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Article

Output Feedback Robust H_∞ Control for Discrete 2D Switched Systems

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Abstract: The two-dimensional (2-D) system has a wide range of applications in different fields, including satellite meteorological maps, process control, and digital filtering. Therefore, the research on the stability of 2-D systems is of great significance. Considering that multiple systems exist in switching and alternating work in the actual production process, but the system itself often has external perturbation and interference. To solve the above problems, this paper investigates the output feedback robust H_∞ stabilization for a class of discrete-time 2-D switched systems, which the Roesser model with uncertainties represents. First, sufficient conditions for exponential stability are derived via the average dwell time method, when the system's interference and external input are zero. Furthermore, in the case of introducing the external interference, the weighted robust H_∞ disturbance attenuation performance of the underlying system is further analyzed. An output feedback controller is then proposed to guarantee that the resulting closed-loop system is exponentially stable and has a prescribed disturbance attenuation level γ. All theorems mentioned in the article will also be given in the form of linear matrix inequalities (LMI). Finally, a numerical example is given, which takes two uncertain values respectively and solves the output feedback controller's parameters by the theorem proposed in the paper. According to the required controller parameter values, the validity of the theorem proposed in the article is compared and verified by simulation.

Keywords: 2-D systems; robust H_∞ control; LMI; output feedback; switched systems; Roesser model

1 Introduction

The issue of stability analysis and controller synthesis is a hot research topic. Reference Medvedeva et al. [1–3] investigated the stability of one-dimensional (1-D) continuous-time or discrete-time systems. Considering the complexity of many manufacturing processes and physical phenomena, a 2-D continuous-time or discrete-time system that depends on two independent variables has its irreplaceable application area. 2-D systems have attracted considerable research attention in control theory and practice over the past few decades due to their wide applications. Reference Du et al. [4–7] showed multi-dimensional digital filtering, linear image processing, signal processing, and process control. Different models such as the Roesser model and Fornasini–Marchesini model can represent 2-D systems, and the stability issues concerning these two models can be found [8,9].

On the other hand, considerable interest has been devoted to the research of switched systems during the recent decades. A switched system comprises a family of subsystems described by continuous or discrete-time dynamics and a switching law that specifies the active subsystem at each instant of time. The switching strategy improves control performance [10–13] and arise many engineering applications, such as in motor engine control, constrained robotics, and satellite image control systems [14]. As far as time-dependent switching is concerned, the average dwell time (ADT) switching is employed in most references owing to its flexibility [15,16].

However, perturbations and uncertainties widely exist in practical systems. In some cases, the perturbations can be merged into the disturbance, which can be bounded in the appropriate norms. The main advantage of robust H_∞ control is that its performance specification considers the system's worst-case performance in terms of energy gain. This is more appropriate for system robustness analysis and robust control under modeling disturbances than other performance specifications. Recently, the problems of robust H_∞ control and filtering for 2-D systems have been studied by many researchers [17–20], and so do the same problems of switched systems [21–23]. However, to the best of our knowledge, the output feedback robust H_∞ control problem of 2-D switched systems in the Roesser model with uncertainties has not yet been thoroughly investigated, which motivates this present study.

In this paper, we confine our attention to the robust H_∞ control problem of discrete 2-D switched systems described by the Roesser model with uncertainties. The main theoretical contributions are threefold: (1) We contribute to the development of stabilization for a class of 2-D switched systems that are exponentially stable, which the Roesser model with uncertainties represents. (2) A sufficient condition is presented to ensure a 2-D switched system's exponential stability at a given disturbance attenuation level robust weighted H_∞. (3) Based on the above two points, this article further designs the output feedback controller of the closed-loop system.

The paper is organized as follows. According to the current research results, we first study the exponential stability of 2-D switched systems described by the Roesser model with uncertainties. Further, when the system contains perturbations, we analyze the robust H_∞ performance index of the system. Finally, we design an output feedback controller for the open-loop system, and an example is given to illustrate the effectiveness of the proposed method.

2 Notation

The following notations are used throughout the paper: the superscript "T" denotes the transpose, and the notation $X \geq Y$ ($X > Y$) means that matrix $X - Y$ is positive semi-definite (positive definite, respectively). $\|\cdot\|$ denotes the Euclidean norm. I represents the identity matrix. Diag$\{a_i\}$ denotes a diagonal matrix with the diagonal elements a_i, $i = 1, 2, \ldots, n$. X^{-1} denotes the inverse of X. R^n denotes the n dimensional vector. Z^+ represents the set of all non-negative integers. The l_2 norm of a 2-D signal $w(i,j)$ is given by $\|w\|_2 = \sqrt{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\|w(i,j)\|^2}$, where w($i$,$j$) belongs to $l_2\{[0,\infty),[0,\infty)\}$.

3 Problem Formulation and Preliminaries

The uncertain Roesser model for a 2-D switched system $G: u \rightarrow y$ is given by the following state equation:

$$\begin{aligned} \bar{x}(k,l) &= A^{\delta(\mathrm{k,l})}(\Delta)x(k,l) + B^{\delta(\mathrm{k,l})}w(k,l) \\ z(k,l) &= C^{\delta(\mathrm{k,l})}x(k,l) + D^{\delta(\mathrm{k,l})}w(k,l), k, l = 0, 1, 2\cdots \end{aligned} \tag{1}$$

with

$$\bar{x}(k,l) = \begin{bmatrix} x^h(k+1,l) \\ x^v(k,l+1) \end{bmatrix}, x(k,l) = \begin{bmatrix} x^h(k,l) \\ x^v(k,l) \\ \end{bmatrix}$$

$$A^{\delta(\mathrm{k,l})}(\Delta) = A^{\delta(\mathrm{k,l})} + F^{\delta(\mathrm{k,l})}G^{\delta(\mathrm{k,l})}(k,l)H^{\delta(\mathrm{k,l})}$$

where $G^{\delta(\mathrm{k,l})}(k,l)$is an unknown uncertain matrix and satisfies the norm bounded condition $G^{T\delta(\mathrm{k,l})}(k,l)G^{\delta(\mathrm{k,l})}(k,l) \leq I$, and where $x^h(k,l) \in R^{n_1}$and $x^v(k,l) \in R^{n_2}$ denote the system horizontal state and vertical state, respectively. Furthermore, $x(k,l)$ is the whole state in R^n with $n = n_1 + n_2$, and $w(k,l) \in R^q$ is the interference input which belongs to $w(k,l) \in l_2\{[0,\infty),[0,\infty)\}$. $u(k,l) \in R^p$ and $z(k,l) \in R^q$ are control input and control output respectively; k and l are integers in Z. $\delta(\mathrm{k,l}) : Z_+ \times Z_+ \to N = \{1,2,3\cdots,N\}$ is the switching signal. N is the number of subsystems. $\delta(\mathrm{k,l}) = i, i \in N$ denotes that the ith subsystem is activated., A^i, B^i, C^i, D^i, F^i, G^i, H^i, are constant matrices with appropriate dimensions.

The boundary condition satisfies:

$$X(0) = \left[x^{hT}(0,0), x^{hT}(0,1), \cdots, x^{vT}(0,0), x^{vT}(1,0), \cdots\right]^T \tag{2}$$

It is easy to know from the above formula $\|X(0)\|_2 < \infty$.

Remark 1. “In this paper, it is assumed that switching occurs only at each sampling point of k or l. The switching sequence can be described as

$$((k_0,l_0),\delta(k_0,l_0)),\cdots,((k_k,l_k),\delta(k_k,l_k)),\cdots \tag{3}$$

with (k_k,l_k) denoting the kth switching instantly. It should be noted that the value of $\delta(k,l)$ only depends on $k+l$ [24,25].

Definition 1. System (1) is said to be exponentially stable under $\delta(k,l)$ if for a given j ≥ 0, there exist positive constants c and f, such that

$$\sum_{k+l=D} \|x(k,l)\|^2 \leq fe^{-c(D-j)} \sum_{k+l=j} \|x(k,l)\|_r^2 \tag{4}$$

holds for all D ≥ j [26].

Remark 2. From Definition 1, it is easy to see that when j is given, $\sum\limits_{k+l=j} \|x(k,l)\|_r^2$ will be bounded, and $\sum\limits_{k+l=D} \|x(k,l)\|_r^2$ will tend to be zero exponentially as D goes to infinity, which also means that $\|x(k,l)\|$ will tend to be zero exponentially.

Definition 2. For a given scalar $\gamma > 0$, system (1) is said to have a weighted disturbance attenuation level γ under switching signal $\delta(k,l)$ if it satisfies the following conditions [4]:

(1) when $w(k,l)$=0, system (1) is asymptotically stable or exponentially stable;

(2) under the zero-boundary condition, we have

$$\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left(\phi^{k+l}\|z\|_2^2\right) < \gamma^2\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\|w\|_2^2), \forall 0 \neq w(k,l) \in l_2\{[0,\infty),[0,\infty)\} \tag{5}$$

where $0 < \phi < 1$ and the l_2-norm of 2D discrete signal $z(k,l)$ and $w(k,l)$ are defined as

$$\|z\|_2^2 = \|z(k+1,l)\|_2^2 + \|z(k,l+1)\|_2^2, \|w\|_2^2 = \|w(k+1,l)\|_2^2 + \|w(k,l+1)\|_2^2 \tag{6}$$

Definition 3. For any $k+l = D \geq j = k_z + l_z$, let $N_\delta(j,D)$ denote the switching number of $\delta(\cdot)$ on an interval $[z,D)$. If

$$N_\delta(\mathrm{j},D) \leq N_0 + \frac{D-j}{\tau_a} \tag{7}$$

holds for given $N_0 \geq 0$ and $\tau_a \geq 0$, then the constant τ_a is called the average dwell time and N_0 is the chatter bound [27].

Lemma 1. For a given matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where S_{11} and S_{22} are square matrices, the following conditions are equivalent [28].

(i)$S < 0$;

(ii)$S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(iii)$S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2. Assuming that $x \in R^p, y \in R^q$ and U, V, W are a suitable dimension matrix, then inequality $x^T UVWy + y^T W^T V^T U^T x \leq \varepsilon_1 x^T UU^T x + \varepsilon_2 y^T WW^T y$ is true for any $V^T V \leq I$, if and only if there exist positive scalars $\varepsilon_1, \varepsilon_2$ and $\varepsilon_1 \geq \varepsilon_2$.

Proof.

$$\begin{aligned} 0 &\leq \left(\sqrt{\varepsilon_1}U^T x - \tfrac{1}{\sqrt{\varepsilon_1}}VWy\right)^T\left(\sqrt{\varepsilon_1}U^T x - \tfrac{1}{\sqrt{\varepsilon_1}}VWy\right) \\ &= \varepsilon_1 x^T UU^T x - x^T UVWy - y^T W^T V^T U^T x + \tfrac{1}{\varepsilon_1}y^T WW^T y \\ &\leq \varepsilon_1 x^T UU^T x - x^T UVWy - y^T W^T V^T U^T x + \tfrac{1}{\varepsilon_2}y^T WW^T y \end{aligned}$$

4 Exponential Stability Analysis

This section focuses on the exponential stability analysis of the 2D switched systems. The following theorem presents sufficient conditions that can guarantee that system (1) is exponentially stable.

Theorem 1. Consider 2D discrete switched system (1) with $w(k,l) = 0$, for a given positive constant $\phi < 1$, if there exist a set of positive-definite symmetric matrices $P^i \in R^{n\times n}, i \in N$ and two positive scalars $\varepsilon_1, \varepsilon_2$, such that

$$\begin{bmatrix} -\phi P^i & A^{iT}P^i & 0 & \varepsilon_1 H^{iT} \\ P^i A^i & -P^i & P^i F^i & 0 \\ 0 & F^{iT}P^i & -\varepsilon_2 I & 0 \\ \varepsilon_1 H^i & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0 \tag{8}$$

$$\varepsilon_1 \geq \varepsilon_2 \tag{9}$$

Then, the system is exponentially stable for any switching signal with the average dwell time satisfying

$$\tau_a > \tau_a^* = \frac{\ln \mu}{-\ln \phi} \tag{10}$$

where $\mu \geq 1$ satisfies

$$P^i \leq \mu P^j, \forall i, j \in N \tag{11}$$

Proof. Without loss of generality, we assume that the ith subsystem is active. For the ith subsystem, we consider the following Lyapunov function candidate:

$$V^i(x(k,l)) = x^T(k,l)P^i x(k,l) \tag{12}$$

where P^i is an $n \times n$ positive-definite matrix for any $i \in N$, and thus $V^i(x(k,l)) > 0, \forall x(k,l) \neq 0$ and $V^i(k,l) = 0$ only when $x(k,l) = 0$. Then we have $V^i(\bar{x}(k,l)) - \phi V^i(x(k,l)) = x^T(k,l)(A^{iT}(\Delta)P^i A^i(\Delta) - \phi P^i)$.

Using Lemma 1 to (8), we can get the equivalent inequality as follows:

$$\begin{bmatrix} -\phi P^i & A^{iT}P^i & 0 & \varepsilon_1 H^{iT} \\ P^i A^i & -P^i & P^i F^i & 0 \\ 0 & F^{iT}P^i & -\varepsilon_2 I & 0 \\ \varepsilon_1 H^i & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0 \Rightarrow \begin{bmatrix} -\phi P^i + \varepsilon_1 H^{iT}H^i & A^{iT}P^i \\ P^i A^i & -P^i + \frac{1}{\varepsilon_2}P^i F^i F^{iT}P^i \end{bmatrix} < 0$$

$$\Rightarrow \begin{bmatrix} -\phi P^i & A^{iT}P^i \\ P^i A^i & -P^i \end{bmatrix} + \varepsilon_1 \begin{bmatrix} H^{iT} \\ 0 \end{bmatrix} \begin{bmatrix} H^i & 0 \end{bmatrix} + \frac{1}{\varepsilon_2} \begin{bmatrix} 0 \\ P^i F^i \end{bmatrix} \begin{bmatrix} 0 & F^{iT}P^i \end{bmatrix} < 0 \tag{13}$$

Then using Lemma 2 to (13), we can get

$$\begin{bmatrix} -\phi P^i & A^{iT}P^i \\ P^i A^i & -P^i \end{bmatrix} + \begin{bmatrix} H_a^{iT} \\ 0 \end{bmatrix} G^{iT} \begin{bmatrix} 0 & F^{iT}P^i \end{bmatrix} + \begin{bmatrix} 0 \\ P^i F^i \end{bmatrix} G^i \begin{bmatrix} H_a^i & 0 \end{bmatrix} < 0 \tag{14}$$

Using Lemma 1 to (14), we can get

$$A_i^T(\Delta)P^i A_i(\Delta) - \phi P^i < 0 \tag{15}$$

Form (15), we know

$$V^i(\bar{x}(k,l)) \leq \phi V^i(x(k,l)) \tag{16}$$

The equality holds only if $V^i(\bar{x}(k,l)) = V^i(x(k,l)) = 0$.

It follows from (16) that

$$\sum_{k+l=N+1} V^i(k,l) \leq \phi \sum_{k+l=N} V^i(k,l) \leq \phi^{N-N_0+1} \sum_{k+l=N_0} V^i(k,l) \tag{17}$$

Now, let $n = N_\delta(j,D)$ denote the switching number of $\delta(\cdot)$ on an interval $[j,D)$, and let $m_{k-n+1} < m_{k-n+2} < \cdots < m_{k-1} < m_k$ denote the switching points of $\delta(\cdot)$ over the interval $[j,D)$, thus, for $D \in [m_k, m_{k+1})$, we have from (16)

$$\sum_{k+l=D} V^{\delta(m_k)}(k,l) < \phi^{D-m_k} \sum_{k+l=m_k} V^{\delta(m_k)}(k,l) \tag{18}$$

Using (11) and (12), at switching instant $m_k = k + l$, we have

$$\sum_{k+l=m_k} V^{\delta(m_k)}(k,l) \leq \mu \sum_{k+l=m_k} V^{\delta(m_{k-1})}(k,l) \tag{19}$$

Also, according to Definition 3, it follows that

$$n = N_\delta(\mathrm{j}, D) \leq N_0 + \frac{D-j}{\tau_a} \tag{20}$$

Therefore, the following inequality can be obtained easily:

$$\begin{aligned}&\sum_{k+l=D} V^{\delta(m_k)}(k,l) < \phi^{D-m_k} \sum_{k+l=m_k} V^{\delta(m_k)}(k,l) \leq \mu\phi^{D-m_k} \sum_{k+l=m_k^-} V^{\delta(m_{k-1})}(k,l) < \mu\phi^{D-m_k} \sum_{k+l=m_{k-1}} V^{\delta(m_{k-1})}(k,l)\phi^{m_k-m_{k-1}} \leq \cdots \\ &< \mu^{n-1}\phi^{D-m_{k-n+1}} \sum_{k+l=m_{k-n+1}} V^{\delta(m_{k-n+1})}(k,l) \leq \mu^n\phi^{D-m_{k-n+1}} \sum_{k+l=m_{k-n+1}^-} V^{\delta(j)}(k,l) < \mu^n\phi^{D-j} \sum_{k+l=j} V^{\delta(j)}(k,l)\end{aligned} \tag{21}$$

Combining (20), inequality (21) can be written as follows:

$$\sum_{k+l=D} V^{\delta(m_k)}(k,l) \leq e^{-\left(-\frac{\ln\mu}{\tau_a}-\ln\phi\right)(D-j)} \sum_{k+l=j} V^{\delta(j)}(k,l) \tag{22}$$

There exist two positive constants a and b *(*$a \leq b$*)* such that

$$\sum_{k+l=D} \|x(k,l)\|^2 \leq \frac{b}{a} e^{-\left(-\frac{\ln\mu}{\tau_a}-\ln\phi\right)(D-j)} \sum_{k+l=j} \|x(k,l)\|^2 \tag{23}$$

By Definition 1, we know that if $-\frac{\ln\mu}{\tau_a} - \ln\phi > 0$, that is $\tau_a > \tau_a^* = \frac{\ln\mu}{-\ln\phi}$, the 2D discrete switched system is exponentially stable.

The proof is completed.

Remark 3. Note that when $\mu = 1$ in (10), (11) turns out to be $P^i = P^j, \forall i,j \in N$. In this case, we have $\tau_a > \tau_a^* = \frac{\ln\mu}{-\ln\phi}$, which means that the switching signal can be arbitrary.

5 Robust H_∞ Performance Analysis

This section focuses on the Robust H_∞ stabilization problem for a class of discrete-time 2D switched systems represented by a Roesser model with uncertainties. The following theorem presents sufficient conditions that can guarantee that system (1) is exponentially stable and has a prescribed weighted H_∞ disturbance attenuation level γ.

Theorem 2. For given positive scalars γ and $\phi < 1$, there exist symmetric and positive-definite matrices $P^i \in R^{n\times n}, i \in N$, and two positive scalars $\varepsilon_1, \varepsilon_2$,such that

$$\begin{bmatrix} -\phi P^i & 0 & A^{iT}P^i & 0 & C^{iT} & \varepsilon_1 H^{iT} \\ 0 & -\gamma^2 & B^{iT}P^i & 0 & D^{iT} & 0 \\ P^iA^i & P^iB^i & -P^i & P^iF^i & 0 & 0 \\ 0 & 0 & F^iP^i & -\varepsilon_2 I & 0 & 0 \\ C^i & D^i & 0 & 0 & -I & 0 \\ \varepsilon_1 H^i & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0 \tag{24}$$

$$\varepsilon_1 \geq \varepsilon_2 \tag{25}$$

Then, 2D switched system (1) is exponentially stable and has a prescribed weighted H_∞ disturbance attenuation level γ for any switching signals with average dwell time satisfying (10), where $\mu \geq 1$ satisfies (11).

Proof. It is an obvious fact that (24) implies that inequality (8) holds. By Lemma 2, we can find that system (1) is exponentially stable when $w(k,l)=0$. Now we are able to prove that system (1) has a prescribed weighted H_∞ performance γ for any nonzero $w(k,l) \in l_2\{[0,\infty),[0,\infty)\}$.

To establish the weighted H_∞ performance, we choose the same Lyapunov functional candidate as in (12) for the system (1). Following the proof line of Theorem 1, we can get

$$V^i(\bar{x}(k,l)) \leq \phi V^i(x(k,l)) + \gamma^2 w^T w - z^T z$$

if

$$\Phi = \begin{bmatrix} A^{iT}(\Delta)P^iA_i(\Delta) - \phi P^i + C^{iT}C^i & A^{iT}(\Delta)P^iB^i + C^{iT}D^i \\ B^{iT}P^iA^i(\Delta) + D^{iT}C^i & B^{iT}P^iB^i - \gamma^2 + D^{iT}D^i \end{bmatrix} < 0 \tag{26}$$

Using Lemma 1 to (26), we can get the equivalent inequality as follows:

$$\begin{bmatrix} -\phi P^i + C^{iT}C^i & C^{iT}D^i & A^{iT}(\Delta) \\ D^{iT}C^i & -\gamma^2 + D^{iT}D^i & B^{iT} \\ A^i(\Delta) & B^i & -(P^i)^{-1} \end{bmatrix} < 0$$

$$\Leftrightarrow \begin{bmatrix} -\phi P^i + C^{iT}C^i & C^{iT}D^i & A^{iT} \\ D^{iT}C^i & -\gamma^2 + D^{iT}D^i & B^{iT} \\ A^i & B^i & -(P^i)^{-1} \end{bmatrix} + \begin{bmatrix} H_a^{iT} \\ 0 \\ 0 \end{bmatrix} G^{iT} \begin{bmatrix} 0 & 0 & F^{iT} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ F^i \end{bmatrix} G^i \begin{bmatrix} H_a^i & 0 & 0 \end{bmatrix} < 0 \tag{27}$$

Pre- and post-multiplying (24) by $diag\{\begin{matrix} I & I & (P^i)^{-1} & I & I & I \end{matrix}\}$, we obtain

$$\begin{bmatrix} -\phi P^i & 0 & A^{iT} & 0 & C^{iT} & \varepsilon_1 H^{iT} \\ 0 & -\gamma^2 & B^{iT} & 0 & D^{iT} & 0 \\ A^i & B^i & -(P^i)^{-1} & F^i & 0 & 0 \\ 0 & 0 & F^i & -\varepsilon_2 I & 0 & 0 \\ C^i & D^i & 0 & 0 & -I & 0 \\ \varepsilon_1 H^i & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0 \tag{28}$$

Using Lemma 1 to (28), we can get

$$\begin{bmatrix} -\phi P^i + C^{iT}C^i + \varepsilon_1 H_a^{iT}H_a^i & C^{iT}D^i & A^{iT} \\ D^{iT}C^i & -\gamma^2 + D^{iT}D^i & B^{iT} \\ A^i & B^i & -(P^i)^{-1} + \frac{1}{\varepsilon_2}F^iF^{iT} \end{bmatrix} < 0 \tag{29}$$

Then using Lemma 2, we find that (29) is equivalent to (27).

Thus it can be obtained from (24) that

$$V^i(\bar{x}(k,l)) - \phi V^i(x(k,l)) + \gamma^2 w^T w - z^T z < 0 \tag{30}$$

Then we have

$$V^i(\bar{x}(k,l)) \leq \phi V^i(x(k,l)) + \gamma^2 w^T w - z^T z \tag{31}$$

Let

$$\mathbb{F}(k+l) = \|z\|_2^2 - \gamma^2\|w\|_2^2 = \left\| \begin{matrix} z^h(k,l) \\ z^v(k,l) \end{matrix} \right\|_2^2 - \gamma^2 \left\| \begin{matrix} w^h(k,l) \\ w^v(k,l) \end{matrix} \right\|_2^2 \tag{32}$$

Summing up both sides of (31) from (D-1) to 0 with respect to l and 0 to (D-1) with respect to k, respectively, and applying the zero-boundary condition, one gets

$$
\begin{aligned}
&\sum_{k+l=D} V^{\delta(m_k)}(k,l) < \phi^{D-1} \sum_{k+l=D-1} V^{\delta(m_k)}(k,l) - \sum_{k+l=D-1} \mathbb{F}(k,l) < \phi^{D-m_k} \sum_{k+l=m_k} V^{\delta(m_k)}(k,l) - \sum_{m=m_k}^{D-1} \sum_{k+l=m} \phi^{D-1-k-l} \mathbb{F}(k,l) \\
&\leq \mu\phi^{D-m_k} \sum_{k+l=m_k^-} V^{\delta(m_{k-1})}(k,l) - \sum_{m=m_k}^{D-1} \sum_{k+l=m} \phi^{D-1-k-l} \mathbb{F}(k,l) \\
&< \mu\phi^{D-\left(m_k^- -1\right)} \sum_{k+l=m_k^- -1} V^{\delta(m_{k-1})}(k,l) - \mu\phi^{D-m_k} \sum_{k+l=m_k^- -1} \mathbb{F}(k,l) - \sum_{m=m_k}^{D-1} \sum_{k+l=m} \phi^{D-1-k-l} \mathbb{F}(k,l) \\
&< \mu^{N_\delta(k+l,\, D)} \phi^{D-\left(m_k^- -1\right)} \sum_{k+l=m_k^- -1} V^{\delta(m_{k-1})}(k,l) - \mu^{N_\delta(k+l,\, D)} \sum_{m=m_k^- -1}^{D-1} \sum_{k+l=m} \phi^{D-1-k-l} \mathbb{F}(k,l) \\
&< \mu^{N_\delta(k+l,\, D)} \phi^{D-\left(m_k^- -1\right)} \sum_{k+l=m_{k-1}} V^{\delta(m_{k-1})}(k,l) - \mu^{N_\delta(k+l,\, D)} \sum_{m=m_{k-1}}^{D-1} \sum_{k+l=m} \phi^{D-2-k-l} \mathbb{F}(k,\ l) \\
&\leq \mu^{N_\delta(k+l-1,\, D)} \phi^{D-m_{k-1}} \sum_{k+l=m_{k-1}^-} V^{\delta(m_{k-2})}(k,l) - \sum_{m=m_{k-1}^-}^{D-1} \sum_{k+l=m} \mu^{N_\delta(k+l+1,\, D)} \phi^{D-1-k-l} \mathbb{F}(k,l) \\
&\cdots \\
&< \sum_{k+l=0} \mu^{N_\delta(k+l,\, D)} \phi^D V^{\delta(1)}(k,l) - \sum_{m=0}^{D-1} \sum_{k+l=m} \mu^{N_\delta(k+l,\, D)} \phi^{D-1-k-l} \mathbb{F}(k,l)
\end{aligned} \tag{33}
$$

Under the zero-initial condition, we have

$$\sum_{k+l=0} \mu^{N_\delta(k+l,\, D)} \phi^D V^{\delta(1)}(k,l) = 0 \tag{34}$$

Thus, we have

$$\sum_{m=0}^{D-1} \sum_{k+l=m} \mu^{N_\delta(k+l,\, D)} \phi^{D-1-k-l} \mathbb{F}(k,l) < -\sum_{k+l=D} V^{\delta(m_k)}(k,l) \tag{35}$$

Multiplying both sides of (35) by $\mu^{-N_\delta(0,\, D)}$, we can get the following inequality:

$$\sum_{m=0}^{D-1} \sum_{k+l=m} \mu^{-N_\delta(0,\, k+l)} \phi^{D-1-k-l} \|z\|_2^2 < \gamma^2 \sum_{m=0}^{D-1} \sum_{k+l=m} \mu^{-N_\delta(0,\, k+l)} \phi^{D-1-k-l} \|w\|_2^2 \tag{36}$$

Noting $N_\delta(0, k+l) \leq \frac{k+l}{\tau_a}$, and using (10), we have

$$\mu^{-N_\delta(0,\, k+l)} = e^{\mu^{-N_\delta(0,\, k+l)} \ln \mu} \geq e^{(k+l) \ln \phi} \tag{37}$$

Thus

$$
\begin{aligned}
&\sum_{m=0}^{D-1} \sum_{k+l=m} e^{(k+l) \ln \phi} \phi^{D-1-k-l} \|z\|_2^2 < \gamma^2 \sum_{m=0}^{D-1} \sum_{k+l=m} \mu^{-N_\delta(0,k+l)} \phi^{D-1-k-l} \|w\|_2^2 \\
&\Rightarrow \sum_{m=0}^{D-1} \sum_{k+l=m} \phi^{D-1} \|z\|_2^2 < \gamma^2 \sum_{m=0}^{D-1} \sum_{k+l=m} \phi^{D-1-k-l} \|w\|_2^2 \\
&\Rightarrow \sum_{m=0}^{D-1} \sum_{k+l=m} \phi^{D-1-k-l} \phi^{k+l} \|z\|_2^2 < \gamma^2 \sum_{m=0}^{D-1} \sum_{k+l=m} \phi^{D-1-k-l} \|w\|_2^2 \\
&\Rightarrow \sum_{m=0}^{\infty} \sum_{k+l=m} \phi^{k+l} \|z\|_2^2 < \gamma^2 \sum_{m=0}^{\infty} \sum_{k+l=m} \|w\|_2^2 \\
&\Rightarrow \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \phi^{k+l} \|z\|_2^2 < \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \gamma^2 \|w\|_2^2
\end{aligned} \tag{38}
$$

According to Definition 3, we can see that system (1) is exponentially stable and has a prescribed weighted robust H_∞ disturbance attenuation level γ.

The proof is completed.

6 Robust H_∞ Control Problem

This subsection will deal with the Robust H_∞ control problem of 2D switched systems via dynamic output feedback. Our purpose is to design a dynamic output feedback controller such that the closed-loop system is exponentially stable and has a specified robust weighted H_∞ disturbance attenuation level γ.

Consider the following discrete 2D switched systems in the Roesser model with uncertainties:

$$\begin{aligned}\bar{x}(k,l) &= A^{1\delta(\mathrm{k,l})}(\Delta)x(k,l) + B^{1\delta(\mathrm{k,l})}w(k,l) + B^{2\delta(\mathrm{k,l})}u(k,l)\\ z(k,l) &= C^{1\delta(\mathrm{k,l})}x(k,l) + D^{11\delta(\mathrm{k,l})}w(k,l) + D^{12\delta(\mathrm{k,l})}u(k,l), k,l = 0,1,2\cdots\\ \mathrm{y}(k,l) &= C^{2\delta(\mathrm{k,l})}x(k,l) + D^{21\delta(\mathrm{k,l})}w(k,l)\end{aligned} \tag{39}$$

where $x(k,l) \in R^n, w(k,l) \in R^{n_w}, u(k,l) \in R^u, z(k,l) \in R^z$ and $y(k,l) \in R^y$ are, respectively, the state, the disturbance input, the control input, the controlled output, and the measurement output of the plant, k and l are integers in Z_+. ${A_1}^i, {B_1}^i, {B_2}^i, {C_1}^i, {C_2}^i, {D_{11}}^i, {D_{12}}^i$ and ${D_{21}}^i$ with $i \in N$ are constant matrices with appropriate dimensions. We do not assume the disturbance input signal's statistics $w(k,l)$ other than that its energy is bounded, i.e., $\|w\|_2 < \infty$.

Introduce the following output feedback controller of order n_c:

$$\begin{aligned}\bar{x}_c(k,l) &= {A_c}^{\delta(\mathrm{k,l})}x_c(k,l) + {B_c}^{\delta(\mathrm{k,l})}y(k,l)\\ u(k,l) &= {C_c}^{\delta(\mathrm{k,l})}x_c(k,l) + {D_c}^{\delta(\mathrm{k,l})}y(k,l)\end{aligned} \tag{40}$$

where

$$\bar{x}_c(k,l) = \begin{bmatrix} x_c^h(k+1,l)\\ x_c^v(k,l+1)\end{bmatrix}, x_c(k,l) = \begin{bmatrix} x_c^h(k,l)\\ x_c^v(k,l)\end{bmatrix}$$

The closed-loop system consisting of the plant (39) and the controller (40) is of the form

$$\begin{aligned}\dot{\hat{x}}(k,l) &= \bar{A}^{\delta(\mathrm{k,l})}\hat{x}(k,l) + \bar{B}^{\delta(\mathrm{k,l})}w(k,l)\\ \bar{\mathrm{z}}(k,l) &= \bar{C}^{\delta(\mathrm{k,l})}\hat{x}(k,l) + \bar{D}^{\delta(\mathrm{k,l})}w(k,l)\end{aligned} \tag{41}$$

with $\dot{\hat{x}}(k,l) = \begin{bmatrix}\bar{x}(k,l)\\ \bar{x}_c(k,l)\end{bmatrix}, \hat{x}(k,l) = \begin{bmatrix} x(k,l)\\ x_c(k,l)\end{bmatrix}$ and

$$\begin{aligned}\tilde{A}^{\delta(\mathrm{k,l})} &= \begin{bmatrix} {A_1}^{\delta(\mathrm{k,l})}(\Delta) + {B_2}^{\delta(\mathrm{k,l})}{D_c}^{\delta(\mathrm{k,l})}{C_2}^{\delta(\mathrm{k,l})} & {B_2}^{\delta(\mathrm{k,l})}{C_c}^{\delta(\mathrm{k,l})}\\ {B_c}^{\delta(\mathrm{k,l})}{C_2}^{\delta(\mathrm{k,l})} & {A_c}^{\delta(\mathrm{k,l})}\end{bmatrix}\\ &= \begin{bmatrix} {A_1}^{\delta(\mathrm{k,l})} + {B_2}^{\delta(\mathrm{k,l})}{D_c}^{\delta(\mathrm{k,l})}{C_2}^{\delta(\mathrm{k,l})} & {B_2}^{\delta(\mathrm{k,l})}{C_c}^{\delta(\mathrm{k,l})}\\ {B_c}^{\delta(\mathrm{k,l})}{C_2}^{\delta(\mathrm{k,l})} & {A_c}^{\delta(\mathrm{k,l})}\end{bmatrix} + \begin{bmatrix} F^{\delta(\mathrm{k,l})}\\ 0\end{bmatrix}\mathrm{G}^{\delta(\mathrm{k,l})}\begin{bmatrix} H^{\delta(\mathrm{k,l})} & 0\end{bmatrix}\\ \bar{B}^{\delta(\mathrm{k,l})} &= \begin{bmatrix} {B_2}^{\delta(\mathrm{k,l})}{D_c}^{\delta(\mathrm{k,l})}{D_{21}}^{\delta(\mathrm{k,l})} + {B_1}^{\delta(\mathrm{k,l})}\\ {B_c}^{\delta(\mathrm{k,l})}{D_{21}}^{\delta(\mathrm{k,l})}\end{bmatrix}\end{aligned}$$

$$\bar{C}^{\delta(\mathrm{k,l})} = \begin{bmatrix} {C_1}^{\delta(\mathrm{k,l})} + {D_{12}}^{\delta(\mathrm{k,l})}{D_c}^{\delta(\mathrm{k,l})}{C_2}^{\delta(\mathrm{k,l})} & {D_{12}}^{\delta(\mathrm{k,l})}{C_c}^{\delta(\mathrm{k,l})}\end{bmatrix}, \bar{D}^{\delta(\mathrm{k,l})} = {D_{11}}^{\delta(\mathrm{k,l})} + {D_{12}}^{\delta(\mathrm{k,l})}{D_c}^{\delta(\mathrm{k,l})}{D_{21}}^{\delta(\mathrm{k,l})}$$

where

$$\bar{A}^{\delta(\mathrm{k,l})} = \begin{bmatrix} {A_1}^{\delta(\mathrm{k,l})} + {B_2}^{\delta(\mathrm{k,l})}{D_c}^{\delta(\mathrm{k,l})}{C_2}^{\delta(\mathrm{k,l})} & {B_2}^{\delta(\mathrm{k,l})}{C_c}^{\delta(\mathrm{k,l})}\\ {B_c}^{\delta(\mathrm{k,l})}{C_2}^{\delta(\mathrm{k,l})} & {A_c}^{\delta(\mathrm{k,l})}\end{bmatrix}, \bar{F}^{\delta(\mathrm{k,l})} = \begin{bmatrix} F^{\delta(\mathrm{k,l})}\\ 0\end{bmatrix}, \bar{H}^{\delta(\mathrm{k,l})} = \begin{bmatrix} H^{\delta(\mathrm{k,l})} & 0\end{bmatrix}.$$

For the closed-loop system (41), we state the 2D Robust H_∞ control problem as finding a 2D dynamic output feedback controller of the form in (40) for the 2D systems (39) such that the closed-loop system (41) has a specified weighted robust H_∞ disturbance attenuation level γ. The controller design procedure is provided in the following theorem.

Theorem 3. For given positive scalars $\bar{\phi} < 1$ and γ, if there exist symmetric positive definite matrices $M_{11}^i > 0, \bar{M}_{11}^i > 0$, two positive scalars $\bar{\varepsilon}_2, \beta$ and appropriate dimensions matrices $D_c{}^i, Z^i, \Phi^i, \Gamma^i, i \in N$ such that

$$\begin{bmatrix} -\bar{\phi} Y_M^i & 0 & Y_A^i & 0 & Y_C^i & Y_H^i \\ 0 & -\gamma^2 & Y_B^i & 0 & \bar{D}^{iT} & 0 \\ Y_A^{iT} & Y_B^{iT} & -Y_M^i & Y_F^i & 0 & 0 \\ 0 & 0 & Y_F^{iT} & -\bar{\varepsilon}_2 I & 0 & 0 \\ Y_C^{iT} & \bar{D}^i & 0 & 0 & -I & 0 \\ Y_H^{iT} & 0 & 0 & 0 & 0 & -\beta I \end{bmatrix} < 0 \tag{42}$$

$$\frac{1}{\beta} > \bar{\varepsilon}_2 \tag{43}$$

with

$$Y_M^i = \begin{bmatrix} \bar{M}_{11}^i & I \\ I & M_{11}^i \end{bmatrix}, Y_A^i = \begin{bmatrix} A^{iT}\bar{M}_{11}^i + C_2{}^{iT}\Gamma & A^{iT} + C_2{}^{iT} D_c{}^{iT} B_2{}^{iT} \\ \Phi & M_{11}^i A^{iT} + Z B_2{}^{iT} \end{bmatrix}, Y_C^i = \begin{bmatrix} \left(C_1{}^i + D_{12}{}^i D_c{}^i C_2{}^i\right)^T \\ M_{11}^i C_1{}^{iT} + Z D_{12}{}^{iT} \end{bmatrix}, Y_H^i = \begin{bmatrix} \bar{H}^{iT} \\ M_{11}^i \bar{H}^{iT} \end{bmatrix}$$

$$Y_B^i = \begin{bmatrix} B_1{}^{iT}\bar{M}_{11}^i + D_{21}{}^{iT}\Gamma & (B_2{}^i D_c{}^i D_{21}{}^i + B_1{}^i)^T \end{bmatrix}, Y_F^i = \begin{bmatrix} \bar{M}_{11}^i F^i \\ F^i \end{bmatrix}$$

then 2D switched closed-loop system (41) is exponentially stable and has a prescribed weighted robust H_∞ disturbance attenuation level γ for any switching signals with the average dwell time satisfying

$$\tau_a > \tau_a^* = \frac{\ln \mu}{-\ln \bar{\phi}} \tag{44}$$

where $M_{12}^i \bar{M}_{12}^{iT} = I - M_{11}^i \bar{M}_{11}^i, M_{11}^i \bar{M}_{12}^i + M_{12}^i \bar{M}_{22}^i = 0, M_{12}^{iT} \bar{M}_{12}^i + M_{22}^i \bar{M}_{12}^{iT} = 0$ and $\mu \geq 1$ satisfies

$$\begin{bmatrix} M_{11}^i & M_{12}^i \\ M_{12}^{iT} & M_{22}^i \end{bmatrix} < \mu \begin{bmatrix} M_{11}^j & M_{12}^j \\ M_{12}^{jT} & M_{22}^j \end{bmatrix} \tag{45}$$

and the controller parameters can be obtained as follows:

$$\begin{aligned} C_c{}^i &= \left(Z^{iT} - D_c{}^i C_2{}^i M_{11}^i\right)\left(M_{12}^{iT}\right)^{-1}, B_c{}^i = \bar{M}_{12}^i\left(\Gamma^{iT} - \bar{M}_{11}^i B_2{}^i D_c{}^i\right) \\ A_c{}^i &= \left(\left(\bar{M}_{12}^i\right)^{-1}\left(\Phi^{iT} - \bar{M}_{11}^i\left(\left(A^i + B_2{}^i D_c{}^i C_2{}^i\right) M_{11}^i + B_2{}^i C_c{}^i M_{12}^{iT}\right)\right) - B_c{}^i C_2{}^i M_{11}^i\right)\left(M_{12}^{iT}\right)^{-1} \end{aligned} \tag{46}$$

Proof. Given Theorem 2 to the closed-loop system (41), the controller solves the 2D switched robust H_∞ control problem if the following matrix inequalities hold

$$\begin{bmatrix} -\bar{\phi} X^i & 0 & \bar{A}^{iT} X^i & 0 & \bar{C}^{iT} & \bar{\varepsilon}_1 \bar{H}^{iT} \\ 0 & -\gamma^2 & \bar{B}^{iT} X^i & 0 & \bar{D}^{iT} & 0 \\ X^i \bar{A}^i & X^i \bar{B}^i & -X^i & X^i \bar{F}^i & 0 & 0 \\ 0 & 0 & \bar{F}^i X^i & -\bar{\varepsilon}_2 I & 0 & 0 \\ \bar{C}^i & \bar{D}^i & 0 & 0 & -I & 0 \\ \bar{\varepsilon}_1 \bar{H}^i & 0 & 0 & 0 & 0 & -\bar{\varepsilon}_1 I \end{bmatrix} < 0 \tag{47}$$

Pre- and post-multiplying (47) by $diag\left\{ \begin{matrix} (X^i)^{-1} & I & (X^i)^{-1} & I & I & \frac{1}{\bar{\varepsilon}_1}I \end{matrix} \right\}$ leads to

$$\begin{bmatrix} -\bar{\phi}(X^i)^{-1} & 0 & (X^i)^{-1}\bar{A}^{iT} & 0 & (X^i)^{-1}\bar{C}^{iT} & (X^i)^{-1}\bar{H}^{iT} \\ 0 & -\gamma^2 & \bar{B}^{iT} & 0 & \bar{D}^{iT} & 0 \\ \bar{A}^i(X^i)^{-1} & \bar{B}^i & -(X^i)^{-1} & \bar{F}^i & 0 & 0 \\ 0 & 0 & \bar{F}^i & -\bar{\varepsilon}_2 I & 0 & 0 \\ \bar{C}^i(X^i)^{-1} & \bar{D}^i & 0 & 0 & -I & 0 \\ \bar{H}^i(X^i)^{-1} & 0 & 0 & 0 & 0 & -\frac{1}{\bar{\varepsilon}_1}I \end{bmatrix} < 0 \tag{48}$$

Definite $M^i = (X^i)^{-1}, \beta = \frac{1}{\bar{\varepsilon}_1}$, we can obtain

$$\begin{bmatrix} -\bar{\phi}M^i & 0 & M^i\bar{A}^{iT} & 0 & M^i\bar{C}^{iT} & \bar{\varepsilon}_1 M^i\bar{H}^{iT} \\ 0 & -\gamma^2 & \bar{B}^{iT} & 0 & \bar{D}^{iT} & 0 \\ \bar{A}^i M^i & \bar{B}^i & -M^i & \bar{F}^i & 0 & 0 \\ 0 & 0 & \bar{F}^i & -\bar{\varepsilon}_2 I & 0 & 0 \\ \bar{C}^i M^i & \bar{D}^i & 0 & 0 & -I & 0 \\ \bar{\varepsilon}_1\bar{H}^i M^i & 0 & 0 & 0 & 0 & -\beta I \end{bmatrix} < 0 \tag{49}$$

Partition M^i and $(M^i)^{-1}$ as

$$M^i = \begin{bmatrix} M_{11}^i & M_{12}^i \\ M_{12}^{iT} & M_{22}^i \end{bmatrix}, \left(M^i\right)^{-1} = \begin{bmatrix} \bar{M}_{11}^i & \bar{M}_{12}^i \\ \bar{M}_{12}^{iT} & \bar{M}_{22}^i \end{bmatrix} \tag{50}$$

It is easy to show from (50) that $M_{12}^i\bar{M}_{12}^{iT} = I - M_{11}^i\bar{M}_{11}^i$.

set

$$J^i = \begin{bmatrix} \bar{M}_{11}^i & I \\ \bar{M}_{12}^{iT} & 0 \end{bmatrix}, \bar{J}^i = \begin{bmatrix} I & M_{11}^i \\ 0 & M_{12}^{iT} \end{bmatrix}$$

Then it follows that $M^iJ^i = \bar{J}^i, J^{iT}M^iJ^i = \begin{bmatrix} \bar{M}_{11}^i & I \\ I & M_{11}^i \end{bmatrix} > 0$ and

$$Z^i = M_{11}^i\left(D_c{}^iC_2{}^i\right)^T + M_{12}^iC_c{}^{iT}, \Gamma^i = D_c{}^{iT}B_2{}^{iT}\bar{M}_{11}^i + B_c{}^{iT}\bar{M}_{12}^{iT}$$
$$\Phi^i = \left(M_{11}^i\left(A^i + B_2{}^iD_c{}^iC_2{}^i\right)^T + M_{12}^i\left(B_2{}^iC_c{}^i\right)^T\right)\bar{M}_{11}^i + \left(M_{11}^i\left(B_c{}^iC_2{}^i\right)^T + M_{12}^iA_c{}^{iT}\right)\bar{M}_{12}^{iT}$$

Pre- and post-multiplying (49) $diag\left\{ \begin{matrix} J^{iT} & I & J^{iT} & I & I & I \end{matrix} \right\}$ and $diag\left\{ \begin{matrix} J^i & I & J^i & I & I & I \end{matrix} \right\}$, respectively, we have

$$\begin{bmatrix} -\bar{\phi}J^{iT}M^iJ^i & 0 & J^{iT}M^i\bar{A}^{iT}J^i & 0 & J^{iT}M^i\bar{C}^{iT} & J^{iT}M^i\bar{H}^{iT} \\ 0 & -\gamma^2 & \bar{B}^{iT}J^i & 0 & \bar{D}^{iT} & 0 \\ J^{iT}\bar{A}^iM^iJ^i & J^{iT}\bar{B}^i & -J^{iT}M^iJ^i & J^{iT}\bar{F}^i & 0 & 0 \\ 0 & 0 & \bar{F}^iJ^i & -\bar{\varepsilon}_2 I & 0 & 0 \\ \bar{C}^iM^iJ^i & \bar{D}^i & 0 & 0 & -I & 0 \\ \bar{H}^iM^iJ^i & 0 & 0 & 0 & 0 & -\beta I \end{bmatrix} < 0 \tag{51}$$

with

$$J^{iT}M^iJ^i = \begin{bmatrix} \bar{M}_{11}^i & I \\ I & M_{11}^i \end{bmatrix}, J^{iT}M^i\bar{C}^{iT} = \begin{bmatrix} \left(C_1{}^i + D_{12}{}^iD_c{}^iC_2{}^i\right)^T \\ M_{11}^iC_1{}^{iT} + ZD_{12}{}^{iT} \end{bmatrix}, J^{iT}M^i\bar{A}^{iT}J^i = \begin{bmatrix} A^{iT}\bar{M}_{11}^i + C_2{}^{iT}\Gamma & A^{iT} + C_2{}^{iT}D_c{}^{iT}B_2{}^{iT} \\ \Phi & M_{11}^iA^{iT} + ZB_2{}^{iT} \end{bmatrix}$$

$$\bar{\varepsilon}_1J^{iT}M^i\bar{H}_a^{iT} = \begin{bmatrix} \bar{H}^{iT} \\ M_{11}^i\bar{H}^{iT} \end{bmatrix}, \bar{B}^{iT}J^i = \begin{bmatrix} B_1{}^{iT}\bar{M}_{11}^i + D_{21}{}^{iT}\Gamma & \left(B_2{}^iD_c{}^iD_{21}{}^i + B_1{}^i\right)^T \end{bmatrix}, J^{iT}\bar{F} = \begin{bmatrix} \bar{M}_{11}^iF^i \\ F^i \end{bmatrix}$$

then we take

$$Y_M^i = J^{iT}M^iJ^i, Y_C^i = J^{iT}M^i\bar{C}^{iT}, Y_A^i = J^{iT}M^i\bar{A}^{iT}J^i, Y_H^i = J^{iT}M^i\bar{H}_a^{iT}, Y_B^i = \bar{B}^{iT}J^i, Y_F^i = J^{iT}.$$

The condition (42) can be obtained.

Remark 4. It is worth noting that in Theorems 1 and 2, the results are derived from the assumption that the switching rule is not known *a priori*, but its value is available in each sampling period. In other words, the switching sequence considered here does not include the random switching one.

In what follows, we present an algorithm for the design of a dynamic output controller.

Algorithm 1

Step 1. Given γ and $\bar{\phi}$, solve the LMI (42) to obtain matrices $M_{11}^i, \bar{M}_{11}^i, Z^i, \Phi^i, \Gamma^i, \bar{\varepsilon}_2, \beta$ and the dynamic output feedback controller parameters $D_c{}^i$,with $i \in N$.

Step 2. The invertible matrices M_{12}^i and $\bar{M}_{12}^i$ can be computed in terms of the nonsingularity of $M_{11}^i\bar{M}_{11}^i = I - M_{12}^i\bar{M}_{12}^{iT}$.

Step 3. The invertible matrices $\bar{M}_{22}^i$ and M_{22}^i also can be computed by equations $M_{11}^i\bar{M}_{12}^i + M_{12}^i\bar{M}_{22}^i = 0$ and $M_{12}^{iT}\bar{M}_{12}^i + M_{22}^i\bar{M}_{12}^{iT} = 0$.

Step 4. By solving inequality (45), the constant μ is obtained, and the average residence time τ_a can be calculated from (44).

Step 5. By solving (46), the remaining controller parameters $A_c{}^i, B_c{}^i$ and $C_c{}^i, i \in N$ can be obtained.

This completes the proof.

Remark 5. If there is only one subsystem in the system (39), it will degenerate into being a general 2D Roesser model, which is a special model of 2D switched systems. Theorem 3 is also applicable for 2D Roesser systems, which means that our results are more general than that just for 2D Roesser systems.

7 Numerical Example

In this section, we shall illustrate the results developed earlier via an example.

Subsystem 1

$$A_1^1 = \begin{bmatrix} 0.31 & 0.43 \\ 0.35 & 0.41 \end{bmatrix}, B_1^1 = \begin{bmatrix} 0.33 \\ 0.43 \end{bmatrix}, B_2^1 = \begin{bmatrix} 0.1 \\ 0.43 \end{bmatrix}, F^1 = \begin{bmatrix} 0.46 & 0 \\ 0 & 0.48 \end{bmatrix}, H^1 = \begin{bmatrix} 0.64 & 0 \\ 0 & 0.47 \end{bmatrix}$$

$$C_2^1 = [0.52 \quad 0.11], D_{21}^1 = 0.2, C_1^1 = [0.53 \quad 0.34], D_{11}^1 = 0.02, D_{12}^1 = 0.5$$

Subsystem 2

$$A_1^2 = \begin{bmatrix} 0.39 & 0.35 \\ 0.42 & 0.38 \end{bmatrix}, B_1^2 = \begin{bmatrix} 0.15 \\ 0.46 \end{bmatrix}, B_2^2 = \begin{bmatrix} 0.1 \\ 0.46 \end{bmatrix}, F^2 = \begin{bmatrix} 0.48 & 0 \\ 0 & 0.39 \end{bmatrix}, H^2 = \begin{bmatrix} 0.54 & 0 \\ 0 & 0.49 \end{bmatrix}$$

$$C_1^2 = [0.45 \quad 0.16], D_{11}^2 = 0.03, D_{12}^2 = 0.6, C_2^2 = [0.75 \quad 0.17], D_{21}^2 = 0.2$$

Taking $\bar{\phi} = 0.6, \gamma = 5$, according to Theorem 3, solving inequality (42) gives rise to the following solutions:

$$M_{11}^1 = \begin{bmatrix} 2.2703 & 0.3591 \\ 0.3591 & 2.4689 \end{bmatrix}, \bar{M}_{11}^1 = \begin{bmatrix} 1.4772 & 0.3845 \\ 0.3845 & 1.5167 \end{bmatrix}, M_{11}^2 = \begin{bmatrix} 4.2613 & -1.0701 \\ -1.0701 & 3.6358 \end{bmatrix}, \bar{M}_{11}^2 = \begin{bmatrix} 0.9256 & -0.1493 \\ -0.1493 & 1.3213 \end{bmatrix}$$

$$\beta^1 = 1.9370, \bar{\varepsilon}_2^1 = 0.4846, \beta^2 = 2.9542, \bar{\varepsilon}_2^2 = 0.2916, \Gamma^1 = \begin{bmatrix} 0.9396 & 0.9915 \end{bmatrix}, \Gamma^2 = \begin{bmatrix} -0.4323 & -0.7422 \end{bmatrix}$$

$$Z^1 = \begin{bmatrix} 2.0984 \\ 1.6106 \end{bmatrix}, \Phi^1 = \begin{bmatrix} 0.0403 & 0.0794 \\ 0.0794 & 0.1787 \end{bmatrix}, \Phi^2 = \begin{bmatrix} 0.0921 & 0.0917 \\ 0.0917 & 0.1607 \end{bmatrix}, Z^2 = \begin{bmatrix} -3.1152 \\ -0.6319 \end{bmatrix}$$

$$D_c^2 = -1.2152, D_c^1 = -2.5800$$

Then, $M_{12}^i, \bar{M}_{12}^i, \bar{M}_{22}^i$ and M_{22}^i can be computed.

$$M_{12}^1 = \begin{bmatrix} -2.7127 & 0.9273 \\ 3.1397 & 0.8012 \end{bmatrix}, \bar{M}_{12}^1 = \begin{bmatrix} 0.6625 & -0.7491 \\ -0.7491 & -0.6625 \end{bmatrix}, M_{12}^2 = \begin{bmatrix} -3.4868 & 1.2961 \\ 4.1048 & 1.1009 \end{bmatrix}, \bar{M}_{12}^2 = \begin{bmatrix} 0.5901 & -0.8073 \\ -0.8073 & -0.5901 \end{bmatrix}$$

$$M_{22}^1 = \begin{bmatrix} 2.7127 & -3.1397 \\ -0.9273 & -0.8012 \end{bmatrix}, \bar{M}_{22}^1 = \begin{bmatrix} 0.6600 & 0.0188 \\ 0.0188 & 1.6322 \end{bmatrix}, M_{22}^2 = \begin{bmatrix} 3.4868 & -4.1048 \\ -1.2961 & -1.1009 \end{bmatrix}, \bar{M}_{22}^2 = \begin{bmatrix} 0.9108 & -0.1563 \\ -0.1563 & 1.7467 \end{bmatrix}$$

The positive scalar $\mu = 3.8438$ can be obtained by solving inequality (45), then $\tau_a^* = \frac{\ln \mu}{-\ln \bar{\phi}} = 2.6359$ can be obtained from (44). And the rest of the controller parameters ${A_c}^i, {B_c}^i, {C_c}^i, i \in N$ can be obtained by solving (46).

$$A_c^1 = \begin{bmatrix} -0.1992 & -0.1156 \\ 0.1686 & 0.0933 \end{bmatrix}, A_c^2 = \begin{bmatrix} 0.0213 & 0.0082 \\ 0.1834 & 0.0779 \end{bmatrix}, B_c^1 = \begin{bmatrix} -1.0960 \\ 0.3458 \end{bmatrix}, B_c^2 = \begin{bmatrix} -0.2204 \\ 0.3385 \end{bmatrix}$$
$$C_c^1 = \begin{bmatrix} -0.3870 & -0.2203 \end{bmatrix}, C_c^2 = \begin{bmatrix} -0.1870 & -0.0806 \end{bmatrix}$$

Choosing $\tau_a = 3$, the simulation results are shown in Figs. 1–5, where the boundary condition of the system is

$x(k, l) = \frac{5}{k+1}, \forall 0 \le k \le 16, l = 0; x(k, l) = \frac{5}{l+1}, \forall 0 \le l \le 16, k = 0,$

and $w(k, l) = 0.5 \exp(-0.045\pi(k + l))$. It can be seen from Figs. 1 and 2 that the system is exponentially stable. Furthermore, when the boundary condition is zero, When $G_1^1 = G_1^2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$ by computing, we get $\sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \phi^{k+l} \|z\|_2^2 = 4.0871$ and $\sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \|w\|_2^2 = 8.2411$, and it satisfies the condition (2) in Definition 2. It can be seen that the system has a weighted robust H_∞ disturbance attenuation level $\gamma = 5$. If $G_1^1 = G_1^2 = \begin{bmatrix} 0.99 & 0 \\ 0 & 0.99 \end{bmatrix}$ by computing, we can get $\sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \phi^{k+l} \|z\|_2^2 = 4.0053$ and $\sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \|w\|_2^2 = 8.2411$, and it also can be seen from Figs. 3 and 4 that the system is exponentially stable.

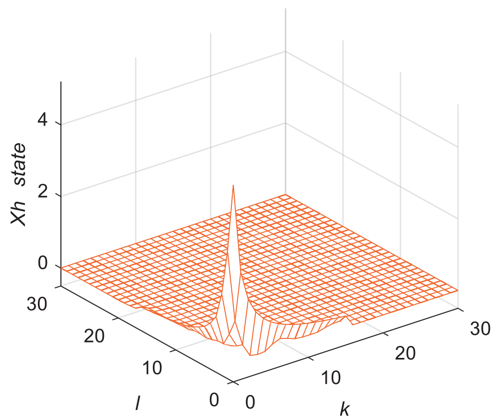

Figure 1: Response of state $x^h(k, l)$

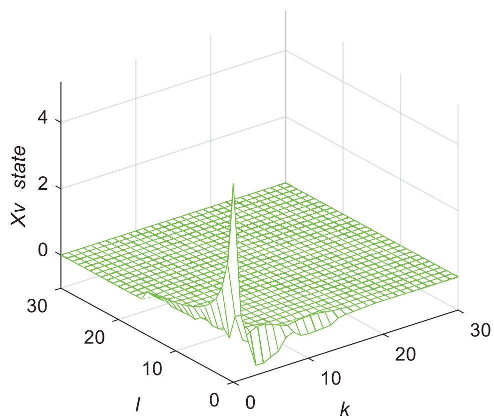

Figure 2: Response of state $x^v(k, l)$

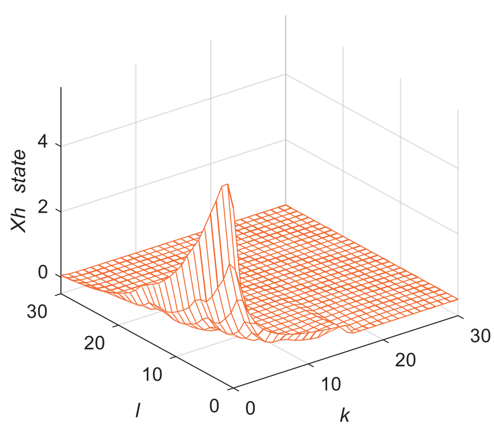

Figure 3: Response of state $x^h(k, l)$

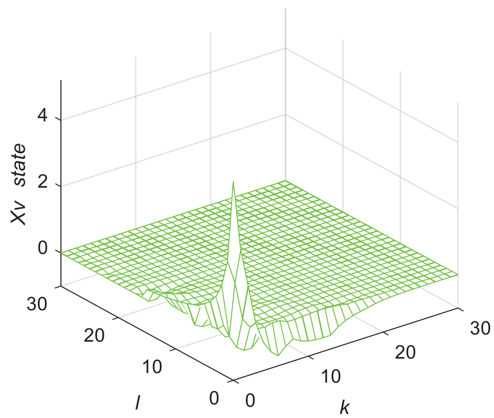

Figure 4: Response of state $x^v(k, l)$

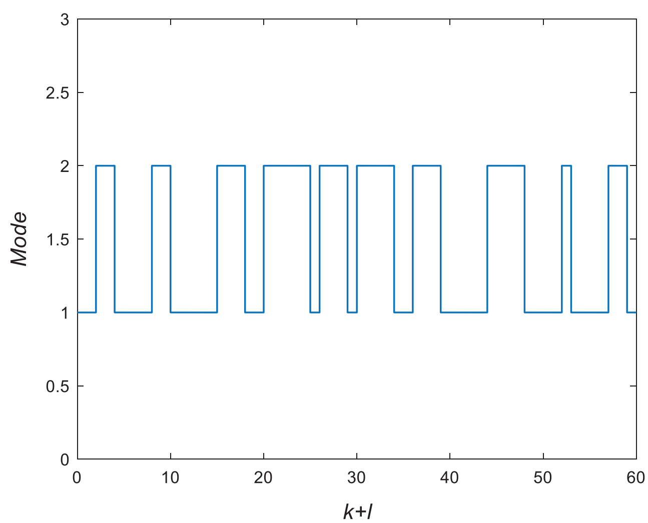

Figure 5: Switching signal

8 Conclusions

This paper has investigated the problems of stability and weighted robust H_∞ disturbance attenuation performance analyses for 2D discrete switched systems described by the Roesser model with uncertainties. An exponential stability criterion is obtained via the average dwell time method. Some sufficient conditions for the existence of weighted robust H_∞ disturbance attenuation level γ for the considered system are derived from LMIs. Besides, a 2D output feedback controller is designed to solve the robust H_∞ control problem. Finally, an example is also given to illustrate the applicability of the proposed results. The future work will be associated with the following directions: 1) stability analysis and stabilization for nonlinear continuous-time descriptor semi-Markov jump systems; and 2) finite-time stabilization for nonlinear discrete-time singular Markov jump systems with piecewise-constant transition probabilities subjected to average dwell time.

Data Availability: The authors declare that they have no conflicts of interest to report regarding the present study.

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