



**ARTICLE**

# Comparative Analysis of Pythagorean MCDM Methods for the Risk Assessment of Childhood Cancer

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## ABSTRACT

According to the World Health Organization (WHO), cancer is the leading cause of death for children in low and middle-income countries. Around 400,000 kids get diagnosed with this illness each year, and their survival rate depends on the country in which they live. In this article, we present a Pythagorean fuzzy model that may help doctors identify the most likely type of cancer in children at an early stage by taking into account the symptoms of different types of cancer. The Pythagorean fuzzy decision-making techniques that we utilize are Pythagorean Fuzzy TOPSIS, Pythagorean Fuzzy Entropy (PF-Entropy), and Pythagorean Fuzzy Power Weighted Geometric (PFPWG). Our model is fed with nineteen symptoms and it diagnoses the risk of eight types of cancers in children. We develop an algorithm for each method and calculate its complexity. Additionally, we consider an example to make a clear understanding of our model. We also compare the final results of various tests that prove the authenticity of this study.

## KEYWORDS

Risk assessment; Pythagorean fuzzy sets; TOPSIS method; entropy; power weighted geometric operators

## 1 Introduction

Childhood cancer is the leading cause of death in children, especially in low and middle-income countries. Their likelihood of survival heavily rests on the country where they live. The chance of curing childhood cancer in high-income countries is above eighty percent, whereas in low and in middle-income countries, it is a mere forty-five percent. This difference in curing rate owes to many factors, such as late diagnosing and cancer diagnosed at its late stages due to unavailability of resources, cost of treatment (the treatment cost is rather high in later stages), or incorrect diagnostics and inappropriate treatment. The survival rate can be increased if low and in middle-income countries improve the access to necessary medicines and technologies. Generally speaking, the most productive way to reduce the effects of childhood cancer is by effective and evidence-based therapy with appropriate nurturing care.

The chances of curing childhood cancer, and the cost of treatment with lesser suffering, can be improved if it is identified early, and appropriate treatment is provided immediately. A correct



diagnosis is required to treat childhood cancer efficiently with the right treatment, which may include surgeries, radiotherapy, and chemotherapy. For early diagnosing, we should consider the following three aspects:

- 1) Parents should know about childhood cancer so that they can perceive its symptoms and consult a medical expert.
- 2) The medical expert should be skilled enough and investigate the case promptly in order to cater for the right treatment.
- 3) The patient has accessibility to the right treatment at right time.

When someone is diagnosed with cancer, the chances of recovery and survival increase if it is detected in an early stage, even with the least amount of financial and physical suffering. The low- and middle-income countries should run education campaigns for parents with the help of competent doctors so that in the presence of symptoms in any child, their parents can respond without delay. This task requires the joint effort of civil society and non-governmental organizations. In 2018, the World Health Organization launched a global initiative on cancer in children. As part of this initiative, they provided governments with professional guidance and support to maintain high-quality childhood cancer programs. They aim to increase childhood cancer survival rates, and by 2030 that rate should be at least sixty percent.

Medical information is very sensitive and contains many uncertainties. Every expert may have their personal opinion on a health record, and it may become rather difficult to make an accurate diagnostic based on these reports. In such uncertain situations, fuzzy logic can play an important role in making decisions among different alternative diagnostics. Many extensions of fuzzy sets theory have been proposed [1,2]. In fact research has provided many medical applications that have taken advantage of different fuzzy approaches [3,4], and of models of vague knowledge born from alternative narratives [5]. Feng et al. [6,7] generalized intuitionistic fuzzy soft sets and related multi attribute decision making methods. By inspiration of these applied studies, we have selected a powerful and flexible model for the representation of uncertain knowledge. It has been popularized by Yager et al. [8,9], who coined the term Pythagorean fuzzy sets (PFSs). PFSs improve the performance of intuitionistic fuzzy sets, which for each alternative, provide a fuzzy assessment of both membership ( $\mu$ ) and nonmembership ( $\nu$ ) in such a way that  $\mu + \nu \leq 1$ . In PFSs however, assessments that meet the relaxed condition  $\mu^2 + \nu^2 \leq 1$  are acceptable. The PFS model has become one of the most influential notions in fuzzy modeling, since it allows us to accept more uncertain appraisals than the intuitionistic fuzzy or fuzzy sets, and consequently, its applications are potentially wider. The next summary of recent results testifies to this claim.

Yucesan et al. [10] presented the ideas of Pythagorean fuzzy analytic hierarchy method and Pythagorean fuzzy method for order decision by comparison to the ideal solution, in order to present an exact decision-making technique for estimating hospital service quality. Guleria et al. [11] proposed a new  $(R, S)$ -norm discriminant measure of Pythagorean fuzzy sets and proved some interesting features. Their monotonicity with respect to the parameters  $R$  &  $S$  were studied too. This information measure is utilized in some problems related to medical diagnosis or pattern recognition. Extensions and hybrid models based on PFSs have been put forward too. Rahman [12] extended the spirit of some popular aggregation operators to the more general framework of interval-valued Pythagorean fuzzy numbers, thus producing the so-called GIVPFWA, GIVPFOWA, and GIVPFHA operators. They discussed their properties, and argued that their generalized operators are more reliable and accurate than the existing aggregation operators. Zulqarnain et al. [13] proposed the averaging and geometric operators of Pythagorean fuzzy hypersoft sets. They also introduced a novel TOPSIS

method in this new environment. They applied this methodology to a case-study of selection of multipurpose masks for protection against COVID-19. Yue et al. [14] proposed novel score function of hesitant fuzzy numbers and prove the validity of this function using an example. Yue [15] proposed a novel bilateral matching (BM) decision-making method for knowledge innovation management considering the matching willingness of bilateral enterprises. Ejegwa [16] improved composite relation for Pythagorean fuzzy sets and applied it to medical diagnosis. Many other decision-making methods have been suggested in the literature [17–25].

The following targets motivate the research contained in this article:

- 1) Early diagnosis of cancer in children can reduce overall mortality and expense of treatment, which ultimately reduces the patient's suffering.
- 2) Effective handling of vague and uncertain data in a medical context is required.
- 3) Get opinions of available medical experts and decide the final treatment.
- 4) Contribution towards WHO's goal of increasing survival from childhood cancer by least sixty percent before 2030.

Concerning these issues, our contribution to this study is described below:

- 1) We develop a novel decision-making system to determine childhood cancer risk at its early stages, thus increasing the survival rate.
- 2) We use the Pythagorean fuzzy sets (PFS) for decision-making because it is very close to human thinking. It is characteristic of simultaneously focusing on the degree of truth, the degree of non-membership, and the degree of indeterminacy of each alternative to make it more powerful.
- 3) We design algorithms to demonstrate the entire performance of the model. In addition, we determine their respective time complexities.

The rest of this paper is structured as follows. [Section 2](#) discusses preliminary work. [Section 3](#) describes the main contributions of the paper. Then [Section 4](#) performs a comparative analysis. [Section 5](#) concludes the proposed work and lays out some future directions for research.

## 2 Preliminaries

This section summarizes some of the introductory concepts that need to be followed to completely benefit from this study. First we overview technical concepts that will help us formulate our theoretical model. Then we summarize some facts about its prospective application (namely, identification of childhood cancer).

### 2.1 Pythagorean Fuzzy Set [8]

Let  $Z$  be a universal set. Then, a Pythagorean fuzzy set  $S$  over  $Z$  is a set of ordered triples indexed by  $Z$ , which adopts the following form:  $S = \{ \langle z, \mu_S(z), \nu_S(z) \rangle \mid z \in Z \}$ , where the functions  $\mu_S(z): Z \rightarrow [0, 1]$  and  $\nu_S(z): Z \rightarrow [0, 1]$  respectively define the degree of membership and the degree of non-membership of  $z \in Z$  to  $S$ , and the inequalities  $0 \leq (\mu_S(z))^2 + (\nu_S(z))^2 \leq 1$  hold for each  $z \in Z$ . The figure  $\pi_S(z) = \sqrt{1 - [(\mu_S(z))^2 + (\nu_S(z))^2]}$  defines the degree of indeterminacy of  $z \in Z$  to  $S$ . Observe  $\pi_S(z) \in [0, 1]$ , and  $\pi_S(z) = 0$  whenever  $(\mu_S(z))^2 + (\nu_S(z))^2 = 1$ . We represent the set of all PFSs over  $Z$  by  $\text{PFS}(Z)$ .

## 2.2 Pythagorean Fuzzy Relation [16]

Let  $U$  and  $V$  be two nonempty sets. A Pythagorean fuzzy relation (PFR),  $L$ , from  $U$  to  $V$  is a PFS over  $U \times V$ . It is characterized by a membership function,  $\mu_R$ , and a nonmembership function,  $\nu_L$  that meet the corresponding bounds. A PF relation or PFR from  $U$  to  $V$  is denoted by  $L(U \rightarrow V)$ .

## 2.3 Pythagorean Fuzzy-Technique for Order of Preference by Similarity to Ideal Solution (PF-TOPSIS) Method [20]

The PF-TOPSIS method uses linguistic terms and Pythagorean fuzzy numbers (PFNs) to represent the relative importance of experts and criteria. These linguistic terms and PFNs are predefined and used to rate any expert or criteria. The following equation is used to calculate the weight in crisp form for any Pythagorean fuzzy evaluation. Assume that  $P_n = [\mu_n, \nu_n, \pi_n]$  is a Pythagorean Fuzzy Number (PFN), then its weight can be calculated by using the following formula:

$$\sigma_n = \frac{\mu_n + \pi_n(\mu_n/(\mu_n + \nu_n))}{\sum_{l=1}^k (\mu_l + \pi_l(\mu_l/(\mu_l + \nu_l)))}. \quad (1)$$

Notice that the sum of all weights should be equal to 1.

Suppose that  $Z^n = (z_{ij}^n)_{l \times m}$  denotes the Pythagorean Fuzzy Decision Matrix (PFDM) of the  $n$ th expert having weight  $\sigma_n$ . The following formula is used to aggregate all PFDMs with the assistance of the Pythagorean Fuzzy Aggregated Averaging (PFWA) operator: for each  $i = 1, 2, 3, \dots, l, j = 1, 2, 3, \dots, m$ ,

$$z_{ij} = \left( \sqrt{1 - \prod_{n=1}^k (1 - (\mu_{ij}^n)^2)^{\sigma_n}}, \prod_{n=1}^k (\nu_{ij}^n)^{\sigma_n}, \sqrt{\prod_{n=1}^k (1 - (\mu_{ij}^n)^2)^{\sigma_n} - \left( \prod_{n=1}^k (\nu_{ij}^n)^{\sigma_n} \right)^2} \right). \quad (2)$$

Let  $P_j^n = [\mu_j^n, \nu_j^n, \pi_j^n]$  be the Pythagorean fuzzy number assigned to the criteria  $R_j$ , then the weighted aggregated PFDM against each criteria  $R_j$  can be calculated as

$$z_{ij} = (\mu_{S_i}(R_j) \cdot \mu_P(R_j), \sqrt{\nu_{S_i}^2(R_j) + \nu_P^2(R_j) - \nu_{S_i}^2(R_j) \cdot \nu_P^2(R_j)}), \quad (3)$$

$$\text{and } \pi_{S_i P}(R_j) = \sqrt{1 - \mu_{S_i}(R_j) \cdot \mu_P(R_j) - \nu_{S_i}^2(R_j) - \nu_P^2(R_j) + \nu_{S_i}^2(R_j) \cdot \nu_P^2(R_j)}.$$

Let  $B_1$  and  $B_2$  be the sets of benefit-type and cost-type criteria, respectively.

The Pythagorean Fuzzy Positive Ideal Solution (PFPIS)  $S^+$  and Pythagorean Fuzzy Negative Ideal Solution (PFNIS)  $S^-$  can be obtained as:  $S^+ = \{\langle R_j, \mu_{S^+P}, \nu_{S^+P} \rangle | R_j \in C, j = 1, 2, \dots, m\}$ ,

$S^- = \{\langle R_j, \mu_{S^-P}, \nu_{S^-P} \rangle | R_j \in C, j = 1, 2, \dots, m\}$ , where

$$\mu_{S^+P}(R_j) = \begin{cases} \max_{1 \leq i \leq l} \mu_{S_i P}(R_j), & \text{if } R_j \in B_1, \\ \min_{1 \leq i \leq l} \mu_{S_i P}(R_j), & \text{if } R_j \in B_2, \end{cases} \quad (4)$$

$$\lambda_{S^+P}(R_j) = \begin{cases} \min_{1 \leq i \leq l} v_{S_iP}(R_j), & \text{if } R_j \in B_1, \\ \max_{1 \leq i \leq l} v_{S_iP}(R_j), & \text{if } R_j \in B_2, \end{cases} \quad (5)$$

$$\mu_{S^-P}(R_j) = \begin{cases} \min_{1 \leq i \leq l} \mu_{S_iP}(R_j), & \text{if } R_j \in B_1, \\ \max_{1 \leq i \leq l} \mu_{S_iP}(R_j), & \text{if } R_j \in B_2, \end{cases} \quad (6)$$

$$\lambda_{S^-P}(R_j) = \begin{cases} \max_{1 \leq i \leq l} v_{S_iP}(R_j), & \text{if } R_j \in B_1, \\ \min_{1 \leq i \leq l} v_{S_iP}(R_j), & \text{if } R_j \in B_2. \end{cases} \quad (7)$$

The following formula is used to calculate the distance of each alternative from PFPIS and PFNIS:

$$E(S_i, S^+) = \sqrt{\frac{1}{2m} \sum_{j=1}^m [(\mu_{S_iP}^2(R_j) - \mu_{S^+P}^2(R_j))^2 + (v_{S_iP}^2(R_j) - v_{S^+P}^2(R_j))^2 + (\pi_{S_iP}^2(R_j) - \pi_{S^+P}^2(R_j))^2]}, \quad (8)$$

$$E(S_i, S^-) = \sqrt{\frac{1}{2m} \sum_{j=1}^m [(\mu_{A_iP}^2(R_j) - \mu_{S^-P}^2(R_j))^2 + (v_{S_iP}^2(R_j) - v_{S^-P}^2(R_j))^2 + (\pi_{S_iP}^2(R_j) - \pi_{S^-P}^2(R_j))^2]}. \quad (9)$$

The relative closeness value of each choice  $S_i$  is calculated as follows:

$$Y_{i^+} = \frac{E(S_i, S^-)}{E(S_i, S^+) + E(S_i, S^-)}, i = 1, 2, \dots, l. \quad (10)$$

The maximum relative closeness value is the best choice among all possible choices.

#### 2.4 Pythagorean Fuzzy-Entropy (PF-Entropy) Method [21]

According to [21], the following equation computes the entropy  $T(P)$  of any criteria represented by a Pythagorean Fuzzy Number (PFN)  $P$ :

$$T(P) = \frac{1}{n} \sum_{i=1}^n [T^*(P_i) + \pi_p(z_i) - \pi_p(z_i)T^*(P_i)], \quad (11)$$

where,  $T^*(p_i) = 1 - |\mu_p(z_i) - v_p(z_i)|$ .

The score function  $K(P)$  of such  $P$  can be defined as follows:

$$K(P) = (\mu_p)^2 - (v_p)^2. \quad (12)$$

The weighted entropy of each criteria is calculated using the next equation:

$$w_j = \frac{1 - T_j}{n - \sum_{j=1}^n T_j}. \quad (13)$$

#### 2.5 Pythagorean Fuzzy Power Weighted Average (PFPWA) [21,22]

The support of two PFNs is calculated using the next formula:

$$Support(P_{ij}, P_{ik}) = 1 - d(P_{ij}, P_{ik}), \quad j, k = 1, 2, \dots, n. \quad (14)$$

The distance between two PFNs can be calculated using the following normalized Hamming distance:

$$d(P_{ij}, P_{ik}) = \frac{(|(\mu_{ij})^2 - (\mu_{ik})^2| + |(v_{ij})^2 - (v_{ik})^2|)}{2}, \quad j, k = 1, 2, \dots, n. \quad (15)$$

The formula for the weighted support is as follows:

$$M(P_{ij}) = \sum_{k=1}^n \omega_k \text{Support}(P_{ij}, P_{ik}), \quad (16)$$

and we compute the weights  $\gamma_{ij}$  associated with the PFN  $P_{ij}$  as

$$\gamma_{ij} = \frac{\omega_j(1 + M(P_{ij}))}{\sum_{j=1}^n \omega_j + (1 + M(P_{ij}))}, \quad (17)$$

where  $i = 1, 2, \dots, m, j = 1, 2, \dots, n, \gamma_{ij} \geq 0$ , and  $\sum_{j=1}^n \gamma_{ij} = 1$ .

The Pythagorean Fuzzy Power Weighted Geometric (PFPWG) operator is as follows:

$$PFPWG = \left( \prod_{j=1}^n (\mu_{ij})^{\gamma_{ij}}, \sqrt{1 - \prod_{j=1}^n (1 - (v_{ij})^2)^{\gamma_{ij}}} \right). \quad (18)$$

Let  $P = (\mu, v)$  be a PFN. Then a score function  $S$  of  $P$ , a PFN, can be defined by the expression:

$$\text{Score}(P) = \frac{1}{2}(1 + \mu^2 + v^2) \in [0, 1]. \quad (19)$$

## 2.6 Major Factors of Childhood Cancer [26,27]

Many studies have tried to identify the causes of childhood cancer. Some factors are related to the environment, such as radiation exposure and chemical exposure. Some are lifestyle-related, such as drugs, alcohol, cell phone use, and smoking. Some children inherit DNA changes from a parent that increase their risk of a certain type of cancer. Here we list possible risk factors for childhood cancer with a small description of each factor.

*Gender (S1)*: Gender can be male or female.

*Age (S2)*: The age of a child is considered between 0 and 19 years.

*Height (S3)*: The height of a child.

*BMI (S4)*: The body mass index (BMI) is a measure of body fat according to height and weight.

*Drugs (S5)*: A medication is a drug used to diagnose, cure, treat, or prevent disease.

*Alcohol (S6)*: It is a substance that contains the recreational drug ethanol, alcohol is made by fermentation of fruits, grains, or any source of sugar.

*Cell Phone Usage (S7)*: The use of cell phones on a daily basis.

*Pagets Disease (S8)*: It is a bone disease that disrupts the body's normal recycling process, in which new bone tissue gradually replaces old bone tissue. Over time, the disease can cause compromised bones to become weak and distorted.

*Genetic Disposition (S9)*: There is an increased chance of acquiring a specific disease based on a person's ancestral genes.

*Smoking (S10)*: The habit of inhaling and exhaling tobacco or drug smoke.

*Blood Disorder (S11)*: These are conditions that affect the blood's ability to function.

*Birth Defects (S12)*: It is a disease that, despite its cause, is present at birth. Birth defects can appear as disabilities that can be physical, mental, or developmental in nature.

*Immunity (S13)*: Immunity is the capability of multi-cellular organisms to resist harmful microorganisms.

*Auto Immune Diseases (S14)*: It is a disease in which your immune system unintentionally attacks your body.

*Certain Syndromes(S15)*: Any syndrome already present in children such as Down syndrome, Li-Fraumeni syndrome, etc.

*Race (S16)*: Identification of a group of people.

*Certain Radiation Exposure (S17)*: Exposed to certain electromagnetic radiation, or living in the vicinity of a source of electromagnetic radiation.

*Certain Chemical Exposure (S18)*: Exposure to certain chemicals or polluted groundwater used for drinking.

*Socioeconomic Status (S19)*: A family's financial status in society.

## 2.7 Types of Childhood Cancers

Children and teenagers tend to get different types of childhood cancers. The most common childhood cancers are discussed below:

*Leukemia (D1)*: It is bone marrow and blood cancer. Twenty-eight percent of childhood cancer cases fall into this category.

*Brain and spinal cord tumors (D2)*: The second most common cancer in children is the brain and spinal cord cancer. In this type of cancer, abnormal growth in tissues of the brain and spinal cord is seen causing headache, nausea, vomiting, blurred vision, and difficulty in walking and holding objects. About 26 children develop this type of cancer every year.

*Neuroblastoma (D3)*: Neuroblastoma begins in the early forms of nerve cells seen in a developing egg or fetus. About 6 percent of cancers in adolescents are neuroblastomas. This type of cancer occurs in newborns and adolescents. It is uncommon in children over 10 years of age. Neuroblastomas mostly occur in and around the adrenal glands. However, neuroblastomas can develop in other areas of the stomach and ribs, neck, and near the spine where there are clusters of nerve cells.

*Wilms Tumor (D4)*: Wilms' tumor begins in one or, rarely, both kidneys. It is usually found in children around 3 to 4 years of age and is rare in more mature children and adults. Wilms' tumor accounts for around 5 percent of childhood cancers. Its symptoms are fever, pain, nausea, or loss of appetite.

*Lymphomas (D5)*: It is a disease that attacks infection-fighting cells in the immune system. These cells are called lymphocytes. These cells are found in the lymph nodes, spleen, thymus gland, bone marrow, and other parts of the body. In this disease, abnormal growth of lymphocytes has been observed. Symptoms include weight loss, fever, sweats, fatigue, and lumps under the skin in the neck, armpits, or groin area.

*Retinoblastoma (D6)*: This type of cancer is related to the eyes. It is a rare type of cancer in which a child could not distinguish the colors of light, also had impaired vision and sensitive eyes. The pupil of the eyes becomes large.

*Rhabdomyosarcoma (D7)*: It is an intrusive and very dangerous cancer that originates from skeletal muscle cells. It is widely believed to be a childhood disease as the vast majority of cases found are under the age of 18. It is about 3 percent of childhood cancers.

*Bone Cancer (D8)*: This type of cancer usually occurs in older children. This type of cancer causes severe bone pain all the time. The bones become weak and can also be broken. In some cases, weight loss is also observed.

### 3 Pythagorean Fuzzy Model of Childhood Cancer

To make the proposed Pythagorean model more understandable, consider the block diagram shown in Fig. 1. The proposed model uses nineteen symptoms as inputs. For each input, a linguistic variable is defined in the Pythagorean fuzzy number. There may be  $n$  experts, but we're only picking three experts here. Their expertise is represented by PFNs. Symptom PFNs and expert weights are input to PF-TOPSIS, PF-entropy, and PFPWG blocks. In these blocks, the algorithm of each approach is executing and generating its final outputs. We compare the results of each method and highlight the signal output. The results of these three methods should be the same.

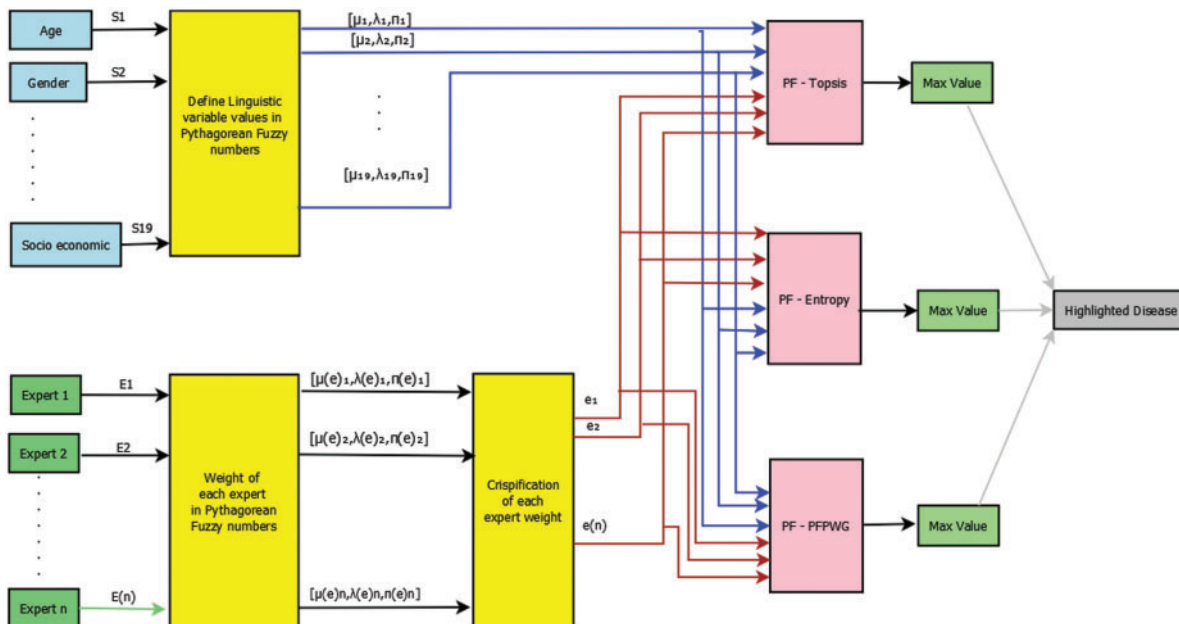


Figure 1: Block diagram of risk assessment of childhood cancer

#### 3.1 Algorithm for Risk Assessment of Childhood Cancer

In this subsection, we write all of the instructions that must be followed to obtain final results for any input. Each algorithm takes certain inputs and produces certain outputs. There are seven sub-algorithms of the PF-TOPSIS algorithm, namely, Algorithm A, Algorithm B, Algorithm C, Algorithm D, Algorithm E, Algorithm F, and Algorithm G. Each sub-algorithm shows each step of the TOPSIS process. We also write the net time complexity of every algorithm.



3.1.1 Algorithm: PF-TOPSIS

*Input:* Two-dimensional arrays containing expert’s weights, and decision matrix of each expert. *Output:* Highlighted type of cancer.

1.  $EW[a] \leftarrow \text{Weight of experts}(EWmem[a][b], EWnmem[a][b], \text{ and } EWpi[a][b])$
2.  $PDMmem[a][b], PDMnmem[a][b], PDMpi[a][b] \leftarrow PDM(PDM_1[a][b], PDM_2[a][b], \dots, PDM_l[a][b])$  and  $EW[a][b]$
3.  $Smem[a][b], Snmem[a][b], Spi[a][b] \leftarrow \text{Symptom's Weight}(S_1mem[a][b], S_1nmem[a][b], S_1pi[a][b], \dots, S_imem[a][b], S_inmem[a][b], S_ipi[a][b])$  and  $EW[a][b]$
4.  $PDMmem[a][b], PDMnmem[a][b], PDMpi[a][b] \leftarrow \text{Weighted Aggregated PDM}(PDMmem[a][b], PDMnmem[a][b], PDMpi[a][b], Smem[a][b], Snmem[a][b], Spi[a][b])$
5.  $PISmem[a][b], PISnmem[a][b], PISpi[a][b] \leftarrow \text{PFIS and PFNIS}(PDMmem[a][b], PDMnmem[a][b], PDMpi[a][b])$
6.  $PD[a], ND[a] \leftarrow \text{Disance PIS NIS}(PISmem[a][b], PISnmem[a][b], PISpi[a][b], NISmem[a][b], NISnmem[a][b], NISpi[a][b], PDMmem[a][b], PDMnmem[a][b], PDMpi[a][b])$
7.  $RClose[a] \leftarrow \text{Relative Closeness}(PD[a], ND[a])$
8. Final disease=Maximun(RClose)

*Algorithm-A: Weights of experts*

*Input:* Two dimensional arrays  $EWmem[a][b]$ ,  $EWnmem[a][b]$ , and  $EWpi[a][b]$  containing Experts weights, and membership, non-membership, and indeterminate parts of PFNs.

*Output:* Two dimensional arrays  $EW[a]$  containing Experts weight in crisp form. *Time complexity*

```

for a ← 1 to l l
  for b ← 1 to o (l - 1)o
    Sum ← Sum +  $\left( EWmem[a][b] + EWpi[a][b] \times \frac{EWmem[a][b]}{EWmem[a][b] + EWnmem[a][b]} \right)$  (l - 1)
  (o - 1)
  end for
end for
for a ← 1 to l l
   $EW[a] \leftarrow \left( \frac{\left( EWmem[a] + EWpi[a] \frac{EWmem[a]}{EWmem[a] + EWnmem[a]} \right)}{Sum} \right)$  l - 1
end for
return  $EW[a][b]$ ; Net time complexity=O(lo)

```

*Algorithm-B: Pythagorean decision matrix (PDM)*

*Input:* Pythagorean decision matrices of all experts, and weight of each expert.

*Output:* Aggregated Pythagorean decision matrix. *Time complexity*

```

for a ← 1 to l l
  for b ← 1 to o (l - 1)o
    for c ← 1 to e (l - 1)(o - 1)e

```

```

    PDMmem[i][j] ← PDMcmem[a][b] * (1 - (PDMcmem[a][b])2)EWc
    PDMnmem[a][b] ← PDMcnmem[a][b] * (1 - (PDMcnmem[a][b])2)EWc
    PDMpi[a][b] ← PDMcmem[a][b] - (PDMcnmem[a][b])2 (l - 1)(o - 1)(e - 1)
  end for
  PDMmem[a][b] ← √(1 - PDMmem[a][b]); (l - 1)(o - 1)
  PDMpi[a][b] ← √PDMpi[a][b]; (l - 1)(o - 1)
end for
end for
return PDMmem[ ][ ],PDMnmem[ ][ ],PDMpi[ ][ ] Net time complexity=O(loe)

```

*Algorithm-C: Symptom’s weight*

*Input:* Pythagorean symptom’s weight matrix, and expert’s weights.

*Output:* Aggregated Pythagorean symptoms weight matrix. *Time Complexity*

```

for a ← 1 to l (l)
  for b ← 1 to o (l - 1)o
    for c ← 1 to e (l - 1)(o - 1)e
      Smem[a][b] ← Scmem[a][b] * (1 - (Scmem[a][b])2)EWc (l - 1)(o - 1)(e - 1)
      Snmem[a][b] ← Scnmem[a][b] * (1 - (Scnmem[a][b])2)EWc
      Spi[a][b] ← Scmem[a][b] - (Scnmem[a][b])2
    end for
    Smem[a][b] ← √(1 - Smem[a][b]); (l - 1)(o - 1)
    Spi[a][b] ← √Spi[a][b]; (l - 1)(o - 1)
  end for
end for
return Smem[ ][ ],Snmem[ ][ ],Spi[ ][ ] Net time complexity=O(loe)

```

*Algorithm-D: Weighted aggregated PDM*

*Input:* Pythagorean Symptom’s weight matrix, and Pythagorean decision matrices.

*Output:* Weighted aggregated decision matrix. *Time complexity*

```

for a ← 1 to l (l)
  for b ← 1 to o (l - 1)o
    PDMnmem[a][b] ← (((PDMnmem[a][b])2 + (Snmem[a][b])2 - PDMnmem[a][b])2
    *(Snmem[a][b])2)0.5 (l - 1)(o - 1)

    PDMpi[a][b] ← ((1 - PDMmem[a][b] * Smem[a][b] - (PDMnmem[a][b])2 - (Snmem[a][b])2 +
    (PDMnmem[a][b])2 * (Snmem[a][b])2)0.5 (l - 1)(o - 1)
  end for
end for
return PDMmem[ ][ ],PDMnmem[ ][ ],PDMpi[ ][ ]; Net time complexity=O(lo)

```

*Algorithm-E: PFPIS and PFNIS*

*Input:* Weighted aggregated decision matrix.

*Output:* Positive ideal and negative ideal solution. *Time complexity*

*Bmax, Bmin, Cmin, Cmax = 0*

```

for  $a \leftarrow 1$  to  $l$  do  $l$ 
  for  $b \leftarrow 1$  to  $o$  do  $(l - 1)o$ 

    if  $Symptom = Benefit$  then  $(l - 1)(o - 1)$ 

      if  $Bmax < PDMmem[b][a]$  then
         $Bmax \leftarrow PDMmem[b][a]$ 
      end if
       $PISmem[b][a] \leftarrow Bmax$ 

      if  $Bmin > PDMnmem[b][a]$  then
         $Bmin \leftarrow PDMnmem[b][a]$ 
      end if
       $PISnmem[b][a] \leftarrow Bmin$ 

      if  $Bmin > PDMmem[b][a]$  then
         $Bmin \leftarrow PDMmem[b][a]$ 
      end if
       $NISmem[b][a] \leftarrow Bmin$ 

      if  $Bmax < PDMnmem[b][a]$  then
         $Bmax \leftarrow PDMnmem[b][a]$ 
      end if
       $NISnmem[b][a] \leftarrow Bmax$ 
    end if

    if  $Symptom = Cost$  then  $(l - 1)(o - 1)$ 

      if  $Cmin > PDMmem[b][a]$  then
         $Cmin \leftarrow PDMmem[b][a]$ 
      end if
       $PISmem[b][a] \leftarrow Cmin$ 

      if  $Cmax < PDMnmem[b][a]$  then
         $Cmax \leftarrow PDMnmem[b][a]$ 
      end if
       $PISnmem[b][a] = Cmax$ 

      if  $Cmax < PDMmem[b][a]$  then
         $Cmax \leftarrow PDMmem[b][a]$ 
      end if
       $NISmem[b][a] \leftarrow Cmax$ 

      if  $Cmin > PDMnmem[b][a]$  then
         $Cmin \leftarrow PDMnmem[b][a]$ 
      end if
       $NISnmem[b][a] \leftarrow Cmin$ 
    end if
  end do
end do

```

```

end if
end for
end for
return PISmem[ ][ ],PISnmem[ ][ ],NISmem[ ][ ],NISnmem[ ][ ] Net time complexity=(lo)

```

*Algorithm-F: Distance PIS NIS*

*Input:* Positive ideal and negative ideal solutions. Weighted aggregated decision matrix.

*Output:* Distance of each disease from positive ideal and negative ideal solutions. *Time complexity*  
*PISsum, NISsum = 0*

```

for a ← 1 to l do l
for b ← 1 to o do (l - 1)o
PISsum ←
PISsum + ((PDMmem[b][a])2 - (PISmem[b][a])2) +
((PDMnmem[b][a])2 - (PISnmem[b][a])2) + ((PDMpi[b][a])2 - (PISpi[b][a])2) (l - 1)(o - 1)
end for
PD[a] ← √( $\frac{PISsum}{2 * b}$ ) (l - 1)
end for
for a ← 1 to l do l
for b ← 1 to o do (l - 1)o
NISsum ←
NISsum + ((PDMmem[b][a])2 - (NISmem[b][a])2) +
((PDMnmem[b][a])2 - (NISnmem[b][a])2) + ((PDMpi[b][a])2 - (NISpi[b][a])2) (l - 1)(o - 1)
end for
ND[a] ← √( $\frac{NISsum}{2 * b}$ ) (l - 1)
end for
return PD[ ], ND[ ] Net time complexity=O(lo)

```

*Algorithm-G: Relative Closeness*

*Input:* Distance of each disease from positive and negative ideal solutions.

*Output:* Relative closeness. *Time complexity*  
*sum = 0*

```

for a ← 1 to l do l
RClose[a] ←  $\frac{NIS[a]}{PIS[a] + NIS[a]}$  (l - 1)
end for
Net time complexity=O(l)

```

After aggregating all complexities, we get the final time complexity of the PF-TOPSIS algorithm, which is  $O(lo)$ . If  $l \approx o \approx e \approx n$  then we can say that net time complexity is  $O(n^3)$ .

*3.1.2 Algorithm: Entropy*

Algorithm PF-Entropy shows the set of instructions that need to follow to find the final results of each childhood cancer.

*Input:* Aggregated decision matrix and symptom's weight matrix.

*Output:* Highlighted disease. *Time complexity*  
*sum, absolute, Esum*

```

for  $a \leftarrow 1$  to  $l$  do l
  for  $b \leftarrow 1$  to  $o$  do (l-1)o
     $absolute \leftarrow 1 - |PDMmem[a][b] - PDMnmem[a][b]|$  (l-1)(o-1)
     $sum \leftarrow sum + (absolute + PDMpi[a][b] * absolute * PDMpi[a][b])$  (l-1)(o-1)
  end for
   $Entropy[a] \leftarrow \frac{sum}{m}$  (l-1)
   $Esum \leftarrow Esum + Entropy[a]$  (l-1)
end for
for  $a \leftarrow 1$  to  $l$  do l
   $W[a] \leftarrow \frac{1-E[a]}{8-Esum}$  (l-1)
end for
Call algorithm PIS NIS (Algrithm-E) O(lo)
Call distance algorithm (Algrithm-F) O(lo)
Call relative closeness (Algrithm-G) O(l)
Find max value O(l)
Net time complexity=O(lo)

```

### 3.1.3 Algorithm: PFPWG

Algorithm-PFPWG shows the set of instructions of PFPWG decision-making techniques.

*Input:* Aggregated decision matrix and symptom’s weight matrix.

*Output:* Highlighted disease.

*Time complexity*

Call expert weight algorithm to calculate crisp weight of each symptom  $S1[], \dots, S[]$ .  
 $sum, absolute, Esum$

$O(lo)$

```

for  $a \leftarrow 1$  to  $l$  do l
  for  $b \leftarrow 1$  to  $o$  do (l-1)o
     $mem[i] \leftarrow |PDMmem[1][b]^2 - PDMmem[b][1]^2|$  (l-1)(o-1)
     $nmem[i] \leftarrow |PDMnmem[1][b]^2 - PDMnmem[b][a]^2|$  (l-1)(o-1)
     $Tp[i] \leftarrow S[a] * (1 - (\frac{mem[a]+nmem[a]}{20}))$  (l-1)(o-1)
     $sum = sum + Tp[a]$  (l-1)(o-1)
  end for
end for
for  $a \leftarrow 1$  to  $l$  do l
  for  $b \leftarrow 1$  to  $o$  do (l-1)o
     $Dmem \leftarrow Dmem * (PDMmem[b][a])^{Tp[a]}$  (l-1)(o-1)
     $Dnmem \leftarrow Dnmem * (1 - (PDMnmem[b][a])^2)^{Tp[a]}$  (l-1)(o-1)
  end for
   $Dnmem = \sqrt{1 - Dnmem}$  (l-1)
   $D[a] \leftarrow \frac{1}{2}(1 + Dmem^2 - Dnmem^2)$  (l-1)
end for
Find max value (l-1)
Net time complexity=O(loe)

```

### 3.2 Example of Risk Assessment of Childhood Cancer

To understand the working of the above algorithm, consider the following example and apply the PF-TOPSIS method to it. The PFNs against each linguistic variable are shown in Table 1. Table 1 shows the linguistic variables for all inputs.

**Table 1:** Linguistic variables

<i>Linguistic variables</i>	<i>PFNs</i>
Very low	[0,1,0]
Low	[0.2,0.9,0.39]
Below medium	[0.4,0.6,0.69]
Medium	[0.65,0.50,0.57]
Above medium	[0.8,0.45,0.4]
High	[0.9,0.2,0.39]
Very high	[1,0,0]

#### 3.2.1 Pythagorean Fuzzy Topsis

We shall first show how a decision can be made with the application of the steps described in Section 3.1.1.

*Step 1:* We are taking opinions from three medical experts, E1, E2, and E3, and the credibility of each expert is high, above medium, and medium, depending upon their experience, qualification, and research. The rating of each medical expert is calculated using Eq. (1) and Table 1. The ratings of E1, E2, and E3 are 0.375, 0.325, and 0.3, respectively [20].

*Step 2:* The Pythagorean Fuzzy Decision Matrix shows ratings of eight diseases by the experts in relation to nineteen symptoms. This step is completed using Tables 1 and 2, and Eq. (2). The result of this step is shown in Tables 3 and 4.

**Table 2:** Ratings of eight diseases

<i>Symptom</i>	<i>Disease</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>	<i>Symptom</i>	<i>Disease</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>
S1	D1	VL	LO	LO	S11	D1	VH	HI	HI
	D2	LO	LO	VL		D2	LO	LO	VL
	D3	MED	MED	BM		D3	VL	LO	LO
	D4	MED	MED	BM		D4	LO	LO	VL
	D5	MED	BM	BM		D5	VL	LO	LO
	D6	LO	LO	VL		D6	LO	VL	VL
	D7	AM	MED	MED		D7	LO	LO	VL
	D8	VL	LO	LO		D8	LO	LO	VL
	D1	VH	VH	HI	D1	VH	HI	HI	
	D2	LO	LO	VL	D2	LO	LO	VL	
	D3	MED	MED	BM	D3	MED	BM	BM	
	D4	MED	MED	BM	D4	LO	LO	VL	

(Continued)

**Table 2 (continued)**

<i>Symptom</i>	<i>Disease</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>	<i>Symptom</i>	<i>Disease</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>
S2	D5	MED	MED	BM	S12	D5	VL	LO	LO
	D6	MED	BM	BM		D6	LO	VL	VL
	D7	MED	BM	BM		D7	LO	LO	VL
	D8	MED	BM	BM		D8	LO	LO	VL
S3	D1	VH	HI	HI	S13	D1	VH	HI	HI
	D2	LO	LO	VL		D2	LO	LO	VL
	D3	VL	LO	LO		D3	VL	LO	LO
	D4	LO	VL	VL		D4	LO	LO	VL
	D5	LO	LO	VL		D5	MED	BM	BM
	D6	VL	LO	LO		D6	LO	VL	VL
	D7	LO	VL	VL		D7	LO	LO	VL
	D8	MED	BM	BM		D8	LO	LO	VL
S4	D1	VH	VH	HI	S14	D1	VH	HI	HI
	D2	LO	LO	VL		D2	LO	LO	VL
	D3	VL	LO	LO		D3	VL	LO	LO
	D4	LO	VL	VL		D4	LO	LO	VL
	D5	LO	LO	VL		D5	BM	BM	LO
	D6	VL	LO	LO		D6	LO	VL	VL
	D7	LO	VL	VL		D7	LO	LO	VL
	D8	LO	LO	VL		D8	LO	LO	VL
S5	D1	LO	LO	VL	S15	D1	VL	LO	LO
	D2	VL	LO	LO		D2	BM	MED	MED
	D3	LO	LO	VL		D3	MED	BM	MED
	D4	VL	LO	LO		D4	BM	BM	MED
	D5	LO	VL	VL		D5	LO	LO	VL
	D6	LO	LO	VL		D6	LO	LO	VL
	D7	MED	BM	BM		D7	LO	LO	LO
	D8	LO	LO	LO		D8	VL	LO	LO
S6	D1	VH	HI	HI	S16	D1	VL	LO	LO
	D2	VL	LO	LO		D2	LO	LO	VL
	D3	LO	LO	VL		D3	BM	BM	MED
	D4	VL	LO	LO		D4	LO	VL	VL
	D5	LO	VL	VL		D5	LO	LO	VL
	D6	LO	LO	VL		D6	LO	LO	VL
	D7	LO	LO	VL		D6	LO	LO	VL
	D8	LO	LO	LO		D8	MED	BM	BM
	D1	LO	LO	VL		D1	VH	VH	VH
	D2	MED	MED	MED		D2	MED	BM	BM
	D3	LO	LO	VL		D3	LO	VL	VL
	D4	VL	LO	LO		D4	LO	LO	VL

(Continued)

**Table 2 (continued)**

<i>Symptom</i>	<i>Disease</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>	<i>Symptom</i>	<i>Disease</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>
S7	D5	LO	VL	VL	S17	D5	BM	BM	BM
	D6	LO	LO	VL		D6	MED	BM	BM
	D7	LO	LO	VL		D7	BM	BM	MED
	D8	LO	LO	LO		D8	MED	BM	BM
S8	D1	LO	LO	VL	S18	D1	VH	HI	HI
	D2	VL	LO	LO		D2	VL	LO	LO
	D3	LO	LO	VL		D3	LO	VL	VL
	D4	VL	LO	LO		D4	LO	LO	VL
	D5	LO	VL	VL		D5	LO	LO	VL
	D6	LO	LO	VL		D6	LO	LO	LO
	D7	LO	LO	VL		D7	VL	LO	LO
	D8	MED	BM	BM		D8	LO	LO	VL
S9	D1	VH	HI	HI	S19	D1	VL	LO	LO
	D2	MED	MED	BM		D2	LO	VL	VL
	D3	BM	BM	BM		D3	LO	LO	VL
	D4	MED	BM	BM		D4	LO	LO	VL
	D5	MED	BM	MED		D5	MED	BM	MED
	D6	BM	BM	BM		D6	VL	LO	LO
	D7	MED	BM	BM		D7	LO	LO	VL
	D8	MED	BM	MED		D8	VL	LO	LO
S10	D1	VH	HI	VH					
	D2	LO	LO	VL					
	D3	VL	LO	LO					
	D4	LO	LO	VL					
	D5	VL	LO	LO					
	D6	LO	VL	VL					
	D7	LO	LO	VL					
	D8	LO	LO	VL					

**Table 3:** Aggregated Pythagorean fuzzy decision matrix

<i>Symptom</i>	<i>D1</i>	<i>D2</i>	<i>D3</i>	<i>D4</i>
S1	[0.158717,0.93621, 0.3133777]	[0.167842,0.928902, 0.330107]	[0.594807,0.52811, 0.606056]	[0.594807,0.52811, 0.606056]
S2	[0.167842,0.928902, 0.330107]	[0.594807,0.52811, 0.606056]	[0.594807,0.52811, 0.606056]	[0.594807,0.52811, 0.606056]
S3	[1,0,0]	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]	[0.123255,0.96126, 0.2465508]
S4	[1,0,0]	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]	[0.123255,0.96126, 0.2465508]

(Continued)



**Table 3 (continued)**

<i>Symptom</i>	<i>D1</i>	<i>D2</i>	<i>D3</i>	<i>D4</i>
S5	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]
S6	[1,0,0]	[0.158717,0.936271, 0.313377]	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]
S7	[0.167842,0.928902, 0.330107]	[0.65,0.5,0.572276]	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]
S8	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]
S9	[1,0,0]	[0.594807,0.52811, 0.6060566]	[0.4,0.6,0.6928203]	[0.519722,0.560349, 0.6449015]
S10	[1,0,0]	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]	[0.167842,0.928909, 0.330107]
S11	[1,0,0]	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]	[0.167842,0.928909, 0.330107]
S12	[1,0,0]	[0.167842, 0.928902, 0.330107]	[0.519722, 0.560349, 0.644901]	[0.167842, 0.928909, 0.330107]
S13	[1,0,0]	[0.167842, 0.928902, 0.330107]	[0.158717, 0.936271, 0.313377]	[0.167842, 0.928909, 0.330107]
S14	[1,0,0]	[0.167842, 0.928902, 0.330107]	[0.158717, 0.936271, 0.313377]	[0.167842, 0.928909, 0.330107]
S15	[0.158717, 0.93621, 0.3133777]	[0.579118, 0.535381, 0.614808247]	[0.589672, 0.530523, 0.6089599]	[0.499309, 0.568063, 0.654213]
S16	[0.158717, 0.93621, 0.3133777]	[0.167842, 0.928902, 0.330107]	[0.499309, 0.568063, 0.654213]	[0.263522, 0.814217, 0.517307]
S17	[1,0,0]	[0.519722, 0.560349, 0.6449015]	[0.123255, 0.96126, 0.2465508]	[0.167842, 0.928909, 0.330107]
S18	[1,0,0]	[0.158717, 0.936271, 0.313377]	[0.123255, 0.96126, 0.2465508]	[0.167842, 0.928909, 0.330107]
S19	[0.158717, 0.93621, 0.3133777]	[0.123255, 0.96126, 0.24655]	[0.167842, 0.928902, 0.330107]	[0.167842, 0.928909, 0.330107]

**Table 4:** Aggregated Pythagorean fuzzy decision matrix

<i>Symptom</i>	<i>D1</i>	<i>D2</i>	<i>D3</i>	<i>D4</i>
S1	[0.519722,0.560349, 0.6449015]	[0.167842,0.928909, 0.330107]	[0.718534,0.48063, 0.502695]	[0.158717,0.936271, 0.313377]
S2	[0.519722,0.560349, 0.644901]	[0.519722,0.560349, 0.6449015]	[0.519722,0.560349, 0.6449015]	
S3	[0.167842,0.928909, 0.330107]	[0.158717,0.936271, 0.313377]	[0.123255,0.96126, 0.2465508]	[0.519722,0.560349, 0.6449015]
S4	[0.167842,0.928909, 0.330107]	[0.158717,0.936271, 0.313377]	[0.123255,0.96126, 0.2465508]	[0.167842,0.928902, 0.330107]

(Continued)

**Table 4 (continued)**

<i>Symptom</i>	<i>D1</i>	<i>D2</i>	<i>D3</i>	<i>D4</i>
S5	[0.123255,0.96126, 0.2465508]	[0.167842,0.928909, 0.330107]	[0.519722,0.560349, 0.6449015]	[0.2,0.9,0.387298]
S6	[0.123255,0.96126, 0.2465508]	[0.167842,0.928909, 0.330107]	[0.167842,0.928902, 0.330107]	[0.2,0.9,0.387298]
S7	[0.123255,0.96126, 0.2465508]	[0.167842,0.928909, 0.330107]	[0.167842,0.928902, 0.330107]	[0.2,0.9,0.387298]
S8	[0.123255,0.96126, 0.2465508]	[0.167842,0.928909, 0.330107]	[0.167842,0.928902, 0.330107]	[0.519722,0.560349, 0.6449015]
S9	[0.589672,0.530523, 0.6089599]	[0.4,0.6,0.69282]	[0.519722,0.560349, 0.6449015]	[0.589672,0.530523, 0.608959]
S10	[0.158717,0.936271, 0.313377]	[0.123255,0.96126, 0.2465508]	[0.167842,0.928902, 0.330107]	[0.167842,0.928902, 0.330107]
S11	[0.158717,0.936271, 0.313377]	[0.123255,0.96126, 0.2465508]	[0.167842,0.928902, 0.330107]	[0.167842,0.928902, 0.330107]
S12	[0.158717,0.936271, 0.31337]	[0.123255,0.96126, 0.2465508]	[0.167842,0.928902, 0.330107]	[0.167842,0.928902, 0.330107]
S13	[0.519722,0.560349, 0.6449015]	[0.123255,0.96126, 0.2465508]	[0.167842,0.928902, 0.330107]	[0.167842,0.928902, 0.330107]
S14	[0.354495,0.677608, 0.644344]	[0.123255,0.96126, 0.2465508]	[0.167842,0.928902, 0.330107]	[0.167842,0.928902, 0.330107]
S15	[0.167842,0.928909, 0.330107]	[0.167842,0.928909, 0.330107]	[0.2,0.9,0.387298]	[0.158717,0.936271, 0.313377]
S16	[0.167842,0.928909, 0.33010]	[0.167842,0.928909, 0.330107]	[0.2,0.9,0.387298]	[0.519722,0.560349, 0.6449015]
S17	[0.4,0.6,0.69282]	[0.519722,0.560349, 0.6449015]	[0.499309,0.568063, 0.654213]	[0.519722,0.560349, 0.6449015]
S18	[0.167842,0.928909, 0.330107]	[0.2,0.9,0.387298]	[0.1518717,0.936271, 0.313377]	[0.167842,0.928902, 0.330107]
S19	[0.65,0.5,0.572276]	[0.158717,0.936271, 0.313377]	[0.167842,0.928902, 0.330107]	[0.158717,0.936271, 0.313377]

Step 3: Determine the weight of each symptom using Table 5, and Eq. (2). The result of this step is shown in Table 6.

**Table 5:** The importance of criteria in linguistic terms

<i>Symptom</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>	<i>Symptom</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>
S1	H	AM	AM	S11	M	M	H
S2	M	H	M	S12	M	H	M
S3	M	H	M	S13	M	H	M
S4	M	M	H	S14	M	M	H
S5	M	M	H	S15	M	H	M
S6	M	H	M	S16	H	AM	AM
S7	M	H	M	S17	H	M	AM

(Continued)

**Table 5 (continued)**

<i>Symptom</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>	<i>Symptom</i>	<i>E1</i>	<i>E2</i>	<i>E3</i>
S8	H	AM	AM	S18	AM	M	H
S9	H	AM	AM	S19	M	H	AM
S10	M	H	M				

**Table 6: Weights of each symptom**

<i>Symptoms</i>	<i>Rating</i>
S1	(0.846591,0.332005,0.416001)
S2	(0.773055,0.371227,0.51437)
S3	(0.773055,0.371227,0.51437)
S4	(0.765686,0.379829,0.519091)
S5	(0.765686,0.379829,0.519091)
S6	(0.773055,0.371227,0.51437)
S7	(0.773055,0.371227,0.51437)
S8	(0.846591,0.332005,0.416001)
S9	(0.846591,0.332005,0.416001)
S10	(0.773055,0.371227,0.51437)
S11	(0.765686,0.379829,0.519091)
S12	(0.773055,0.371227,0.51437)
S13	(0.773055,0.371227,0.51437)
S14	(0.765686,0.379829,0.519091)
S15	(0.773055,0.371227,0.51437)
S16	(0.846591,0.332005,0.416001)
S17	(0.818343,0.34357,0.460732)
S18	(0.808373,0.365114,0.461762)
S19	(0.806725,0.359677,0.468858)

*Step 4:* Constructing aggregated weighted PFDM using Eq. (3), and Tables 3, 4, and 6. The results are shown in Tables 7 and 8.

*Step 5:* Table 9 shows the results of PFPIS and PFNIS using Eqs. (4)–(7), and Tables 7 and 8.

**Table 7: Aggregated weighted Pythagorean fuzzy decision matrix**

<i>Symptom</i>	<i>D1</i>	<i>D2</i>	<i>D3</i>	<i>D4</i>
S1	[0.1343,0.9434, 0.3030]	[0.1420,0.937, 0.3191]	[0.5035,0.5986, 0.6229]	[0.5035,0.5986, 0.6229]
S2	[0.7730,0.3712, 0.5143]	[0.1297,0.9390, 0.3184]	[0.4598,0.6150, 0.6405]	[0.4598,0.6150, 0.6405]

(Continued)

**Table 7 (continued)**

<i>Symptom</i>	<i>D1</i>	<i>D2</i>	<i>D3</i>	<i>D4</i>
S3	[0.7730,0.3712, 0.5143]	[0.1297,0.9390, 0.3184]	[0.1229,0.9453, 0.3022]	[0.0952,0.9666, 0.2375]
S4	[0.7656,0.3798, 0.5190]	[0.1285,0.9394, 0.3175]	[0.1215,0.9457, 0.3013]	[0.0943,0.9664, 0.2368]
S5	[0.1285,0.9394, 0.3175]	[0.1215,0.9457, 0.3013]	[0.1285,0.9394, 0.3175]	[0.1215,0.9457, 0.3013]
S6	[0.7730,0.3712, 0.5143]	[0.1226,0.9453, 0.3021]	[0.1297,0.9390, 0.3184]	[0.1226,0.9453, 0.3022]
S7	[0.1297,0.9390, 0.3184]	[0.5024,0.5944, 0.6278]	[0.1297,0.9390, 0.3184]	[0.1226,0.9453, 0.3022]
S8	[0.142,0.9370, 0.3191]	[0.1343,0.9435, 0.3028]	[0.1420,0.9370, 0.3191]	[0.1343,0.9435, 0.3028]
S9	[0.8465,0.332, 0.4160]	[0.5035,0.598, 0.622]	[0.338,0.6561, 0.6743]	[0.4399,0.6241, 0.6455]
S10	[0.7730,0.3712, 0.5143]	[0.1297,0.9390, 0.3184]	[0.1226,0.9453, 0.3022]	[0.1297,0.9390, 0.3184]
S11	[0.7656,0.3798, 0.5190]	[0.1285,0.9394, 0.31755]	[0.1215,0.9457, 0.3013]	[0.1285,0.9394, 0.3175]
S12	[0.7730,0.3712, 0.5143]	[0.1297,0.9390, 0.3184]	[0.4017,0.6391, 0.6557]	[0.1297,0.9390, 0.3184]
S13	[0.7730,0.3712, 0.5143]	[0.1297,0.9390, 0.3184]	[0.1226,0.9453, 0.3022]	[0.1297,0.9390, 0.3184]
S14	[0.7656,0.3798, 0.5190]	[0.1285,0.9394, 0.3175]	[0.1215,0.9457, 0.3013]	[0.1285,0.9394, 0.3175]
S15	[0.1226,0.9452, 0.3023]	[0.4476,0.6204, 0.6439]	[0.4558,0.6168, 0.6416]	[0.3859,0.6450, 0.6595]
S16	[0.1343,0.9434, 0.3030]	[0.1420,0.9370, 0.3191]	[0.4227,0.6303, 0.6511]	[0.223,0.8367, 0.5001]
S17	[0.8183,0.343, 0.4607]	[0.4253,0.6284, 0.6512]	[0.1008,0.9659, 0.2384]	[0.1373,0.9375, 0.319]
S18	[0.8083,0.3651, 0.4617]	[0.1283,0.9450, 0.3008]	[0.0996,0.9665, 0.2364]	[0.1356,0.9386, 0.3169]
S19	[0.1280,0.9447, 0.3018]	[0.0994,0.9663, 0.2371]	[0.1354,0.938, 0.31790]	[0.1354,0.9384, 0.3178]

**Table 8:** Aggregated weighted Pythagorean fuzzy decision matrix

<i>Symptom</i>	<i>D5</i>	<i>D6</i>	<i>D7</i>	<i>D8</i>
S1	[0.4399,0.6241, 0.6455]	[0.1420,0.9370, 0.3190]	[0.6083,0.5619, 0.5605]	[0.1343,0.9435, 0.3028]

(Continued)

**Table 8 (continued)**

<i>Symptom</i>	<i>D5</i>	<i>D6</i>	<i>D7</i>	<i>D8</i>
S2	[0.4598,0.6150, 0.6405]	[0.4017,0.6391, 0.6557]	[0.4017,0.6391, 0.6557]	[0.4017,0.6391, 0.6557]
S3	[0.1297,0.9390, 0.3184]	[0.1226,0.9453, 0.3022]	[0.0952,0.9666, 0.2375]	[0.4017,0.6391, 0.6557]
S4	[0.1285,0.9394, 0.3175]	[0.1215,0.9457, 0.3013]	[0.0943,0.9669, 0.2368]	[0.1285,0.9394, 0.3175]
S5	[0.0943,0.9669, 0.236]	[0.1285,0.9394, 0.3175]	[0.3979,0.6426, 0.6547]	[0.1531,0.9151, 0.3730]
S6	[0.0952,0.9666, 0.2375]	[0.1297,0.9390, 0.3184]	[0.1297,0.9390, 0.3184]	[0.1546,0.9144, 0.3740]
S7	[0.0952,0.9666, 0.2375]	[0.1297,0.9390, 0.3184]	[0.1297,0.9390, 0.3184]	[0.1546,0.9144, 0.3740]
S8	[0.1043,0.9656, 0.2381]	[0.1420,0.9370, 0.3190]	[0.1420,0.9370, 0.3191]	[0.4399,0.6241, 0.6455]
S9	[0.4992,0.6005, 0.6246]	[0.3386,0.6561, 0.6743]	[0.4399,0.6241, 0.6455]	[0.4992,0.6005, 0.6240]
S10	[0.1226,0.9453, 0.3022]	[0.0952,0.9666, 0.2375]	[0.1297,0.9390, 0.3184]	[0.1297,0.9390, 0.3184]
S11	[0.1215,0.9457, 0.3013]	[0.0943,0.9667, 0.2368]	[0.1285,0.9394, 0.3175]	[0.1285,0.9394, 0.3175]
S12	[0.1226,0.9453, 0.3022]	[0.0952,0.9667, 0.2375]	[0.1297,0.9390, 0.318]	[0.1297,0.9390, 0.3184]
S13	[0.4017,0.6391, 0.6557]	[0.0952,0.9666, 0.2375]	[0.1297,0.9390, 0.3184]	[0.1297,0.9390, 0.3184]
S14	[0.2714,0.7329, 0.6238]	[0.09437,0.9669, 0.236]	[0.1285,0.9394, 0.3175]	[0.1285,0.9394, 0.3175]
S15	[0.1297,0.9390, 0.3184]	[0.1297,0.9390, 0.3184]	[0.1546,0.9144, 0.3740]	[0.1226,0.9453, 0.3022]
S16	[0.1420,0.937, 0.3190]	[0.1420,0.9370, 0.3190]	[0.1693,0.9115, 0.3746]	[0.4399,0.624, 0.6455]
S17	[0.327,0.6594, 0.6762]	[0.4253,0.6284, 0.6512]	[0.4086,0.6345, 0.6560]	[0.4253,0.6284, 0.6512]
S18	[0.1356,0.9386, 0.3169]	[0.1616,0.9139, 0.3721]	[0.1227,0.9450, 0.3031]	[0.1356,0.9386, 0.3169]
S19	[0.5243,0.5890, 0.6148]	[0.1280,0.9447, 0.301726]	[0.1354,0.938, 0.3179]	[0.128,0.9447, 0.3017]

**Table 9:** PIS and NIS

<i>Symptom</i>	<i>PIS</i>	<i>NIS</i>
S1	(0.6083044176, 0.5619336555, 0.56053216)	(0.1343683837, 0.9435067904, 0.3028862392)
S2	(0.773055, 0.371227, 0.514369985)	(0.1297510973, 0.9390198435, 0.318443694)
S3	(0.773055, 0.371227, 0.514369985)	(0.09528289403, 0.9666909764, 0.2375494187)
S4	(0.765686, 0.379829, 0.5190904354)	(0.09437462793, 0.9669448349, 0.2368778499)
S5	(0.3979438593, 0.6426208583, 0.6547359142)	(0.09437462793, 0.9669448349, 0.2368778499)
S6	(0.773055, 0.371227, 0.514369985)	(0.09528289403, 0.9666909764, 0.2375494187)
S7	(0.50248575, 0.5944384864, 0.6278144287)	(0.09437462793, 0.9669448349, 0.2368778499)
S8	(0.4399919677, 0.6241858178, 0.6455998243)	(0.1043465737, 0.9656064274, 0.2381512544)
S9	(0.846591, 0.332005, 0.4160004311)	(0.3386364, 0.6561596489, 0.6743737123)
S10	(0.773055, 0.371227, 0.514369985)	(0.09528289403, 0.9666909764, 0.2375494187)
S11	(0.765686, 0.379829, 0.5190904354)	(0.09437462793, 0.9669448349, 0.2368778499)
S12	(0.773055, 0.371227, 0.514369985)	(0.09528289403, 0.9666909764, 0.2375494187)
S13	(0.773055, 0.371227, 0.514369985)	(0.09528289403, 0.9666909764, 0.2375494187)
S14	(0.765686, 0.379829, 0.5190904354)	(0.09437462793, 0.9669448349, 0.2368778499)
S15	(0.455848888, 0.6168281917, 0.6416578319)	(0.1226969704, 0.9453087376, 0.3022198604)
S16	(0.4399919677, 0.6241858178, 0.6455998243)	(0.1343683837, 0.9434529307, 0.3030539637)
S17	(0.818343, 0.34357, 0.4607324489)	(0.1008648665, 0.9659137643, 0.2384048629)
S18	(0.808373, 0.365114, 0.4617627745)	(0.09963601412, 0.9665140672, 0.2364809137)
S19	(0.52437125, 0.5890888373, 0.6148244741)	(0.09943288988, 0.9663591629, 0.2371983741)

*Step 6:* Find distance of each disease with respect to PIS and NIS using Eqs. (8) and (9), and Tables 7 and 8.

*Step 7:* Now apply Eq. (10) on Table 10 to calculate the relative closeness of each disease.

**Table 10:** Numerical results

<i>Disease</i>	$D(L_i, L^+)$	$D(L_i, L^-)$	$(L_i)$	<i>Rating of disease</i>
D1	0.275112	0.557659	0.669642	1
D2	0.562568	0.194540	0.256951	6
D3	0.546636	0.233754	0.299534	4
D4	0.562210	0.184700	0.247285	7
D5	0.527542	0.255102	0.325949	2
D6	0.579636	0.149774	0.205336	8
D7	0.547882	0.217842	0.284491	5
D8	0.525929	0.240877	0.314130	3

*Step 8:* The maximum value is *D1* disease.

3.2.2 *Pythagorean Fuzzy Entropy Method*

Now we evaluate the same inputs with our second methodology (cf., [Section 3.1.2](#)). The step-by-step calculations of this algorithm are as follows:

*Step 1:* Compute the overall entropy of each criterion using [Tables 3 and 4](#), and [Eq. \(11\)](#).

*Step 2:* Compute the overall weight of each symptom: we use [Tables 6 and 11](#), and [Eq. \(13\)](#), to get [Table 12](#).

**Table 11:** Entropy of each Symptom

<i>Entropy</i>	<i>Value</i>	<i>Entropy</i>	<i>Value</i>	<i>Entropy</i>	<i>Value</i>	<i>Entropy</i>	<i>Value</i>
E1	0.7159104962	E6	0.4178064273	E11	0.4076892564	E16	0.6504636906
E2	0.7960083765	E7	0.5377938017	E12	0.4726217253	E17	0.7162852648
E3	0.4544188736	E8	0.5308859753	E13	0.4726217431	E18	0.4172182446
E4	0.3924935986	E9	0.8469676038	E14	0.4600604293	E19	0.5216621254
E5	0.5410031461	E10	0.4076892564	E15	0.6774897253		

**Table 12:** Weight of each Symptom

<i>Weight</i>	<i>Value</i>	<i>Weight</i>	<i>Value</i>	<i>Weight</i>	<i>Value</i>	<i>Weight</i>	<i>Value</i>
W1	0.03317680263	W6	0.06799014839	W11	.0691716591	W16	.04081980058
W2	0.02382269643	W7	0.05397769656	W12	.06158866884	W17	.03313297819
W3	0.06371444735	W8	0.05478441131	W13	0.06158866884	W18	0.06805883796
W4	0.07094624934	W9	0.01787153932	W14	0.06305561114	W19	0.05586159757
W5	0.05360290059	W10	0.0691716591	W15	0.03766362677		

*Step 3:* Determine the Pythagorean fuzzy PIS  $\gamma^+$  and NIS  $\gamma^-$  of each alternative using [Eqs. \(4\)–\(7\)](#), and [Tables 3 and 4](#). The PIS and NIS results are shown in [Table 13](#).

**Table 13:** PIS & NIS

<i>Symptoms</i>	$\gamma^+(mem)$	$\gamma^+(non - mem)$	$\gamma^-(mem)$	$\gamma^-(non - mem)$
S1	0.7185344956	0.4806300777	0.1587167962	0.9362709927
S2	1	0	0.1678421306	0.9289016977
S3	1	0	0.123255	0.96126
S4	1	0	0.123255	0.96126
S5	0.519722	0.560349	0.123255	0.96126
S6	1	0	0.123255	0.96126
S7	0.65	0.5	0.123255	0.96126
S8	0.519722	0.560349	0.123255	0.96126
S9	1	0	0.4	0.6
S10	1	0	0.123255	0.96126
S11	1	0	0.123255	0.96126

(Continued)

**Table 13 (continued)**

<i>Symptoms</i>	$\gamma^+(mem)$	$\gamma^+(non - mem)$	$\gamma^-(mem)$	$\gamma^-(non - mem)$
S12	1	0	0.123255	0.96126
S13	1	0	0.123255	0.96126
S14	1	0	0.123255	0.96126
S15	0.5896724049	0.5305226244	0.158717	0.936271
S16	0.519722	0.560349	0.1587167962	0.9362709927
S17	1	0	0.1232545023	0.9612601555
S18	1	0	0.1232545023	0.9612601555
S19	0.65	0.5	0.1232545023	0.9612601555

*Step 4:* Calculate the distance between alternatives using [Tables 12](#) and [13](#).

*Step 5:* Calculate the relative degree of closeness of each alternative using [Eq. \(10\)](#), and [Table 14](#). [Table 15](#) shows the relative closeness of each disease.

**Table 14:** Distance between alternatives

<i>Disease</i>	$D(A_i, \gamma^+)$	$D(A_i, \gamma^-)$
D1	0.2673218011	0.7632174342
D2	0.7701354677	0.188677577
D3	0.7588156448	0.2131948401
D4	0.7738775789	0.1547205578
D5	0.7495318939	0.2463499314
D6	0.7855832056	0.1222792276
D7	0.7662546068	0.1935045705
D8	0.7451043189	0.2317462283

**Table 15:** Degree of closeness

<i>Disease</i>	<i>Value</i>	<i>Ranking</i>
D1	0.7406000743	1
D2	0.1967824469	6
D3	0.2193338893	4
D4	0.1666173468	7
D5	0.2473686387	2
D6	0.1346891589	8
D7	0.2016178381	5
D8	0.237238162	3

*Step 6:* Rank all alternatives  $A_i$  according to relative closeness  $\phi(A_i)$  of each alternative. The highest value is of D1.



3.2.3 *Pythagorean Fuzzy Power Weighted Geometric Method (PFPWG)*

We consider the same example again, but now we follow the PFPWG algorithm (cf., Section 3.1.3) to find the final result.

*Step 1:* Calculate the supports using distance and support formulas–Eqs. (15) and (14), with Tables 3 and 4.

*Step 2:* Calculate the weighted support using Tables 12, 16, and Eq. (16).

*Step 3:* Use Table 2 and Eq. (1) to get crisp weight of each symptom.

*Step 4:* Use Eq. (17), plus Tables 16, 17 and 18, to determine the weight associated with PFNs.

**Table 16:** Support of each symptom

Symptom	D1	D2	D3	D4	D5	D6	D7	D8
S1	1	1	1	1	1	1	1	1
S2	0.07429382481	1	1	0.9406123802	0.6045895256	0.8354170216	0.5962338726	
S3	0.07429382481	1	0.5368457814	0.5081379061	0.6045895256	0.991644347	0.4029425476	0.5962338726
S4	0.07429382481	1	0.5368457814	0.5081379061	0.6045895256	0.991644347	0.4029425476	0.9916378446
S5	0.9916375164	0.9916375164	0.545208265	0.5368462527	0.567525526	1	0.8354170216	0.9592938503
S6	0.07429382481	0.9916375164	0.545208265	0.5368462527	0.567525526	1	0.4400130496	0.9592938503
S7	0.9916375164	0.4964063084	0.545208265	0.5368462527	0.567525526	1	0.4400130496	0.9592938503
S8	0.9916375164	0.9916375164	0.545208265	0.5368462527	0.567525526	1	0.4400130496	0.5962338726
S9	0.07429382481	0.5452087848	0.8625519566	0.9406123802	0.9449307707	0.6826495033	0.8354170216	0.5411646433
S10	0.07429382481	1	0.5368457814	0.5452019058	0.5962338726	0.9629360004	0.4400130496	0.9916378446
S11	0.07429382481	1	0.5368457814	0.5452019058	0.5962338726	0.9629360004	0.4400130496	0.9916378446
S12	0.07429382481	1	0.9406124118	0.5452019058	0.5962338726	0.9629360004	0.4400130496	0.9916378446
S13	0.07429382481	1	0.5368457814	0.5452019058	1	0.9629360004	0.4400130496	0.9916378446
S14	0.07429382481	1	0.5368457814	0.5452019058	0.8551970739	0.9629360004	0.4400130496	0.9916378446
S15	1	0.5582837365	0.9956816017	0.9258593552	0.6045895256	1	0.4723570439	1
S16	1	1	0.9258584902	0.6657996631	0.6045895256	1	0.4723570439	0.5962338726
S17	0.07429382481	0.6045958532	0.5081372495	0.5452019058	0.9219400223	0.6045895256	0.8206639966	0.5962338726
S18	0.07429382481	0.9916375164	0.5081372495	0.5452019058	0.6045895256	0.9676495033	0.4305879033	0.9916378446
S19	1	0.9629289845	0.545208265	0.5452019058	0.8918099777	0.991644347	0.4400130496	1

**Table 17:** Weighted support

Disease	Weighted support
D1	0.1194823486
D2	0.2670615473
D3	0.196587252
D4	0.1866896644
D5	0.2115133567
D6	0.2750992294
D7	0.1617538985
D8	0.2559919525

**Table 18:** Weights of each symptom

<i>Symptom</i>	<i>Weight</i>	<i>Symptom</i>	<i>Weight</i>
S1	0.01396530372	S11	0.01621589327
S2	0.01624052684	S12	0.01624052684
S3	0.01624052684	S13	0.01624052684
S4	0.01621589327	S14	0.01621589327
S5	0.01621589327	S15	0.01624052684
S6	0.01624052684	S16	0.01396530372
S7	0.01624052684	S17	0.01516553483
S8	0.01396530372	S18	0.0148661582
S9	0.01396530372	S19	0.01515533886
S10	0.01624052684		

*Step 5:* Apply the PFPWG operator using Eq. (18), and Tables 19, 3, and 4.

*Step 6:* Calculate the scores of the overall PFNs using Eq. (19) and Tables 20 and 21.

**Table 19:** Weights associated with the PFN'S

<i>Symptom</i>	<i>D1</i>	<i>D2</i>	<i>D3</i>	<i>D4</i>	<i>D5</i>	<i>D6</i>	<i>D7</i>	<i>D8</i>
S1	0.04757226825	0.04719906613	0.04737572284	0.04740046969	0.04733855366	0.04717920819	0.04746287952	0.04722336609
S2	0.05462660171	0.05501188798	0.05521778653	0.05524662971	0.05512210018	0.05464126637	0.05517386925	0.05468505886
S3	0.05462660171	0.05501188798	0.05480908316	0.05481236679	0.05482581458	0.05498140024	0.05479153667	0.05468505886
S4	0.05454364459	0.0549271147	0.05472523149	0.05472854815	0.05474184762	0.05489668488	0.05470788958	0.05494806034
S5	0.05535403485	0.05491978515	0.05473258846	0.05475381779	0.05470926579	0.05490400536	0.0550890632	0.0549196969
S6	0.05462660171	0.05500453615	0.05481646251	0.05483771326	0.05479313368	0.05498874297	0.05482430915	0.05500440529
S7	0.05543945597	0.05456915647	0.05481646251	0.05483771326	0.05479313368	0.05498874297	0.05482430915	0.05500440529
S8	0.04756678906	0.04719362992	0.04707896944	0.04709810219	0.04705658376	0.04717920819	0.04709681361	0.04696075437
S9	0.04696573542	0.04690341962	0.04728603745	0.0473616988	0.04730264896	0.04697299468	0.04735529091	0.04692493704
S10	0.05462660171	0.05501188798	0.05480908316	0.05484509043	0.05481844704	0.05495617208	0.05482430915	0.05503285497
S11	.05454364459	0.0549271147	0.05472523149	0.0547611726	0.05473450242	0.0548715332	0.05474056271	0.05494806034
S12	0.05462660171	0.05501188798	0.05516538086	0.05484509043	0.05481844704	0.05495617208	0.05482430915	0.05503285497
S13	0.05462660171	0.05501188798	0.05480908316	0.05484509043	0.05517446476	0.05495617208	0.05482430915	0.05503285497
S14	0.05454364459	0.0549271147	0.05472523149	0.0547611726	0.0549621491	0.0548715332	0.05474056271	0.05494806034
S15	0.05544686593	0.05462355566	0.05521397583	0.05518117125	0.05482581458	0.05498874297	0.05485290312	0.0550402103
S16	0.04757226825	0.04719906613	0.04732734522	0.04718228875	0.04708074919	0.04717920819	0.04711795702	0.04696075437
S17	0.0510066918	0.05101308249	0.05112979126	0.0511850345	0.05140784028	0.05099161504	0.05146478003	0.05103293237
S18	0.04999867869	0.05028219687	0.05011289268	0.05016648925	0.05014483695	0.05024337859	0.05014763937	0.05030808439
S19	0.05168666771	0.05125280746	0.05112364047	0.05115034017	0.05134966675	0.0512532187	0.05113670654	0.05130758994

**Table 20:** Results after applying PFPWG operator

<i>Disease</i>	<i>Results</i>
<i>D1</i>	[0.5194678133, 0.7224007634]
<i>D2</i>	[0.2114815744, 0.9036659282]

(Continued)

**Table 20 (continued)**

<i>Disease</i>	<i>Results</i>
<i>D3</i>	[0.227599577, 0.8913426691]
<i>D4</i>	[0.2089763783, 0.9037153385]
<i>D5</i>	[0.2324989425, 0.8878193748]
<i>D6</i>	[0.1809496915, 0.9227676788]
<i>D7</i>	[0.2225946569, 0.8940073968]
<i>D8</i>	[0.242201083, 0.8764918485]

**Table 21:** Scores of the overall PFN's

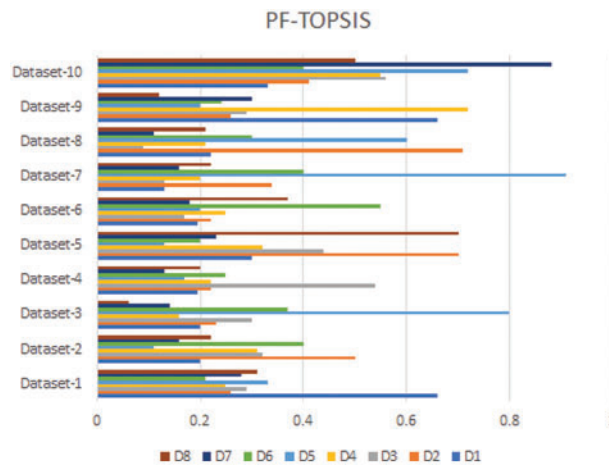
<i>Disease</i>	<i>Score</i>
<i>D1</i>	0.373991973
<i>D2</i>	0.1140561733
<i>D3</i>	0.1286549069
<i>D4</i>	0.1134848568
<i>D5</i>	0.132916258
<i>D6</i>	0.09062130094
<i>D7</i>	0.1251495779
<i>D8</i>	0.1452117021

*Step 7:* Disease *D1* is pinpointed again.

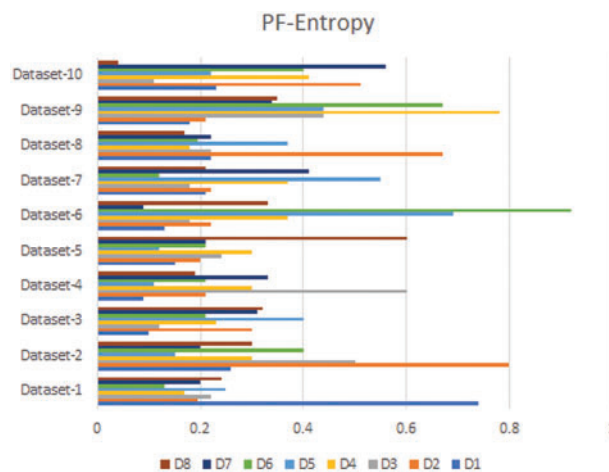
We confirm that each approach highlights the same disease based on the patient record provided.

#### 4 Comparative Analysis

In this section, we compare the results of the Pythagorean Fuzzy TOPSIS (PF-TOPSIS), the Pythagorean Fuzzy Entropy (PF-Entropy), and the PFPWG method. To do this, we took ten different data sets and applied PF-TOPSIS, PF-Entropy, and the PFPWG techniques. The results obtained from each technique are represented by drawing bar graphs. In [Fig. 2](#), the results of PF-TOPSIS are displayed. The eight diseases are shown in different eight colors. The length of each bar shows its value obtained from the PF-TOPSIS method. In [Fig. 3](#), the results of the PF entropy are displayed. In this figure, eight diseases are also represented by different colors, and the length of the bar shows the value of each disease for each data set obtained from PF entropy method. [Fig. 4](#) shows the results of the PFPWG method.

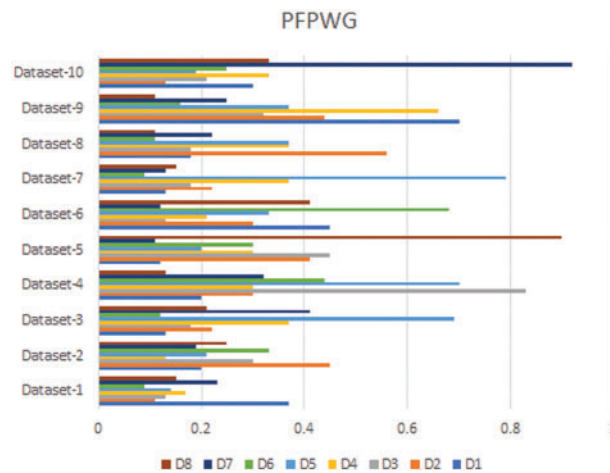


**Figure 2: PF-TOPSIS Results**



**Figure 3: PF-Entropy Results**

Now we compare these three bar charts for each data set. We can see that the data set 1,2,3,4,5,6,7,8,9, highlighted *D1*, *D2*, *D1*, *D4*, *D8*, *D6*, *D1*, *D2*, *D1*, and *D7* diseases, respectively. We can see that each approach highlights the same disease based on the provided patient’s records, which testifies the authenticity of our model.



**Figure 4: PFPWG Results**

## 5 Conclusion and Future Work

According to the World Health Organization (WHO), around 400,000 children are diagnosed with cancer each year and the rate of cure in low and middle-income countries is only 45 percent, which is highly unsatisfactory. To improve this percentage, WHO has launched a global initiative and provided appropriate professional guidance and resources. Their goal is to increase the survival rate up to sixty percent by the end of 2030. To help achieve this goal, we have proposed a novel model that allows doctors to diagnose the type of childhood cancer early, so that appropriate treatment can be given at the right time. This ultimately reduces the physical and financial suffering of the patient and their parents. Our model takes nineteen symptoms as inputs and determines the type of cancer. We have used Pythagorean fuzzy decision-making techniques for diagnostic purposes. We designed three algorithms, namely, Pythagorean fuzzy TOPSIS method, Pythagorean fuzzy entropy, and PFPWG. We have determined their respective time complexities. To test them, we have taken ten data sets and compared the results of the different approaches. Also, we have set forth a numerical example to make each of their steps understandable.

There are many other applications where decision-making takes place and our approaches can provide assistance. Our system is applicable when data is fuzzy and decisions must be made. So, some future directions of our work are discussed below:

*Industrial automation and Industry 4.0:* In Industry 4.0, we connect the devices through the internet to make a network of different things. Then through the proposed approach, the different manufacturing parts of the machines can be controlled without human intervention. We can use our proposed model in Industry 4.0 to make intelligent decisions by taking into account all parameters and making the manufacturing process more productively and efficiently.

*Precision agriculture:* We need various decision-making systems to automate traditional farming to increase the yields and reduce the potential risks. With our model, precision farming can be made more efficient and productive. We can make timely decisions, and automate the decision process. Through this approach, an irrigation system can be improved, and water wastage could be reduced. Our proposed approach can be useful for designing a pest control system that helps the farmer to save

crops from pests timely. This approach can also be useful to monitor soil pH and other ingredients, which require for the proper growth of the crops. Through this procedure, farmers can decide the right amount of fertilizers for the field.

*Computer aided diagnosis:* These systems help doctors to analyze the medical images and highlight diseases based on symptoms. The proposed approach can help doctors to early detect the chances of any disease, which could happen in the future due to the patient's routine or changes in his body and enable the doctor to prevent it from spreading more by proper medication or therapies.

*Classroom monitoring:* The proposed approach could be beneficial for monitoring students' activities in a large classroom and concluding which student is not attentive in class or how much students in the class are attentive.

**Data Availability:** No data were used to support this study.

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