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New Class of Doubt Bipolar Fuzzy Sub Measure Algebra

Shadia Majeed Noori[1,2], **Abd Ghafur Ahmad**[1] **and Shuker Mahmood Khalil**[3,*]

[1]Department of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, Bangi Selangor, 43600, Malaysia

[2]Department of Mathematics, College of Education for Pure Science, University of Tikrit, Tikrit, 34001, Iraq

[3]Department of Mathematics, College of Science, University Basrah, Basrah, 61004, Iraq

*Corresponding Author: Shuker Mahmood Khalil. Email: shuker.khalil@uobasrah.edu.iq

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ABSTRACT

The ideas of ambiguous bipolar skepticism under algebra and closed skepticism ambiguous bipolar ideals and related features have been developed. The fuzzy measure ideal is described in terms of bipolar ambiguous measure algebra and bipolar skepticism, and the linkages between bipolar fuzzy measure algebra are determined. A bipolar misty ideal's skepticism is examined. In BCW and BCL-measure algebra, homogeneous ideas and dubious pictures of fuzzy bipolar measure ideas are examined. Also, we gave the relationship between these concepts. Finally, it is given the perfect terms for an occult bipolar doubt to be a measure of ideal fuzzy bipolar closed doubt.

KEYWORDS

Fuzzy measure algebra; fuzzy measure; BCW-idials; BCF-ideals

1 Introduction

The BCK-algebras class has been identified as an appropriate subclass of the BCI-algebra class. Iséki et al. [1–4] proposed BCK/BCI-algebras as two key classes of Boolean algebra. BCK/BCI-algebra theory has spawned a plethora of literature since then. This gave a useful mathematical tool for modeling systems that were overly complicated or inadequately specified. Bipolar sets have been used in a variety of fields of mathematics since then. What is now known as fuzzy mathematics is the research of fuzzy sets and various implications in mathematics. Zhang et al. [5] was the first to propose bipolar fuzzy groups as a generalization of regular fuzzy groups [0, 1] in 1994. In addition, Lee [6] proposed a bipolar fuzzy group extension for fuzzy groups. Fuzzy dipole groups confer both a positive and negative membership score, while fuzzy groups confer a degree of membership in an element in a particular group. The interval [0, 1] corresponds to positive membership degrees, while the interval [−1, 0] corresponds to negative membership degrees. The range of degrees of membership in bipolar fuzzy groups is raised from period [0, 1] to period [−1, 1]. Many operations and relations have been proposed to dipole fuzzy sets as a basis for investigating the enigmatic pole set theory [−1, 1]. Fuzzy dipole group theory has recently gained traction in a variety of fields, including group theory, half group theory, semifinal theorems, statistics, medicine, and among others. Rosenfeld [7] introduced a fuzzy subgroup of fuzzy algebraic structures.

There have been many contributions to the concepts of fuzzy subgroups and fuzzy ideal of doubt BCK/BCI-algebras by several researchers such as: Biswas [8] later proposed the concept of non-fuzzy subgroups of groups. The related characteristics of BCK-algebras using the fuzzy group notion are examined [9]. Jun et al. have also researched fuzzy features of numerous ideas in BCK/BCI/RHO-algebras [10–21]. Huang [22], on the other hand, are concerned with BCI algebra in other ways. To avoid confusion resulting from, Huang's definition of perturbation BCI-algebra [23], Jun [24] proposed the definition of the fuzzy sub-algebra of doubt and the fuzzy ideal of doubt BCK/BCI-algebras and offered some conclusions concerning them. Following that, Zhan et al. [25] added the concept of ambiguous skepticism to BCI-algebra ideals, as well as the concept of doubt opaque H ideals in BCK algebra. As a result of what was accomplished by the previous works, new concepts were proposed and circulated to fuzzy measure algebra. The objective of this work is to introduce the concept of BCW and BCM measure algebras in a bipolar fuzzy measure array with discussed properties in this study. Also, we introduce fuzzy bipolar measure algebra and bipolar fuzzy measure proverb, as well as examine associated aspects. Then, by doubting the set of positive segments at the t-level and doubting the set of negative s-level segments, we describe the doubtful fuzzy dipole measure sub-algebra and the questionable fuzzy dipole measure ideal. Also examined are the connections between the ambiguous dipole measure sub-algebra and the ambiguous dipole perfect measure sub-algebra. Finally, we study homogenous and doubt images of putative fuzzy bipolar measure ideals in BCW and BCM-measure algebras. Moreover, we define the conditions under which a perfect fuzzy dipole measure model becomes a fuzzy closed dipole model.

2 Preliminaries

Definition 2.1. [26] A functional $\mu{:}T \to R^{+}$ is called a σ-additive measure if whenever a set $A \in T$ is a disjoint union of an at most countable sequence $\{A_k\}_{k=1}^{M}$ (where M is either finite or $M = \infty$) then $u(A) = \sum_{k=1}^{M} u(A_k)$. If $M = \infty$ then the above sum is understood as a string. If this property applies only to the finite values of M, then μ is a final additive measure.

Definition 2.2. [27] Let W be a universe of discourse, then a fuzzy set (FS) T is characterized by a membership function $\mu_T(w)$ that takes values in [0, 1].

Definition 2.3. [28]. Let $\varnothing \neq J \subseteq F$, where F is BCK/BCI algebra. Then J is a sub algebra of F if $\forall \zeta, \eta \in J$ then $\zeta * \eta \in J$.

Definition 2.4. [28]. Let $\varnothing \neq J \subseteq F$, where F is BCK/BCI algebra. Then J is an ideal of F if it achieves the following:

1) $0 \in J$
2) $\forall \zeta, \eta \in F, \zeta * \eta \in J, \eta \in J \Rightarrow \zeta \in J$.

Definition 2.5. [29]. Let $F \neq \varnothing$. An M-polar fuzzy measure set is a map $\psi : F \to [0, 1]^{M}$. The membership value of any ζ in F is defined by:

$\psi(\zeta) = \left(\mathcal{P}_1^{\circ}\psi(\zeta), \mathcal{P}_2^{\circ}\psi(\zeta), \ldots, \mathcal{P}_M^{\circ}\psi(\zeta)\right)$ where $\mathcal{P}_k^{\circ}\psi(\zeta) : [0, 1]^{M} \to [0, 1]$ is defined as the k-th function of projection.

Definition 2.6. [30] A fuzzy ordered set (X, μ_R) is called a fuzzy well-ordered set if it is a totally fuzzy ordered set in which every non-empty subset has a fuzzy least element.

Definition 2.7. [30] (Zorn's lemma) Let P be a partially ordered set. If every chain in P has an upper bound, then X has a maximal element.

Proposition 2.8. [30] If Zorn's lemma holds, then fuzzy Zorn's lemma holds.

3 BCW and BCM in Polar Fuzzy Measure Sub-Algebras

In this part, we give three concepts of BCW and BCM in fuzzy measure algebra and with a study of its most prominent characteristics.

Definition 3.1. Let $\varnothing \neq J \subseteq F$ where F is fuzzy measure algebra. Then J is a $BCW-$ sub algebra of F if $(\zeta * \eta) - 1 \in J, \forall \zeta, \eta \in J$.

Example 3-1: A 3-polar fuzzy measure set ψ : F = {0, 1, j, 2} → $[0, 1]^3$ by:

$$\psi\,(\mathrm{x}) = \begin{cases} (0.6, 0.7, 0.4)\,, & \mathit{if}\ x = 0 \\ (0.3, 0.5, 0.6)\,, & \mathit{if}\ x = 1 \\ (0.1, 0.4, 0.5)\,, & \mathit{if}\ x = 2 \\ (0.0, 0.3, 0.7)\,, & \mathit{if}\ x = j \end{cases}$$

So ψ is a 3-polar fuzzy measure ideal of F.

For each 3-polar fuzzy measure set ψ on F with $\breve{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\} \in [0, 1]^3$, the set $\psi_{[\breve{\sigma}]} = \left\{\mathrm{x} \in \mathrm{F} : \psi\,(\mathrm{x}) \geq \breve{\sigma}\right\}$ is BCM-sub algebra set of F because $(J * \eta)^2 \in \psi_{[\breve{\sigma}]}$, $\forall J, \eta \in \psi_{[\breve{\sigma}]}$ for the $*$-operation we can see the following Cayley table:

*	0	1	2	J
0	0	0	2	1
J	J	0	2	J
1	1	1	0	2
2	2	2	1	0

Let K={0, 1}, then K is BCM-sub algebra of F.

Definition 3.2. Let $\varnothing \neq J \subseteq F$, where F is fuzzy measure algebra. Then J is a $BCM-$ sub algebra of F if $(\zeta * \eta)^2 \in J \forall \zeta, \eta \in J$.

Definition 3.3. Let $\varnothing \neq J \subseteq F$, where F is BCW and BCM fuzzy measure algebra. Then J is an ideal of F if it achieves the following:

1) $0, 1 \in F$.
2) $\forall \zeta, \eta \in F, (\zeta * \eta)^2 \in J, \eta \in J \Rightarrow \zeta^2 \in J$.

Definition 3.4. A fuzzy measure effect algebra is a system $(F, M, 0_F, u, \oplus)$ consisting of a set F, M is fuzzy measure on bolean algebra, special elements 0_F and u are called the zero and the unit respectively, and a totally defined binary operation $\oplus$ on F, called the ortho sum if for all $h, \ell, \daleth \in F$:

1) If $h \oplus \ell$ and $(h \oplus \ell)^2 \oplus \daleth$ are defined, then $\ell \oplus \daleth$ and $h \oplus (\ell \oplus \daleth)$ are defined and $h \oplus (\ell \oplus \daleth)^2 = (h \oplus \ell)^2 \oplus \daleth$.
2) If $h \oplus \ell^2$ is defined, then $(\ell \oplus \daleth)^2 = (\ell \oplus h)^2$, also $\ell^2 \oplus h$ is fuzzy measure set.

3) $\forall h \in F$, there is a unique $\ell \in F$ such that $h \oplus \ell^2$ is fuzzy measure set and $h \oplus \ell^2 = u$.

4) If $h \oplus u$ is fuzzy measure set defined, then $h = 0_F$.

Definition 3.5. Let $\breve{\Theta}$ be M-polar fuzzy measure set of F . We say $\breve{\Theta}$ is an M-polar fuzzy measure sub-algebra if: $\forall \mu, \nu \in F, \left(\breve{\Theta}(\mu * \nu)\right) \geq \inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\nu)\right\}$, where $\breve{\Theta}(\mu), \breve{\Theta}(\nu)$ are fuzzy measure point of μ and ν, respectively. So $\forall\ \mu, \nu \in F$

$$p_i \circ \breve{\Theta}(\mu * \nu) \geq \inf\left\{p_i \circ \breve{\Theta}(\mu), p_i \circ \breve{\Theta}(\nu)\right\} \forall i = 1, 2, \ldots, m.$$

Lemma 3.6. If $\breve{\Theta}$ is a polar fuzzy measure sub-algebra of F, then $\breve{\Theta}(0) \geq \breve{\Theta}(\mu), \forall \mu \in F$.

Proof. Let $\breve{\Theta}$ be a polar fuzzy measure sub-algebra of F. Then, we have $\breve{\Theta}(\mu * \nu) \geq \inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\nu)\right\}, \forall \mu, \nu \in F$. *Take* $\mu = \nu$, we have $\breve{\Theta}(0) = \breve{\Theta}(\mu * \mu) \geq \inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\mu)\right\} = \breve{\Theta}(\mu) \rightarrow \breve{\Theta}(0) \geq \breve{\Theta}(\mu), \forall \mu \in F$.

Example 3.7. Let $F = \left\{0, \iota^2, \kappa^2\right\}$ be BCW and BCM- fuzzy measure algebra. Define a mapping $\breve{\Theta} : F \rightarrow [0, 1]^3$ by:

$$\breve{\Theta}(\mu) = \begin{cases} (0.4, 0.4, 0.8)\ if\ \ \mu = 0. \\ (0.1, 0.2, 0.4)\ if\ \ \mu = \iota^2. \\ (0.2, 0.3, 0.3)\ if\ \ \mu = \kappa^2. \end{cases}$$

Then $\breve{\Theta}$ is 3-polar fuzzy measure sub-algebra of F.

Definition 3.8. A polar fuzzy measure set $\breve{\Theta}$ of F is said to be an a polar fuzzy measure ideal (PFMI, for short) if satisfies:

$$\forall \mu, \nu \in F, (p_i \circ \breve{\Theta}(0) \geq p_i \circ \breve{\Theta}(\mu)^2 \leq \sup\left\{p_i \circ \breve{\Theta}(\mu * \nu)^2, p_i \circ \breve{\Theta}(\mu)^2\right\})$$
$$\forall i = 1, 2, \ldots, \zeta.$$

Theorem 3.9. Suppose that $\breve{\Theta}$ is an M-polar fuzzy measure set of F. Then $\breve{\Theta}_{[\breve{\sigma}]}$ is BCM-sub algebra of F for any $\breve{\Theta}_{[\breve{\sigma}]} \neq \varnothing$, where $\breve{\sigma} = \{\sigma_1, \sigma_2, \ldots, \sigma_m\} \in [0, 1]^m$ if and only if $\breve{\Theta}$ is an M-polar fuzzy measure sub-algebra of F.

Proof. Suppose that $\breve{\Theta}$ is an polar fuzzy measure sub-algebra of F and assume $\breve{\sigma} \in [0, 1]^m$ with $\breve{\Theta}_{[\breve{\sigma}]} \neq \varnothing$. Let $\mu, \nu \in \breve{\Theta}_{[\breve{\sigma}]}$. Then $\breve{\Theta}(\mu) \geq \tilde{\sigma}$ and $\breve{\Theta}(\nu) \geq \tilde{\sigma}$. It follows that $\breve{\Theta}(\mu * \nu) \geq \inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\nu)\right\} \geq \tilde{\sigma}$ and hence $\breve{\Theta}(((\mu * \nu)^2)) = \breve{\Theta}((\mu * \nu) * (\mu * \nu)) \geq \inf\left\{\breve{\Theta}((\mu * \nu)), \breve{\Theta}((\mu * \nu))\right\} \geq \tilde{\sigma}$, thus $(\mu * \nu)^2 \in \breve{\Theta}_{[\breve{\sigma}]}$. Therefore $\breve{\Theta}_{[\breve{\sigma}]}$ is BCM-sub algebra of F.

Conversely, assume that $\breve{\Theta}_{[\breve{\sigma}]}$ is BCM-sub algebra of F, i.e., $\breve{\Theta}(\mu * \nu) \geq \inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\nu)\right\}, \forall \mu, \nu \in F$. Now, suppose that there exist $\mu, \nu \in F$ s.t, $\breve{\Theta}(\mu * \nu) < \inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\nu)\right\}$. Thus there exist $\breve{\sigma} = \{\sigma_1, \sigma_2, \ldots, \sigma_m\} \in [0, 1]^m$ such that, $\breve{\Theta}(\mu * \nu) < \tilde{\sigma} \leq \inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\nu)\right\}$. However, $\inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\nu)\right\} \leq \breve{\Theta}(\mu)$ and $\inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\nu)\right\} \leq \breve{\Theta}(\nu)$. Hence $\mu, \nu \in \breve{\Theta}_{[\breve{\sigma}]}$, but $(\mu * \nu) \notin \breve{\Theta}_{[\breve{\sigma}]}$ and it is a contradiction. So $\breve{\Theta}(\mu * \nu) \geq \inf\left\{\breve{\Theta}(\mu), \breve{\Theta}(\nu)\right\}, \forall \mu, \nu \in F$. Thus $\breve{\Theta}$ is an polar fuzzy measure sub- algebra of F.

Proposition 3.10. A polar fuzzy measure sub-algebra $\breve{\Theta}$ of F is a PFMI if such that $\breve{\Theta}(\mu * \nu)^2 \leq \breve{\Theta}(\nu)^2$ implies $\breve{\Theta}(\mu) = \breve{\Theta}(0), \forall \mu, \nu \in F$.

Example 3.11. Let $F = \{0, \iota^2, 3, 6\}$ be BCW and BCM-fuzzy measure algebra with Cayley table defined by a mapping $\check{\Theta} : F \to [0, 1]^3$:

$$\check{\Theta}(\mu) = \begin{cases} (0.7, 0.3, 0.6) \text{ if } \mu = 0, 3. \\ (0.4, 0.6, 0.8) \text{ if } \mu = \iota^2. \\ (0.3, 0.3, 0.7) \text{ if } \mu = 6. \end{cases}$$

Then $\check{\Theta}$ is a (PFMI) of F.

Proposition 3.12. If $\check{\Theta}$ is an PFMI of F, then

$\forall \mu, \nu \in F, \mu \geq \nu \Rightarrow \check{\Theta}(\mu)^2 \leq \check{\Theta}(\nu)^2$.

Proof. Let $\mu,\ \nu\ \in\ F$ be s.t, $\mu\ \geq\ \nu$. Then $\mu\ *\ \nu = 1$ and so

$\check{\Theta}(\mu)^2 \leq \sup\left\{\check{\Theta}(\mu * \nu)^2, \check{\Theta}(\nu)^2\right\} = \sup\left\{\check{\Theta}(1), \check{\Theta}(\nu)^2\right\} = \check{\Theta}(\nu)^2$. Thus $\check{\Theta}(\mu)^2 \leq \check{\Theta}(\nu)^2$.

Theorem 3.13. Let $\omega \in F$. If $\check{\Theta}$ is a PFMI of F, then F_ω is an fuzzy measure ideal of F.

Proof. Let $\omega \in F$. Let $\mu, \nu \in F$ be s.t, $(\mu * \nu)^2 \in F_\omega$ and $\nu \in F_\omega$. Then $\check{\Theta}(\mu * \nu)^2 \leq \check{\Theta}(\omega)^2$ and $\check{\Theta}(\mu\ )^2 \leq \check{\Theta}(\omega)^2$. Since $\check{\Theta}$ is an M-polar fuzzy measure ideal (M-PFMI, for short) of F, so $\check{\Theta}(\mu)^2 \leq \sup\left\{\check{\Theta}(\mu * \nu)^2, \check{\Theta}(\nu)^2\right\} \geq \check{\Theta}(\omega)^2, \omega^2 \in F_\omega$. Hence, F_ω is an fuzzy measure ideal of F.

Defi1q`nition 3.14. Let F be a BCW and BCM fuzzy measure algebra. Then a PFMI $\check{\Theta}$ of F is closed if it is a polar fuzzy measure sub-algebra of F.

Proposition 3.15. Every closed a PFMI $\check{\Theta}$ of a BCW-fuzzy measure algebra F satisfies:

$\forall \mu \in F, \check{\Theta}(1 * \mu)^2 \leq \check{\Theta}(\mu)^2$.

Proof. For any $\mu \in F$, we have $\check{\Theta}(1 * \mu)^2 \leq \sup\left\{\check{\Theta}(1), \check{\Theta}(\mu)^2\right\} \leq \sup\left\{\check{\Theta}(1), \check{\Theta}(\mu)^2\right\} = \check{\Theta}(\mu)$. Thus, we get the result.

Proposition 3.16. Every closed M-PFMI $\check{\Theta}$ of a BCM-fuzzy measure algebra F such that:

$\forall \mu \in F\check{\Theta}(1 * \mu)^2 \leq \check{\Theta}(\mu)^2$.

Proof. For any $\mu \in F$, we have $\check{\Theta}(1 * \mu)^2 \geq \sup\left\{\check{\Theta}(0), \check{\Theta}(\mu)\right\} \geq \sup\left\{\check{\Theta}(x), \check{\Theta}(\mu)^2\right\} = \check{\Theta}(\mu)^2$. Therefore $\check{\Theta}(1 * \mu)^2 \geq \check{\Theta}(\mu)^2$.

4 *M*-Polar (*α*, *β*)-Fuzzy Measure Ideals

In this part, we suggest and discussion this concept M-polar (α, β)-BCM and $BCI2$ fuzzy measure ideals, where: $\alpha, \beta \in \{\in, \delta, \in \vee \delta, \in \wedge \delta\}, \alpha \neq\in \wedge q$.

Proposition 4-1. Let $\wp$ be an M-polar fuzzy measure M-PFM of F, the set $\wp_1 \neq \varnothing, \forall \hat{\iota} \in (0.25, 1]^m$ is an ideal **of** F if and only if $\wp$ such that the assertions below: for all $x, y \in F$,

(1) $\text{Inf}\left\{\wp(0), 0.\breve{25}\right\} \geq \wp(x)$.

(2) $\text{Inf}\left\{\wp(x), 0.\breve{25}\right\} \geq \inf\{\wp(x * y), \wp(y)\}$.

Proof. Suppose that $\wp_1 \neq \varnothing$ is an ideal of F with $\sup\left\{\wp(0), 0.\breve{25}\right\} < \wp(\nu)$, for some $\upsilon \in F$. Then, $\wp(\nu) \in (0.25, 1]^m$, so $\upsilon \in \wp_{\wp(\nu)}$, hence $\wp(0) > \wp(\nu)$, thus $0 \notin \wp_{\wp(\nu)}$ and its a contradiction. So that (1) holds.

Now, suppose $\sup\left\{\wp(x), 0.\breve{25}\right\} > \inf\{\wp(x*y), \wp(y)\} = \hat{\iota}$ for some $x, y \in F$. So, $\hat{\iota} \in (0.25, 1]^m$ m and $y, x*y \in \wp_1$. Let, $x \notin \wp_1$ since $\wp(x) > \hat{\iota}$, a contradiction. Hence, (2) holds.

Conversely, assume that (1) and (2) hold. Let $\hat{\iota} \in (0.25, 1]^m$ be such that $\wp_1 \neq \varnothing$, for any $x \in \wp$, then $0.\breve{25} > \hat{\iota} \leq \wp(x) \geq \sup\left\{\wp(x), 0.\breve{25}\right\}$ So that, $\wp(0) = \sup\left\{\wp(x), 0.\breve{25}\right\} \leq \hat{\iota}$. Thus, $0 \in \wp_{\hat{\iota}}$. Let $x, y \in F$ be such that $x * y, y \in \wp_{\hat{\iota}}$.

Therefore $\sup\left\{\wp(x), 0.\breve{25}\right\} \leq \inf\{\wp(x*y), \wp(y)\} \leq \hat{\iota} < 0.\breve{25}$, hence, $\wp(x) = \sup\left\{\wp(x), 0.\breve{25}\right\} \leq \hat{\iota}$, that is, $x \in \wp_{\hat{\iota}}$. Thus $\wp_{\hat{\iota}}$ is an ideal of F.

Definition 4.2. Let $\wp$ be an M-PFMI of F. We say $\wp$ is an M-polar $(\alpha, \beta)-BCK2-$ fuzzy measure ideal [M–P-(α, β)-$BCK2-$ FMI, for short] of F if for all $x, y \in F$ and $\hat{\iota}, \hat{\kappa} \in (0.5, 1]^m$,

(1) If $x_{\hat{\iota}}\alpha\wp$ then $0.5_{\hat{\iota}}\beta\wp$.

(2) If $(x * y)^2{}_{\hat{\iota}}\alpha\wp$ and $y_{\hat{\kappa}}\alpha\wp$ then $x_{sup\{\hat{\iota},\hat{\kappa}\}}\beta\wp$.

Definition 4.3. Let $\wp$ be an M-PFMI of F. We say $\wp$ is an [M–P-(α, β)-$BCK2-$ FMI] of F if for all $x, y \in F$ and $\hat{\iota}, \hat{\kappa} \in (0.5, 1]^m$,

(1) If $x_{\hat{\iota}}\alpha\wp$ then $0.075_{\hat{\iota}}\beta\wp$

(2) If $(x * y)^2{}_{\hat{\iota}}\alpha\wp$ and $y_{\hat{\kappa}}\alpha\wp$ then $x_{inf\{\hat{\iota},\hat{\kappa}\}}\beta\wp$.

Definition 4.4. Let $\wp$ be an M-PFMI of F. Then $\wp$ is called an [M–P-(α, β)-$BCK2-$ FMI] of F if for all $x, y \in F$ and $\hat{\iota}, \hat{\kappa} \in (0.5, 1]^m$,

(1) If $x_{\hat{\iota}}\alpha\wp$ then $0.25_{\hat{\iota}}\beta\wp$.

(2) If $(x * y)^2{}_{\hat{\iota}}\alpha\wp$ and $y_{\hat{\kappa}}\alpha\wp$ then $x_{inf\{\hat{\iota},\hat{\kappa}\}}\beta\wp$.

Theorem 4.5. Let $\wp$ be an M-PFMI subset of F and ξ be an ideal of F such that,

(1) $\wp(x) = \breve{1}$, for all $x \notin \xi$.

(2) $\wp(x) \leq 1$, for all $x \in J$.

Then, $\wp(x)$ is an $M-$ polar $-(\alpha, \in \vee q) - BCK2-$ fuzzy measure ideal $(M - P - (\alpha, \in \vee q) - BCK2-$ FMI, for short) of F.

Proof. (1) (For $\alpha = q$) Let $x \in F$ and $\hat{\iota} \in (0.5, 1]^m$ such that $x_{\hat{\iota}}q\wp$.

Then, $\wp(x) + \hat{\iota} > \breve{1}$. Since $0.5 \in J$, so $\wp(0.5) \geq 0.\breve{75}$. If $\hat{\iota} \leq 0.\breve{75}$, then $\wp(0.5) \leq \hat{\iota}$ and so $0.5 \in \wp$. $\hat{\iota} \geq 0.\breve{75}$, then $\wp(0.5) + \hat{\iota} < \breve{1}$. Hence $0.5_{\hat{\iota}} \in \vee q\wp$.

Let $x, y \in F$ and $\hat{\iota}, \hat{\kappa} \in (0.5, 1]^m$ be such that $(x * y)_{\hat{\iota}}\, q\wp$ and $y_{\hat{\kappa}}q\wp$ thus, $\wp(x*y) + \hat{\iota} < \breve{1}$ and $\wp(y) + \hat{\kappa} < \breve{1}$. Therefore $x * y, y \in J$, and $x \in J$, $\wp(x) \leq 0.\breve{75}$. If $\inf\left\{\hat{\iota}, \hat{\kappa}\right\} \geq 0.\breve{75}$, then $\wp(x) \leq 0.\breve{75} \leq \inf\left\{\hat{\iota}, \hat{\kappa}\right\}$ and so $x_{inf\{\hat{\iota},\hat{\kappa}\}} \in q\wp$. If $\inf\left\{\hat{\iota}, \hat{\kappa}\right\} < 0.\breve{75}$, then $\wp(x) + \inf\left\{\hat{\iota}, \hat{\kappa}\right\} < \breve{1}$ and we have $x_{inf\{\hat{\iota},\hat{\kappa}\}} \in \vee q\wp$. Therefore, $\wp(x)$ is a $[M - P - (\alpha, \in \vee q) - BCK2-$ FMI] of F.

Theorem 4.6. Suppose that $\wp$ is an-ideal subset of F and ξ be an ideal of F such that

(1) $\wp(x) = \breve{1}$, for all $x \notin \xi$.

(2) $\wp(x) \leq \breve{1}$, for all $x \in J$.

Then, $\wp(x)$ is an $(M - P - (\alpha, \in \vee q) - BCK2-$ FMI, for short) of F.

Proof. Similarly, to the proof of Theorem 4.5.

Theorem 4.7 Let $\wp$ be an-ideal subset of F and ξ be an ideal of F such that

(1) $\wp(x) = \breve{1}$, for all $x \notin \xi$.

(2) $\wp(x) \leq 1$, for all $x \in J$.

Then, $\wp(x)$ is an $[M - P - (\alpha, \in \vee q) - BCK2-$FMI] of F.

Proof. Similarly, to the proof of Theorem 4.5.

5 Conclusions

We got a new class of BCW and BCM measure algebra based on polar fuzzy measure sets. We looked at characterizations of the blurry polar measure sub-algebra and fuzzy (commutative) measure ideals of polarity. We also have discussed the relationships among polar fuzzy measure sub algebras, and M-polar ambiguous and ambiguous pole reciprocal ideals. Concepts suggested in this article can be extended to different types from the ideals in BCW and BCM-measure algebras, for instance, a-ideal, implicated, n-fold and n-fold ideals commutative measure ideals. We also deduced a new concept called M-polar (α, β)-BCW and BCM-fuzzy measure algebras and some specific results for it. We investigated M-polar (α, β)-BCW and BCM-fuzzy measure algebras measure ideas with relationships related to them. Finally, we defined the criteria under which a closed doubt bipolar fuzzy idea can exist. We feel our findings in this paper will serve as a foundation for further research into the algebraic structure of BCW and BCM-algebras. In future work, the concepts of M-polar (α, β)-BCW and BCM can be generalized in other topics such as graph topology, graph algebra and other topics.

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