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Vertex-Edge Degree Based Indices of Honey Comb Derived Network

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Abstract: Chemical graph theory is a branch of mathematics which combines graph theory and chemistry. Chemical reaction network theory is a territory of applied mathematics that endeavors to display the conduct of genuine compound frameworks. It pulled the research community due to its applications in theoretical and organic chemistry since 1960. Additionally, it also increases the interest the mathematicians due to the interesting mathematical structures and problems are involved. The structure of an interconnection network can be represented by a graph. In the network, vertices represent the processor nodes and edges represent the links between the processor nodes. Graph invariants play a vital feature in graph theory and distinguish the structural properties of graphs and networks. In this paper, we determined the newly introduced topological indices namely, first ve-degree Zagreb α index, first ve-degree Zagreb β index, second ve-degree Zagreb index, ve-degree Randic index, ve-degree atom-bond connectivity index, ve-degree geometric-arithmetic index, ve-degree harmonic index and ve-degree sum-connectivity index for honey comb derived network. In the analysis of the quantitative structure property relationships (QSPRs) and the quantitative structure-activity relationships (QSARs), graph invariants are important tools to approximate and predicate the properties of the biological and chemical compounds. Also, we give the numerical and graphical representation of our outcomes.

Keywords: Honey comb derived network; *ev*-degree; topological indices

1 Introduction

A structural molecular diagram is a basic diagram in the study of structural chemical graph theory where atoms are spoken to by nodes and chemical bonds are spoken to by lines. A diagram is associated if there is an association between any pair of nodes. A network is an associated diagram that has no various lines between two nodes and loop. The number of nodes which are associated with a fixed node v is known as the degree of v and is denoted by d_v. The collection of all the adjacent nodes to the node v is referred to open neighborhood of v and can be represented by $N(v)$. The open neighborhood became the closed neighborhood when we include the node v in the collection and is represented by $N[v]$. The shortest distance between two vertices $u,v \in V(G)$ is denoted by $d(u,v)$, and the maximum value of $d(u,v)$ in G is called the diameter of G, denoted as $diam(G)$. For basic definition, see West [1].

The relation between the ($QSPR$) and ($QSAR$) predict the properties and natural exercises of unstudied material. In these materials, the topological indices and some physico-chemical properties are utilized to anticipate bioactivity for chemical compounds [2–5]. A number represents a topological index in a diagram of a chemical compound, which can be utilized to portray the underlined chemical compound and help to foresee its physio-chemical properties. In 1947 Weiner established the framework of topological index. He was approximated the breaking point of alkanes and presented the Weiner index [6]. In 1975, Milan Randic presented Randic index [7]. In 1998, Bollobas et al. [8] and Amic et al. [9] proposed the general Randic index and has been concentrated by both scientist and mathematicians [10]. The Randic index is one of the most important and generally considered and applied topological index. Numerous surveys, papers and books [11–16] are composed on graph invariant. For detail of different topological indices, see [17–22]. Chellali et al. [23] introduced two novel degree thoughts which they called "*ve*-degree and *ev*-degree". Horoldagva et al. [24] contributed to the study related to "*ve*-degree and *ev*-degree". The new style degree base indices have been applied to already existing indices and found the better results in [25–27]. It has been found that the *ve*-degree Zagreb index has more grounded estimate power than the old-style Zagreb index.

2 The *ve*-degree and *ev*-degree Based Topological Indices

Chellali et al. [23] gave the definition of *ev*-degree of an edge $e = uv \in E$ which is denoted by $d_{ev}(e)$, and is the cardinality of nodes of the union of the closed neighborhoods of u and v. The *ve*-degree of the node $v \in V$, denoted by $d_{ve}(v)$, and is the cardinality of lines of different lines that are incident to any node from the closed neighborhood of v. Throughout this paper we consider G is a connected graph, $e = uv \in E(G)$ and $v \in V$. For some basic definitions regarding "*ev*-degree and *ve*-degree topological indices" [28–30]. The topological indices related to *ev*-degree are: The *ev*-degree Zagreb index, *ev*-degree Randic index, The topological indices related to *ev*-degree are: The first *ve*-degree Zagreb α index, first *ve*-degree Zagreb β index, second *ve*-degree Zagreb index, *ve*-degree Randic index, *ve*-degree atom-bond connectivity index, *ve*-degree geometric-arithmetic index, *ve*-degree harmonic index and *ve*-degree sum-connectivity index.

3 Main Results

In the present section, we determined our computational results for Honey Comb derived network (see Fig. 1), which is a planar graph. The number of nodes and lines in $HcDN1(n)$ are $9n^2 - 3n + 1$ and $27n^2 - 21n + 6$ respectively.

There are five types of lines in $HcDN1(n)$ based on degrees of end nodes of each line. Tab. 1 shows line partition of $HcDN1(n)$. Tab. 2 represents the number of nodes corresponding their degrees.

In Tab. 3, We partition the lines, based on *ev*-degree of the $HcDN1$. In Tabs. 4 and 5, we partition the nodes, based on *ve*-degree of $HcDN1$.

4 Computing Indices for $HcDN1$ Formulas

In this section, we will calculate *ev*-degree and *ve*-degree based indices of the different types of indices which are given as under;

- $ev-$degree Zagreb Index

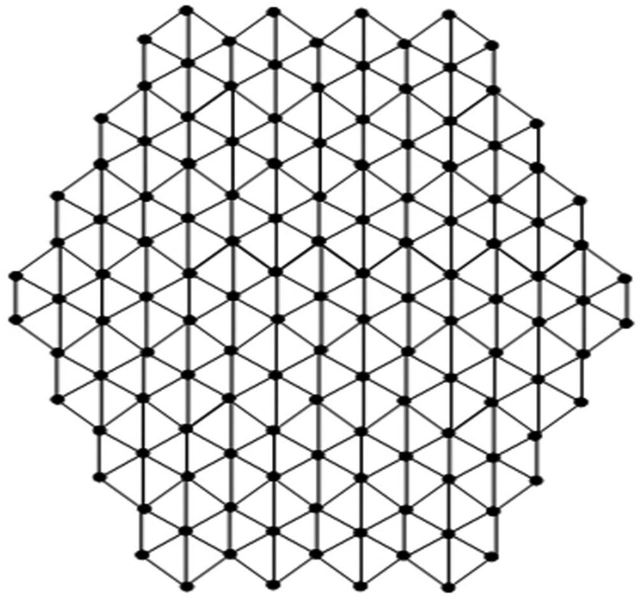

Figure 1: $HcDN1(n)$ network with $n = 4$

Table 1: Line partition $HcDN1$

$(d(u), d(v))$	Number of lines
$(3, 3)$	6
$(3, 5)$	$12(n - 1)$
$(3, 6)$	$6n$
$(5, 6)$	$18(n - 1)$
$(6, 6)$	$27n^2 - 57n + 30$

Table 2: Number of nodes with corresponding degrees

$d(u)$	Number of nodes
3	$6n$
5	$6(n - 1)$
6	$9n^2 - 15n + 7$

Table 3: Line partition of $HcDN1$

Number of lines	Degree of its end nodes	*ev*-degrees
6	$(3, 3)$	6
$12(n - 1)$	$(3, 5)$	8
$6n$	$(3, 6)$	9
$18(n - 1)$	$(5, 6)$	11
$27n^2 - 57n + 30$	$(6, 6)$	12

Table 4: Node partition of $HcDN1$

Number of nodes	*ve*-degrees
12	12
$6(n-2)$	14
$6(n-1)$	20
6	22
$6(n-2)$	25
$6(n-1)$	29
$9n^2-27n+19$	30

Table 5: The *ve*-degree of the end nodes of lines of $HcDN1$

Number of lines	*ve*-degrees of its end nodes
6	$(12, 12)$
12	$(12, 20)$
12	$(12, 22)$
12	$(20, 22)$
12	$(22, 29)$
6	$(29, 29)$
$6(3n-5)$	$(29, 30)$
$6(n-1)$	$(20, 29)$
$12(n-2)$	$(14, 20)$
$6(n-2)$	$(14, 25)$
$12(n-2)$	$(20, 25)$
$12(n-2)$	$(25, 29)$
$6(n-2)$	$(25, 30)$
$27n^2-93n+78$	$(30, 30)$

Now with the help of Tab. 3, we compute the *ev*-degree based Zagreb index of $HcDN1$ as:

$$\mathcal{M}^{ev}(HcDN1) = \sum_{e\in E(HcDN1)} d_{ev}(e)^2,$$

$$\begin{aligned}\mathcal{M}^{ev}(HcDN1) &= 6\times 6^2 + 12(n-1)\times 8^2 + 6n\times 9^2 + 18(n-1)\times 11^2 + (27n^2-57n+30)\times 12^2\\ &= 216 + 768n - 768 + 486n + 2178n - 2178 + 3888n^2 - 8208n + 4320\\ &= 3888n^2 - 4776n + 1590.\end{aligned}$$

- The first *ve*-degree Zagreb α index

Now with the help of Tab. 4, we compute the first *ve*-degree Zagreb α index of $HcDN1$ as:

$$\mathcal{M}_1^{\alpha ve}(HcDN1) = \sum_{v \in V(HcDN1)} d_{ve}(v)^2,$$

$$\begin{aligned}\mathcal{M}_1^{\alpha ve}(HcDN1) &= 12 \times 12^2 + 6(n-2) \times 14^2 + 6(n-1) \times 20^2 + 6 \times 22^2 + 6(n-2) \times 25^2 \\ &\quad + 6(n-1) \times 29^2 + (9n^2 - 27n + 19) \times 30^2 \\ &= 1728 + 1176n - 2352 + 2400n - 2400 + 2904 + 3750n - 7500 \\ &\quad + 5046n - 5046 + 8100n^2 - 24300 + 17100 \\ &= 8100n^2 - 11928n + 4434.\end{aligned}$$

- The first *ve*-degree Zagreb β index

Now with the help of Tab. 5, we compute the first *ve*-degree Zagreb β index of $HcDN1$ as:

$$\mathcal{M}_1^{\beta ve}(HcDN1) = \sum_{uv \in E(HcDN1)} (d_{ve}(u) + d_{ve}(v)),$$

$$\begin{aligned}\mathcal{M}_1^{\beta ve}(HcDN1) &= 6 \times 24 + 12 \times 32 + 12 \times 34 + 12 \times 42 + 12 \times 51 + 6 \times 58 + 6(3n-5) \times 59 \\ &\quad +6(n-1) \times 49 + 12(n-2) \times 34 + 6(n-2) \times 39 + 12(n-2) \times 45 + 12(n-2) \\ &\quad \times 54 + 6(n-2) \times 55 + (27n^2 - 93n + 78) \times 6 \\ &= 144 + 384 + 408 + 504 + 612 + 348 + 1062n - 1770 + 294 + 408n - 816 + 234n \\ &\quad -468 + 540n - 1080 + 648n - 1296 + 330n - 660 + 1620n^2 - 5580n + 4680 \\ &= 1620n^2 - 2064n + 696.\end{aligned}$$

- The second ve-degree Zagreb index

Now with the help of Tab. 5, we compute the second *ve*-degree based Zagreb index of $HcDN1$ as:

$$\mathcal{M}_2^{ve}(HcDN1) = \sum_{uv \in E(HcDN1)} (d_{ve}(u) \times d_{ve}(v)),$$

$$\begin{aligned}\mathcal{M}_2^{ve}(HcDN1) &= 6 \times 144 + 12 \times 240 + 12 \times 264 + 12 \times 440 + 12 \times 638 + 6 \times 841 + 6(3n-5) \\ &\quad \times 870 + 6(n-1) \times 580 + 12(n-2) \times 280 + 6(n-2) \times 350 + 12(n-2) \times 500 \\ &\quad + 12(n-2) \times 725 + 6(n-2) \times 750 + \left(27n^2 - 93n + 78\right) \times 900 \\ &= 864 + 2880 + 3168 + 5280 + 7656 + 5046 + 15660n - 26100 + 3480n - 3480 \\ &\quad + 3360n - 6720 + 2100n - 4200 + 6000n - 12000 + 8700n - 17400 + 4500n \\ &\quad - 9000 + 24300n^2 - 83700n + 70200 \\ &= 24300n^2 - 39900n + 16194.\end{aligned}$$

- The *ve*-degree Randic index

Now with the help of Tab. 5, we compute the *ve*-degree Randic index of $HcDN1$ as:

$$\mathcal{R}^{ve}(HcDN1) = \sum_{uv \in E(HcDN1)} (d_{ve}(u) \times d_{ve}(v))^{-\frac{1}{2}},$$

$$\begin{aligned}\mathcal{R}^{ve}(HcDN1) &= 6 \times 144^{-\frac{1}{2}} + 12 \times 240^{-\frac{1}{2}} + 12 \times 264^{-\frac{1}{2}} + 12 \times 440^{-\frac{1}{2}} + 12 \times 638^{-\frac{1}{2}} \\ &\quad + 6 \times 841^{-\frac{1}{2}} + 6(3n-5) \times 870^{-\frac{1}{2}} + 6(n-1) \times 580^{-\frac{1}{2}} + 12(n-2) \times 280^{-\frac{1}{2}} \\ &\quad + 6(n-2) \times 350^{-\frac{1}{2}} + 12(n-2) \times 500^{-\frac{1}{2}} + 12(n-2) \times 725^{-\frac{1}{2}} + 6(n-2) \times 750^{-\frac{1}{2}} \\ &\quad + \left(27n^2 - 93n + 78\right) \times 900^{-\frac{1}{2}}\end{aligned}$$

$$=\frac{6}{12}+\frac{12}{4\sqrt{15}}+\frac{12}{2\sqrt{66}}+\frac{12}{2\sqrt{110}}+\frac{12}{\sqrt{638}}+\frac{6}{\sqrt{841}}+\frac{18}{\sqrt{870}}n-\frac{30}{\sqrt{870}}+\frac{6}{2\sqrt{145}}n-\frac{6}{2\sqrt{145}}+\frac{12}{2\sqrt{70}}n$$
$$-\frac{24}{2\sqrt{70}}+\frac{6}{5\sqrt{14}}n-\frac{12}{5\sqrt{14}}+\frac{12}{5\sqrt{20}}n-\frac{24}{5\sqrt{20}}+\frac{12}{5\sqrt{29}}n-\frac{24}{5\sqrt{29}}+\frac{6}{5\sqrt{30}}n-\frac{12}{5\sqrt{30}}+\frac{27}{30}n^2$$
$$-\frac{93}{30}n+\frac{78}{30}$$
$$=\frac{9}{10}n^2+\left(\frac{18}{\sqrt{870}}+\frac{3}{\sqrt{145}}+\frac{6}{\sqrt{70}}+\frac{6}{5\sqrt{14}}+\frac{12}{5\sqrt{20}}+\frac{12}{5\sqrt{29}}+\frac{6}{5\sqrt{30}}-\frac{31}{10}\right)n+\frac{1}{2}+\frac{3}{\sqrt{15}}+\frac{6}{\sqrt{66}}$$
$$+\frac{6}{\sqrt{110}}+\frac{12}{\sqrt{638}}+\frac{6}{\sqrt{841}}-\frac{30}{\sqrt{870}}-\frac{3}{\sqrt{145}}-\frac{12}{\sqrt{70}}-\frac{12}{5\sqrt{14}}-\frac{24}{5\sqrt{20}}-\frac{24}{5\sqrt{29}}-\frac{12}{5\sqrt{30}}+\frac{13}{5}$$
$$=0.9n^2-0.0013n+0.122$$

• The *ev*-degree Randic index

Now with the help of Tab. 3, we compute the *ev*-degree based Randic index of $HcDN1$ as:

$$\mathcal{R}^{ev}(HcDN1)=\sum_{e\in E(HcDN1)} d_{ev}(e)^{-\frac{1}{2}},$$

$$\mathcal{R}^{ev}(HcDN1)=6\times 6^{-\frac{1}{2}}+12(n-1)\times 6^{-\frac{1}{2}}+6n\times 9^{-\frac{1}{2}}+18(n-1)\times 11^{-\frac{1}{2}}+\left(27n^2-57n+30\right)\times 12^{-\frac{1}{2}}$$
$$=\frac{27}{\sqrt{12}}n^2+\left(\frac{6}{\sqrt{2}}+2+\frac{18}{\sqrt{11}}-\frac{57}{\sqrt{12}}\right)n+\left(\sqrt{6}-\frac{6}{\sqrt{2}}-\frac{18}{\sqrt{11}}+\frac{15}{\sqrt{3}}\right).$$
$$=7.794n^2-4.785+1.44.$$

• The atom-bond connectivity index

Now with the help of Tab. 5, we compute the atom-bond connectivity index of $HcDN1$ as:

$$\mathcal{ABC}^{ve}(HcDN1)=\sum_{uv\in E(HcDN1)}\sqrt{\frac{d_{ve}(u)+d_{ve}(v)-2}{d_{ve}(u)\times d_{ve}(v)}},$$

$$\mathcal{ABC}^{ve}(HcDN1)=6\times\sqrt{\frac{24-2}{144}}+12\times\sqrt{\frac{32-2}{240}}+12\times\sqrt{\frac{34-2}{264}}+12\times\sqrt{\frac{42-2}{440}}$$
$$+12\times\sqrt{\frac{51-2}{638}}+6\times\sqrt{\frac{58-2}{841}}+6(3n-5)\times\sqrt{\frac{59-2}{870}}+6(n-1)\times\sqrt{\frac{49-2}{580}}$$
$$+12(n-2)\times+6(n-2)\times\sqrt{\frac{39-2}{350}}+12(n-2)\times\sqrt{\frac{45-2}{500}}$$
$$+12(n-2)\times\sqrt{\frac{54-2}{725}}+6(n-2)\times\sqrt{\frac{55-2}{750}}+\left(27n^2-93n+78\right)\times\sqrt{\frac{60-2}{900}}$$

$$= \frac{6\sqrt{22}}{12} + \frac{12\sqrt{30}}{4\sqrt{15}} + \frac{48\sqrt{2}}{2\sqrt{66}} + \frac{24\sqrt{10}}{2\sqrt{110}} + \frac{84}{\sqrt{638}} + \frac{12\sqrt{14}}{\sqrt{841}} + \frac{18\sqrt{57}}{\sqrt{870}}n - \frac{30\sqrt{57}}{\sqrt{870}} + \frac{6\sqrt{47}}{2\sqrt{145}}n - \frac{6\sqrt{47}}{2\sqrt{145}}$$
$$+ \frac{48\sqrt{2}}{2\sqrt{70}}n - \frac{96\sqrt{2}}{2\sqrt{70}} + \frac{6\sqrt{37}}{5\sqrt{14}}n - \frac{12\sqrt{37}}{5\sqrt{14}} + \frac{12\sqrt{43}}{5\sqrt{20}}n - \frac{24\sqrt{43}}{5\sqrt{20}} + \frac{24\sqrt{13}}{5\sqrt{29}}n - \frac{48\sqrt{13}}{5\sqrt{29}} + \frac{6\sqrt{53}}{5\sqrt{30}}n$$
$$- \frac{12\sqrt{53}}{5\sqrt{30}} + \frac{27\sqrt{58}}{30}n^2 - \frac{93\sqrt{58}}{30}n + \frac{78\sqrt{58}}{30}$$

$$= \frac{9\sqrt{58}}{10}n^2 + \left(\frac{18\sqrt{57}}{\sqrt{870}} + \frac{3\sqrt{47}}{\sqrt{145}} + \frac{24}{\sqrt{35}} + \frac{6\sqrt{37}}{\sqrt{14}} + \frac{12\sqrt{43}}{\sqrt{20}} + \frac{24\sqrt{13}}{5\sqrt{29}} + \frac{6\sqrt{53}}{5\sqrt{30}} - \frac{31\sqrt{58}}{10}\right)n + \frac{\sqrt{22}}{2}$$
$$+ 3\sqrt{2} + \frac{24}{\sqrt{33}} + \frac{12}{\sqrt{11}} + \frac{84}{\sqrt{638}} + \frac{12\sqrt{14}}{\sqrt{841}} - \frac{30\sqrt{57}}{\sqrt{870}} - \frac{3\sqrt{47}}{2\sqrt{145}} - \frac{48}{\sqrt{35}} - \frac{12\sqrt{37}}{5\sqrt{14}} - \frac{24\sqrt{43}}{5\sqrt{20}} - \frac{48\sqrt{13}}{5\sqrt{29}}$$
$$- \frac{12\sqrt{53}}{5\sqrt{30}} + \frac{13\sqrt{58}}{5}.$$

$$= 6.854n^2 + 18.92n + 1.855.$$

• The geometric-arithmetic index

Now with the help of Tab. 5, we compute the geometric-arithmetic index of $HcDN1$ as:

$$\mathcal{GA}^{ve}(HcDN1) = \sum_{uv \in E(HcDN1)} \frac{2\sqrt{deg_{ve}(u) \times d_{ve}(v)}}{d_{ve}(u) + d_{ve}(v)},$$

$$\mathcal{GA}^{ve}(HcDN1) = 6 \times \frac{2\sqrt{144}}{24} + 12 \times \frac{2\sqrt{240}}{32} + 12 \times \frac{2\sqrt{264}}{34} + 12 \times \frac{2\sqrt{440}}{42} + 12 \times \frac{2\sqrt{638}}{51}$$
$$+ 6 \times \frac{2\sqrt{841}}{58} + 6(3n-5) \times \frac{2\sqrt{870}}{59} + 6(n-1) \times \frac{2\sqrt{580}}{49} + 12(n-2) \times \frac{2\sqrt{280}}{34}$$
$$+ 6(n-2) \times \frac{2\sqrt{350}}{39} + 12(n-2) \times \frac{2\sqrt{500}}{45} + 12(n-2) \times \frac{2\sqrt{725}}{54}$$
$$+ 6(n-2) \times \frac{2\sqrt{750}}{55} + \left(27n^2 - 93n + 78\right) \times \frac{2\sqrt{900}}{60}$$

$$= 6 + 3\sqrt{15} + \frac{24\sqrt{66}}{17} + \frac{8\sqrt{110}}{7} + \frac{8\sqrt{638}}{17} + \frac{6\sqrt{841}}{29} + \frac{36\sqrt{870}}{59}n - \frac{60\sqrt{870}}{59} + \frac{24\sqrt{145}}{49}n - \frac{24\sqrt{145}}{49}$$
$$+ \frac{24\sqrt{70}}{17}n - \frac{48\sqrt{70}}{17} + \frac{20\sqrt{14}}{13}n - \frac{40\sqrt{14}}{13} + \frac{8\sqrt{20}}{3}n - \frac{16\sqrt{20}}{3} + \frac{20\sqrt{29}}{9}n - \frac{40\sqrt{29}}{9} + \frac{12\sqrt{30}}{11}n$$
$$- \frac{24\sqrt{30}}{11} + 27n^2 - 93n + 78$$

$$= \left(\frac{36\sqrt{870}}{59} + \frac{24\sqrt{145}}{49} + \frac{24\sqrt{70}}{17} + \frac{20\sqrt{14}}{13} + \frac{8\sqrt{20}}{3} + \frac{20\sqrt{29}}{9} + \frac{12\sqrt{30}}{11} - 93\right)n + 6 + 3\sqrt{15} + \frac{24\sqrt{66}}{17}$$
$$+ \frac{8\sqrt{110}}{7} + \frac{8\sqrt{638}}{17} + \frac{6\sqrt{841}}{29} - \frac{60\sqrt{870}}{59} - \frac{24\sqrt{145}}{49} - \frac{48\sqrt{70}}{17} - \frac{40\sqrt{14}}{13} - \frac{16\sqrt{20}}{3} - \frac{40\sqrt{29}}{9} - \frac{24\sqrt{30}}{11}$$
$$+ 78 + 27n^2$$

$$= 27n^2 - 21.669n + 6.195.$$

• The harmonic index

Now with the help of Tab. 5, we compute the Harmonic index of $HcDN1$ as:

$$\mathcal{H}^{ve}(HcDN1) = \sum_{uv \in E(HcDN1)} \frac{2}{d_{ve}(u) + d_{ve}(v)},$$

$$\mathcal{H}^{ve}(HcDN1) = 6 \times \frac{2}{24} + 12 \times \frac{2}{32} + 12 \times \frac{2}{34} + 12 \times \frac{2}{42} + 12 \times \frac{2}{51} + 6 \times \frac{2}{58} + 6(3n-5) \times \frac{2}{59} + 6(n-1) \times \frac{2}{49} + 12(n-2) \times \frac{2}{34} + 6(n-2) \times \frac{2}{39} + 12(n-2) \times \frac{2}{45} + 12(n-2) \times \frac{2}{54} + 6(n-2) \times \frac{2}{55} + \left(27n^2 - 93n + 78\right) \times \frac{2}{60}$$

$$= \frac{1}{2} + \frac{3}{4} + \frac{12}{17} + \frac{4}{7} + \frac{8}{17} + \frac{6}{29} + \frac{36}{59}n - \frac{60}{59} + \frac{12}{49} - \frac{12}{49} + \frac{12}{17}n - \frac{24}{17} + \frac{4}{13}n - \frac{8}{13} + \frac{8}{15}n - \frac{16}{15} + \frac{4}{9}n - \frac{8}{9} + \frac{12}{55}n - \frac{24}{55} + \frac{9}{10}n^2 - \frac{31}{10}n + \frac{13}{5}$$

$$= 0.90n^2 - 0.036n + 0.122.$$

• The sum-connectivity index

Now with the help of Tab. 5, we compute the Sum-connectivity index of $HcDN1$ as:

$$\chi^{ve}(HcDN1) = \sum_{uv \in E(HcDN1)} (d_{ve}(u) + d_{ve}(v))^{-\frac{1}{2}},$$

$$\chi^{ve}(HcDN1) = 6 \times 24^{-\frac{1}{2}} + 12 \times 32^{-\frac{1}{2}} + 12 \times 34^{-\frac{1}{2}} + 12 \times 42^{-\frac{1}{2}} + 12 \times 51^{-\frac{1}{2}} + 6 \times 58^{-\frac{1}{2}} + 6(3n-5) \times 59^{-\frac{1}{2}} + 6(n-1) \times 49^{-\frac{1}{2}} + 12(n-2) \times 34^{-\frac{1}{2}} + 6(n-2) \times 39^{-\frac{1}{2}} + 12(n-2) \times 45^{-\frac{1}{2}} + 12(n-2) \times 54^{-\frac{1}{2}} + 6(n-2) \times 55^{-\frac{1}{2}} + \left(27n^2 - 93n + 78\right) \times 60^{-\frac{1}{2}}$$

$$= \frac{3}{\sqrt{6}} + \frac{3}{\sqrt{2}} + \frac{12}{\sqrt{34}} + \frac{12}{\sqrt{42}} + \frac{12}{\sqrt{51}} + \frac{6}{\sqrt{58}} + \frac{18}{\sqrt{59}}n - \frac{30}{\sqrt{59}} + \frac{6}{\sqrt{7}}n - \frac{6}{7} + \frac{12}{\sqrt{34}}n - \frac{24}{\sqrt{34}} + \frac{6}{\sqrt{39}}n - \frac{12}{\sqrt{39}} + \frac{4}{\sqrt{5}}n - \frac{8}{\sqrt{5}} + \frac{4}{\sqrt{6}}n - \frac{8}{\sqrt{6}} + \frac{6}{\sqrt{55}}n - \frac{12}{\sqrt{55}} + \frac{27}{2\sqrt{15}}n^2 - \frac{93}{2\sqrt{15}}n + \frac{39}{\sqrt{15}}$$

$$= \frac{27}{2\sqrt{15}}n^2 + \left(\frac{18}{\sqrt{59}} + \frac{6}{7} + \frac{12}{\sqrt{34}} + \frac{6}{\sqrt{39}} + \frac{4}{\sqrt{5}} + \frac{4}{\sqrt{6}} + \frac{6}{\sqrt{55}} - \frac{93}{2\sqrt{15}}\right)n + \frac{3}{\sqrt{6}} + \frac{3}{\sqrt{2}} + \frac{12}{\sqrt{34}} + \frac{12}{\sqrt{42}} + \frac{12}{\sqrt{51}} + \frac{6}{\sqrt{58}} - \frac{30}{\sqrt{59}} - \frac{6}{7} - \frac{24}{\sqrt{34}} - \frac{12}{\sqrt{39}} - \frac{8}{\sqrt{5}} - \frac{8}{\sqrt{6}} - \frac{12}{\sqrt{55}} + \frac{39}{\sqrt{15}}$$

$$= 3.486n^2 - 1.556n + 0.531.$$

5 Numerical and Graphical Representation and Discussion

The *ve* and *ev* for ten different types od degree base topological descriptors for the $HcDN1$ are calculated both numerically and graphically. From Fig. 2 it is clearly noted that the behavior of first Zegreb alpha index, first Zagreb beta index and second Zagreb index is almost same in the increasing direction as the value of n increases while *ev* Zagreb index value has a very rapid increase with the increase value of n. From Fig. 3 it is clearly noted that the behavior of atom bond connectivity index and geometric arithmetic index are almost closely increasing with the increase value of n while ev Randic index value has a very rapid increase with the increase value of n. The numerical representation of $HcDN1$ is shown in Tabs. 6–8. The graphical representation of $HcDN1$ are shown in Figs. 2–4.

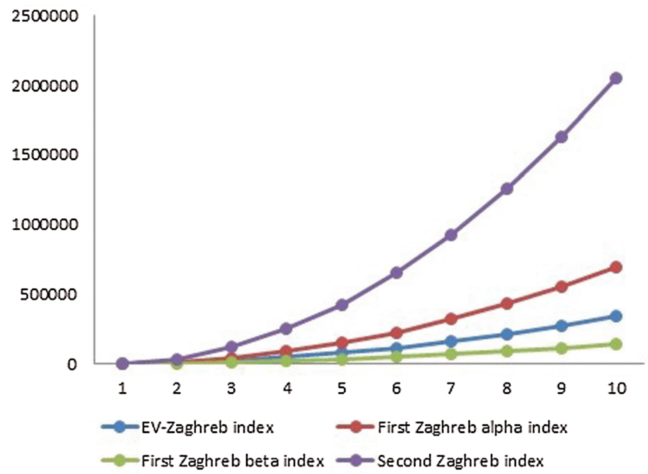

Figure 2: Graphical comparison of M^{ev}, $M_1^{\alpha ve}$, $M_1^{\beta ve}$ and M_2^{ve}

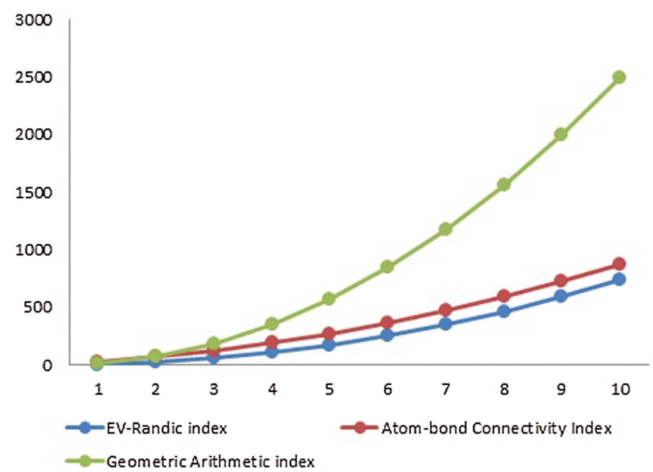

Figure 3: Graphical comparison of R^{ev}, ABC^{ve} and GA^{ve}

Table 6: Numerical comparison of $M^{ev}, M_1^{\alpha ve}, M_1^{\beta ve}$ and M_2^{ve}

n	M^{ev}	$M_1^{\alpha ve}$	$M_1^{\beta ve}$	M_2^{ve}
1	702	606	252	594
2	7590	12978	3048	33594
3	22254	41550	9084	115194
4	44694	86322	18360	245394

(Continued)

Table 6 (continued).

n	M^{ev}	$M_1^{\alpha ve}$	$M_1^{\beta ve}$	M_2^{ve}
5	74910	147294	30876	424194
6	112902	224466	46632	651594
7	158670	317838	65628	927594
8	212214	427410	87864	1252194
9	273534	553182	113340	1625394
10	342630	695154	142056	2047194

Table 7: Numerical comparison of R^{ev}, ABC^{ve} and GA^{ve}

n	R^{ev}	ABC^{ve}	GA^{ve}
1	4.449	27.63	11.526
2	23.046	67.113	70.857
3	57.231	120.304	184.188
4	107.004	187.203	351.519
5	172.365	267.81	572.85
6	253.314	362.125	848.181
7	349.851	470.148	1177.512
8	461.976	591.879	1560.843
9	589.689	727.318	1998.174
10	732.99	876.465	2489.505

Table 8: Numerical comparison of R^{ve}, H^{ve} and χ^{ve}

n	R^{ve}	H^{ve}	χ^{ve}
1	1.02	0.986	2.461
2	3.719	3.65	11.363
3	8.218	8.114	27.237
4	14.517	14.378	50.083
5	22.615	22.442	79.901
6	32.514	32.306	116.691
7	44.213	43.97	160.453
8	57.116	57.434	211.187
9	73.01	72.698	268.893
10	90.109	89.762	333.571

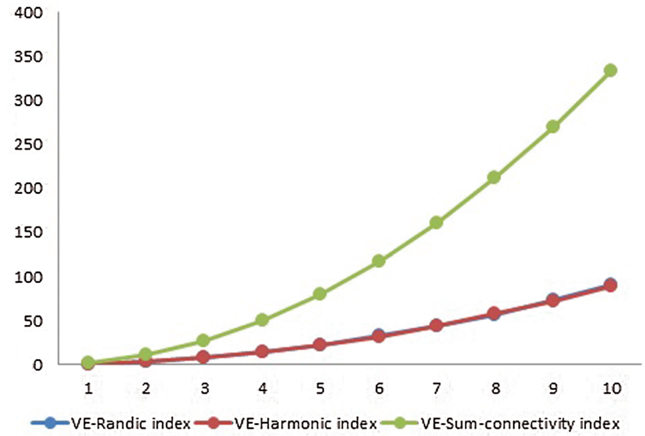

Figure 4: Graphical comparison of R^{ve}, H^{ve} and χ^{ve}

6 Conclusion

There are many applications of topological descriptors in computer science, networks, agriculture and chemical graph theory etc. These descriptors help in finding the behavior of their structures. We dealt with the honey comb derived network and computed ten different types of topological descriptors which are base on *ev* and *ve* degree. We have computed their explicit formulas and then computed their numerical values for different values of n. Further we plotted their graphs for comparison and discussed their behavior. We observe that the values of all descriptors increases with the increase value of n. In the analysis of the quantitative structure property relationships (QSPRs) and the quantitative structureactivity relationships (QSARs), graph invariants are important tools to approximate and predicate the properties of the biological and chemical compounds. In this paper, we study the vertex-edge based topological indices for honey comb derived network.

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