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ARTICLE

An Uncertainty Analysis and Reliability-Based Multidisciplinary Design Optimization Method Using Fourth-Moment Saddlepoint Approximation

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ABSTRACT

In uncertainty analysis and reliability-based multidisciplinary design and optimization (RBMDO) of engineering structures, the saddlepoint approximation (SA) method can be utilized to enhance the accuracy and efficiency of reliability evaluation. However, the random variables involved in SA should be easy to handle. Additionally, the corresponding saddlepoint equation should not be complicated. Both of them limit the application of SA for engineering problems. The moment method can construct an approximate cumulative distribution function of the performance function based on the first few statistical moments. However, the traditional moment matching method is not very accurate generally. In order to take advantage of the SA method and the moment matching method to enhance the efficiency of design and optimization, a fourth-moment saddlepoint approximation (FMSA) method is introduced into RBMDO. In FMSA, the approximate cumulative generating functions are constructed based on the first four moments of the limit state function. The probability density function and cumulative distribution function are estimated based on this approximate cumulative generating function. Furthermore, the FMSA method is introduced and combined into RBMDO within the framework of sequence optimization and reliability assessment, which is based on the performance measure approach strategy. Two engineering examples are introduced to verify the effectiveness of proposed method.

KEYWORDS

Reliability-based multidisciplinary design optimization; moment method; saddlepoint approximate; sequence optimization and reliability assessment; performance measure approach

1 Introduction

The progress of science and technology has put forward higher reliability requirements for engineering structural systems [1,2]. The actual engineering structure system always has various uncertainties to varying degrees [3–5]. Uncertainty can be roughly divided into two categories: aleatory uncertainty and epistemic uncertainty [6]. The aleatory uncertainty is objective and irreducible. It can be modeled by probabilistic methods. While the epistemic uncertainty is subjective and reducible. It is caused by incomplete information.

Uncertainty-based design optimization problems are mainly divided into two categories: robust design optimization and reliability design optimization. Robust design optimization focuses on guaranteed performance, looking for designs that are relatively insensitive to changes in uncertain variables. The purpose is to make the design solution robust when the design variables are degraded. The main observation is the tail of the PDF. Reliability design optimization focuses on the possibility of system failure, mainly to obtain a design that satisfies a given reliability. The main observation is the center of the PDF [7–13]. The definition of reliability can be given as follows: the probability that a structure completes a specified function within a specified time and under specified conditions [14–17].

Among reliability calculation methods, the moment method approximates the distribution of random response by fitting the first few random moments based on a type of hypothetical distribution. It reduces the calculation difficulty of design under uncertainty [18,19]. As one of the basic methods based on the moment method, the first order second moment (FOSM) is effective, but not accurate [20,21]. Therefore, it is necessary to use higher-order moments to improve the calculation accuracy. The use of the saddlepoint approximation (SA) for progressive analysis is efficient and practical [22–28]. The SA method has the following characteristics: simple calculation and strong operability; the overall approximation effect of the function is excellent, especially the tail probability distribution; when the density function is known and the calculation of the cumulative distribution function (CDF) is difficult, the SA method is useful. However, the SA method requires that the cumulative generating functions (CGF) exist. Moreover, when using the SA method, it is necessary to solve the saddlepoint equation to obtain the saddlepoint. But when the probability distribution type CGF is complicated, the saddlepoint equation is highly nonlinear. In this case it is difficult to solve.

Based on these problems, this study combines the moment method with the SA. It proposes an improved reliability-based multidisciplinary design and optimization (RBMDO) combined with fourth-moment saddlepoint approximation (FMSA) method (RBMDO-FMSA). In the FMSA method, the CGF is constructed based on the first four moments of limit state function (LSF) [29,30]. Then, this study uses the CGF to approximate the probability density function (PDF) and the CDF. To further improve the efficiency of RBMDO [31–35], this study also uses the FMSA method while adopting the sequence optimization and reliability assessment (SORA) [36,37] based on the performance measure approach (PMA) strategy.

The structure of the study can be briefly summarized as: in Section 2, the moment method and the SA method are briefly reviewed. Section 3 introduces the FMSA method. Section 4 applies the FMSA method to RBMDO. Section 5 uses two examples to verify the proposed method. Section 6 gives the conclusion.

2 The Moment Method and Saddlepoint Approximation

2.1 The Moment Method

Compared with FORM and SORM, the moment method does not have the problem of finding the derivative and design point of the performance function. It directly uses the value of the performance function at some feature points to approximate the failure probability. The moment method can also be directly used for reliability analysis of single and multiple failure mode systems [38–40].

Here, $Y = g(x)$ is assumed to follow a normal distribution. If the first two orders of Y are obtained, namely μ_Y (or α_{1Y}) and σ_Y (or α_{2Y}), the reliability index β_{2M} of the matrix of the first two orders can be approximately expressed as Eq. (1).

$$\beta_{2M} = \frac{\mu_Y}{\sigma_Y} = \frac{\alpha_{1Y}}{\alpha_{2Y}} \tag{1}$$

where μ_Y denotes the mean, σ_Y denotes the standard deviation.

Normalize the random variable Y standard in Eq. (2).

$$Y_u = \frac{Y - \mu_Y}{\sigma_Y} \tag{2}$$

Then in Eq. (3).

$$P_{f2M} = P\{Y \leq 0\} = P\left\{\frac{Y - \mu_Y}{\sigma_Y} \leq -\frac{\mu_Y}{\sigma_Y}\right\} = P\{Y_u \leq -\beta_{2M}\} = \Phi(-\beta_{2M}) \tag{3}$$

where $\Phi(\cdot)$ denotes the CDF of the standard normal variable.

When the first four-order center distance of the performance function is known, according to the high-order moment standardization technique (HOMST), the reliability index β_{4M} can be obtained as Eq. (4).

$$\beta_{4M} = \frac{3(\alpha_{4Y} - 1)\beta_{2M} + \alpha_{3Y}\left(\beta_{2M}^2 - 1\right)}{\sqrt{\left(5\alpha_{3Y}^2 - 9\alpha_{4Y} + 9\right)(1 - \alpha_{4Y})}} \tag{4}$$

where α_{3Y} and α_{4Y} denote the third-order and fourth-order dimensionless center distances respectively, also known as, the skewness and kurtosis.

The failure probability is expressed as Eq. (5).

$$P_{4M} = \Phi(-\beta_{4M}) \tag{5}$$

when $\alpha_{3Y} = 0$, $\beta_{4M} = \beta_{2M}$.

A new higher accuracy reliability index takes into account the parameters ignored by HOMST, which is expressed as Eq. (6).

$$\beta_{4M}^{*} = \frac{3(\alpha_{4Y} + 1)\beta_{2M} + 5\alpha_{3Y}\left(\beta_{2M}^2 - 1\right)}{\sqrt{9(3\alpha_{4Y} + 1)^2 - 5\alpha_{3Y}^2(13\alpha_{4Y} + 11)}} \tag{6}$$

2.2 The Saddlepoint Approximation

The use of the SA method for progressive analysis is efficient and practical. The SA has the following characteristics: (1) convenient calculation and strong operability; (2) good overall approximation effect to the function, especially the tail probability distribution; (3) SA method is useful when the PDF is known but the calculation of the CDF is difficult [41,42].

Firstly, given the performance function $g(Y)$, then, the moment generating function (MGF) is defined by Eq. (7).

$$M_Y(t) = \int_{-\infty}^{+\infty} e^{ty} g(y)\, dy \tag{7}$$

Take the logarithm of MGF to get CGF in Eq. (8).

$$K_Y(t) = \ln M_Y(t) \tag{8}$$

By deriving CGF, the saddlepoint t_s can be obtained by Eq. (9).

$$K'_Y(t) = y \tag{9}$$

According to the t_s, the failure probability $P_{f,\text{SPA}}$ and the PDF can be calculated as Eqs. (10) and (11).

$$P_{f,\text{SPA}} = \Pr\{Y \le y\} = \Phi(\omega) + \varphi(\omega)\left(\frac{1}{\omega} - \frac{1}{\upsilon}\right) \tag{10}$$

$$f_{Y,\text{SPA}}(y) = \left(\frac{1}{2\pi K''_Y(t_s)}\right)^{\frac{1}{2}} \exp\left(K_Y(t_s) - t_s y\right) \tag{11}$$

where $\omega = \text{sgn}(t_s)\left[2\left(t_s y - K_Y(t_s)\right)\right]^{1/2}$, $\upsilon = t_s\left[K''_Y(t_s)\right]^{1/2}$.

Alternative equation for calculating the failure probability is as Eq. (12).

$$P^*_{f,\text{SPA}} = \Pr\{Y \le y\} = \Phi\left[\omega + \frac{1}{\omega}\ln\left(\frac{\upsilon}{\omega}\right)\right] \tag{12}$$

3 The FMSA

This study combines the moment method and the saddlepoint approximation, then introduces an improved FMSA.

Assuming that Y is a random variable, its MGF and CGF are denoted by M_Y and K_Y, respectively. It is known that K_Y has the following relationship with the first four moments in Eqs. (13a)–(13d).

$$K_Y^{(1)}(0) = \mu_Y \tag{13a}$$

$$K_Y^{(2)}(0) = \sigma_Y^2 \tag{13b}$$

$$K_Y^{(3)}(0) = \alpha_{3Y} \tag{13c}$$

$$K_Y^{(4)}(0) = \alpha_{4Y} - 3\sigma_Y^2 \tag{13d}$$

where $K_Y^{(1)}(0)$, $K_Y^{(2)}(0)$, $K_Y^{(3)}(0)$ and $K_Y^{(4)}(0)$ denote the first, second, third and fourth derivatives of the function K_Y, respectively. μ_Y, σ_g, α_{3Y} and α_{4Y} denote the mean, standard deviation, third-order and fourth-order center distances of random variables, respectively.

Then, the first four moments of the standard variable $Y_s = \dfrac{Y - \mu_Y}{\sigma_Y}$ are 0, 1, α_{3Y_s} and $\alpha_{4Y_s} - 3$, where $\alpha_{3Y_s} = \dfrac{\alpha_{3Y}}{\sigma_Y^3}$, $\alpha_{4Y_s} = \dfrac{\alpha_{4Y}}{\sigma_Y^4}$.

According to the saddlepoint approximation method, the CGF can be modeled as Eq. (14).

$$K_{Y_s}(t) = m_1 t + m_2 t^2 - m_3 \ln\left\{(1 - nt)^2\right\} \tag{14}$$

where m_1, m_2, m_3 and n are undetermined coefficients.

Therefore, the first four derivatives of CGF can be derived as Eqs. (15a)–(15d) in turn according to the Eqs. (13a)–(13d).

$$K_{Y_s}^{(1)}(t) = m_1 + 2m_2 t + \frac{2m_3 n}{1 - nt} \tag{15a}$$

$$K_{Y_s}^{(2)}(t) = 2m_2 + \frac{2m_3 n^2}{(1 - nt)^2} \tag{15b}$$

$$K_{Y_s}^{(3)}(t) = \frac{4m_3 n^2}{(1 - nt)^3} \tag{15c}$$

$$K_{Y_s}^{(4)}(t) = \frac{12m_3 n^4}{(1 - nt)^4} \tag{15d}$$

Then combine with the Eqs. (13a)–(13d) to get Eq. (16).

$$\begin{cases} m_1 + 2m_3 n = 0 \\ 2m_2 + 2m_3 n^2 = 1 \\ 4m_3 n^3 = \alpha_{3Y_s} \\ 12m_3 n^4 = \alpha_{4Y_s} - 3 \end{cases} \tag{16}$$

If $\alpha_{3Y_s} = 0$, then in Eq. (17)

$$\begin{cases} m_1 = 0 \\ m_2 = 0.5 \\ m_3 n = 0 \\ \alpha_{4Y_s} = 3 \end{cases} \tag{17}$$

Then in Eq. (18)

$$K_{Y_s}(t) = 0.5t^2 \tag{18}$$

This is the CGF of a standard normal variable whose second derivative is $K_{Y_s}^{(2)}(t) = 1$.

When $\alpha_{3Y_s} \neq 0$, $\alpha_{4Y_s} \neq 3$, we can get Eq. (19).

$$\begin{cases} m_1 = -\dfrac{9\alpha_{3Y_s}^3}{2(\alpha_{4Y_s} - 3)^2} \\ m_2 = \dfrac{-3\alpha_{3Y_s}^3 + 2\alpha_{4Y_s} - 6}{4(\alpha_{4Y_s} - 3)^2} \\ m_3 = \dfrac{27\alpha_{3Y_s}^4}{4(\alpha_{4Y_s} - 3)^3} \\ n = \dfrac{\alpha_{4Y_s} - 3}{3\alpha_{3Y_s}} \end{cases} \tag{19}$$

According to the Eq. (9), by deriving CGF can get Eq. (20).

$$m_1 + 2m_2 t + \frac{2m_3 n}{1 - nt} - y = 0 \tag{20}$$

The saddlepoint t_s can be obtained as Eq. (21).

$$t_s = \frac{\pm\sqrt{\left(16m_2m_3 + (y - m_1)^2\right) n^2 - 4m_2 (y - m_1) n + 4m_2{}^2} + (y - m_1) n + 2m_2}{4nm_2} \tag{21}$$

Bring the Eq. (19) into Eqs. (14) and (15b), the approximate CGF and second moment can be obtained as Eqs. (22) and (23).

$$K_{Y_s}(t) = \frac{-3\alpha_{3Y_s}^3\psi^2t^2 - 27\alpha_{3Y_s}^4 \ln\left(\frac{(\psi t - 3\alpha_{3Y_s})^2}{9\alpha_{3Y_s}^2}\right) + 2\psi^3t^2 - 18\alpha_{3Y_s}^3\psi t}{4\psi^3} \tag{22}$$

$$K_{Y_s}{}^{(2)}(t) = \frac{-3\alpha_{3Y_s}^3 + 2\psi}{2\psi} + \frac{27\alpha_{3Y_s}^4}{2\psi\,(\psi t - 3\alpha_{3Y_s})^2} \tag{23}$$

where $\psi = \alpha_{4Y_s} - 3$.

According to the Eq. (2), the CGF can be expressed as Eq. (24).

$$\begin{aligned} K_Y(t) =& K_{Y_s}(\sigma_Y t) + \mu_Y t \\ =& \frac{-3\alpha_{3Y_s}^3\psi^2\sigma_Y{}^2t^2 - 27\alpha_{3Y_s}^4 \ln\left(\frac{(\psi\sigma_Y t - 3\alpha_{3Y_s})^2}{9\alpha_{3Y_s}^2}\right) + 2\psi^3\sigma_Y{}^2t^2 - \left(18\alpha_{3Y_s}^3\psi\sigma_Y - 4\psi^3\mu_Y\right) t}{4\psi^3} \end{aligned} \tag{24}$$

Then according to the Eq. (11), the failure probability and the PDF are expressed as Eqs. (25) and (26).

$$P_f = F_Y(0) = \Pr\{Y \le 0\} = \Pr\{Y_s \le -\beta_{2M}\} = \Phi\left[\omega_y + \omega_y{}^{-1}\ln\left(\frac{\upsilon_y}{\omega_y}\right)\right] \tag{25}$$

$$f_Y(y) = \left(2\pi K_Y''(t_s)\right)^{-\frac{1}{2}} \exp\left(K_Y(t_s) - t_s y\right) \tag{26}$$

where $\omega_y = \mathrm{sgn}(t_s)\,[2(-\beta_2 t_s - K_{Y_s}(t_s))]^{1/2}$, $\upsilon = t_s\left[K_{Y_s}''(t_s)\right]^{1/2}$.

Therefore, the CDF can be expressed as Eq. (27).

$$F_Y(0) = \Pr\{Y_s \le 0\} = \frac{1}{2} + \frac{K_Y^{(3)}(0)}{6\sqrt{2\pi}} \tag{27}$$

When $y = 0$,

$$P_f = \Pr\{Y \le \mu_Y\} = \Pr\{Y_s \le 0\} = \frac{1}{2} + \frac{\alpha_{3Y_s}}{6\sqrt{2\pi}} \tag{28}$$

4 The RBMDO-FMSA

This section first introduces the SORA and the PMA. Finally, the RBMDO-FMSA model is proposed.

4.1 The SORA Strategy

The SORA method is a decoupling method that can efficiently solve reliability design optimization problems [43,44]. It serializes the reliability design optimization process of the traditional nested loop based on decoupling. It divides the reliability design optimization process into deterministic design optimization and reliability analysis, forming a recursive optimization loop, as shown in the Fig. 1. The SORA method transforms the RBMDO problem into an approximate MDO problem by means of equivalent constraints. Then use the deterministic MDO method to solve it. It can make the equivalent constraint gradually shift toward the direction of the probability constraint. Then, the optimal solution can be quickly obtained [45].

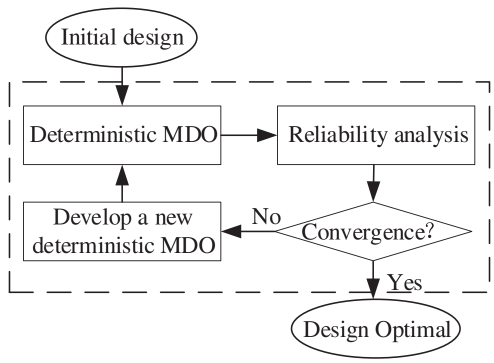

Figure 1: SORA process diagram [43]

The SORA method uses a single-cycle strategy to perform continuous deterministic optimization cycles and reliability analysis. In each cycle, optimization and reliability analysis do not interfere with each other. Reliability analysis is used to verify the feasibility of probabilistic constraints after optimization. The key of this method is to continuously revise the constraints in the optimization with the results obtained through the reliability analysis. Keep it close to the expected probability constraints, realize the optimal design as quickly as possible, reduce the number of optimizations. Thereby reducing the number of reliability analyses.

4.2 The PMA

In RBMDO, the PMA has the advantages of high calculation efficiency, good stability and wide application range [46,47]. It takes the specified reliability index β as the radius of the hypersphere as the search area. It uses the performance function value at the searched extreme point to determine whether the target system is reliable. The equation $\Pr[Y(\cdot) \geq Y_{\mathrm{MLP}}] = 1 - [P_{fi}]$ can be obtained by PMA, where Y_{MLP} denotes the value of the LSF at most likelihood point (MLP) [48,49]. As shown in the Fig. 2, if $Y_{\mathrm{MLP}} \geq 0$, the reliability requirements can be met.

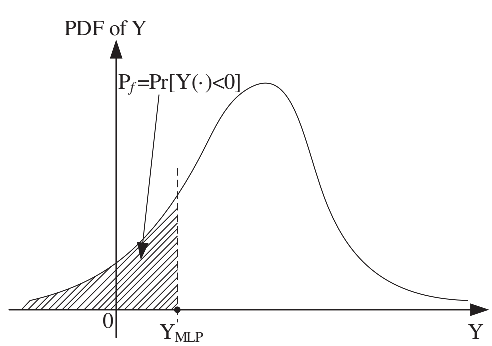

Figure 2: PMA schematic [47]

4.3 The Improved RBMDO Using FMSA

The process of RBMDO-FMSA using SORA is as follows:

Step 1. Deterministic MDO.

$$\begin{aligned}
&\min_{d_S, d_i, x_S, x_i} f\left(d_S, d_i, x_i, x_S, p, z\right) \\
&\text{s.t.}\, Y_i\left(d_S, d_i, x_i, x_S, p, z_{\cdot i}\right) \geq 0, \\
&g_i\left(d_S, d_i, x_i, x_S, p, z_{\cdot i}\right) \geq 0, \\
&d_S \in \left[d_S^L, d_S^U\right], d_i \in \left[d_i^L, d_i^U\right], \\
&x_S \in \left[x_S^L, x_S^U\right], x_i \in \left[x_i^L, x_i^U\right], \\
&z \in \left[z^L, z^U\right], i = [1, n], i \in N^*
\end{aligned} \tag{29}$$

where d_S and x_S denote shared deterministic design variables and shared random input variables, respectively; d_i and x_i denote local deterministic design variables and local random variables of discipline i, respectively; p denotes the vector of independent random design parameters; z denotes a coupling variable; the superscripts L denotes the lower limit, U denotes the upper limit.

The initial values $d_S^{(0)}$, $d_i^{(0)}$, $x_S^{(0)}$, $x_i^{(0)}$ of the design variable in SORA are given when $k = 1$. Then solve the MDO problem according to the Eq. (29).

Step 2. Reliability analysis.

I. Linearization of the LSF by first-order Taylor expansion at the MLP point to minimize the accuracy of reliability analysis. When $k = 2$, if any reliability constraints are not met, the deterministic MDO constraints will use the MLP information based on the previous cycle. Joint PDF has the maximum value at MLP, so the MLP can be obtained by the following Eq. (30):

$$\begin{aligned}
&\max_{x_S, x_i, p} \prod f_{x_i}\left(x_i\right) f_{x_S}\left(x_S\right) f_p\left(p\right) \\
&\text{s.t.}\, Y_i\left(d_s, d_i, x_S, x_i, p, z_{\cdot i}\right) = 0 \\
&i = [1, n], i \in N^*
\end{aligned} \tag{30}$$

II. Solve the percent performance $Y_i^{1-\left[P_{fi}\right]}$ and MLP. According to PMA, the following Eq. (31) can be obtained

$$\Pr\left[\hat{Y}_i\left(d_s, d_i, x_i, x_S, p, z_{\cdot i}\right) < Y_i^{1-\left[P_{fi}\right]}\right] = \left[P_{fi}\right] \tag{31}$$

Solving Eq. (31) can get $Y_i^{1-\left[P_{fi}\right]}$. Define $\hat{Y}_i$ as Eq. (32).

$$\hat{Y}_i = \hat{Y}_i\left(d_s, d_i, x_i, x_S, p, z_{\cdot i}\right) - Y_i^{1-\left[P_{fi}\right]} \tag{32}$$

In FMSA, the CGF $K_{\hat{Y}}(t)$ can be obtained by Eq. (24). Then, the reliability $\Pr\left[\hat{Y}_i - Y_i^{1-\left[P_{fi}\right]} \geq 0\right] = \Pr\left[\hat{Y}_i \geq 0\right]$ can be obtained by Eq. (12).

The MLP needs the corresponding percent performance $Y_i^{1-[P_{fi}]}$ to solve. Then construct the reliability constraint Y_i for the next $(k+1)$ cycles. It can be obtained by the following Eq. (33).

$$\begin{aligned}&\max_{x_S,x_i,p} \prod f_{x_i}(x_i) f_{x_S}(x_S) f_p(p)\\&\text{s.t.}\, Y_i(d_s, d_i, x_i, x_S, p, z_{\cdot i}) - Y_i^{1-[P_{fi}]} = 0\\&i=[1,n], i\in N^*\end{aligned} \tag{33}$$

Step 3. Modified MDO.

Through the obtained MLP, the shift vector can be derived as Eq. (34).

$$\begin{cases} S_{x_S}^{(k+1)} = x_S^{(k)} - \hat{x}_S^{(k)}\\ S_{x_i}^{(k+1)} = x_i^{(k)} - \hat{x}_i^{(k)}\end{cases} \tag{34}$$

Then, the RBMDO problem is transformed into an MDO problem. Deterministic optimization can be performed in the next cycle of SORA. The optimization process is as Eq. (35).

$$\begin{aligned}&\min_{d_S,d_i,\mu_{X_S},\mu_{X_i}} f(d_s, d_i, x_i, x_S, p, \mathrm{z})\\&\text{s.t.}\, Y_i\left(d_s, d_i, x_i - S_{x_i}^{(k+1)}, x_S - S_{x_S}^{(k+1)}, \hat{p}^{(k)}, z\right) \geq 0, g_i(d_s, d_i, x_i, x_S, p, z_{\cdot i}) \geq 0,\\&d_S \in \left[d_S^L, d_S^U\right], d_i \in \left[d_i^L, d_i^U\right], x_S \in \left[x_S^L, x_S^U\right], x_i \in \left[x_i^L, x_i^U\right], z \in [z^L, z^U], i=[1,n], i \in N^*\end{aligned} \tag{35}$$

In the next cycle of SORA, after the MDO problem of the equation is solved, the reliability analysis is performed again. This process is repeated until the optimization converges.

5 Examples

This section gives two examples to verify the proposed method. The results obtained by the proposed method will be compared with existing methods. Other methods include mean first-order second-moment method (MVFOSM), first-order reliability method (FORM), second-order reliability method (SORM) and Monte Carlo simulation method (MCS) [50].

5.1 The Simple Wellhead Platform Calculation Example

This is a one-leg wellhead platform composed of a deep well foundation, a tower body and two upper and lower decks [22]. Its simplified model is shown in Fig. 3. In the figure, $X_i\,(i=2,\ldots,11)$ denotes the failure point, $L_i\,(i=1,\ldots,5)$ denotes the wave force at each point of action, $Y_i\,(i=1,\ldots,5)$ denotes the wall thickness design variable, $Y_i\,(i=6,\ldots,8)$ denotes the diameter design variable, $Y_i\,(i=9,\ldots,12)$ denotes the height design variable, $\boldsymbol{F}$ denotes the wind force, W denotes the weight of the platform equipment.

The example has 7 basic random variables. The specific information is shown in Table 1, where H_m, U_c and V_F are completely correlated, C_D and C_M are negatively correlated, with a correlation coefficient of -0.8, other variables are independent and uncorrelated. The above-mentioned related variables can be converted into linear independent variables through $Z_i = a_i \times H_m + b_i \times U_c + c_i \times V_F + d_i \times C_D + e_i \times C_M$, as shown in Table 2.

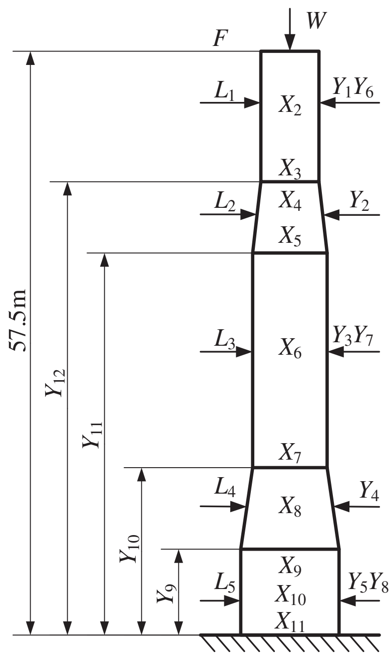

Figure 3: The Simple wellhead platform model

Table 1: The relevant information of each variable in example 1

Variable	Name	Distribution	Mean	Coefficient
W(N)	Equipment weight	Normality	1.94×10^6	0.1
H_m(m)	Limit wave height	Extreme value type I	15.4	0.07
U_c(m/s)	Flow rate	Extreme value type I	1.10	0.13
V_F(m/s)	Limit wind speed	Lognormal	67.5	0.1
C_D	Drag coefficient	Normality	1.83	0.1
C_M	Mass force coefficient	Normality	2.90	0.1
σ_y(Pa)	Bow to extremes	Lognormal	1.88×10^8	0.13

Here only the yield failure of each failure point is considered. The load effect takes into account the axial force $\boldsymbol{N}$ and the bending moment $\boldsymbol{M}$, so the LSF is:

$$g_i(Z) = 1 - N_i/N_{Fi} - |M_i/M_{Fi}| \tag{36}$$

where M_i denotes the bending moment borne by each section; N_i denotes the axial force borne by each section; $M_{Fi} = \frac{\sigma_y \left[d_i^3 - (d_i - wt_i)^3\right]}{6}$; $N_{Fi} = \frac{\sigma_y \pi \left[d_i^2 - (d_i - 2t_i)^2\right]}{4}$; d_i denotes the diameter of each section; t_i denotes the wall thickness of each section.

Table 2: The coefficients of related variables

	a_i	b_i	c_i	d_i	e_i
Z_1	0.656	0.716	−0.240	0	0
Z_2	−0.673	0.697	0.245	0	0
Z_3	0.343	0.001	0.940	0	0
Z_4	0	0	0	0.856	0.516
Z_5	0	0	0	−0.516	0.856

The mathematical model of the optimal design is as Eq. (37).

$$\begin{aligned} &\min F(Y) \\ &\text{s.t.}\beta_i \geq \beta_{\alpha i}, i = 2, 3, \ldots, 11 \end{aligned} \tag{37}$$

where $F(Y)$ is the volume of the platform steel, the unit is m^3; Y_i $(i = 1, 2, \ldots, 12)$ is the design variable; β_i is the reliability index at each failure point; $\beta_{\alpha i}$ is the target value of the reliability index at each failure point, which is taken as 4.0 in this example.

The results obtained by different methods are shown in Table 3. The minimum volume of the optimized platform steel is 16.26 m^3, which is nearly 14% smaller than the original design. For more details, the length of truncated cones in two transition sections is obviously increased. Among different methods, the increase obtained by FMSA is the largest one, followed by MCS. While MVFOSM and FORM have relatively smaller length increases. The wall thickness increase of the transition section obtained by FMSA is less. While the increase obtained by FORM is the most. In addition, the diameters of the non-transition sections away from the fixed surface obtained by FMSA increased more than those obtained by the other methods. However, the diameters of the parts contacting the fixed surface decreased.

Table 3: The optimization results of different methods in example 1

Variable	FMSA	MCS	MVFOSM	FORM	SORM
Y_1	0.025	0.025	0.025	0.025	0.025
Y_2	0.032	0.030	0.027	0.025	0.028
Y_3	0.028	0.031	0.035	0.040	0.033
Y_4	0.028	0.031	0.034	0.040	0.032
Y_5	0.028	0.031	0.034	0.040	0.033
Y_6	2.26	2.23	2.08	2.00	2.19
Y_7	3.66	3.59	3.24	3.00	3.40
Y_8	4.20	4.25	4.96	5.00	4.86
Y_9	9.8	9.0	8.0	7.0	8.6
Y_{10}	17.0	14.0	10.8	9.0	13.7
Y_{11}	24.0	25.4	32.6	34.0	28.3
Y_{12}	38.4	38.6	37.5	37.0	37.7

5.2 The Engineering Speed Reducer Example

This is an engineering evaluation example of NASA standard MDO test [51,52]. The model of this example is shown in Fig. 4. Table 4 gives the relevant information of each variable. This example involves three disciplines: discipline 1 (Bearing set A and Shaft A), discipline 2 (Bearing set B and Shaft B) and discipline 3 (Gear A and Gear B). Table 5 shows the test results of different RBMDO methods.

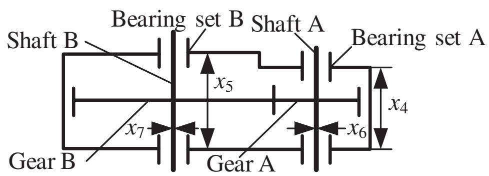

Figure 4: Speed reducer design

Table 4: The information about related variables in example 2

Variable	Description	Distribution	Mean	Standard deviation	Lower bound	Upper bound
x_1	Gear face width	–	–	–	2.6	3.6
x_2	Teeth module	–	–	–	0.3	1.0
x_3	Number of teeth of pinion	–	–	–	17	28
x_4	Distance between Bearings A	Gumbel	μ_{x_4}	0.001 μ_{x_4}	7.3	8.3
x_5	Distance between Bearings B	Gumbel	μ_{x_5}	0.001 μ_{x_5}	7.3	8.3
x_6	Diameter of Shaft A	Gumbel	μ_{x_6}	0.001 μ_{x_6}	2.9	3.9
x_7	Diameter of Shaft B	Gumbel	μ_{x_7}	0.001 μ_{x_7}	5	5.5

Table 5: The optimization results of different methods in example 2

	FMSA	MCS	FORM	SORM	MVFOSM
x_1	3.426	3.427	3.424	3.425	3.423
x_2	0.649	0.650	0.645	0.646	0.644
x_3	18	18	18	18	18
μ_{x_4}	7.300	7.300	7.300	7.300	7.300
μ_{x_5}	7.688	7.688	7.686	7.687	7.685
μ_{x_6}	3.322	3.320	3.323	3.322	3.323
μ_{x_7}	5.264	5.264	5.263	5.263	5.263
f	2882	2888	2866	2877	2859

It can be seen from Table 5 that the values of each variable obtained by different optimization methods are still within the constraints. The value of each variable varies little. The optimized value obtained by FMSA is the closest to MCS, indicating that FMSA has higher accuracy. The objective function f is to minimize the volume of the gear system. Compared with other methods, FMSA has higher accuracy and more conservative results, so the volume is larger. However, the volume obtained by FMSA is not much larger than that obtained by other methods. It illustrates that FMSA can enjoy an effective balance between accuracy and cost. The smallest volume is obtained by MVFOSM. However, its accuracy is relatively low.

6 Conclusions

In uncertainty analysis and RBMDO of engineering structures, the SA method can be utilized to enhance the accuracy and efficiency of reliability evaluation. However, the random variables in SA should be easy to handle. Moreover, the corresponding saddlepoint equation should not be complicated. Both of them limit the application of SA for engineering problems. The moment method can construct an approximate cumulative distribution function of the performance function based on the first few statistical moments. However, the traditional moment matching method is not very accurate generally. To solve these problems, SA is combined with the moment method. An improved RBMDO-FMSA method is proposed to take the advantage of above methods. In FMSA, the approximate CGF is constructed based on the first four moments of the LSF. Then, the PDF and CDF are estimated based on this approximate CGF. Furthermore, the FMSA method is introduced and combined into RBMDO within the framework of SORA, which is based on the PMA strategy. The corresponding formulation RBMDO-FMSA effectively improves the efficiency and accuracy.

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