

NOMA with Adaptive Transmit Power Using Intelligent Reflecting Surfaces

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Abstract: In this article, we use Intelligent Reflecting Surfaces (IRS) to improve the throughput of Non Orthogonal Multiple Access (NOMA) with Adaptive Transmit Power (ATP). The results are valid for Cognitive Radio Networks (CRN) where secondary source adapts its power to generate low interference at primary receiver. In all previous studies, IRS were implemented with fixed transmit power and previous results are not valid when the power of the secondary source is adaptive. In CRN, secondary nodes are allowed to transmit over the same band as primary users since they adapt their power to minimize the generated interference. Each NOMA user has a subset of dedicated reflectors. At any NOMA user, all IRS reflections have the same phase. CRN-NOMA using IRS offers 7, 13, 20 dB gain *vs.* CRN-NOMA without IRS for N = 8, 16, 32 reflectors. We also evaluate the effects of primary interference. The results are valid for any number of NOMA users, Quadrature Amplitude Modulation (QAM) and Rayleigh channels.

Keywords: IRS; 6G; CRN; NOMA; adaptive transmit power (ATP)

1 Introduction

Intelligent Reflecting Surfaces (IRS) are a good candidate for sixth generation 6G networks [1-3]. IRS phases are adjusted so that reflections have a zero-phase at all users [4-7]. IRS has been suggested for optical communications [8-10] and Millimeter Wave (mmWave) systems [11,12]. Experimental results of IRS have been presented in [13-15]. A practical implementation of IRS can be found in [16,17] were phase shifts can be continuous or quantized. Machine learning algorithms can be used to optimize the performance of IRS [18,19]. A real time cutting model using finite element was proposed in [20]. A fast and accurate tissue simulation model was discussed in [21]. Device to Device (D2D) communications for the fifth generation and beyond was studied in [22].

IRS can be used to reflect the transmitted signal to NOMA users. The source combines of symbols of K NOMA users. This signal is reflected by RIS toward K users. The weakest user detects its signal and considers the rest of signals as noise. The strongest user detects weakest user signal. Then, it removes it and continue the detection process of remaining users that are ranked from the weakest to the strongest one. IRS was implemented when the transmitter has a fixed transmit power in [1-19]. In all previous



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studies [1–19], IRS were implemented with fixed transmit power and previous results are not valid when the power of the secondary source is adaptive. In CRN, secondary nodes are allowed to transmit over the same band as primary users since they adapt their power to minimize the generated interference. In this paper, we derive the throughput of NOMA using IRS and adaptive transmit power.

In this article, we propose to:

- Compute the throughput of CRN using NOMA and IRS where the secondary source has an adaptive power. Each secondary NOMA user has a given set of reflectors.
- We derive the statistics of Signal to Noise Ratio (SNR) as well as Signal to Interference plus Noise Ratio (SINR). We study the effects of primary interference. CRN-NOMA using IRS offers 7, 13, 20 dB gain versus CRN-NOMA without IRS for N = 8, 16, 32 reflectors.
- Two algorithms are discussed to rank the NOMA users.

Next section gives the throughput when there are two users. Section 3 generalizes the results to CRN-NOMA with K users. Section 4 discusses the obtained results. The paper is concluded in last section.

2 CRN-NOMA with Two Users

Fig. 1 depicts the network model with two secondary users, a Source (SS), a Primary Receiver and Transmitter and PR and PT. SS adapts its power to have a small interference at PR. We consider Rayleigh channels. Let $\sqrt{\lambda}a_k$ be the channel from SS to k-th reflector of IRS. $\lambda = 1/dSS$, IRS^{ple} dX, Y is the distance from X to Y and ple is the path loss exponent. We can write $a_k = c_k e^{-j \Phi k}$ where $c_k = |a_k|$.

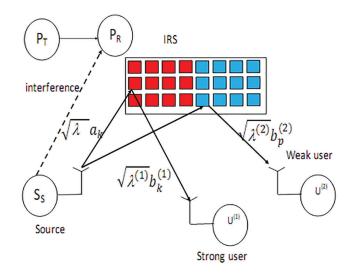


Figure 1: A network with two users

Let $\sqrt{\lambda^{(i)}}b_k^{(i)}$ be the channel from k-th reflector to i-th user $U^{(i)}$. $\lambda^{(i)} = 1/dIRS$, $U(i)^{\text{ple}}$. We can write $b_k^{(i)} = e_k^{(i)}e^{-jf_k^{(i)}}$ where $e_k^{(i)} = |b_k^{(i)}|$. Let $I^{(i)}$ be the set of reflector's of $U^{(i)}$. The phase of k-th reflector dedicated to $U^{(i)}$ in set $I^{(i)}$ given by

$$v_k^{(i)} = f_k^{(i)} + \Phi \mathbf{k},\tag{1}$$

The transmitted symbol by SS is written as

$$s = \sqrt{po_1}s^{(1)} + \sqrt{po_2}s^{(2)},\tag{2}$$

where $s^{(i)}$ is the symbol of $U^{(i)}$, po_i is the power of $U^{(i)}$, $po_1 + po_2 = 1$ and $1 > po_2 > po_1 > 0$.

The signal at U⁽ⁱ⁾ given by

$$r^{(i)} = s \sqrt{\lambda \lambda^{(i)} E_{SS}} \sum_{k \in I^{(i)}} a_k b_k^{(i)} e^{i v_k^{(i)}} + n^{(i)},$$
(3)

 $n^{(i)}$ is an additive Gaussian r.v. with variance N_0 and E_{SS} is SS symbol energy defined as

$$E_{SS} = \min\left(E_{max}, \frac{I}{|g_{SSP_R}|^2}\right) \tag{4}$$

 E_{max} is the maximum symbol energy, I is the interference threshold and g_{SSPR} is channel coefficient between SS and PR. SS verifies interference constraints as

$$E_{SS}|g_{SSP_R}|^2 \le I,\tag{5}$$

Using (1), we obtain

$$r^{(i)} = A^{(i)} \sqrt{\lambda \lambda^{(i)} E_{SS} \left[\sqrt{po_1} s^{(1)} + \sqrt{po_2} s^{(2)} \right]} + n^{(i)}, \tag{6}$$

where

$$A^{(i)} = \sum_{k \in I^{(i)}} c_k e_k^{(i)}, \tag{7}$$

Weak user $U^{(2)}$ estimates $s^{(2)}$ with SINR

$$\Gamma^{(2)} = \frac{po_2 B^{(2)}}{po_1 B^{(2)} + N_0},\tag{8}$$

where

$$B^{(i)} = [A^{(i)}]^2 \lambda \lambda^{(i)} E_{SS},.$$
(9)

The probability of an outage event at $U^{(2)}$ is given by

$$P_{outage}^{(2)}(x) = P_{B^{(2)}}\left(\frac{N_0 x}{po_2 - po_1 x}\right)$$
(10)

where the Cumulative Distribution Function (CDF) of $B^{(i)}$, $P_{B^{(i)}}(x)$, is provided in Appendix A. U⁽¹⁾ detects s⁽²⁾ as po₂>po₁ with SINR

$$\Gamma^{(1)\to(2)} = \frac{po_2 B^{(1)}}{po_1 B^{(1)} + N_0},\tag{11}$$

Then $U^{(1)}$ removes $s^{(2)}$ and demodulates $s^{(1)}$ with SNR

$$\Gamma^{(1)\to(1)} = \frac{po_1 B^{(1)}}{N_0},\tag{12}$$

The probability of an outage event at U⁽¹⁾ is computed as

$$P_{outage}^{(1)}(x) = P(\min[\Gamma^{(1)\to(1)}, \Gamma^{(1)\to(2)}] \le x) = P_{B^{(1)}}\left(\max\left[\frac{N_0 x}{po_1}, \frac{N_0 x}{po_2 - po_1 x}\right]\right)$$
(13)

The Packet Error Probability (PEP) of U⁽ⁱ⁾ is given by

$$PEP^{(i)}(po_1, po_2) \le P^{(i)}_{outage}(W_0), \tag{14}$$

where

$$W_0 = \int_0^{+\infty} 1 - \left[1 - SEP(w)\right]^L dw,$$
(15)

L is packet length and

$$SEP(w) = 2\left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3w}{M-1}}\right),\tag{16}$$

The throughput of U⁽ⁱ⁾ is given by

 $Thr^{(i)}(po_1, po_2) = \log_2(M)[1 - PEP^{(i)}(po_1, po_2)],$ (17)

The total throughput (TThr) is given by

$$TThr(po_1, po_2) = Thr^{(1)}(po_1, po_2) + Thr^{(2)}(po_1, po_2)$$
(18)

We maximize the total throughput as follows

$$TThr^{maximized} = max_{0 < po_1 < po_2 < 1} TThr(po_1, po_2)$$

$$\tag{19}$$

3 CRN-NOMA with K Users

3.1 Ranking Using Average Gains

The network model is depicted in Fig. 2. It contains PT, PR, SS and K secondary NOMA users. U⁽ⁱ⁾ has the i-th maximum average channel gain between SS and NOMA users. Let P be defined as

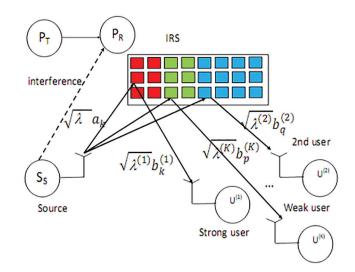


Figure 2: A network with K users

$$P = \sum_{j=1}^{K} N^{(j)},$$
(20)

N^(j) is the number of IRS reflectors of U^(j).

NOMA symbol is written as

$$s = \sum_{i=1}^{K} \sqrt{po_i} s^{(i)},$$
(21)

where $po_1 + po_2 = 1$ and $1 > po_2 > po_1 > 0$.

$$\sum_{i=1}^{K} po_i = 1 \tag{22}$$

The received signal at U⁽ⁱ⁾ is written as

$$r^{(i)} = A^{(i)} \sqrt{\lambda \lambda^{(i)} E_{SS}} \left[\sum_{i=1}^{K} \sqrt{po_i} s^{(i)} \right] + n^{(i)},$$
(23)

 $U^{(i)}$ detects s_K as $po_K > po_i$ with SINR

$$\Gamma^{(i)\to(K)} = \frac{po_K B^{(i)}}{B^{(i)} \sum_{l=1}^{K-1} po_l + N_0},$$
(24)

Then $U^{(i)}$ performs Successive Interference Cancelation (SIC), removes s_K to detect s_{K-1} with SINR

$$\Gamma^{(i)\to(K-1)} = \frac{po_{K-1}B^{(i)}}{B^{(i)}\sum_{l=1}^{K-2} po_l + N_0},$$
(25)

U⁽ⁱ⁾ detects sp with SINR

$$\Gamma^{(i)\to(p)} = \frac{po_p B^{(i)}}{B^{(i)} \sum_{l=1}^{p-1} po_l + N_0},$$
(26)

The probability of an outage event at U⁽ⁱ⁾ is computed as

$$P_{outage}^{(i)}(x) = P(\Gamma^{(i)\to(K)} \le x, \ \dots, \ \Gamma(i)\to(i)\le x) = P_{B^{(i)}}\left(\max_{1\le p\le K}\left[\frac{N_0x}{po_p - x\sum_{l=1}^{p-1}po_l}\right]\right),\tag{27}$$

The PEP at U(i) is equal to

$$PEP^{(i)}(po_1, \ldots, po_K) \le P^{(i)}_{outage}(W_0),$$

$$(28)$$

where W0 is defined in (15)

The throughput of U⁽ⁱ⁾ is given by

$$Thr^{(i)}(po_1, \ldots, po_K) = log_2(M)[1 - PEP^{(i)}(po_1, \ldots, po_K)],$$
 (29)

The total throughput (TThr) is given by

$$TThr(po_1, \ldots, po_K) = \sum_{i=1}^{K} Thr^{(i)}(po_1, \ldots, po_K),$$
 (30)

We maximize the total throughput as follows

$$TThr^{maximized} = max_{0 < po_1 < \dots < po_K < 1} TThr(po_1, \dots, po_K).$$

$$(31)$$

3.2 Ranking Using Instantaneous Gains

Let Ui⁽¹⁾ be the strongest user with largest instantaneous channel gain B⁽ⁱ⁾:

$$Bi^{(1)} = max_{1 \le p \le K} B^{(p)},$$
(32)

Let $Ui^{(K)}$ be the weakest user :

$$Bi^{(K)} = min_{1 \le p \le K} B^{(p)},$$
(33)

Let Ui^(q) be q-th ranked user:

$$Bi^{(q)} = q - th - max_{1 \le p \le K} B^{(p)},$$
(34)

The CDF of $Bi^{(q)}$ is given by

$$P_{Bi^{(q)}}(x) = \sum_{j=1}^{q} \sum_{m1,m2,\dots,mj-1} \prod_{l=1}^{j-1} [1 - P_{B^{(m_l)}}(x)] \sum_{mj,\dots,mN} \prod_{p=j}^{K} P_{B^{(m_p)}}(x)$$
(35)

where $1 \le mi \le N$ for i = 1, ..., N. $m1^{1}m_{2}^{1}...mN$, $mq \le mq + 1 \le ... \le mK$ and $P_{B^{\wedge}(i)}(x)$ is given in Appendix A.

The PEP and throughput can be computed as Section 3.1 where we have to replace $P_{B(q)}(x)$ by $P_{Bi(q)}(x)$ given in (35).

4 Effects of Primary Interference

The SINR is computed as

$$\Gamma^{(i)\to(p)} = \frac{po_p B^{(i)}}{B^{(i)} \sum_{l=1}^{p-1} po_l + N_0 + I_{PT,i}},$$
(36)

The probability of an outage at U⁽ⁱ⁾ is computed as

$$P_{outage}^{(i)}(x) = \int_{0}^{+\infty} P_{B^{(i)}}\left(\max_{1 \le p \le K} \left[\frac{(N_0 + y)x}{po_p - x \sum_{l=1}^{p-1} po_l} \right] \right) p_{I_{PT,i}}(y) dy,$$
(37)

where

$$p_{I_{PT,i}}(y) = \frac{e^{-\frac{y}{I_{PT,i}}}}{\overline{I_{PT,i}}}$$
(38)

 $\overline{I_{PT,i}}$ is the average interference. The PEP and throughput are computed using (37).

5 Numerical Results

Fig. 3 shows the total throughput for CRN-NOMA for K = 2, for I = 1 16 Quadrature Amplitude Modulation (QAM), dIRSU⁽ⁱ⁾ = 1,1.5 i = 1,2 Packet length is L = 200.. IRS allows 6, 12, 18 dB *vs.* CRN-NOMA without IRS for N = 8, 16, 32.

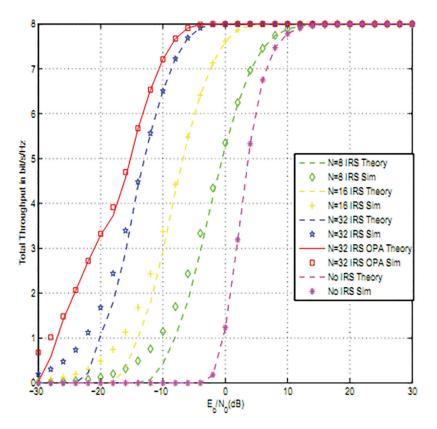


Figure 3: Total throughput for 2 users for 16QAM

Fig. 4 shows the throughput for different values of I, 16QAM modulation and N = 8 reflectors. As I increases as the throughput improves since SS can increase its power while verifying interference constraints.

Fig. 5 shows that NOMA with IRS for N = 64 offers better performance than Orthogonal Multiple Access (OMA) and NOMA without IRS for 16QAM and two users. At high average SNR, the throughput of OMA is half that of NOMA.

Fig. 6 depicts the effects of primary interference when there are two users, N = 8, 16 reflectors per user for 16QAM Modulation. The parameters are dPTU(i) = 1,0.9, 0.5,0.6. We notice that the performance degrades as PT is close to NOMA users since there is more interference.

Fig. 7 depicts the total throughput for 16-QAM modulations for two users and N=8, 16 reflectors. Ranking using instantaneous channel gains offers the best throughput.

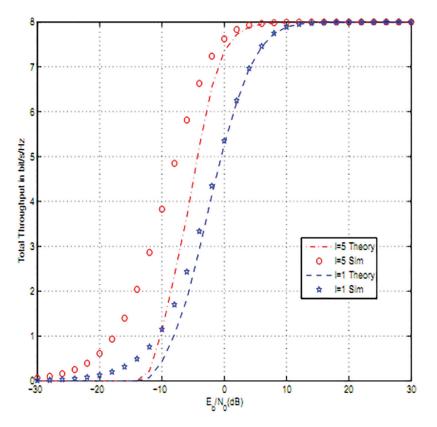


Figure 4: Effect of interference threshold I: 2 users, 16QAM and N = 8 reflectors

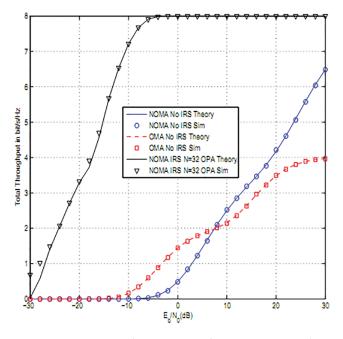


Figure 5: OMA and NOMA performance comparison

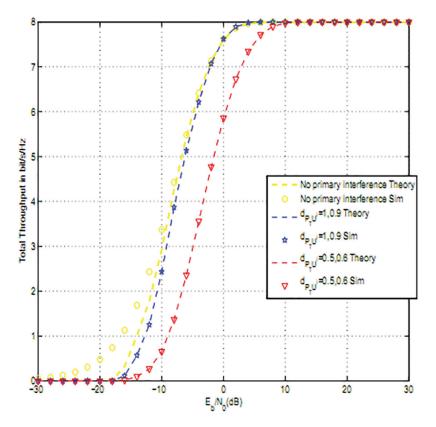


Figure 6: Effects of primary interference on Total throughput of NOMA: 2 users, 16QAM modulation and N = 16

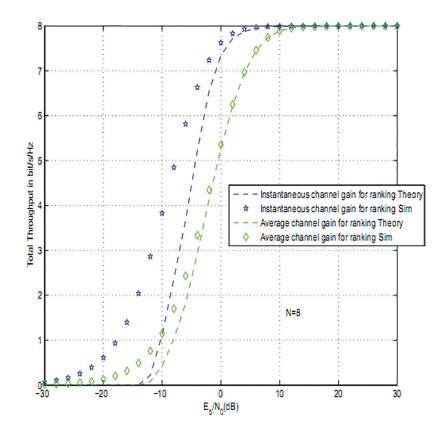


Figure 7: Secondary throughput for 16QAM modulation and different ranking strategies: N = 8

6 Conclusions

In this article, we computed the PEP and throughput of NOMA with adaptive transmit power and IRS. IRS are deployed to enhance data reception at all users. CRN-NOMA using IRS offers 7, 13, 20 dB gain *vs.* the absence of IRS for N = 8, 16, 32. We have also derived the SNR and SINR statistics.

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Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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Appendix A: CDF of B⁽ⁱ⁾

The variance and mean of A(i) are $\sigma_A^2 = N\left(1 - \frac{\pi^2}{16}\right)$ and mA = N $\pi/4$. B⁽ⁱ⁾ is equal to

$$B^{(i)} = [A^{(i)}]^2 \lambda \lambda^{(i)} E_{SS}$$
(39)

We deduce

$$P_{B^{(i)}}(x) = P\left(B^{(i)} \le x \left|\frac{I}{\left|g_{SSPR}\right|^{2}} < E^{max}\right) P\left(\frac{I}{\left|g_{SSPR}\right|^{2}} < E^{max}\right) + P\left(B^{(i)} \le x \left|\frac{T}{\left|g_{SSPR}\right|^{2}} > E^{max}\right) P\left(\frac{I}{\left|g_{SSPR}\right|^{2}} > E^{max}\right),$$
(40)

where

$$P\left(\frac{I}{\left|g_{SSPR}\right|^{2}} < E^{max}\right) = e^{-\frac{I}{\lambda_{SSPR}E^{max}}},\tag{41}$$

where $\lambda_{\text{SSPR}} = E(|g_{\text{SSPR}}|^2)$, E(.) is the expectation operation and gSSPR is the channel coefficient between SS and PR. When $I/|g_{SSPR}|^2 > E^{max}$, we have

$$B^{(i)} = \mathrm{E}^{\mathrm{max}} [A^{(i)}]^2 \lambda \lambda^{(i)}$$
and
$$(42)$$

$$P\left(B^{(i)} \le x \left|\frac{I}{\left|g_{SSPR}\right|^{2}} > E^{max}\right) = 1 - Q_{0.5}\left(\frac{m_{A}}{\sigma_{A}}, \sqrt{\frac{x}{E^{max}\lambda\lambda^{(i)}\sigma_{A}^{2}}}\right),\tag{43}$$

where $Q_m(.,.)$ is the Generalized Marcum Q-function.

When $I/|g_{SSPR}|^2 < E^{max}$, $E_{SS} = \frac{I}{|g_{SSPR}|^2}$ and we have

$$P\left(B^{(i)} \le x \left|\frac{I}{\left|g_{SSPR}\right|^{2}} < E^{max}\right) = \int_{\frac{I}{E^{max}}}^{+\infty} 1 - Q_{0.5}\left(\frac{m_{A}}{\sigma_{A}}, \sqrt{\frac{xy}{E^{max}I\lambda\lambda^{(i)}\sigma_{A}^{2}}}\right) e^{-\frac{y}{SSPR}} \frac{1}{\lambda_{SSPR}} dy$$
(44)

 $P_{B^{(i)}}(x)$ is computed using (40),(41) and (43),(44).