function $\mu_A(\chi) \rightarrow [0, 1]$ in the universe of discourse X are respectively defined as Eqs. (5) and (6) [44]:

$$\mu_A = \begin{cases} \frac{\chi - a}{b - a} & a \leq \chi \leq b \\ \frac{c - \chi}{c - b} & b \leq \chi \leq c \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

$$\mu_A = \begin{cases} \frac{\chi - a}{b - a} & a \leq \chi \leq b \\ \frac{d - \chi}{d - c} & c \leq \chi \leq d \\ 1 & b \leq \chi \leq c \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Definition 2.2.4 In Fig. 1, $a, b, c,$ and d are real numbers. The fuzzy number A in general is defined as $A = (a, b, c, d; \omega_A)_{LR}$. When $\omega_A = 1$, the notation is simplified as $A = (a, b, c, d)_{LR}$. L^{-1} and R^{-1} , respectively, are the inverse functions of the functions L and R . $h(L_{(h)}^{-1} + R_{(h)}^{-1})/2$ is the graded mean h -level value of number A in general. Then, the graded mean integration of A is represented as Eq. (7) [47]:

$$\begin{aligned} P(A) &= \frac{\int_0^{\omega_A} (h (L_{(h)}^{-1} + R_{(h)}^{-1}) / 2) dh}{\int_0^{\omega_A} h dh} \\ &= \frac{\int_0^{\omega_A} (h (a + (b - a)h + d + (c - d)h) / 2) dh}{\int_0^{\omega_A} h dh} \\ &= \frac{1}{6}(a + 2b + 2c + d) \end{aligned} \quad (7)$$

where $h \in [0, \omega_A]$ and $\omega_A \in [0, 1]$.

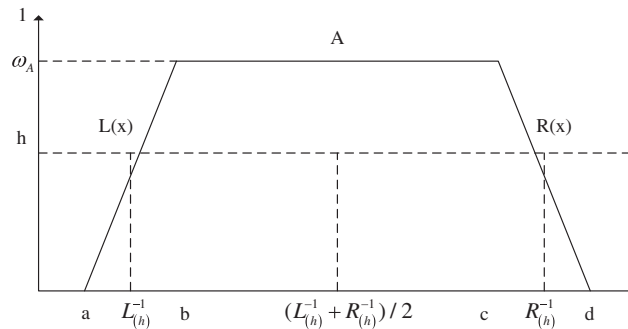


Figure 1: The graded mean h-level value of generalized number $A = (a; b; c; d; \omega_A)_{LR}$ [47]

2.3 Z-Number

Zadeh proposed Z-numbers’ contents in 2011 [39]. According to Zadeh, the Z-number is a pair of ordered fuzzy numbers expressed as $Z = (A, B)$, the first component A limits values of uncertain variable X and the second component B measures the reliability of component A .

If X is a random variable with probability density function, $p(X)$ and $Z = (A, B)$, where the component A is a continuous fuzzy number to limit the possibilistic of variable X and the component B is a fuzzy constraint on the probability (reliability) of the component A . In effect, the Z-valuation (X, A, B) is a limitation on X expressed as $\text{Prob}(X \text{ is } A) \text{ is } B$. Thus, probability of the event $p(X)$ is expressed as Eq. (8):

$$\text{Prob}(X \text{ is } A) = P(A) = \int_R \mu_A(u)p_x(u)du \tag{8}$$

where $\mu_A(u)$ is the degree to which u satisfies the constraint, $p_x(u)$ is the probability density of X .

Some examples of Z-valuations are given below:

- (dogs’ life, nearly 15 years, likely);
- (size of German, about 36 million hectares, very sure);
- (level of Sam’s effort, low, not sure).

3 Improved FMEA Method Based on Z-Numbers and GMIR

In this section, a new method is proposed to rank failure modes. There are much uncertain information to deal with in FMEA. With the development of Z-number, as it can express both possibility and reliability, large numbers of studies apply the Z-number to the risk priority numbers in the FMEA. However, the use of fuzzy weighted mean to integrate the Z-valuations of failure mode are not greatly suitable.

This article uses the graded mean integration representation (GMIR) method to integrate Z-valuations and propose a new method for calculating the risk priority numbers.

As is shown in Fig. 2, assume the potential failure modes in the system are previously identified based on historical data, the procedures of the new method are as follows.

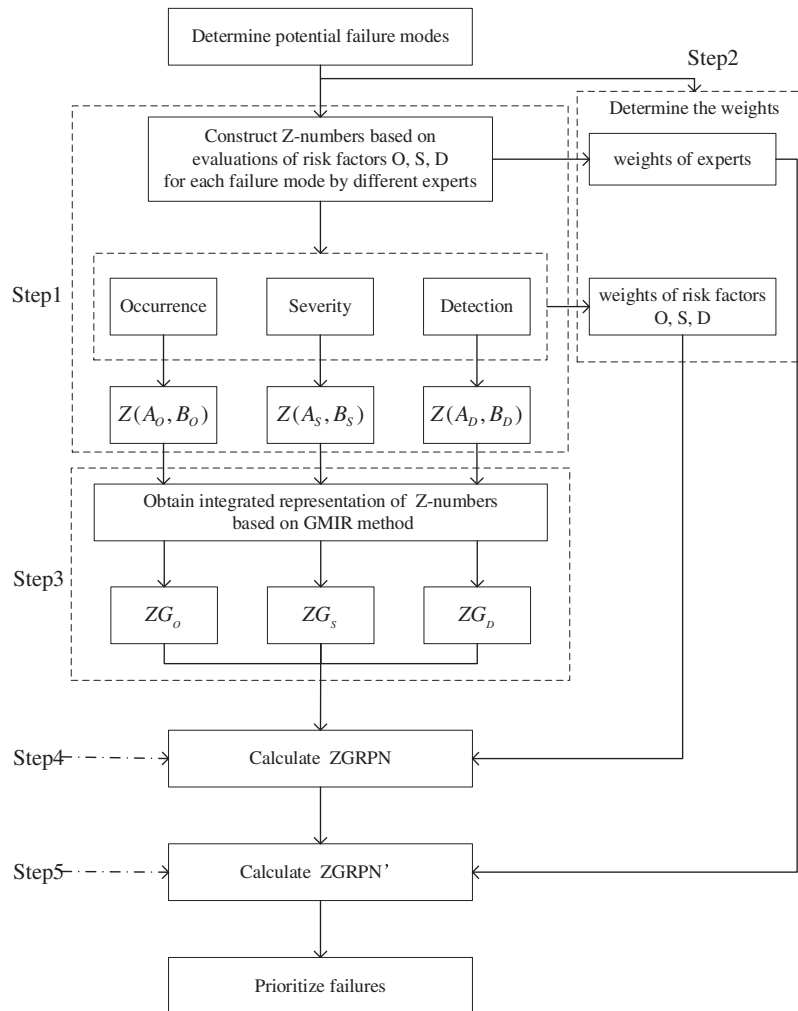


Figure 2: Procedures of the new method

Step 1: Construct Z-numbers based on evaluations of risk factors (O, S, D) for each failure mode by different experts.

Assume that there are M potential failure modes in FMEA and each potential failure mode includes three risk factors. And, there are K experts making evaluations. In a Z-number, the component A and B are modeled as trapezoidal fuzzy number set and triangle fuzzy number set, respectively.

The risk factor u_i in failure mode m evaluated by the k th expert can be represented with a triplet as Eq. (9):

$$\tilde{Z}_{u_i}^k = (FM_{mu_i}, A_{u_i}^k, B_{u_i}^k) \tag{9}$$

where $m = 1, 2, \dots, M, i = 1, 2, 3, u_i|_{i=1,2,3} = \{O, S, D\}, k = 1, 2, \dots, K, A_{u_i}^k = (a_{u_i1}^k, a_{u_i2}^k, a_{u_i3}^k, a_{u_i4}^k)$ represents the possibility of risk and $B_{u_i}^k = (b_{u_i1}^k, b_{u_i2}^k, b_{u_i3}^k)$ represents the reliability of risk. FM_{mu_i} represents the risk factor u_i of failure mode m , where $a_{u_i^r}^k (r = 1, 2, 3, 4)$ is defined within the interval

$[0, 10]$ and $b_{u_i g}^k (g = 1, 2, 3)$ is within the interval $[0, 1]$. Then the Z-evaluation represented as $(FM_{mu_i}$ is $A_{u_i}^k$) is $B_{u_i}^k$ can be written as $Z_{m_i}^k (A_{u_i}^k, B_{u_i}^k)$ with the fuzzy number.

Step 2: Determine the weights of experts and risk factors.

Weights of experts and weights of risk factors O, S, D in FMEA are usually investigated by decision support group by using weight determination methods such as Analytic Hierarchy Process [48].

Step 3: Obtain integrated representation of Z-numbers based on GMIR method.

In this step, we obtain integrated representation of Z-numbers (ZG_O, ZG_S, ZG_D) on the basis of the graded mean integration method as defined in Eq. (7).

Definition 3.1 The integrated representation of Z-numbers ZG_O, ZG_S and ZG_D are defined as Eq. (10):

$$\begin{cases} ZG_O &= P_A(O) \cdot P_B(O) + R \cdot (1 - P_B(O)) \\ ZG_S &= P_A(S) \cdot P_B(S) + R \cdot (1 - P_B(S)) \\ ZG_D &= P_A(D) \cdot P_B(D) + R \cdot (1 - P_B(D)) \end{cases} \quad (10)$$

$$R = \frac{1}{p - q + 1} \sum_{w=q}^p r_w \quad (11)$$

where ZG_O, ZG_S and ZG_D are the integrated representation of Z-numbers based on evaluations of risk factors O, S, D , respectively; $P_A(O), P_A(S)$ and $P_A(D)$ are the graded mean integration representation of part A in risk factors O, S, D ($\frac{1}{6} (a_{u_i 1}^k + 2a_{u_i 2}^k + 2a_{u_i 3}^k + a_{u_i 4}^k)$) according to Eq. (7); R in Eq. (11) is the mean value of ratings for risk factors O, S, D , q is the minimum sequence number of ratings and p is the maximum sequence number of ratings, r_w represented variables changed between minimum value and maximum value of the ratings. Take ‘‘Occurrence’’ for example, $q = 1, p = 10, r_w = 1, 2, \dots, 9, 10$, and $R = 5.5$. R is adopted here to handle the unreliable (unconfident) part. $P_B(O), P_B(S)$ and $P_B(D) \in [0, 1]$ are the graded mean integration representation of the reliability part B in evaluations of risk factors O, S, D ($\frac{1}{6} (b_{u_i 1}^k + 4b_{u_i 2}^k + b_{u_i 3}^k)$) according to Eq. (7).

Step 4: Calculate the ZGRPNs of failure modes.

In this step, the integrated representations of three risk factors (ZG_O, ZG_S, ZG_D) , are multiplied to gain the ZGRPN, which means the RPN are derived from Z-numbers and the graded mean integration representation. The definition of ZGRPN are as follows:

Definition 3.2 Assume the system has M failure modes, K experts and three risk factors, the ZGRPN of the m th failure mode from the k th expert is defined as Eq. (12):

$$ZGRPN_m^k = \prod_{i=1}^3 (\omega_{u_i mk} ZG_{u_i m}^k) \quad (12)$$

where $i = 1, 2, 3, u_i |_{i=1,2,3} = \{O, S, D\}$; $\omega_{u_i mk}$ ($i = 1, 2, 3$) represents the weight of risk factor u_i of failure mode m from the k th expert and $\sum_{i=1}^3 \omega_{u_i mk} = 1$; $ZG_{u_i m}^k$ represents the integrated representation of the Z-number based on evaluations of risk factor u_i in failure mode m from the k th expert.

Step 5: Calculate the ZGRPN’s of failure modes and give the failure modes’ rankings in descending order of the ZGRPN’s.

In this step, evaluations from k different experts about one specific failure mode m ($ZGRPN_m^k$) are combined to gain the comprehensive evaluation ($ZGRPN_m'$) based on the weighted average method.

Definition 3.3 Assume that there are K experts, the ZGRPN' of the m th failure mode from k experts is defined as Eq. (13):

$$ZGRPN'_m = \sum_{k=1}^K \omega_k ZGRPN_m^k \tag{13}$$

where ω_k is the weight of the k th expert and $k = 1, 2, \dots, K$ and $\sum_{k=1}^K \omega_k = 1$.

Then give the failure modes' rankings in descending order according to the ZGRPN's. The failure modes with higher ZGRPN' are riskier (have higher priority) and should be paid more attention to.

4 Numerical Examples

To show the validity of modified method, two examples are given in this section.

To make the calculation simple, we assume that risk factors O, S, D have the same weight in this paper.

Example 1. Suppose that Z-numbers based on evaluations of risk factors O, S, D have the same component A (possibility) and the component B (reliability part) is equal to zero or a specialized trapezoidal fuzzy number with four same coordinate values which means that the fuzzy number degenerates into crisp number (as shown in Table 2). \tilde{V} is the integrated representation of Z-numbers in Jiang et al.'s method [43]; ZRPN is the RPN derived by Jiang et al.'s method [43]; (ZG_o, ZG_s, ZG_d) is the proposed integrated representation of Z-numbers of three risk factors O, S, D based on GMIR method; ZGRPN is the RPN derived by the proposed method.

Table 2: The integrated representation of Z-numbers and the RPN in the situation that B is a specialized trapezoidal fuzzy number

O	S	D	B	Jiang et al.'s method [43]		Proposed method	
				\tilde{V}	ZRPN	(ZG_o, ZG_s, ZG_d)	ZGRPN
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.0, 0.0, 0.0, 0.0)	/	/	(5.5, 5.5, 5.5)	166.38
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.1, 0.1, 0.1, 0.1)	(10, 10, 10, 10)	0.67	(6.0, 6.0, 6.0)	210.64
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.2, 0.2, 0.2, 0.2)	(10, 10, 10, 10)	0.67	(6.4, 6.4, 6.4)	262.14
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.3, 0.3, 0.3, 0.3)	(10, 10, 10, 10)	0.67	(6.9, 6.9, 6.9)	321.42
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.4, 0.4, 0.4, 0.4)	(10, 10, 10, 10)	0.67	(7.3, 7.3, 7.3)	389.02
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.5, 0.5, 0.5, 0.5)	(10, 10, 10, 10)	0.67	(7.8, 7.8, 7.8)	465.48
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.6, 0.6, 0.6, 0.6)	(10, 10, 10, 10)	0.67	(8.2, 8.2, 8.2)	551.37

(Continued)

Table 2 (continued)

O	S	D	B	Jiang et al.'s method [43]		Proposed method	
				\tilde{V}	ZRPN	(ZG_o, ZG_s, ZG_d)	ZGRPN
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.7, 0.7, 0.7, 0.7)	(10, 10, 10, 10)	0.67	(8.7, 8.7, 8.7)	647.21
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.8, 0.8, 0.8, 0.8)	(10, 10, 10, 10)	0.67	(9.1, 9.1, 9.1)	753.57
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(0.9, 0.9, 0.9, 0.9)	(10, 10, 10, 10)	0.67	(9.6, 9.6, 9.6)	870.98
(10, 10, 10, 10)	(10, 10, 10, 10)	(10, 10, 10, 10)	(1.0, 1.0, 1.0, 1.0)	(10, 10, 10, 10)	0.67	(10.0, 10.0, 10.0)	1000

As can be seen from Table 2, in the situation that the component *B* (reliability part) is a specialized trapezoidal fuzzy number with four same coordinate values, the value of ZRPN does not change when part *B* changes. This means that, in such case, the part *B* does not work. Especially, when the part *B* is equal to zero, Jiang et al.'s method [43] cannot handle such situation since the denominator in the fuzzy weighted mean formula becomes zero. Different from ZRPN, the value of ZGRPN obtained by the proposed method increased with increasing value of part *B* (reliability). This is reasonable: as can be seen from Table 2, part *A* of risk factors *O*, *S*, *D* belongs to the highest rating (the most risk), the priority of such failure mode should be promoted with a higher confidence level (reliability).

Example 2. Assume that the Z-number constructed based on the evaluation of risk factor is $Z = (A, B) = ((7, 8, 8, 9), (6, 6, 6.3, 6.3))$. Now assume that part *A* does not change, part *B* changes from $(6, 6, 6.3, 6.3)$ to $(5.7, 6, 6.3, 6.3)$. In such case, since part *A* shows high risk (higher than average rating “5”) for a certain risk factor, if part *B* (reliability of part *A*) decreases, the risk priority of such case will decrease accordingly. The integrated representations of Z-numbers under these two cases are given in Table 3. \tilde{V} is the integrated fuzzy number of Z-number derived by Jiang et al.'s method [43] and ZG is the integrated crisp number of Z-number based on GMIR derived by the proposed method.

Table 3: The integrated representations of Z-numbers under two cases

Case	A	B	Jiang et al.'s method [43] \tilde{V}	Proposed method ZG
Case 1	(7, 8, 8, 9)	(6, 6, 6.3, 6.3)	(6.67, 7.62, 8.4, 9.45)	20.875
Case 2	(7, 8, 8, 9)	(5.7, 6, 6.3, 6.3)	(6.33, 7.62, 8.4, 9.95)	20.8

As can be seen in Table 4, the integrated representation of Z-number by Jiang et al.'s method [43] is in the form of a fuzzy number which is difficult to compare. Thus, we first defuzzify the fuzzy set by the centroid defuzzification method. Set α levels as 0, 0.1, 0.2, ..., 1.0 (as shown in Fig. 3). \tilde{V}_α is the centroids of the \tilde{V}_α computed by Eq. (4). The defuzzified centroid values of \tilde{V}_α are shown in Table 4.

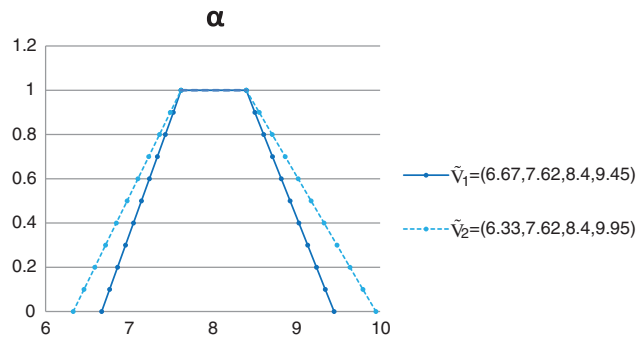


Figure 3: α -level cuts of the integrated valuations \tilde{V} in two cases

Table 4: The centroids of \tilde{V}_α of the two cases

α	\tilde{V}_α	
	Case 1	Case 2
0	8.055	8.140
0.1	8.050	8.127
0.2	8.045	8.114
0.3	8.040	8.101
0.4	8.035	8.088
0.5	8.030	8.075
0.6	8.025	8.062
0.7	8.020	8.049
0.8	8.015	8.036
0.9	8.010	8.023
1.0	8.010	8.010

It can be seen from Tables 3 and 4, in Jiang et al.’s method [43] that when the part A shows high risk evaluation, with the reliability of part A (part B) decreases, the integrated valuation \tilde{V} (which combine parts A and B) of risk level even increases (in other words, the value of ZRPN increases). Such phenomenon is contrary to common sense. By using the proposed method, when part B (reliability of part A) decreases, the integrated representation of risk evaluation (ZG, or ZGRPN since no more experts are considered here) will decrease accordingly. This is in accordance with the common sense.

5 Case Study

In this section, a case study of prioritizing risk levels of failure modes in an aircraft turbine’s rotor blades system from reference [43] is given to show the use and effectiveness of the proposed method.

Seven failure modes are identified and analyzed in the aircraft turbine’s rotor blades system. Assume that there are three experts and their weights are 0.4, 0.3, 0.3, respectively. And suppose the Z-numbers based on evaluations of risk factors O, S, D have the same reliability parts $B, B = 0.8$, and the possibility parts A are shown in Table 5. The integrated representation of Z-numbers ZG_o, ZG_s

and ZG_D are also shown in Table 5. The ZGRPN's and the rankings of seven failure modes obtained by the proposed method are given in the last two columns of Table 6.

Table 5: The possibility parts A and the integrated representation of Z-numbers based on evaluations of three risk factors

Experts	Failure mode	O	S	D	(ZG_O, ZG_S, ZG_D)
Expert 1	FM ₁	(6, 7, 8, 9)	(5, 6, 7, 8)	(6, 8, 8, 10)	(7.1, 6.3, 7.5)
	FM ₂	(2, 3, 4, 5)	(7, 8, 9, 10)	(3, 4, 5, 7)	(3.9, 7.9, 4.8)
	FM ₃	(0, 2, 3, 5)	(1, 3, 3, 5)	(0, 1, 2, 5)	(3.1, 3.5, 2.6)
	FM ₄	(1, 2, 3, 5)	(5, 6, 7, 9)	(2, 4, 5, 7)	(3.2, 6.4, 4.7)
	FM ₅	(6, 8, 9, 10)	(3, 4, 5, 6)	(6, 8, 9, 10)	(7.8, 4.7, 7.8)
	FM ₆	(0, 1, 1, 3)	(0, 1, 2, 3)	(5, 7, 7, 9)	(2.0, 2.3, 6.7)
	FM ₇	(5, 7, 8, 10)	(3, 4, 4, 5)	(2, 4, 4, 6)	(7.1, 4.3, 4.3)
Expert 2	FM ₁	(4, 6, 6, 8)	(6, 7, 7, 8)	(6, 8, 8, 10)	(5.9, 6.7, 7.5)
	FM ₂	(2, 4, 4, 5)	(6, 7, 7, 8)	(3, 4, 5, 7)	(4.2, 2.0, 4.8)
	FM ₃	(0, 2, 3, 4)	(0, 1, 1, 3)	(0, 1, 2, 5)	(3.0, 6.3, 2.6)
	FM ₄	(3, 4, 4, 5)	(5, 6, 7, 8)	(2, 4, 5, 7)	(4.3, 4.3, 4.7)
	FM ₅	(6, 8, 8, 10)	(2, 4, 4, 6)	(6, 8, 8, 10)	(7.5, 3.5, 7.5)
	FM ₆	(2, 3, 4, 5)	(1, 3, 3, 5)	(5, 7, 7, 9)	(3.9, 4.7, 6.7)
	FM ₇	(5, 6, 6, 7)	(3, 4, 5, 6)	(2, 4, 4, 6)	(5.9, 6.3, 4.3)
Expert 3	FM ₁	(6, 7, 7, 8)	(4, 6, 7, 9)	(5, 8, 8, 10)	(6.7, 6.3, 7.4)
	FM ₂	(1, 3, 4, 6)	(6, 7, 8, 9)	(2, 4, 5, 7)	(3.9, 7.1, 4.7)
	FM ₃	(1, 2, 3, 5)	(0, 2, 3, 5)	(0, 1, 1, 5)	(3.2, 3.1, 2.3)
	FM ₄	(2, 4, 4, 5)	(2, 6, 6, 10)	(2, 3, 4, 5)	(4.2, 5.9, 3.9)
	FM ₅	(5, 7, 8, 10)	(2, 5, 5, 7)	(7, 8, 8, 10)	(7.1, 5.0, 7.7)
	FM ₆	(0, 1, 1, 3)	(0, 1, 1, 4)	(7, 8, 8, 9)	(2.0, 2.2, 7.5)
	FM ₇	(6, 7, 8, 9)	(2, 4, 5, 7)	(1, 3, 3, 5)	(7.1, 4.7, 3.5)

In Table 6, the new proposed method (ZGFMEA method) is compared with the traditional RPN method [49], Wang et al.'s FWGM method [27] and Jiang et al.'s ZFMEA method [43].

It can be seen from Table 6 that the ranking of the failure modes by the proposed method is the same as the traditional RPN. That is $FM_1 > FM_5 > FM_7 > FM_2 > FM_4 > FM_6 > FM_3$. The rankings of failure modes 2 and 7 in the proposed method are opposite to the FWGM method and the FMEA method, with slight difference between RPNs of these two failure modes. The reason may be that in the FWGM method and ZFMEA method, the fuzzy weighted mean is adopted to integrate the evaluations which may lead to unreasonable results under some situations. What's more, the values of RPNs calculated by the proposed method (i.e., ZGRPN') show larger difference among them, which means the proposed method is more sensitive to differences and is more useful for decision making.

Table 6: Comparisons of the results

Failure mode	Traditional RPN [49]		FWGM method [27]		ZFMEA method [43]		Proposed method	
	RPN	Ranking	FRPN	Ranking	ZRPN	Ranking	ZGRPN	Ranking
1	337.68	1	7.16	1	0.70	1	346.06	1
2	97.68	4	5.03	3	0.52	3	110.89	4
3	4.2	7	1.96	7	0.22	7	32.45	7
4	71.04	5	4.49	5	0.45	5	93.94	5
5	264.88	2	6.67	2	0.66	2	253.22	2
6	17.92	6	2.82	6	0.34	6	59.29	6
7	99.16	3	4.82	4	0.48	4	135.5	3

6 Conclusion

This paper proposed a new method based on Z-numbers and the graded mean integration representation (GMIR) to handle the risk priority ranking problem in FMEA. Z-numbers are used to model the uncertainty of the evaluation results of the risk factors O , S , and D . The graded mean integration representation method is used to help integrate the Z-evaluations. Two examples and the case study are used to illustrate the superiority of the proposed method: the proposed method can handle some situations while other methods cannot, e.g., when part A shows high risk (higher than average rating “5”) for a certain risk factor, if part B (reliability of part A) decreases, the risk priority of such failure mode will decrease accordingly with the proposed method which is intuitive; the ranking of failure mode by the proposed method is basically consistent with other methods in the case study, and the result of ZGRPN’ is more sensitive to differences and is more useful for decision making. In the proposed method, part A and part B of a Z-number are constructed directly on experts’ opinions. In subsequent research, we will further investigate the construction of Z-numbers based on numerical data. Also, how to make full use of part B of a Z-number is a promising issue.

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