Computer Systems Science & Engineering
DOI:10.32604/csse.2022.018962
Article

Obtaining Crisp Priorities for Triangular and Trapezoidal Fuzzy Judgments

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Received: 27 March 2021; Accepted: 03 May 2021

Abstract: This paper proposes anoptimal fuzzy-based model for obtaining crisp priorities for Fuzzy-AHP comparison matrices. Crisp judgments cannot be given for real-life situations, as most of these include some level of fuzziness and complexity. In these situations, judgments are represented by the set of fuzzy numbers. Most of the fuzzy optimization models derive crisp priorities for judgments represented with Triangular Fuzzy Numbers (TFNs) only. They do not work for other types of Triangular Shaped Fuzzy Numbers (TSFNs) and Trapezoidal Fuzzy Numbers (TrFNs). To overcome this problem, a sum of squared error (SSE) based optimization model is proposed. Unlike some other methods, the proposed model derives crisp weights from all of the above-mentioned fuzzy judgments. A fuzzy number is simulated using the Monte Carlo method. A threshold-based constraint is also applied to minimize the deviation from the initial judgments. Genetic Algorithm (GA) is used to solve the optimization model. We have also conducted casestudiesto show the proposed approach's advantages over the existingmethods. Results show that the proposed model outperforms other models to minimize SSE and deviation from initial judgments. Thus, the proposed model can be applied in various real time scenarios as it can reduce the SSE value upto 29% compared to the existing studies.

Keywords: Analytic hierarchy process; comparison matrices; priority vectors; fuzzy judgments; triangular fuzzy numbers; triangular-shaped fuzzy numbers; trapezoidal fuzzy numbers

1 Introduction

Multi-Criteria decision-making (MCDM) methods are used to solve complex decision-making systems. Rao [1] has listed many decision-making methods like AHP [2–3] ELECTRE [4], TOPSIS [5], PROMETHEE [6] and GRA [7] etc. AHP is the most popular method among these MCDM methods. Triantaphyllou and Lin [8] discussed advantages of AHP in many real-life selection problems. AHP had been integrated with other theories and practices by researchers to solve many decision-making problems in MCDM scenarios [9–10]. Zhang et al. [11] proposed a method for prioritization using additive and multiplicative relative deviation interconnection in pairwise comparison matrices. Aguarona et al. [12]

proposed a novel method for improving the inconsistency of the GCI of AHP by a reevaluation of judgments. Olabanji et al. [13] proposed MADM method to choose the best design for the Reconfigurable Assembly Fixture (RAF) from the set of choices. They considered the better of two techniques Fuzzy AHP and Fuzzy Weighted Average for choosing among various designs. Gundogdu et al. [14] proposed a fuzzy spherical AHP method to select renewable energy sites. Guler et al. [15] proposed a novel method based on Fuzzy AHP and Geographic Information System to select stations for electric vehicle charging. Sarkar et al. [16] presented a hybrid technique consisting of Fuzzy AHP and TOPSIS to solve transport management's decision-making issues. They have used Pythagorean fuzzy sets to overcome the limitation of existing distance measuring techniques. Crisp judgments cannot be given in many real-life scenarios as most of these include some level of fuzziness and complexity. In such situations, judgments are given in fuzzy numbers [17–19]. Derivation of crisp weights from the fuzzy numbers is a difficult task [20]. Triangular fuzzy weights have been obtained from the fuzzy triangular matrix by the logarithmic least squares method (LLSM) [21]. Buckley [22] derived fuzzy weights from trapezoidal fuzzy numbers. Boender et al. [23] proposed a technique on normalizing the local priorities and showed that this normalizing technique in LLSM might result in irrational weights. Generalized pseudo-inverse method has been applied by Kwiesielewicz [24] to obtain a consistent and geometrically normalized solution. All of the models mentioned above derive fuzzy weights. Therefore, they also require additional aggregation and ranking processes. Extent analysis method [25] removes these requirements and derives crisp priorities for the fuzzy comparison matrices. Wang et al. [26] have found that this method does not estimate true weights and assigns irrational zero weights to useful attributes in some scenarios. Interval priorities weights are derived from fuzzy comparison matrices by some methods [27–34]. But these methods also require additional fuzzy ranking procedures.

Mikhailov [35] has derived crisp weights using a linear fuzzy prioritization programming method using a series of interval judgments. This method removes the requirement of the ranking process but still requires the aggregation process. Therefore, Mikhailov [35] also proposed a non-linear fuzzy prioritization (fuzzy optimization) method to remove the aggregation process. Wang et al. [36] proposed a new optimization model for fuzzy judgments. The objective function's degree was varied from 2 to 10. Javanbarget al. [37] have solved three examples with the same objective function of degree [2]. Goyal et al. [38] proposed a constraint for this objective function to improve its consistency. New methods presented by Goyal et al. in [39] and Goyal et al. [40] also derive more consistent weights than the fuzzy optimization model shown in Javanbarg et al. [37].

All these fuzzy optimization models derive crisp weights for Triangular Fuzzy Numbers (TFNs) only in the literature presented above. Mohtashami [41] proposed a Modified Fuzzy Logarithmic Least Square Model (MFLLSM) to derive priorities from TFNs, Triangular Shaped Fuzzy Numbers (TSFNs), and Trapezoidal Fuzzy Numbers (TrFNs). Evaluation of trapezoidal fuzzy numbers on AHP based solution of multi-objective programming problems for group decision making is given by AkbasandDalkilic [42].

After introducing some preliminary concepts for the fuzzy judgments in comparison pairwise matrices in Section 2, the following contributions are claimed:

• A novel Sum of Squared Error (SSE) based fuzzy optimization model is proposed. The simulation of the fuzzy number using Monte Carlo's method and the threshold-based constraint is also presented.

• Further empirical analysis has been conducted based on eight case studiesto compare the proposed SSE-based model with existing models. It has been proved that the proposed model performs better than the existing Mohtashami's model [41].

To the best of the author's knowledge, no other fuzzy optimization model has been proposed till now to derive crisp weights for these types of fuzzy judgments that can surpass the work done by Mohtashami [41].

2 Materials and Methods

In this section, we have discussed how the various fuzzy judgments can be represented.

2.1 Fuzzy Judgments

For modeling many real-time applications, a decision-maker cannot always give crisp judgments. In these situations, judgments are illustrated by fuzzy numbers [43]. Fuzzy judgments can be of the following forms:

• *Triangular Fuzzy Number (TFN):* A TFN is represented as (l,m,u) where l, m, and u denote the lowest value, the most promising value uppermost possible value, respectively. Membership function of a TFN is represented in Eq. (1) [44].

$$\mu_{\tilde{N}}(x) = \begin{cases} \dfrac{x-l}{m-l}, & l \le x \le m \\ \dfrac{u-x}{u-m}, & m \le x \le u \\ 0, & otherwise \end{cases} \tag{1}$$

• *Triangular Shaped Fuzzy Numbers (TSFNs):* These fuzzy numbers are represented similarly to TFN except the member function does not increase or decrease linearly. A TSFN is defined as shown in Eq. (2). If the values of δ and χ are equal to one, then this membership function becomes the function of TFN. Fig. 1. 1 depicts the asymmetric membership function for TSFN with δ=3 and χ=5. From Fig. 1, it can be visible that there is more intent of the decision-maker towards l rather than u. If the values of χ and δ are equal, then the shape of the membership function is symmetric with the decision-maker's equal intent towards both l and u.

$$\mu_{\tilde{N}}(x) = \begin{cases} \left(\dfrac{x-l}{m-l}\right)^{\delta}, & l \le x \le m \\ \left(\dfrac{u-x}{u-m}\right)^{\chi}, & m \le x \le u \\ 0, & otherwise \end{cases} \tag{2}$$

• *Trapezoidal Fuzzy Number:* In this type of membership function, there are two midpoints between the lower and upper limits and are represented as (l, m1, m2, u). The membership function for the trapezoidal fuzzy numbers (TrFNs) is shown in Eq. (3). Fig. 2. shows the graphical representation of TrFN.

$$\mu_{\tilde{N}}(x) = \begin{cases} \dfrac{x-l}{m1-l}, & l \le x \le m1 \\ \dfrac{u-x}{u-m2}, & m2 \le x \le u \\ 1, & m1 \le x \le m2 \\ 0, & otherwise \end{cases} \tag{3}$$

• *Trapezoidal Shaped Fuzzy Number (TrSFN):* This type of fuzzy number is similar to TrFN except that the membership function does not increase or decrease linearly. The TrSFN membership function is shown in Eq. (4). If the values of δ and χ are equal to one, then this membership function becomes the function of TrFN.

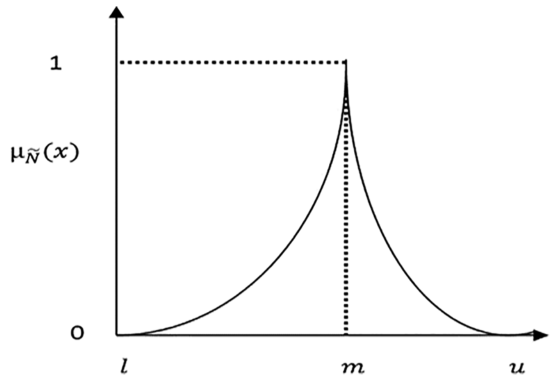

Figure 1: Triangular shaped Fuzzy Number with δ<χ

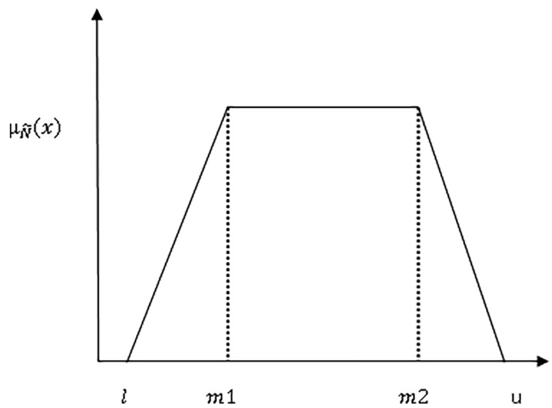

Figure 2: Trapezoidal Fuzzy Number

$$\mu_{\tilde{N}}(x) = \begin{cases} \left(\dfrac{x-l}{m1-l}\right)^{\delta}, & l \le x \le m1 \\ \left(\dfrac{u-x}{u-m2}\right)^{\chi}, & m2 \le x \le u \\ 1, & m1 \le x \le m2 \\ 0, & otherwise \end{cases} \tag{4}$$

The main challenge is to devise a model that can derive crisp priorities from all these types of fuzzy judgments. In the next section, the proposed fuzzy optimization model for obtaining crisp priorities is presented.

2.2 Proposed Work

This section presents a novel fuzzy prioritization method for deriving crisp priorities from fuzzy pairwise comparison matrices. Here, the proposed SSE-based fuzzy optimization model is discussed and the constraint applied to the fuzzy optimization model for minimum deviation from initial judgmentsis also presented. Further, the fuzzy number simulation method is elaborated, and the overall working of the proposed work has been discussed.

2.2.1 Fuzzy Optimization Model

In this paper, a new Sum of Squared Error (SSE) based fuzzy optimization model is presented as a nonlinear system. Crisp weights (w_1, w_2, …, w_n) of n criteria are derived from fuzzy judgments ($\tilde{a}_{ij}$) in the comparison matrices as follows:

$$min\, J(w_1,\ w_2,\ \ldots,\ w_n) =\ min\sum_{\mathrm{i}=1}^{\mathrm{n}}\sum_{\mathrm{j}=1}^{\mathrm{n}}\left(\tilde{a}_{ij}-\left(\frac{w_i}{w_j}\right)\right)^2 (\ i\neq j) \tag{5}$$

subject to $\sum_{k=1}^{n} w_k = 1,\ k = 1,\ 2\ldots,\ n$. where $\mathrm{w_i}$ and $\mathrm{w_j} > 0$

The weight ratio (w_i/w_j) must satisfy the following inequality:

$$l_{ij}\tilde{\leq}w_i/w_j\tilde{\leq}u_{ij} \tag{6}$$

l_{ij} and u_{ij} represent the lower and upper limits of fuzzy judgments. The symbol $\tilde{\leq}$ represents the fuzzy inequalities. For a consistent comparison matrix, solutions satisfying $l_{ij} \leq w_i/w_j \leq u_{ij}$ can be found. But for inconsistent matrices, there will be no solution to satisfying this inequality simultaneously. Therefore, the fuzzy inequality $l_{ij}\tilde{\leq}w_i/w_j\tilde{\leq}u_{ij}$ is applied to find the crisp priorities that satisfy the inequality as well as it can. To find the weight ratios w_i/w_j that deviate minimum for this inequality, a constraint is proposed. $\tilde{a}_{ij}$ represents the fuzzy judgments in the form of TFNs, TSFNs, TrFNs, and TrSFNs. In the case of TFN and TrFN, the value of $\tilde{a}_{ij}$ assumed as the center of the fuzzy number. In other cases, the value of $\tilde{a}_{ij}$ represents the objective function obtained by Monte Carlo simulation.

2.2.2 Monte Carlo's Method for Fuzzy Number Simulating

The value of $\tilde{a}_{ij}$ is obtained, as shown in Algorithm 1. This algorithm is a modified version of the Monte Carlo method for fuzzy number simulation presented by Mohtashami [41]. Monte Carlo simulation is used to generate the fuzzy number based on algorithm 1.

Algorithm 1: Modified Monte Carlo Simulation

Step 1: Divide the fuzzy number into n+1 numbers from l to u as follows with distance$d$: l, $l+d$, $l+2d$, $l+3d$, …, $l+(n-1)\ d$, u

Step 2: Calculate the membership degree of each number as $\mu_N(l)$, $\mu_N(l+d)$, $\mu_N(l+2d)$, …, $\mu_N(l+(n-1)\ d)$, $\mu_N(u)$.

Step 3: Normalize the membership function values ($_N(l)$, $_N(l+d)$, $_N(l+2d)$, …, $_N(l+(n-1)\ d)$, $_N(u)$) as following

$$\chi_1 = \frac{\mu_N(l)}{N}, \chi_2 = \frac{\mu_N(l+d)}{N}, \chi_3 = \frac{\mu_N(l+2d)}{N},\ \ \ldots, \chi_n = \frac{\mu_N(l+(n-1)d)}{N}, \chi_{n+1} = \frac{\mu_N(u)}{N}$$

where $N = \sum_{i=l}^{u} \mu_N(i)$

Step 4: Classify the successive values of χ_i as

$$\lambda_1 = \chi_1 + \chi_2, \lambda_2 = \chi_2 + \chi_3,\ \ \ldots, \lambda_n = \chi_n + \chi_{n+1}$$

Step 5: Generate a random number $r \in (0, 1)$

$if\ \ (r <\ = \lambda_1)$

(continued)

Algorithm 1: (continued)

The fuzzy number = l

$if\,(r > \lambda_1)\ \&\&\ if\ (r < \ = \lambda_2)$

The fuzzy number = $l + d$

$if\,(r > \lambda_2)\ \&\&\ if\ (r \leq = \lambda_3)$

The fuzzy number = $l + 2d$

$\vdots$

$if\,(r > \lambda_{n-1})\ \&\&\ if\,(r < \ = \lambda_n)$

The fuzzy number = $l + (n-1)\ d$

$if\ \ (r >_n)$

The fuzzy number = u

Step 6: Repeat Step 5 for large number of times to simulate fuzzy number.

Step 7: Calculate the average of these simulated fuzzy numbers in order to simulate the fuzzy number $\tilde{a}_{ij}$.

To check the effectiveness of this proposed Model, graphs for different types of fuzzy numbers have been drawn considering the frequency (count) of the simulated fuzzy number. If the values of δ and χ are equal to one, then it is a TFN otherwise, the number is TSFN. The values of (l,m,u) are taken as (1,2,3). Similarly, this method can be extended to Trapezoidal Fuzzy Numbers (TrFN). Fig. 3. shows the shape obtained for a TrFN with values (1, 1.5, 2.5, 3), which is similar to the actual shape.

$$l_{ij} \leq w_i / w_j \leq u_{ij} \tag{7}$$

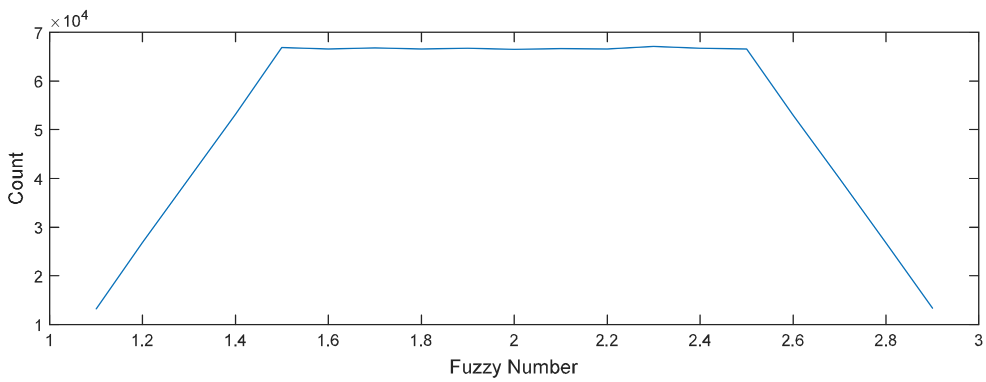

Figure 3: Simulation of trapezoidal number based on algorithm 1

The point to be noted here is that this inequality is $\leq$ *not* $\tilde{\lesssim}$ as mentioned in Eq. (6). But, for inconsistent pairwise matrices, no set of priority vectors can simultaneously satisfy this inequality. Therefore, fuzzy inequality is used in Eq. (6) to allow some deviation. To achieve minimum violation from the Eq. (7), the constraint shown in Eq. (8) is applied.

2.2.3 Violation from Initial Judgments

Each priority vector (crisp weight) should satisfy the constraint defined in Eq. (6). The proposed algorithm first tries to obtain the crisp weights by satisfying the following inequality

$$l_{ij} - \alpha \leq \frac{w_i}{w_j} \leq u_{ij} + \alpha \tag{8}$$

α represents the threshold value of the allowed deviation for the optimization problem. If the solution is feasible, then the crisp weights are derived. Otherwise, the optimization process is repeated and α is increased with a small value of 0.1. After obtaining the feasible crisp weights $(w_1^*, w_2^*, \ldots, w_n^*)$ deviation from initial judgments (ε) can be calculated as follows:

$$\varepsilon = max\left(\frac{w_i^*}{w_j^*} - l_{ij},\ u_{ij} - \frac{w_i^*}{w_j^*}\right) \text{ where } i, j = 1, 2, 3 \ldots, n. \tag{9}$$

Thus, the crisp weights deviate with ε from the initial judgments far less than the existing models shown in Section 4.

2.2.4 Working of the Proposed Model

The working of the proposed model is shown in Fig. 4. Firstly, the fuzzy number is simulated, as mentioned in above mentioned algorithm.

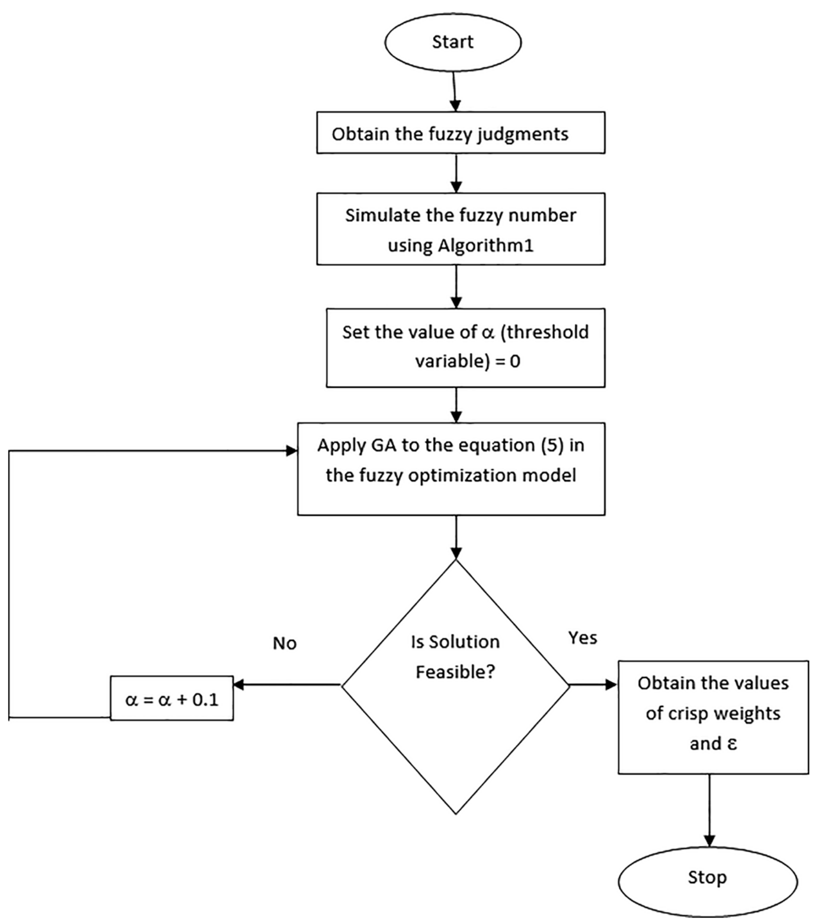

Figure 4: Working of the proposed technique

Then, the crisp weights are derived using the fuzzy optimization model, as shown in Eq. (5). To check the deviation from initial weights, a threshold-based constraint shown in Eq. (8) is applied. The value of α is set to zero initially and is increased by 0.1 until feasible weights are obtained. Genetic Algorithm (GA) [45,46] is used to solve this optimization problem with both population size and number of iterations equal to 300 and 350 respectively. Rank scaling is used as a fitness scaling function, and Uniform Stochastic Sampling is used to select parents. The heuristic operator is applied for crossover operation while adaptive feasible mutation is applied for the mutation process. We have solved this objective function by GA method with significantly large population size of 300 and 350 iterations for each example in Matlab. Any optimization algorithm can be used to solve this optimization problem. The time complexity is increased in case of finding crisp weights only. Obtaining the crisp weights is a one-time process only. So, it will have no impact on the selection process. So, the proposed model can be easily applied in real world problems.

3 Case Studies and Result Analysis

To evaluate the proposed model, eight case studies are solved. As discussed earlier, among the fuzzy optimization models, only Mohtashami's model [41] derive crisp weights for all the TFNs, TSFNs and TrFNs.

Case-1:

In this example, a perfectly consistent comparison pairwise matrix with only two elements is considered. In comparison to the second element, the first element is more important and is represented by a TFN $\tilde{a}_{12}$ = (1,2,3). Crisp weights obtained by different methods are shown in Tab. 1. Results show that the difference between crisp weights obtained by the proposed fuzzy optimization model and the existing models is inconsiderable. Therefore, results obtained by the proposed model are acceptable for this consistent matrix.

Table 1: Crisp weights, deviation from initial judgments (ε) and SSE values for case 1

	Non-Linear FPP [35]	Linear FPP [35]	Mohtashami [41]	Proposed model
w_1	0.6666	0.6666	0.6664	0.6667
w_2	0.3333	0.3333	0.3336	0.3333
ε	0	0	0	0
SSE	0	0	0.0002	0

Case-2:

Consider the comparison matrix of three elements represented by TFNs as $\tilde{a}_{13}$ = (4, 5, 6), $\tilde{a}_{21}$ = (2, 3, 4) and $\tilde{a}_{23}$ = (6, 7, 8). For this, the value of SSE is more as compared to Mohtashami's [41] model as shown in Tab. 2. But the value of ε is not zero with Motashami's model. The value of SSE is more with the proposed model as it minimizes the SSE by constraining the deviation (ε) from the judgments. Therefore, the value of ε is zero with the proposed model. The proposed derives similar priority vectors (crisp weights) as of Non-Linear FPP model. Linear FPP performs poorly in terms of SSE and ε. For illustration purpose, the weights are also derived with α (threshold value) = 0.1. The results of the proposed model are better than Mohtashami's model. Therefore, the proposed model outperforms all the other models for this example.

Table 2: Crisp weights, deviation from initial judgments (ε) and SSE values for case 2

	Non-Linear FPP [35]	Linear FPP [35]	Mohtashami [41]	Proposed model	
w_1	0.3076	0.2738	0.3104	0.3077	0.3103
w_2	0.6153	0.6492	0.6117	0.6154	0.6115
w_3	0.0769	0.0769	0.0777	0.0769	0.0782
ε	0	0.031	0.036	0	0.0293
SSE	3.0313	4.5624	2.8610	3.0306	2.8328

Case-3:

In this example, judgments of the comparison matrices of four elements are given as $\tilde{a}_{12} = (7, 8, 9)$, $\tilde{a}_{13} = (3, 4, 5)$, $\tilde{a}_{14} = (6, 7, 8)$, as $\tilde{a}_{32} = (4, 5, 6)$, $\tilde{a}_{34} = (7, 8, 9)$ and $\tilde{a}_{42} = (3, 4, 5)$. Tab. 3. depicts the values of SSE and ε. Results show the superiority of the proposed model over all the other models. The value of SSE in Mohtashami model is more than the Linear FPP model. But with the proposed model, lesser values of SSE and ε are obtained as compared to all the considered models.

Table 3: Crisp weights, deviation from initial judgments (ε) and SSE values for case 3

	Non-Linear FPP [35]	Linear FPP [35]	Mohtashami [41]	Proposed model
w_1	0.5583	0.6397	0.5262	0.4918
w_2	0.0519	0.0805	0.0487	0.0584
w_3	0.3311	0.2221	0.3568	0.3806
w_4	0.0585	0.0574	0.0680	0.0701
ε	1.8728	3.1307	1.8049	1.7997
SSE	32.2888	52.7075	34.97	24.8729

In all the above examples, triangular fuzzy numbers are used. But, the proposed optimization model can work for trapezoidal numbers as well. The followingcases demonstrate the usefulness of the proposed model by taking the judgments in the comparison matrices in the form of TrFNs. As discussed earlier, only Mohtashami's model [35] can derive crisp weights from the TrFN and TSFN judgments out of all the fuzzy optimisation models. Therefore, only Linear FPP and Mohtashami's models are considered compared to the proposed fuzzy optimization model.

Case-4:

Consider the comparison matrix of four elements where the judgments are represented in the form of TrFNs as $\tilde{a}_{12} = (4, 9/2, 11/2, 6)$, $\tilde{a}_{14} = (7, 15/2, 17/2, 9)$, $\tilde{a}_{31} = (3, 7/2, 9/2, 5)$, $\tilde{a}_{32} = (7, 15/2, 17/2, 9)$, $\tilde{a}_{34} = (6, 13/2, 15/2, 8)$ and $\tilde{a}_{42} = (3, 7/2, 9/2, 5)$. Results show that the proposed model derives crisp weights with lesser values of SSE and ε as compared to Linear FPP and Mohtashami's model as shown in Tab. 4.

Table 4: Crisp weights, deviation from initial judgments (ε) and SSE values for case 4

	Linear FPP [35]	Mohtashami [41]	Proposed model
w_1	0.2423	0.3249	0.3677
w_2	0.0721	0.511	0.0487
w_3	0.6274	0.5589	0.5147
w_4	0.0579	0.0649	0.0681
ε	2.84	1.99	1.6
SSE	45.614	35.080	34.2698

Case-5:

To demonstrate the superiority of the proposed fuzzy optimization model over the other models, another example where judgments are represented in the form of TrFNs is considered. Suppose the judgments of the comparison matrix are as $\tilde{a}_{12}$ = (2, 5/2, 7/2, 4), $\tilde{a}_{13}$ = (5, 11/2, 13/2, 7), $\tilde{a}_{14}$ = (6, 13/2, 15/2, 8), $\tilde{a}_{32}$ = (6, 13/2, 15/2, 8), $\tilde{a}_{34}$ = (7, 15/2, 17/2, 9) and $\tilde{a}_{42}$ = (3, 7/2, 9/2, 5). Tab. 5. depicts the values of crisp weights, SSE and ε. Tab. 5 depicts the values of crisp weights, SSE and ε. The proposed model shows its superiority to the other models in terms of ε and SSE values. Therefore, these two examples demonstrate that the proposed model performs better than the other models for judgments represented in the form of TrFNs.

Table 5: Crisp weights, deviation from initial judgments (ε) and SSE values for case 5

	Linear FPP [35]	Mohtashami [41]	Proposed model
w_1	0.6544	0.5555	0.5594
w_2	0.0843	0.0784	0.0932
w_3	0.1875	0.2923	0.2797
w_4	0.0728	0.0738	0.0667
ε	4.4245	3.09	3.0
SSE	96.1872	71.0065	69.7089

Till now the fuzzy judgments considered in this manuscript are symmetric like TFNs and TrFNs. But in some situations, a decision-maker makes the judgments of asymmetric nature like TSFNs and TrSFNs. If the values of δ and χ are equal to one, then it demonstrates TFN otherwise it will act as TSFN.

Case-6:

Consider a comparison matrix of four elements in which judgments are represented by $\tilde{a}_{12}$ = (3, 4, 5), $\tilde{a}_{14} = (7,\ 8,\ 9)\ \delta = 3, \chi = 3$ as TSFN; $\tilde{a}_{31}$ = (4, 5, 6) as TFN; $\tilde{a}_{32} = (7,\ 8,\ 9)\ \delta = 3, \chi = 1/3$ as TSFN, $\tilde{a}_{34}$ = (3, 4, 5) as TFN and $\tilde{a}_{42}$ = (1, 2, 3) δ = 1/3, χ = 3 as TSFN. Tab. 6 depicts the values of crisp weights, SSE and ε. The value of SSE with the proposed model is 36.4661 as compared to 37.5998 in Mohtashami's model. Values of SSE and ε in Linear FPP are very high. Results show that the proposed model outperforms all these models as depicted in Tab. 6.

Table 6: Crisp weights, deviation from initial judgments (ε) and SSE values for case 6

	Linear FPP [35]	Mohtashami [41]	Proposed model
w_1	0.2002	0.3341	0.3302
w_2	0.1194	0.0612	0.0690
w_3	0.6135	0.5315	0.5283
w_4	0.0669	0.0732	0.0718
ε	4.1704	2.4358	2.4001
SSE	73.1194	37.5998	36.4661

Case-7:

To show the proposed model's effectiveness, another example of comparison matrix with four elements is considered. Judgments are represented as by $\tilde{a}_{12}$ = $(3,\ 4,\ 5)$ $\delta = 1/3, \chi = 3$ as TSFN, $\tilde{a}_{31}$ = (5, 6, 7) as TFN; $\tilde{a}_{32}$ = $(7,\ 8,\ 9)$ $\delta = 3, \chi = 1/3$ as TSFN; $\tilde{a}_{34}$ = $(3,\ 4,\ 5)$ $\delta = 3, \chi = 3$ as TSFN, $\tilde{a}_{41}$ = (7, 8, 9) as TFN and $\tilde{a}_{42}$ = $(4,\ 5,\ 6)$ $\delta = 1/3, \chi = 3$ as TSFN. Linear FPP model performs poorly as shown in Tab. 7. The values of SSE and ε with the Mohtashami's model are 39.4676 and 1.9846, respectively. These values are comparatively more than the proposed model's value of 37.4647 and 1.6, respectively. Thus, the proposed model outperforms all the other considered models.

Table 7: Crisp weights, deviation from initial judgments (ε) and SSE values for case 7

	Linear FPP [35]	Mohtashami [41]	Proposed model
w_1	0.0608	0.0652	0.0681
w_2	0.0883	0.0514	0.0487
w_3	0.6263	0.5565	0.5152
w_4	0.2246	0.3269	0.3680
ε	3.4	1.9846	1.6
SSE	57.3787	39.4676	37.4647

Case-8:

In the last example, consider the comparison matrix of four elements as $\tilde{a}_{13}$ = $(3,\ 4,\ 5)$ $\delta = 1/3, \chi = 3$ as TSFN, $\tilde{a}_{14}$ = $(3,\ 4,\ 5)$ $\delta = 1/3, \chi = 3$ as TSFN; $\tilde{a}_{21}$ = $(5,\ 6,\ 7)$ $\delta = 3, \chi = 3$ as TSFN; $\tilde{a}_{23}$ = $(1,\ 2,\ 3)$ $\delta = 1/3, \chi = 3$ as TSFN, $\tilde{a}_{24}$ = (3, 4, 5) as TFN and $\tilde{a}_{43}$ = $(7,\ 8,\ 9)$ $\delta = 1/3, \chi = 3$ as TSFN. Tab. 8. depicts the values of SSE and ε of different models for deriving crisp weights. In this case, the SSE value for the proposed model is more as compared to Mohtashami's model. But the value of ε is lesser with the proposed model. Similarly, when the value of threshold limit (α) is increased, the values of SSE and ε are lesser with the proposed model is compared to Mohtashami's model. Thus the proposed model is superior to the other models for this example also. In all the eightexamples the proposed model performs better than the other models. Therefore, crisp weights can be derived from different types of fuzzy judgments. This model can be applied in various real-life applications like operation research, material selection, and handover in wireless networks, cloud computing, and other managerial and engineering problems.

Table 8: Crisp weights, deviation from initial judgments (ε), and SSE values for case 8

	Linear FPP [35]	Mohtashami [41]	Proposed model	
w_1	0.1705	0.2639	0.1788	0.2270
w_2	0.5015	0.4339	0.3039	0.3631
w_3	0.0761	0.0669	0.0483	0.0571
w_4	0.2518	0.2353	0.4686	0.3530
ε	4.7623	3.4824	3.3	3.4
SSE	102.4446	89.3605	90.2756	80.5218

4 Conclusion

This paper presents a novel Sum of Squared Error (SSE) based fuzzy optimization model to derive crisp weights for fuzzy pairwise comparison matrices. The proposed optimization model can derive priorities for Triangular Fuzzy Numbers (TFNs), Trapezoidal Fuzzy Numbers (TrFNs), Triangular Shaped Fuzzy (TSFNs), and Trapezoidal Shaped Fuzzy Numbers (TSFNs) unlike most of the optimization models that derive weights in the case of TFNs only. Simulation of the fuzzy number is done by Monte Carlo's method. The proposed model minimizes the SSE value and deviation from judgments ischecked by applying a threshold-based constraint.Eightexamples are illustrated that include examples of consistent and inconsistent fuzzy judgments represented by TFNs, TrFNs and TSFNs. Results show that the proposed model outperforms other models for all theabove mentioned cases.

Analytic Hierarchy Process (AHP) can be used in many decision-making problems in various fields like healthcare [45] operation research [46], management [47], engineering [48], etc. Therefore, the proposed model has utility in all these decision-making systems where decision makers cannot give crisp weights. The proposed model's main limitation is that it considers the judgments as type 1 fuzzy sets only. This work can be further extended to obtain crisp priorities from type 2 fuzzy sets. Also methods to measure theconsistency for triangular shaped and trapezoidal fuzzy comparison matrices can be measured.

Funding Statement: The authors received no specific funding for this study.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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