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Two-Sided Stable Matching Decision-Making Method Considering Matching Intention under a Hesitant Fuzzy Environment

Qi Yue[1,2,*] and Zhibin Deng[2]

[1]School of Management, Shanghai University of Engineering Science, Shanghai, 201620, China

[2]School of Information Management, Jiangxi University of Finance and Economics, Nanchang, 330013, China

*Corresponding Author: Qi Yue. Email: yueqichina@126.com

ABSTRACT

In this paper, a stable two-sided matching (TSM) method considering the matching intention of agents under a hesitant fuzzy environment is proposed. The method uses a hesitant fuzzy element (HFE) as its basis. First, the HFE preference matrix is transformed into the normalized HFE preference matrix. On this basis, the distance and the projection of the normalized HFEs on positive and negative ideal solutions are calculated. Then, the normalized HFEs are transformed into agent satisfactions. Considering the stable matching constraints, a multiobjective programming model with the objective of maximizing the satisfactions of two-sided agents is constructed. Based on the agent satisfaction matrix, the matching intention matrix of two-sided agents is built. According to the agent satisfaction matrix and matching intention matrix, the comprehensive satisfaction matrix is set up. Furthermore, the multiobjective programming model based on satisfactions is transformed into a multiobjective programming model based on comprehensive satisfactions. Using the G-S algorithm, the multiobjective programming model based on comprehensive satisfactions is solved, and then the best TSM scheme is obtained. Finally, a terminal distribution example is used to verify the feasibility and effectiveness of the proposed method.

1 Introduction

In the process of two-sided matching decision-making (TSMDM), two-sided agents yield the preference information according to their own needs and produce a reasonable matching scheme. The TSMDM problem widely exists in various fields, such as the freight source matching problem [1], blockchain-based task matching problem [2], cap-trade matching problem in carbon emissions [3], mentor-mentee matching problem [4], emergency rescue personnel and task matching problem [5] and so on. Gale et al. [6] first studied TSMDM problems with ordinal preferences. After the initial study, different types of preference information were used in the TSMDM problems, such as the linguistic term, ordinal number and interval number. For instance, Tong et al. [7] proposed a two-sided matching (TSM) model based on the preference information of linguistic terms. Pittel [8] proposed

a TSM model under strict preference order. Hu et al. [9] presented a stable TSM model with one-side preference. Antler [10] studied TSMDM problems based on the endogenous preference ordinal number and proposed a new TSMDM method. Many scholars at home and abroad are devoting increasing attention to TSMDM because of the important roles of TSMDM theory and methodology in economic activities.

Currently, the preference information provided by people is often vague or uncertain due to the subjectivity and fuzziness of human thinking [11]. Thus, preference information based on different forms of fuzzy sets has been noticed and studied by many scholars, including intuitionistic fuzzy sets, interval intuitionistic fuzzy sets, and triangular fuzzy numbers [12–14]. These types of fuzzy sets mostly contain a membership and nonmembership element. However, in some decision-making scenarios, the decision-maker may not provide a single evaluation value, but rather multiple evaluation values. In addition, multiple decision-makers may give different evaluation information at the same time, and they may not convince each other due to conflicting opinions. For instance, in a venture investment matching problem, five experts in the field of venture capital may assign different evaluation values with respect to venture capital enterprises, such as 0.5, 0.4, 0.6, 0.8 and 0.7. The traditional fuzzy sets mentioned above cannot effectively maintain the integrity of decision information. Compared with the traditional fuzzy sets mentioned above, the hesitant fuzzy set (HFS) proposed by Torra can maximize the retention of initial decision information due to the multiple membership degrees [15]. In the case mentioned above, the preference information given by five experts can be described by the hesitant fuzzy element (HFE) {0.5, 0.4, 0.6, 0.8, 0.7}. Therefore, the TSMDM considering HFS preferences also has important research value.

Relevant studies on HFS have emerged in different fields. First, the basic theory regarding HFS has been generalized. For instance, Xu et al. [16] proposed various hesitant ordered weighted distance measures and corresponding hesitant ordered weighted similarity measures. Following Xu and Xia's [17] research, a brand-new distance measure and similarity measure for higher order HFS were given in Farhadinia's study. Liang et al. [18] developed a novel utility function for HFS. Liao et al. [19] proposed brand-new hesitant fuzzy weighted operators, including the hesitant power average operator, weighted hesitant power average operator, ordered weighted hesitant power average operator and hybrid weighted hesitant power average operator. Moreover, the application scope of HFS has expanded. For instance, a novel multiattribute group decision-making model based on the hesitant fuzzy superiority and inferiority ranking method and the interval-valued hesitant fuzzy superiority and inferiority ranking method was proposed by Ma et al. [20]. Ullah [21] developed a new multiattribute group decision-making model based on the fuzzy picture Maclaurin symmetric mean operator.

In addition, according to the aforementioned literature, there have been many studies on the application of HFS in the TSMDM problem. For instance, Zhang et al. [22] established a stable TSM model considering multi-granular hesitant fuzzy linguistic term information and proposed an example for the matching of green building technology supply and demand to verify the effectiveness of this model. Li et al. [23] studied the complex product manufacturing problem and proposed a TSMDM method considering dual-hesitant fuzzy information. Yue et al. [24] developed a new HFS score function and constructed the corresponding TSMDM model. Zhang et al. [25] proposed a two-sided matching method considering regret aversion psychological behavior and matching aspiration to solve TSM problems with intuitionistic fuzzy preference information. Xiang et al. [26] constructed a novel TSM model considering hesitancy and ambiguity and proposed a novel Pareto refining method, which can be applied in high-end equipment cloud manufacturing platform matching problems.

Although a considerable number of scholars have explored the TSMDM problem under HFS preferences from a variety of perspectives and have achieved some promising results, there are some deficiencies in the existing research.

First, in the TSMDM with HFS preference information, it is vital for decision-makers to find a reasonable tool to transform HFS preferences into exact values. In most of the existing studies, the TOPSIS method based on the distance measure or biprojection technology has been widely used in the TSMDM problem with HFS preference information. However, the above methods may still result in the loss of decision-making information in some cases. Hence, it would be meaningful to create an effective method to compute agent satisfaction.

Second, the matching intention coefficient, which reflects the matching intention of two-sided agents, plays an important role in the TSMDM problem. For some TSMDM problems, the matching intention coefficient may affect the final TSM result. However, it seems that the matching intention coefficient has been neglected in the aforementioned TSMDM model. Therefore, it would be practicable to develop a method to obtain the matching intention coefficient of two-sided agents in the TSMDM problem with HFS preference information.

Third, stability is of great significance for two-sided agents to achieve a reasonable and satisfactory matching result. However, this topic has not been fully discussed by many existing studies on the TSMDM problem in HFS preference environments. Thus, it is necessary to consider the stability of the TSMDM problem based on HFS preference information.

To overcome these deficiencies, this paper aims to develop a reasonable and effective method to cope with the TSMDM problem considering HFS preferences. Moreover, a TSMDM model with HFS preferences considering the matching intention coefficient is constructed. The key contributions of this work are as follows. First, we create an effective method to compute the agent satisfaction, which uses the distance measure and bi-projection technology. Second, we develop an effective method for calculating the matching intention coefficient of two-sided agents. Finally, a TSMDM model with HFS preference information considering stable matching constraints and matching intention is constructed.

The remaining structure of this paper is as follows: In Section 2, some relevant concepts of the TSM, HFS and HFE, and stable matching are presented. The stable TSM problem with HFS preference information considering matching intention is described in Section 3. Section 4 describes the TSMDM method with HFSs using TOPSIS technology. In Section 5, a TSM case is applied to verify the effectiveness and feasibility of the proposed method. Section 6 summarizes this paper.

2 Basic Concepts

In this section, we first introduce the basic concepts of the TSM, HFS and HFE. Then, the stable matching based on satisfaction is briefly described.

2.1 Two-Sided Matching

There are two agent sets in the TSM problem, namely, $U = \{U_1, U_2, \ldots, U_m\}$ and $V = \{V_1, V_2, \ldots, V_n\}$. Assume $2 \leq m \leq n$ and suppose $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

Definition 1. [22] Let $\theta : U \cup V \to U \cup V$ be a mapping; if $\forall U_i \in U, \forall V_j \in V, \theta$ satisfies three constraints: 1) $\theta(U_i) \in V$; 2) $\theta(V_j) \in U \cup \{V_j\}$; 3) $\theta(U_i) = V_j$ if $\theta(V_j) = U_i$; then, $\theta : U \cup V \to U \cup V$ is called a TSM. Here, (U_i, V_j) represents that U_i and V_j match successfully in TSM θ; (U_i, U_i) and (V_j, V_j) represent that U_i and V_j fail to match successfully in TSM θ.

2.2 Hesitant Fuzzy Set and Hesitant Fuzzy Element

Definition 2. [24] Assume $X = \{x_1, x_2, \ldots, x_n\}$ is a fixed set; then, the HFS P in X can be denoted as $P = \{< x, p(x) > | x \in X\}$, where $p(x) = \{p^{\sigma(1)}(x), p^{\sigma(2)}(x), \ldots, p^{\sigma(l_{p(x)})}(x)\}(x = x_1, x_2, \ldots, x_n)$ is a finite set of membership degrees within interval $[0, 1]$, and $l_{p(x)}$ represents the number of elements in HFE $p(x)$. For convenience, note HFE $p = \{p^{\sigma(1)}, p^{\sigma(2)}, \ldots, p^{\sigma(l_p)}\}$.

Definition 3. [16] Let p_A and p_B be two HFEs; then, the distance measures between p_A and p_B can be denoted as:

$$d(p_A, p_B) = \frac{1}{l_p} \sum_{i=1}^{l_p} \left| p_A^{\sigma(j)} - p_B^{\sigma(j)} \right| \tag{1}$$

where $p_A^{\sigma(j)}$ represents the jth element of HFE p_A, $j = 1, 2, \ldots, l_{p_A}$, and $p_B^{\sigma(j)}$ represents the jth element of HFE p_B, l_{p_A} and l_{p_B} are the length of HFEs p_A and p_B, and suppose $l_p = \max\{l_{p_A}, l_{p_B}\}$.

2.3 Stable Matching Based on Satisfaction

In the real TSMDM scenario, we assume that f_{ij} is the satisfaction from U_i to V_j and z_{ij} is the satisfaction from V_j to U_i. The definition of stable matching can be defined as follows.

Definition 4. [22] If neither of the following conditions occurs in a TSM θ:

(i) $\exists U_i, U_l \in U, V_i, V_k \in V$, s.t. $\theta(U_i) = V_k, \theta(U_l) = V_j$, where $f_{ij} > f_{ik}$ and $z_{ij} > z_{lj}$;

(ii) $\exists U_i, U_l \in U, V_i, V_k \in V$, s.t. $\theta(U_i) = V_k, \theta(U_i) = U_i$, where $f_{ij} > f_{ik}$;

then θ is called a stable TSM.

3 Stable Two-Sided Matching Problem Considering Matching Intention

In this section, we introduce the TSMDM problem based on HFE preference information considering the matching intention.

In the TSMDM problem, two-sided agents usually hesitate to assign their own preferences among multiple membership degrees. Therefore, to show the hesitancy and fuzziness of the evaluation information in TSMDM activities, we assume that two-sided agents provide hesitant fuzzy preference information. Furthermore, assume $U = \{U_1, U_2, \ldots, U_m\}$ and $V = \{V_1, V_2, \ldots, V_n\}$ are two sets of TSM agents. Suppose $\tilde{C} = \left[\tilde{c}_{ij}\right]_{m \times n}$ is the HFE matrix from U to V, where $\tilde{c}_{ij} = \left\{\tilde{c}_{ij}^{\mu(1)}, \tilde{c}_{ij}^{\mu(2)}, \ldots, \tilde{c}_{ij}^{\mu\left(l_{\tilde{c}_{ij}}\right)}\right\}$ signifies the HFE information from U to V. Assume $\tilde{D} = \left[\tilde{d}_{ij}\right]_{m \times n}$ is the HFE matrix from V to U, where $\tilde{d}_{ij} = \left\{\tilde{d}_{ij}^{\mu(1)}, \tilde{d}_{ij}^{\mu(2)}, \ldots, \tilde{d}_{ij}^{\mu\left(l_{\tilde{d}_{ij}}\right)}\right\}$ signifies the HFE information from V to U.

The problem studied in this paper is as follows: how to construct a TSM model considering the matching intention and stable matching conditions according to the HFE preference matrices $\tilde{C} = \left[\tilde{c}_{ij}\right]_{m \times n}$ and $\tilde{D} = \left[\tilde{d}_{ij}\right]_{m \times n}$ given by two-sided agents.

4 Two-Sided Matching Decision-Making Based on Hesitant Fuzzy Elements

In this section, the TSMDM method based on HFE considering matching intention is presented. First, the HFE matrices are transformed into standardized HFE matrices. Then, the agent satisfaction

matrices are calculated by using the TOPSIS and biprojection technology-based methods. Third, the TSM model considering the matching intention and stability is constructed by using the multiobject planning method. Finally, the optimal TSM scheme is obtained through the G-S algorithm.

4.1 Standardized Hesitant Fuzzy Element Matrix

According to Definition 2, we know that l signifies the number of elements in the HFE. However, the number of elements in the two HFEs is not the same in many decision scenarios. Therefore, the HFE is normalized based on the method proposed in the reference [23]. Assuming two HFEs p_1 and p_2, their lengths can be expressed as l_1 and l_2. Under the condition of risk preference, the element with the maximum value is added to the HFE with the small value l, so that $l = l_1 = l_2$; under the condition of risk aversion, the elements with minimum value are added to the HFE with the small value l so that $l = l_1 = l_2$. At the same time, the elements in the HFE need to be sorted in descending order.

The normalized HFE matrices $\tilde{E} = \left[\tilde{e}_{ij}\right]_{m \times n}$ and $\tilde{F} = \left[\tilde{f}_{ij}\right]_{m \times n}$ can be obtained by normalizing the HFE elements in matrices $\tilde{C} = \left[\tilde{c}_{ij}\right]_{m \times n}$ and $\tilde{D} = \left[\tilde{d}_{ij}\right]_{m \times n}$. According to Definition 2, the length $l_{e_j}, \forall i = 1, 2, \ldots, m$ of HFE $\tilde{e}_{ij}$ in matrix $\tilde{E} = \left[\tilde{e}_{ij}\right]_{m \times n}$ can be represented as follows:

$$\tilde{E} = \begin{bmatrix} \tilde{e}_{11} & \tilde{e}_{12} & \cdots & \tilde{e}_{1n} \\ \tilde{e}_{21} & \tilde{e}_{22} & \cdots & \tilde{e}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{e}_{m1} & \tilde{e}_{m2} & \cdots & \tilde{e}_{mn} \end{bmatrix}$$
$$\phantom{\tilde{E} = }\; l_{e_1} \quad l_{e_2} \quad \cdots \quad l_{e_n}$$

Therefore, the length $l_{f_i}, \forall j = 1, 2, \ldots, n$ of HFE $\tilde{f}_{ij}$ in matrix $\tilde{F} = \left[\tilde{f}_{ij}\right]_{m \times n}$ can be represented as follows:

$$\tilde{F} = \begin{bmatrix} \tilde{f}_{11} & \tilde{f}_{12} & \cdots & \tilde{f}_{1n} \\ \tilde{f}_{21} & \tilde{f}_{22} & \cdots & \tilde{f}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{f}_{m1} & \tilde{f}_{m2} & \cdots & \tilde{f}_{mn} \end{bmatrix} \begin{matrix} l_{f_1} \\ l_{f_2} \\ \vdots \\ l_{f_m} \end{matrix}$$

4.2 Construction of the Agent Satisfaction Matrix

The normalized HFE matrix is processed according to TOPSIS technology, and the following definitions are given with reference to the reference [24] and Definition 3.

Definition 5. [23] Given a normalized HFE matrix $\tilde{E} = \left[\tilde{e}_{ij}\right]_{m \times n}$, the vectors of positive solution $\tilde{e}^+$ and negative solution $\tilde{e}^-$ in $\tilde{E} = \left[\tilde{e}_{ij}\right]_{m \times n}$ are denoted as follows:

$$\tilde{e}^+ = \left\{\max_i \tilde{e}_{ij}^{\sigma(1)}, \max_i \tilde{e}_{ij}^{\sigma(2)}, \ldots, \max_i \tilde{e}_{ij}^{\sigma(n)}\right\} \tag{2}$$

$$\tilde{e}^- = \left\{\min_i \tilde{e}_{ij}^{\sigma(1)}, \min_i \tilde{e}_{ij}^{\sigma(2)}, \ldots, \min_i \tilde{e}_{ij}^{\sigma(n)}\right\} \tag{3}$$

Definition 6. Let the normalized HFE, the positive ideal solution HFE and negative ideal solution HFE be $\tilde{e}_{ij}, \tilde{e}^+, \tilde{e}^-$, respectively; then, $D\left(\tilde{e}_{ij}, \tilde{e}_i^+\right)$ and $D\left(\tilde{e}_{ij}, \tilde{e}_i^-\right)$ are called the standard Hamming distance of the positive ideal solution HFE $\tilde{e}^+$ and the normalized HFE $\tilde{e}_{ij}$, and the standard Hamming distance of the negative ideal solution HFE $\tilde{e}^-$ and the normalized HFE $\tilde{e}_{ij}$, where $D\left(\tilde{e}_{ij}, \tilde{e}_i^+\right)$ and $D\left(\tilde{e}_{ij}, \tilde{e}_i^-\right)$ are computed by:

$$D\left(\tilde{e}_{ij}, \tilde{e}_i^+\right) = \frac{1}{l_{e_j}} \sum_{k=1}^{l_{e_j}} \left|\tilde{e}_{ij}^{\sigma(k)} - \tilde{e}_i^{+\sigma(k)}\right| \tag{4}$$

$$D\left(\tilde{e}_{ij}, \tilde{e}_i^-\right) = \frac{1}{l_{e_j}} \sum_{k=1}^{l_{e_j}} \left|\tilde{e}_{ij}^{\sigma(k)} - \tilde{e}_i^{-\sigma(k)}\right| \tag{5}$$

where $\tilde{e}^{+\sigma(k)}$ represents the kth element of the positive ideal solution HFE $\tilde{e}^+$, and $\tilde{e}^{-\sigma(k)}$ represents the kth element of the negative ideal solution HFE $\tilde{e}^-$.

Definition 7. [23] Let the normalized HFE, the positive ideal solution HFE and negative ideal solution HFE be $\tilde{e}_{ij}$, $\tilde{e}^+$, $\tilde{e}^-$, respectively; then, $\mathrm{Pr}_{\tilde{e}^-\tilde{e}^+}\left(\tilde{e}^-\tilde{e}_{ij}\right)$ and $\mathrm{Pr}_{\tilde{e}^+\tilde{e}_{ij}}\left(\tilde{e}^-\tilde{e}^+\right)$ are called the projection of the vector formed by HFE $\tilde{e}_{ij}$ and the negative ideal solution $\tilde{e}^-$ on the positive ideal points $\tilde{e}^+$ and negative ideal points $\tilde{e}^-$, and the projection of the vector formed by the positive ideal solution $\tilde{e}^+$ and negative ideal solution $\tilde{e}^-$ on positive ideal points $\tilde{e}^+$ and HFE $\tilde{e}_{ij}$, where $\mathrm{Pr}_{\tilde{e}^-\tilde{e}^+}\left(\tilde{e}^-\tilde{e}_{ij}\right)$ and $\mathrm{Pr}_{\tilde{e}^+\tilde{e}_{ij}}\left(\tilde{e}^-\tilde{e}^+\right)$ are computed by:

$$\Pr_{\tilde{e}^-\tilde{e}^+}\left(\tilde{e}^-\tilde{e}_{ij}\right) = \frac{\sum\limits_{k=1}^{l_{e_j}} \left(\tilde{e}^{+\sigma(k)} - \tilde{e}^{-\sigma(k)}\right)\left(\tilde{e}_{ij}^{\sigma(k)} - \tilde{e}^{-\sigma(k)}\right)}{\sqrt{\sum\limits_{k=1}^{l_{e_j}} |\tilde{e}^{+\sigma(k)} - \tilde{e}^{-\sigma(k)}|^2}} \tag{6}$$

$$\Pr_{\tilde{e}^+\tilde{e}_{ij}}\left(\tilde{e}^-\tilde{e}^+\right) = \frac{\sum\limits_{k=1}^{l_{e_j}} \left(\tilde{e}^{+\sigma(k)} - \tilde{e}^{-\sigma(k)}\right)\left(\tilde{e}^{+\sigma(k)} - \tilde{e}_{ij}^{\sigma(k)}\right)}{\sqrt{\sum\limits_{k=1}^{l_{e_j}} |\tilde{e}^{+\sigma(k)} - \tilde{e}_{ij}^{\sigma(k)}|^2}} \tag{7}$$

According to Definition 6 and Definition 7 and the calculation theory of closeness in reference [24], the definitions of combined closeness are given below.

Definition 8. Let $\tilde{a}_{\tilde{e}_{ij}}$ be the combined closeness between the HFE $\tilde{e}_{ij}$ and the positive and negative ideal solution $\tilde{e}^+$, $\tilde{e}^-$; then, $\tilde{a}_{\tilde{e}_{ij}}$ can be computed as:

$$\tilde{a}_{e_{ij}} = W_D\left(\frac{D\left(\tilde{e}_{ij}, \tilde{e}_i^-\right)}{D\left(\tilde{e}_{ij}, \tilde{e}_i^+\right) + D\left(\tilde{e}_{ij}, \tilde{e}_i^-\right)}\right) + W_{\mathrm{Pr}}\left(\frac{\mathrm{Pr}_{\tilde{e}^-\tilde{e}^+}\left(\tilde{e}^-\tilde{e}_{ij}\right)}{\mathrm{Pr}_{\tilde{e}^-\tilde{e}^+}\left(\tilde{e}^-\tilde{e}_{ij}\right) + \mathrm{Pr}_{\tilde{e}_{ij}\tilde{e}^+}\left(\tilde{e}^-\tilde{e}^+\right)}\right) \tag{8}$$

where W_D and W_{Pr} signify the weights of the distance measure and projection, respectively.

In Eq. (8), the larger the combined closeness $\tilde{a}_{\tilde{e}_{ij}}$ is, the closer $\tilde{e}_{ij}$ is to the positive ideal solution $\tilde{e}^+$, and the more distant $\tilde{e}_{ij}$ is from the negative ideal solution $\tilde{e}^-$: namely, the greater the satisfaction. Hence, the satisfaction matrix $\tilde{A} = \left[\tilde{a}_{e_{ij}}\right]_{m \times n}$ is calculated by Eq. (8).

Definition 9. [23] Given a normalized HFE matrix $\tilde{F} = \left[\tilde{f}_{ij}\right]_{m \times n}$, the vectors of positive solution $\tilde{f}^+$ and negative solution $\tilde{f}^-$ in $\tilde{F} = \left[\tilde{f}_{ij}\right]_{m \times n}$ are denoted as:

$$\tilde{f}^+ = \left\{\max_i \tilde{f}_{ij}^{\sigma(1)}, \max_i \tilde{f}_{ij}^{\sigma(2)}, \ldots, \max_i \tilde{f}_{ij}^{\sigma(m)}\right\} \tag{9}$$

$$\tilde{f}^- = \left\{\min_i \tilde{f}_{ij}^{\sigma(1)}, \min_i \tilde{f}_{ij}^{\sigma(2)}, \ldots, \min_i \tilde{f}_{ij}^{\sigma(m)}\right\} \tag{10}$$

Definition 10. Let the normalized HFE, the positive ideal solution HFE and negative ideal solution HFE be $\tilde{f}_{ij}$, $\tilde{f}^+$, $\tilde{f}^-$, respectively; then, $D\left(\tilde{f}_{ij}, \tilde{f}^+\right)$ and $D\left(\tilde{f}_{ij}, \tilde{f}^-\right)$ are called the standard Hamming distance of the positive ideal solution HFE $\tilde{f}^+$ and the normalized HFE $\tilde{f}_{ij}$, and the standard Hamming distance of the negative ideal solution HFE $\tilde{f}^-$ and the normalized HFE $\tilde{f}_{ij}$, where $D\left(\tilde{f}_{ij}, \tilde{f}^+\right)$ and $D\left(\tilde{f}_{ij}, \tilde{f}^-\right)$ are computed by:

$$D\left(\tilde{f}_{ij}, \tilde{f}^+\right) = \frac{1}{l_{f_i}} \sum_{k=1}^{l_{f_i}} |\tilde{f}_{ij}^{\sigma(k)} - \tilde{f}^{+\sigma(k)}| \tag{11}$$

$$D\left(\tilde{f}_{ij}, \tilde{f}^-\right) = \frac{1}{l_{f_i}} \sum_{k=1}^{l_{f_i}} |\tilde{f}_{ij}^{\sigma(k)} - \tilde{f}^{-\sigma(k)}| \tag{12}$$

where $\tilde{f}^{+\sigma(k)}$ represents the kth element of the positive ideal solution HFE $\tilde{f}^+$, and $\tilde{f}^{-\sigma(k)}$ represents the kth element of the negative ideal solution HFE $\tilde{f}^-$.

Definition 11. [23] Let the normalized HFE, the positive ideal solution HFE and negative ideal solution HFE be $\tilde{f}_{ij}$, $\tilde{f}^+$, $\tilde{f}^-$, respectively; then, $\mathrm{Pr}_{\tilde{f}^-\tilde{f}^+}\left(\tilde{f}^-\tilde{f}_{ij}\right)$ and $\mathrm{Pr}_{\tilde{f}^+\tilde{f}_{ij}}\left(\tilde{f}^-\tilde{f}^+\right)$ are called the projection of the vector formed by the HFE $\tilde{f}_{ij}$ and the negative ideal solution $\tilde{f}^-$ on the positive ideal points $\tilde{f}^+$ and negative ideal points $\tilde{f}^-$, and the projection of the vector formed by the positive ideal solution $\tilde{f}^+$ and the negative ideal solution $\tilde{f}^-$ on the positive ideal points $\tilde{f}^+$ and the HFE $\tilde{f}_{ij}$, where $\mathrm{Pr}_{\tilde{f}^-\tilde{f}^+}\left(\tilde{f}^-\tilde{f}_{ij}\right)$ and $\mathrm{Pr}_{\tilde{f}^+\tilde{f}_{ij}}\left(\tilde{f}^-\tilde{f}^+\right)$ are computed by:

$$\mathrm{Pr}_{\tilde{f}^-\tilde{f}^+}\left(\tilde{f}^-\tilde{f}_{ij}\right) = \frac{\sum\limits_{k=1}^{l_{f_i}} \left(\tilde{f}^{+\sigma(k)} - \tilde{f}^{-\sigma(k)}\right)\left(\tilde{f}_{ij}^{\sigma(k)} - \tilde{f}^{-\sigma(k)}\right)}{\sqrt{\sum\limits_{k=1}^{l_{f_i}} |\tilde{f}^{+\sigma(k)} - \tilde{f}^{-\sigma(k)}|^2}} \tag{13}$$

$$\mathrm{Pr}_{\tilde{f}^+\tilde{f}_{ij}}\left(\tilde{f}^-\tilde{f}^+\right) = \frac{\sum\limits_{k=1}^{l_{f_i}} \left(\tilde{f}^{+\sigma(k)} - \tilde{f}^{-\sigma(k)}\right)\left(\tilde{f}^{+\sigma(k)} - \tilde{f}_{ij}^{\sigma(k)}\right)}{\sqrt{\sum\limits_{k=1}^{l_{f_i}} |\tilde{f}^{+\sigma(k)} - \tilde{f}_{ij}^{\sigma(k)}|^2}} \tag{14}$$

According to Definition 9, Definition 10, and the calculation theory of closeness in reference [24], the definitions of combined closeness are given below.

Definition 12. Let $\tilde{b}_{\tilde{f}_{ij}}$ be the combined closeness between the HFE $\tilde{f}_{ij}$ and the positive and negative ideal solution $\tilde{f}^+$, $\tilde{f}^-$; then, can be computed as:

$$\tilde{b}_{f_{ij}} = W_D\left(\frac{D\left(\tilde{f}_{ij}, \tilde{f}^-\right)}{D\left(\tilde{f}_{ij}, \tilde{f}^+\right) + D\left(\tilde{f}_{ij}, \tilde{f}^-\right)}\right) + W_{\mathrm{Pr}}\left(\frac{\mathrm{Pr}_{\tilde{f}^-\tilde{f}^+}\left(\tilde{f}^-\tilde{f}_{ij}\right)}{\mathrm{Pr}_{\tilde{f}^-\tilde{f}^+}\left(\tilde{f}^-\tilde{f}_{ij}\right) + \mathrm{Pr}_{\tilde{f}^+\tilde{f}_{ij}}\left(\tilde{f}^-\tilde{f}^+\right)}\right) \tag{15}$$

where W_D and W_{Pr} signify the weights of the distance measure and projection.

In Eq. (15), the larger the combined closeness $\tilde{b}_{\tilde{f}_{ij}}$ is, the closer $\tilde{f}_{ij}$ is to the positive ideal solution $\tilde{f}^+$, and the more distant from the negative ideal solution $\tilde{f}^-$: namely, the greater the satisfaction. Then, the satisfaction matrix $\tilde{B} = \left[\tilde{b}_{f_{ij}}\right]_{m \times n}$ is constructed by Eq. (15).

4.3 Construction of a Two-Sided Matching Model Considering the Matching Intention and Stability

Based on the satisfaction matrices $\tilde{A} = \left[\tilde{a}_{e_{ij}}\right]_{m \times n}$ and $\tilde{B} = \left[\tilde{b}_{f_{ij}}\right]_{m \times n}$ constructed in the above section and the stable matching constraints in Definition 4, a TSM model with the objective of maximizing the agent satisfactions can be denoted as:

$$(\text{M-1}) \begin{cases} \text{Max } Z_A = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{a}_{e_{ij}} x_{ij} \\ \text{Max } Z_B = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{b}_{f_{ij}} x_{ij} \\ \text{s.t.} \sum_{j=1}^{n} x_{ij} \leq 1. i = 1, 2, \ldots, m \\ \sum_{i=1}^{m} x_{ij} \leq 1. j = 1, 2, \ldots, n \\ x_{ij} + \sum_{k:\tilde{a}_{e_{ik}} > \tilde{a}_{e_{ij}}} x_{ik} + \sum_{l:\tilde{b}_{f_{lj}} > \tilde{b}_{f_{ij}}} x_{lj} \geq 1 \\ x_{ij} = \{0, 1\}, i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n. \end{cases}$$

where x_{ij} is defined as a variable from $0-1$, $\sum_{j=1}^{n} x_{ij} \leq 1. i = 1, 2, \ldots, m$ and $\sum_{i=1}^{m} x_{ij} \leq 1. j = 1, 2, \ldots, n$ are defined as the one-to-one matching constraints, and $x_{ij} + \sum_{k:\tilde{a}_{e_{ik}} > \tilde{a}_{e_{ij}}} x_{ik} + \sum_{l:\tilde{a}_{e_{lj}} > \tilde{a}_{e_{ij}}} x_{lj} \geq 1$ is defined as the stable matching constraint.

Referring to the references [24,25], we know that the matching intention coefficient is associated with agent satisfaction and matching diversity. Thus, the optimal model for determining the matching intention coefficient considering matching satisfaction and matching diversity is constructed as follows:

$$(\text{M-2}) \begin{cases} \max f(\omega_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} \dfrac{\tilde{a}_{e_{ij}} + \tilde{b}_{f_{ij}}}{\left(1 + |\tilde{a}_{e_{ij}} - \tilde{b}_{f_{ij}}|\right)} \omega_{ij} \\ \text{s.t.} \sum_{i=1}^{m}\sum_{j=1}^{n} \omega_{ij}^2 = 1 \\ 0 \leq w_{ij} \leq 1, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \end{cases}$$

To solve Model (M-2), the Lagrange function is introduced as follows:

$$L(\omega_{ij}, \lambda) = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\tilde{a}_{e_{ij}} + \tilde{b}_{f_{ij}}}{\left(1 + |\tilde{a}_{e_{ij}} - \tilde{b}_{f_{ij}}|\right)} \omega_{ij} - \frac{\lambda}{2}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} \omega_{ij}^2 - 1\right] \tag{16}$$

Then, let

$$\frac{\partial L}{\partial \omega_{ij}} = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\tilde{a}_{e_{ij}} + \tilde{b}_{f_{ij}}}{\left(1 + |\tilde{a}_{e_{ij}} - \tilde{b}_{f_{ij}}|\right)} - \lambda \omega_{ij} = 0 \tag{17}$$

$$\frac{\partial L}{\partial \lambda} = -\frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \omega_{ij}^2 + \frac{1}{2} = 0 \tag{18}$$

Hence, the optimal solution can be obtained on the basis of Eqs. (17) and (18), in which

$$\omega_{ij} = \frac{\dfrac{\tilde{a}_{e_{ij}} + \tilde{b}_{f_{ij}}}{\left(1 + |\tilde{a}_{e_{ij}} - \tilde{b}_{f_{ij}}|\right)}}{\sqrt{\displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n} \left[\dfrac{\tilde{a}_{e_{ij}} + \tilde{b}_{f_{ij}}}{\left(1 + |\tilde{a}_{e_{ij}} - \tilde{b}_{f_{ij}}|\right)}\right]^2}} \tag{19}$$

Furthermore, the standardized matching intention coefficient can be computed by normalizing Eq. (19), in which

$$w_{ij} = \frac{\omega_{ij}}{\displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n} \omega_{ij}} = \frac{\dfrac{\tilde{a}_{e_{ij}} + \tilde{b}_{f_{ij}}}{\left(1 + |\tilde{a}_{e_{ij}} - \tilde{b}_{f_{ij}}|\right)}}{\displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n} \dfrac{\tilde{a}_{e_{ij}} + \tilde{b}_{f_{ij}}}{\left(1 + |\tilde{a}_{e_{ij}} - \tilde{b}_{f_{ij}}|\right)}} \tag{20}$$

To concretely illustrate the calculation method of the matching intention coefficient given above, we take an example with three agents of each side.

Example 1. Let $U = \{U_1, U_2, U_3\}$ and $V = \{V_1, V_2, V_3\}$ be the sets of two-sided agents, where the satisfaction matrices of the two-sided agents are represented as:

$$\tilde{a}_{e_{ij}} = \begin{bmatrix} 0.60 & 0.70 & 0.80 \\ 1.00 & 0.50 & 0.40 \\ 0.30 & 0.60 & 0.90 \end{bmatrix}$$

$$\tilde{b}_{f_{ij}} = \begin{bmatrix} 0.30 & 1.00 & 0.40 \\ 0.60 & 0.80 & 0.50 \\ 0.90 & 0.70 & 0.40 \end{bmatrix}$$

then, the matching intention coefficient can be computed through Eq. (20) as follows:

$$w_{ij} = \begin{bmatrix} 0.08 & 0.15 & 0.10 \\ 0.13 & 0.12 & 0.09 \\ 0.09 & 0.14 & 0.10 \end{bmatrix}$$

According to Eq. (20), the matching intention coefficient matrix $\phi = [w_{ij}]_{m \times n}$ is constructed; then, the comprehensive satisfaction g_{ij} from U_i to V_j and the comprehensive satisfaction h_{ij} from V_j to U_i are calculated as:

$$g_{ij} = \tilde{a}_{e_{ij}} w_{ij} \tag{21}$$

$$h_{ij} = \tilde{b}_{f_{ij}} w_{ij} \tag{22}$$

Therefore, Model (M-1) is transformed into Model (M-3) based on the comprehensive satisfactions g_{ij} and h_{ij}:

$$(\text{M-3}) \begin{cases} \text{Max } Z_A = \sum_{i=1}^{m} \sum_{j=1}^{n} g_{ij} x_{ij} \\ \text{Max } Z_B = \sum_{i=1}^{m} \sum_{j=1}^{n} h_{ij} x_{ij} \\ \text{s.t.} \sum_{j=1}^{n} x_{ij} \leq 1. i = 1, 2, \ldots, m \\ \sum_{i=1}^{m} x_{ij} \leq 1. j = 1, 2, \ldots, n \\ x_{ij} + \sum_{k:g_{ik} > g_{ij}} x_{ik} + \sum_{l:h_{lj} > h_{ij}} x_{lj} \geq 1 \\ x_{ij} = \{0, 1\}. i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \end{cases}$$

4.4 Solution Process of the Two-Sided Matching Model Based on the G-S Algorithm

The framework of the solution process for TSM Model (M-3) based on the G-S algorithm is shown in Algorithm 1; then, the detailed procedures are presented as follows:

Step 1. For each agent U_i $(i = 1, 2, \ldots, m)$, rank all agents of side V according to the comprehensive satisfactions. For each agent $V = \{V_1, V_2, \ldots, V_n\}$, rank all agents of side U according to the comprehensive satisfactions, and then proceed to Step 2.

Step 2. For each agent U_i $(i = 1, 2, \ldots, m)$, send a cooperation invitation to the agent V_r at the top of side U. If agent V_r is not matched and accepts the cooperation invitation, then match (U_i, V_r) is obtained. Otherwise, proceed to Step 3.

Step 3. For agent V_r of side V, choose the matching agent according to the comprehensive satisfaction ranking of side U. If agent V_r prefers agent U_{i+1} rather than U_i, then U_i still remains free. If agent V_r prefers agent U_i rather than U_{i+1}, then match (U_i, V_r) is obtained and U_{i+1} is not matched with V_r; then, proceed to Step 4.

Step 4. For other two-sided agents of side V and side U, rank all remaining agents of side V and side U according to the comprehensive satisfactions. Then, return to Step 2 until all agents of side U are matched.

Algorithm 1: G-S algorithm

Input: TSM θ that is empty
Output: TSM θ that is stable
begin
For each agent $U_i \in U(i = 1, 2, \ldots, m)$
Rank all agents $V = \{V_1, V_2, \ldots, V_n\}$ according to the comprehensive satisfaction $g_{ij}(r = 1, 2, \ldots, n)$
For each agent $V_r \in V(r = 1, 2, \ldots, n)$
Rank all agents $U = \{U_1, U_2, .., U_m\}$ according to the comprehensive satisfaction $h_{ij}(i = 1, 2, \ldots, m)$
While agent $U_i \in U$ did not send a cooperation invitation to the agent V_r at the top of the side with which U_i is not matched **do**
U_i proposes V_r
If V_r is not matched **then** (U_i, V_r)
Else (U_{i+1}, V_r)
If agent V_r prefers agent U_{i+1} rather than agent U_i **then**
Agent U_i is not matched an (U_i, V_r)
Else agent V_{r+1} prefers agent U_i rather than agent U_{i+1} **then**
agent U_{i+1} is not matched and (U_i, V_r)
End if
End if
End while

4.5 Steps of Two-Sided Matching Decision-Making with Hesitation Fuzzy Set Preference

In summary, the steps of stable HFE TSMDM considering matching intention are given below:

Step 1. Transform HFE preference matrices $\tilde{C} = [\tilde{c}_{ij}]_{m \times n}$ and $\tilde{D} = [\tilde{d}_{ij}]_{m \times n}$ into normalized HFE preference matrices $\tilde{E} = [\tilde{e}_{ij}]_{m \times n}$ and $\tilde{F} = [\tilde{f}_{ij}]_{m \times n}$.

Step 2. Transform normalized HFE matrices $\tilde{E} = [\tilde{e}_{ij}]_{m \times n}$ and $\tilde{F} = [\tilde{f}_{ij}]_{m \times n}$ into the agent satisfaction matrices $\tilde{A} = [\tilde{a}_{e_{ij}}]_{m \times n}$ and $\tilde{B} = [\tilde{b}_{f_{ij}}]_{m \times n}$ through Eqs. (2)–(8) and Eqs. (9)–(15).

Step 3. Construct the TSM Model (M-1) based on the satisfaction matrices $\tilde{A} = [\tilde{a}_{e_{ij}}]_{m \times n}$ and $\tilde{B} = [\tilde{b}_{f_{ij}}]_{m \times n}$ under the one-to-one stable matching constraint.

Step 4. Calculate the matching intention coefficient matrix $\phi = [w_{ij}]_{m \times n}$ through Eq. (20) based on the satisfaction matrices $\tilde{A} = [\tilde{a}_{e_{ij}}]_{m \times n}$ and $\tilde{B} = [\tilde{b}_{f_{ij}}]_{m \times n}$.

Step 5. Transform satisfactions $\tilde{a}_{e_{ij}}$ and $\tilde{b}_{f_{ij}}$ into the comprehensive satisfactions g_{ij} and h_{ij} through Eqs. (21) and (22).

Step 6. Transform Model (M-1) into Model (M-3) based on the comprehensive satisfactions g_{ij} and h_{ij}.

Step 7. Determine the stable alternative of TSM by solving Model (M-3) through Algorithm 1.

5 Case Study

In the business activities of enterprises, the terminal distribution problem is the last link in the enterprise supply chain. Currently, the quality of terminal distribution will affect the overall benefit of enterprises, so the terminal distribution plays an important role in the supply chain. At the same time, there are two decision-makers in a terminal distribution problem, namely, freight cars and terminal distribution centers. The decision-making process may be affected by many factors; therefore, we will consider a distribution problem between freight cars and terminal delivery points as an example to verify the validity and feasibility of the TSM model proposed in this paper. The framework of the proposed case is given in Fig. 1. In addition, a comparison analysis is presented to clarify the advantage of the proposed model.
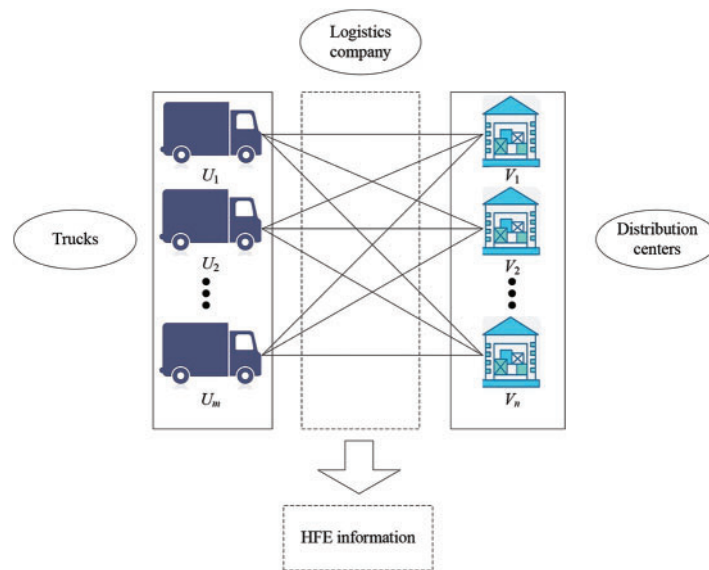

Figure 1: Framework of the terminal distribution problem

5.1 Steps of Two-Sided Matching Decision-Making with Hesitation Fuzzy Set Preference

GH is a third-party logistics enterprise specializing in LCL transportation. In a certain terminal distribution task, GH Company needs to arrange four trucks $\{U_1, U_2, \ldots, U_4\}$ to transport different quantities of goods to five terminal distribution centers $\{V_1, V_2, \ldots, V_5\}$. The business department of the terminal distribution center will conduct a comprehensive evaluation of four trucks in terms of transportation speed, transportation volume, safety measures, movement cost, etc., and then provide the hesitant fuzzy preference information for four trucks by means of questionnaires. The HFE evaluation matrix $\tilde{C} = \left[\tilde{c}_{ij}\right]_{4 \times 5}$ is given, as shown in Table 1. The freight department will conduct a comprehensive evaluation of terminal distribution stations based on fire protection measures, geographical location, informatization degree and other indicators, and then provide evaluation information in the form of an HFE for five terminal distribution stations through statistical analysis. The HFE matrix $\tilde{D} = \left[\tilde{d}_{ij}\right]_{4 \times 5}$ is given, as shown in Table 2. GH Company will optimize matching according to the HFE matrices $\tilde{C} = \left[\tilde{c}_{ij}\right]_{4 \times 5}$ and $\tilde{D} = \left[\tilde{d}_{ij}\right]_{4 \times 5}$ given by two-sided agents and then provide an optimal matching scheme.

Table 1: HFE preference matrix $\tilde{C} = \left[\tilde{c}_{ij}\right]_{4 \times 5}$

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	{0.2, 0.3, 0.4}	{0.2, 0.5, 0.6, 0.7}	{0.1, 0.2, 0.9}	{0.2, 0.6, 0.8}	{0.1, 0.6, 0.7}
U_2	{0.4, 0.6, 0.8}	{0.2, 0.3, 0.5, 0.9}	{0.1, 0.4, 0.6}	{0.1, 0.4, 0.7}	{0.3, 0.6, 0.7}
U_3	{0.1, 0.4, 0.5, 0.6}	{0.2, 0.7, 0.8}	{0.3, 0.5, 0.9}	{0.3, 0.6}	{0.3, 0.7, 0.8}
U_4	{0.2, 0.7, 0.9}	{0.3, 0.5, 0.6}	{0.4, 0.5}	{0.2, 0.4, 0.5}	{0.1, 0.3, 0.5, 0.9}

Table 2: HFE preference matrix $\tilde{D} = \left[\tilde{d}_{ij}\right]_{4 \times 5}$

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	{0.5, 0.6, 0.8}	{0.2, 0.4, 0.7}	{0.1, 0.3, 0.8, 0.9}	{0.2, 0.3, 0.5}	{0.4, 0.5}
U_2	{0.3, 0.5, 0.6}	{0.1, 0.3, 0.6, 0.8}	{0.2, 0.4, 0.5}	{0.3, 0.7}	{0.1, 0.2, 0.7}
U_3	{0.1, 0.3, 0.6}	{0.2, 0.4, 0.9}	{0.4, 0.6}	{0.5, 0.7, 0.8}	{0.3, 0.5, 0.6}
U_4	{0.1, 0.3, 0.7}	{0.2, 0.4, 0.5, 0.6}	{0.2, 0.4, 0.7}	{0, 2, 0.6, 0.9}	{0.3, 0.4, 0.6}

5.2 Solution Process

The proposed method is adopted to solve the problem, and the solution steps are displayed as follows.

Step 1. Transform the HFE preference matrices $\tilde{C} = \left[\tilde{c}_{ij}\right]_{4 \times 5}$ and $\tilde{D} = \left[\tilde{d}_{ij}\right]_{4 \times 5}$ given by the terminal distribution point and freight part into normalized HFE preference matrices $\tilde{E} = \left[\tilde{e}_{ij}\right]_{4 \times 5}$ and $\tilde{F} = \left[\tilde{f}_{ij}\right]_{4 \times 5}$, as shown in Tables 3 and 4.

Table 3: Normalized HFE preference matrix $\tilde{E} = \left[\tilde{e}_{ij}\right]_{4 \times 5}$

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	{0.2, 0.3, 0.4, 0.4}	{0.2, 0.5, 0.6, 0.7}	{0.1, 0.2, 0.9}	{0.2, 0.6, 0.8}	{0.1, 0.6, 0.7, 0.7}
U_2	{0.4, 0.6, 0.8, 0.8}	{0.2, 0.3, 0.5, 0.9}	{0.1, 0.4, 0.6}	{0.1, 0.4, 0.7}	{0.3, 0.6, 0.7, 0.7}
U_3	{0.1, 0.4, 0.5, 0.9}	{0.2, 0.7, 0.8, 0.8}	{0.3, 0.5, 0.9}	{0.3, 0.6, 0.6}	{0.3, 0.7, 0.8, 0.8}
U_4	{0.2, 0.7, 0.9, 0.9}	{0.3, 0.5, 0.6, 0.6}	{0.4, 0.5, 0.7}	{0.2, 0.4, 0.5}	{0.1, 0.3, 0.5, 0.9}

Table 4: Normalized HFE preference matrix $\tilde{F} = \left[\tilde{f}_{ij}\right]_{4\times5}$

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	{0.5, 0.6, 0.8, 0.8}	{0.2, 0.4, 0.7, 0.7}	{0.1, 0.3, 0.8, 0.9}	{0.2, 0.3, 0.5, 0.5}	{0.4, 0.5, 0.5, 0.5}
U_2	{0.3, 0.5, 0.6, 0.6}	{0.1, 0.3, 0.6, 0.8}	{0.2, 0.4, 0.5, 0.5}	{0.3, 0.7, 0.7, 0.7}	{0.1, 0.2, 0.7, 0.7}
U_3	{0.1, 0.3, 0.6}	{0.2, 0.4, 0.9}	{0.4, 0.6, 0.6}	{0.5, 0.7, 0.8}	{0.3, 0.5, 0.6}
U_4	{0.1, 0.3, 0.7, 0.7}	{0.2, 0.4, 0.5, 0.6}	{0.2, 0.4, 0.7, 0.7}	{0, 2, 0.6, 0.9, 0.9}	{0.3, 0.4, 0.6, 0.6}

Step 2. Calculate the positive solutions for two-sided normalized HFE matrices through Eqs. (2) and (9): {0.4, 0.7, 0.9, 0.9}, {0.3, 0.7, 0.8, 0.9}, {0.4, 0.5, 0.9}, {0.4, 0.6, 0.8}, {0.3, 0.7, 0.8, 0.9}, {0.5, 0.6, 0.8, 0.9}, {0.3, 0.7, 0.7, 0.8}, {0.5, 0.7, 0.9}, {0.3, 0.6, 0.9, 0.9}. Calculate the negative solutions for two-sided normalized HFE matrices through Eqs. (3) and (10): {0.1, 0.3, 0.4, 0.4}, {0.2, 0.3, 0.5, 0.6}, {0.1, 0.2, 0.6}, {0.1, 0.4, 0.5}, {0.1, 0.3, 0.5, 0.7}, {0.1, 0.3, 0.5, 0.5}, {0.1, 0.2, 0.5, 0.5}, {0.1, 0.3, 0.6}, {0.1, 0.3, 0.5, 0.6}.

Construct the agent satisfaction matrices $\tilde{A} = \left[\tilde{a}_{e_{ij}}\right]_{4\times5}$ and $\tilde{B} = \left[\tilde{b}_{f_{ij}}\right]_{4\times5}$ through the use of Eqs. (4)–(8) and Eqs. (11)–(15), as shown in Tables 5 and 6, where $W_D = W_{\text{Pr}} = 0.5$.

Table 5: Agent satisfaction matrix $\tilde{A} = \left[\tilde{a}_{e_{ij}}\right]_{4\times5}$

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	0.177	0.321	0.311	0.632	0.435
U_2	0.637	0.256	0.217	0.245	0.551
U_3	0.387	0.734	0.753	0.516	0.811
U_4	0.804	0.310	0.669	0.115	0.147

Table 6: Agent satisfaction matrix $\tilde{B} = \left[\tilde{b}_{f_{ij}}\right]_{4\times5}$

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	0.908	0.367	0.456	0.082	0.324
U_2	0.481	0.360	0.240	0.780	0.269
U_3	0.000	0.390	0.484	0.790	0.326
U_4	0.246	0.142	0.359	0.878	0.280

Step 3. Then, Model (M-1) considering the stable matching constraints is constructed as:

(M-1)
$$
\begin{cases}
\text{Max } Z_A = \sum_{i=1}^{4} \sum_{j=1}^{5} \tilde{a}_{e_{ij}} x_{ij} \\
\text{Max } Z_B = \sum_{i=1}^{4} \sum_{j=1}^{5} \tilde{b}_{f_{ij}} x_{ij} \\
\text{s.t. } \sum_{j=1}^{5} x_{ij} \leq 1. \ i = 1, 2, 3, 4 \\
\sum_{i=1}^{4} x_{ij} \leq 1. \ j = 1, 2, 3, 4, 5 \\
x_{ij} + \sum_{k:\tilde{a}_{e_{ik}} > \tilde{a}_{e_{ij}}} x_{ik} + \sum_{l:\tilde{b}_{f_{lj}} > \tilde{b}_{f_{ij}}} x_{lj} \geq 1 \\
x_{ij} = \{0, 1\}, i = 1, 2, 3, 4, \ j = 1, 2, 3, 4, 5.
\end{cases}
$$

Step 4. Based on the satisfaction matrices $\tilde{A} = \left[\tilde{a}_{e_{ij}}\right]_{4\times5}$ and $\tilde{B} = \left[\tilde{b}_{f_{ij}}\right]_{4\times5}$, the matching intention coefficient matrix $\phi = [w_{ij}]_{4\times5}$ is calculated through Eq. (16), as shown in Table 7; then, we transform satisfactions $\tilde{a}_{e_{ij}}$ and $\tilde{b}_{f_{ij}}$ into the comprehensive satisfactions g_{ij} and h_{ij} through Eqs. (21) and (22), as shown in Tables 8 and 9.

Table 7: Matching intention coefficient matrix $\phi = [w_{ij}]_{4\times5}$

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	0.048	0.050	0.051	0.035	0.052
U_2	0.074	0.043	0.034	0.051	0.049
U_3	0.021	0.064	0.075	0.079	0.059
U_4	0.052	0.030	0.060	0.043	0.029

Table 8: Comprehensive satisfaction g_{ij}

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	0.0085	0.0161	0.0159	0.0221	0.0226
U_2	0.0471	0.0110	0.0074	0.0125	0.0270
U_3	0.0081	0.0470	0.0565	0.0408	0.0478
U_4	0.0418	0.0093	0.0401	0.0049	0.0043

Table 9: Comprehensive satisfaction h_{ij}

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	0.0445	0.0184	0.0233	0.0029	0.0168
U_2	0.0356	0.0155	0.0082	0.0398	0.0132
U_3	0.0000	0.0250	0.0363	0.0624	0.0192
U_4	0.0128	0.0043	0.0215	0.0378	0.0081

Step 5. Transform Model (M-1) into Model (M-3) based on the comprehensive satisfactions g_{ij} and h_{ij}.

Step 6. Determine a stable alternative of TSM by solving Model (M-3), as shown in Table 10.

Table 10: The best TSM scheme

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	0	0	0	0	1
U_2	1	0	0	0	0
U_3	0	0	1	0	0
U_4	0	1	0	0	0

Therefore, the best TSM is $\theta = \theta_1 \cup \theta_2$, where $\theta_1 = \{(U_1, V_5), (U_2, V_1), (U_3, V_3), (U_4, V_2)\}$, $\theta_2 = \{(V_4, V_4)\}$, as shown in Fig. 2. According to Fig. 2, we can see that car U_1 and terminal distribution center V_5 are matched, car U_2 and terminal distribution center V_1 are matched, car U_3 and terminal distribution center V_3 are matched, car U_4 and terminal distribution center V_2 are matched, and terminal distribution center V_4 is not matched.

Furthermore, we reconstruct Model (M-4) without the stable constraint:

$$(M\text{-}4) \begin{cases} \text{Max } Z = \sum_{i=1}^{4} \sum_{j=1}^{5} \left(\tilde{a}_{e_{ij}} + \tilde{b}_{f_{ij}} \right) w_{ij} x_{ij} \\ \text{s.t. } \sum_{j=1}^{5} x_{ij} \leq 1. \ i = 1, 2, \ldots, 4 \\ \quad \sum_{i=1}^{4} x_{ij} \leq 1. \ j = 1, 2, \ldots, 5 \\ x_{ij} = \{0, 1\}. \ i = 1, 2, \ldots, 4; \ j = 1, 2, \ldots, 5 \end{cases}$$
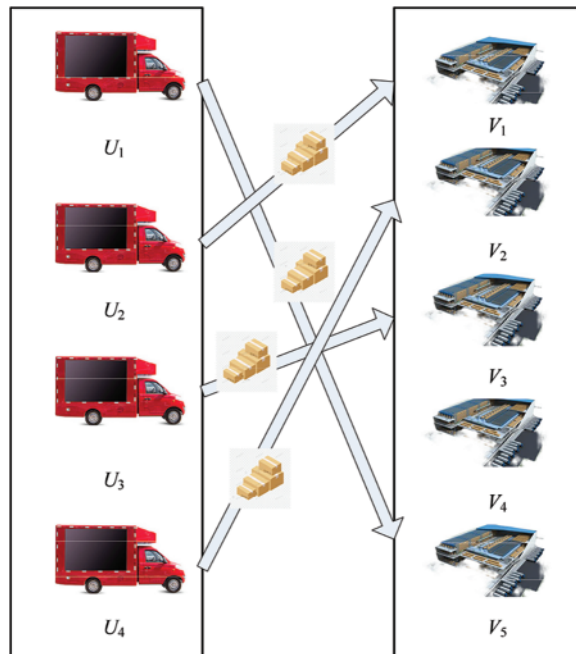
Figure 2: The best TSM scheme between cars and terminal distribution centers

Then, the matching scheme can be obtained by solving TSM Model (M-4), as shown in Table 11.

Table 11: The best TSM scheme without stability constraint

Van	Distribution center				
	V_1	V_2	V_3	V_4	V_5
U_1	0	0	0	0	1
U_2	1	0	0	0	0
U_3	0	0	0	1	0
U_4	0	0	1	0	0

Hence, the best TSM scheme is determined from Table 11, i.e., $\theta_1 = \{(U_1, V_5), (U_2, V_1), (U_3, V_4), (U_4, V_3)\}$, $\theta_2 = \{(V_2, V_2)\}$. Furthermore, two TSM schemes are compared, as shown in Table 12.

Table 12: The comparison analysis for two TSM schemes

TSM method considering stable constraints (Proposed Method I)	$\theta_1 = \{(U_1, V_5), (U_2, V_1), (U_3, V_4), (U_4, V_3)\}$, $\theta_2 = \{(V_4, V_4)\}$
TSM method without stable constraints (Method II)	$\theta_1 = \{(U_1, V_5), (U_2, V_1), (U_3, V_4), (U_4, V_3)\}$, $\theta_2 = \{(V_2, V_2)\}$

As seen from Table 12, in the best TSM schemes obtained through method I and method II, only (U_1, V_5) and (U_2, V_1) are the same, and other matching pairs are different. Furthermore, the model

proposed in this paper better ensures the stability of the matching result and improves the satisfaction of two-sided agents as much as possible due to the stable matching constraints.

5.3 Comparative Analysis

First, there are several kinds of methods for dealing with fuzzy preference information, such as the TOPSIS method based on distance measures or biprojection technology. This paper adopted a brand-new method to deal with fuzzy preference information, which contains distance measures and biprojection technology. To further state the effectiveness of the method proposed in this paper, we compare the proposed method for handling hesitant fuzzy information with two methods given in references [16] and [23]. Reference [23] only adopted the biprojection technology for transforming fuzzy preference information into exact values. Meanwhile, reference [16] adopted only the distance measure for transforming fuzzy preference information into exact values. The TSM results obtained by different methods are displayed in Table 13. From Table 13, it can be concluded that the TSM results obtained by the mentioned methods are different. Meanwhile, the methods proposed in references [23] and [16] may cause deviations in hesitant preference information in the process of conversion. Furthermore, the method proposed in this paper can effectively synthesize the advantages of the two methods proposed in the references [23] and [16]. Therefore, the proposed method can guarantee the integrity of decision-making evaluation information.

Table 13: TSM results of different methods

Method proposed in this paper	$\theta_1 = \{(U_1, V_5), (U_2, V_1), (U_3, V_3), (U_4, V_2)\}$, $\theta_2 = \{(V_4, V_4)\}$
Method proposed in reference [23]	$\theta_1 = \{(U_1, V_4), (U_2, V_1), (U_3, V_5), (U_4, V_3)\}$, $\theta_2 = \{(V_2, V_2)\}$
Method proposed in reference [16]	$\theta_1 = \{(U_1, V_4), (U_2, V_1), (U_3, V_3), (U_4, V_2)\}$, $\theta_2 = \{(V_5, V_5)\}$

Second, many of the existing TSMDM studies ignored the matching intention of two-sided agents. In this paper, the proposed TSMDM method considered the matching intention of matching agents, which can be applied to minimize the diversity of the TSM scheme.

Third, stability plays a significant role in the TSMDM process because the unstable matching pair may hinder the formation of matching results even if the satisfaction of two-sided agents is high. From the aforementioned research, we find that stability has not been fully considered in TSMDM problems in hesitant fuzzy preference environments. Hence, the one-to-one stable matching constraints are integrated into the TSMDM model developed in this paper, which can not only maximize the satisfaction of two-sided agents but also ensure a stable TSM scheme.

6 Conclusion

In this paper, a TSM decision-making model considering the matching intention and stability of two-sided agents under hesitant fuzzy preference is proposed. In this method, the normalized HFE is transformed into the agent satisfaction after normalizing the HFE preference information. A multiobjective programming model considering the agent satisfactions and the stable matching constraint is constructed. Based on the agent satisfaction matrices, the matching intention coefficient matrix is calculated, and then the comprehensive satisfaction matrices are calculated. Furthermore, the

multiobjective programming model based on the satisfactions is transformed into the multiobjective programming model based on the comprehensive satisfactions. The best TSM scheme can be obtained by solving the developed model.

The main innovation of this paper can be denoted as follows: First, the TSM scheme obtained by the method proposed in this paper can reflect the matching intention of two-sided agents; second, the proposed method ensures the stability of the matching scheme. The TSM method proposed in this paper provides a new perspective for studying the TSM problem under the condition of HFS.

Further research work can be conducted as follows:

(1) In the proposed TSMDM method, we present a novel method including distance measures and biprojection technology for dealing with hesitant fuzzy preference information. However, due to the complex decision-making environment, the proposed distance measures may not fully reveal the comments of the decision-makers who have various preferences. Therefore, it could be practicable to develop a brand-new distance measure for HFS and the corresponding TOPSIS method.

(2) For some TSMDM problems, two-sided agents may provide multiple hesitant fuzzy preference information, such as probability hesitant fuzzy sets, interval hesitant fuzzy sets, and probability interval hesitant fuzzy sets. These would be interesting topics for developing a TSMDM model with multiple hesitant fuzzy preference information.

(3) It is meaningful to conduct research on the TSMDM problem considering bipolar hesitant fuzzy sets and T-spherical fuzzy sets [27] which can be used to solve decision-making problems in dynamic scenarios.

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