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ARTICLE

# Navier Slip and Heat Transfer in a Nanofluid Due to a Stretching/Shrinking Sheet: An Analytical Study

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## ABSTRACT

This paper is devoted to the analysis of the heat transfer and Navier's slip effects in a non-Newtonian Jeffrey fluid flowing past a stretching/shrinking sheet. The nanoparticles, namely, Cu and $Al_2O_3$ are used with a water-based fluid with Prandtl number 6.272. Velocity slip flow is assumed to occur when the characteristic size of the flow system is small or the flow pressure is very small. By using the similarity transformations, the governing nonlinear PDEs are turned into ordinary differential equations (ODE's). Analytical results are presented and analyzed for various values of physical parameters: Prandtl number, Radiation parameter, stretching/shrinking parameter and mass transpiration for the flow and heat transfer. The considered problem is relevant to various physical applications in the field of engineering, e.g., the production of certain materials, the preparation of plastic and rubber sheets and glass blowing. It is shown that the considered nanofluid increases the thermal efficiency. The nanoparticles act as a heater by increasing the solid volume fraction and thermal radiation. Vice versa, they can act as a cooler if the strength of magnetic field is increased. The flow strength decreases by increasing the values of Deborah number.

## KEYWORDS

Nanofluid; navier's slip; radiation; ordinary differential equations

**Nomenclature**

| | |
|---|---|
| $A$ and $B$ | coefficients of first and second order slip, respectively |
| $a$ and $b$ | constants |
| $B_0$ | uniform magnetic field (*Tesla*) |
| $C_P$ | specific heat at constant pressure ($JKg^{-1}K^{-1}$) |
| $c$ | constant |
| $Erfc$ | complementary error function |
| $k$ | solution domain |
| $k^*$ | mean absorption coefficient |
| $l_1$ and $l_2$ | first and second order slip, respectively |
| $Pr$ | Prandtl number $\left(\frac{(\mu C_P)_f}{\kappa_f}\right)$ |
| $Q$ | Chandrasekhar's number $\left(\frac{\sigma_f B_0^2}{\rho_f c}\right)$ |

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| | |
|---|---|
| $q_r$ | radiative heat flux |
| $q_w$ | local heat flux ($wm^{-2}$) |
| $R$ | thermal radiation |
| $T_\infty$ | ambient temperature ($K$) |
| $T_w$ | temperature at wall ($K$) |
| $T$ | temperature ($K$) |
| $u_w(x)$ | velocity at wall |
| $u_e(x)$ | potential velocity |
| $v_w$ | mass flux ($ms^{-1}$) |
| $V_C$ | mass transpiration |
| $(u, v)$ | components of velocity ($ms^{-1}$) |
| $(x, y)$ | Cartesian coordinate ($m$) |

**Greek symbols**

| | |
|---|---|
| $\sigma$ | electrical conductivity ($Sm^{-1}$) |
| $\sigma^*$ | Stefan-Boltzmann constant ($Wm^{-2}K^{-4}$) |
| $\lambda$ | stretching/shrinking parameter |
| $\xi_1$ *and* $\xi_2$ | related to relaxation parameter |
| $\beta$ | Deborah number ($c\xi_2$) |
| $\Lambda$ | strength parameter $\left(\frac{a}{c}\right)$ |
| $\eta$ | similarity variable |
| $\nu_{nf}$ | kinematic viscosity of nanofluid ($m^2s^{-1}$) |
| $\nu_f$ | kinematic viscosity of base fluid ($m^2s^{-1}$) |
| $\rho_{nf}$ | density of nanofluid ($kgm^{-3}$) |
| $\rho_f$ | density of base fluid ($kgm^{-3}$) |
| $\alpha_{nf}$ | thermal diffusivity of nanofluid. |
| $\mu_{nf}$ | dynamic viscosity of nanofluid ($kgm^{-1}S^{-1}$) |
| $\mu_f$ | dynamic viscosity of base fluid ($kgm^{-1}S^{-1}$) |
| $\kappa$ | thermal conductivity ($W/mK$) |
| $K$ | temperature jump $\left(K_0\sqrt{\frac{a}{\nu_f}}\right)$ |
| $\tau_w$ | skin friction |
| $\tau$ | inclined angle |

**Subscripts**

| | |
|---|---|
| $w$ | quantities at wall |
| $\infty$ | quantities at freestream |
| $f$ | fluid |
| $nf$ | snanofluid |

**Abbeviations**

| | |
|---|---|
| MHD | magnetohydrodynamics |
| ODE | ordinary differential equation |
| PDE | partial differential equation |

## 1 Introduction

In the recent few eras the researchers show much interest on the flow of non-Newtonian fluids. This interest is growing like a stem of a tree because of its huge technological and industrial applications in many fields such as engineering applications, exchanger of heat, extrusion of metal and polymer process, manufacturing of paper, cable coating and so on. Numerous investigations take place on the flow of a fluid through a stretching/shrinking surface. Most of them agreed that Sakiadis [1] conducted the first investigation on the flow of a fluid induced by a moving surface. The flow of a fluid due to a stretching sheet is analyzed by Crane [2]. In this he studied the flow due to a sheet with constant speed from a slit into a fluid at rest. Motivated by Crane's work, many researchers conducted an experiment on the flow caused by the stretching/shrinking sheet. See the examples of Mahabaleshwar et al. [3,4] and Amkadni et al. [5]. Further, many more investigations take place on the flow of a stretching/shrinking sheet in the presence of MHD. Pavlov [6,7] primarily examined the stretching wall problem in the presence of MHD, this study describes the flow of a conducting fluid through a deformation of a plane elastic surface in a transverse magnetic field and it is analyzed in the approximation of boundary layer theory. Later Mahabaleshwar et al. [8,9] investigated the flow problem in the presence of MHD, this study describes the characteristics of heat and MHD imposed on the stretching sheet and it is solved analytically. Fang et al. [10] examined the magnetohydrodynamic flow in the presence of slip condition through a permeable stretching surface and the solution is solved analytically by inclusion of MHD and radiation. Smith [11] 1$^{st}$ work on the effect of radiation in a boundary layer flow, this work describes the radiation in the boundary layer on aerodynamic heat transfer. After this huge research was conducted on radiation. Siddheshwar et al. [12], worked on MHD flow and heat transfer in a viscoelastic liquid past a stretching sheet in the presence of radiation. Later Mahabaleshwar et al. [13] various methods have been conducted to improve the thermal conductivity of flow fluid by adding nano sized particles inside the liquids: then these liquids are called nanofluids. see the two examples of research on nanofluids [14,15]. Recently many works have been conducted on nanofluids, hybrid nanofluids and carbon nanotubes with a variety of boundary conditions under many situations such as MHD and thermal radiation, mass transpiration and so on. Mahabaleshwar et al. [16–18] and Anusha et al. [19] worked on role of brinkmann model with non-Newtonian fluid flow in the presence of porous medium with heat transfer and the effect of hybrid nanofluid flow with superlinear stretching sheet and Navier's slip effects on flow of a fluid with thermal radiation and mass transpiration.

Apart from these research some of the researchers conducted an experiment on Jeffrey fluid. The magnetohydrodynamics effects and other physical mechanisms show huge significance in many Jeffrey fluid applications. Jeffrey fluid past a Narrow tube with a Magnetic field is examined in [20]. Peristaltic flow of chyme in the small intestine with magnetic field and their characteristics explained in [21]. Kumar et al. [22] described incompressible Jeffrey fluid flow in the presence of a magnetic field with the annulus of rotating concentric.

Inspired by the above research papers the present work is to extend the articles [23,24], this work fulfills the impact of inclined MHD and second order slip in the presence of nanofluid. The governing PDEs of the problems are transformed into ODEs with the help of similarity variables; the whole investigation takes place on *Cu-water* nanofluid. The flow is caused due to stretching/shrinking sheets, with the help of suitable similarity transformation. The resulting ODEs are solved analytically to get the solution domain, this solution domain is used in heat equation and solved direct integration technique to obtain exact solution. The results of $C_f$ and $Nu$ can be attained with the help of graphs.

## 2 Physical Modeling and Governing Equations

In the current work Jeffrey nanofluid flow is taken into account. This flow is a laminar steady state and it is emerging towards the x-axis near the stagnation point in the presence of *Cu-water* nanofluid. The flow is

caused through a permeable stretching/shrinking surface lying at $y = 0$, the flow configuration is physically explained in Fig. 1. Also the nanofluids quantities are represented at Table 1. The uniform magnetic field $B_0$ is inserted at the deforming velocity of the wall $u_w(x)$. Along with potential velocity $u_e(x)$, assume the uniform ambient temperature $T_\infty$ at the for field. The governing Navier's stokes equations are given below:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} - \frac{\sigma_{nf}B_0^2}{\rho_{nf}}Sin[\tau]^2(u - u_e) + \frac{\nu_{nf}}{1+\xi_1}\left[\frac{\partial^2 u}{\partial y^2} + \xi_2\left(u\frac{\partial^3 u}{\partial x \partial y^2} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right)\right], \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_P)_{nf}}\frac{\partial q_r}{\partial y}, \tag{3}$$

the applied boundary conditions (see [25,26])

$$\left.\begin{aligned} u = \lambda cx + A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2} = 0\,,\ v = v_w,\ T = T_w(x) \quad & \text{at } y = 0 \\ u = u_e(x) = \text{ax}, \frac{\partial u}{\partial y} \to 0,\ T \to T_\infty, \quad & \text{as } y \to \infty \end{aligned}\right\}. \tag{4}$$

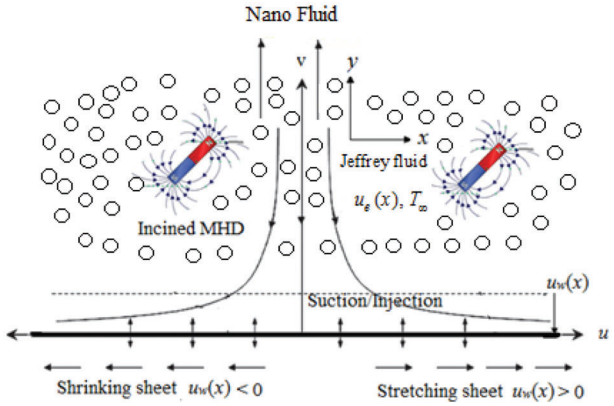


**Figure 1:** Physical representation of jeffrey fluid flow

**Table 1:** Thermophysical properties of nanofluids (see Mahabaleshwar et al. [15])

| Sr. No. | Physical properties | Liquid phase | Copper | Alumina |
|---|---|---|---|---|
| 1 | $C_p$ $(J/kgK)$ | 4179 | 385 | 765 |
| 2 | $\rho$ $(kg/m^3)$ | 997.1 | 8933 | 3970 |
| 3 | $\kappa$ $(W/\text{m}K)$ | 0.613 | 400 | 40 |
| 4 | $\sigma$ $(Sm^{-1})$ | 0.05 | $5.97 \times 10^7$ | |

It is mentioned that there are two boundary conditions exist for wall temperature $T_w(x)$, either it is constant or evolving with $x$ linearly via $T_w(x) = T_w + bx$. $T$ denotes the temperature of the fluid, and $v_w$

denotes the mass flux velocity with $v_w < 0$ represents suction and $v_w < 0$ represents injection. For the purpose of obtaining exact solutions the fluid properties are assumed to be constant, $\xi_1$ and $\xi_2$ are associated to relaxation parameters., at lastly stretching case is represented by $\lambda = 1$ and shrinking case by $\lambda = -1$.

The Rosseland's formulation for the radiative heat flux can be simplified for radiation as (see [27–32])

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial z}, \tag{5a}$$

where, $T^4$ is expressed in the form of linear function of the temperature. Increasing the term $T^4$ in the form of Taylor series and also some terms are ignored.

$$T^4 = 4T_\infty^3 T - 3T_\infty^4, \tag{5b}$$

Substituting Eqs. (5a) and (5b) in Eq. (3) and the following suitable transformation applied to get the ordinary differential equations:

$$\left.\begin{aligned} \eta &= y\sqrt{\frac{c(1+\xi_1)}{\nu_f}}, \quad u = cxf'(\eta), \\ v &= -\sqrt{\frac{c\nu_f}{(1+\xi_1)}}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \end{aligned}\right\}, \tag{5c}$$

on applying these similarity transformations to the Eqs. (1) to (4), then these equations are simplified to

$$C_3 f''' + C_3\beta(f''^2 - ff^{iv}) - f'^2 + ff'' - \frac{C_1}{C_2}QSin^2(\tau)(f' - \Lambda) + \Lambda^2 = 0, \tag{6}$$

$$(C_5 + R)\theta'' + C_4 \Pr f\theta' = 0, \tag{7}$$

$$(C_5 + R)\theta'' + C_4 \Pr(f\theta' - f'\theta) = 0, \tag{8}$$

the constant parameters inserted in the Eqs. (5) to (7), can be defined as

$$C_1 = \frac{\sigma_{nf}}{\sigma_f}, \quad C_2 = \frac{\rho_{nf}}{\rho_f}, \quad C_3 = \frac{\nu_{nf}}{\nu_f}, \quad C_4 = \frac{\kappa_{nf}}{\kappa_f}, \quad C_5 = \frac{(\rho C_P)_{nf}}{(\rho C_P)_f},$$

$$Q = \frac{\sigma B_0^2}{\rho c}, \quad R = \frac{16\sigma^* T_\infty^3}{3k^*\kappa_f}, \quad \Pr = \frac{(\mu C_P)_f}{\kappa_f}.$$

The BC's transformed to

$$\left.\begin{aligned} f(0) = V_C, \quad f'(0) = \lambda + l_1 f''(0) + l_2 f'''(0) \quad \text{at} \quad y = 0 \\ f'(\infty) = \Lambda, \quad f''(\infty) \to 0, \quad \theta(\infty) \to 0 \quad \text{as} \quad \to 0 \end{aligned}\right\}. \tag{9}$$

Here $v_w = -\sqrt{\frac{c\nu_f}{(1+\xi_1)}}$ is the wall transpiration, $l_1 = A\sqrt{\frac{c(1+\xi_1)}{\nu_f}}$, $l_2 = B\frac{c(1+\xi_1)}{\nu_f}$ are the first and second order slip, respectively, $\beta$, $V_C$, Pr, $\Lambda$ are the highly dominating parameters of the Jeffrey fluid flow:

## 3 Physical Quantities of Interest

The skin friction coefficient $C_f$ and Local Nusselt number $Nu_x$ is given by

$$C_f = \frac{\tau_w}{\rho_f u_e^2(x)}, \quad Nu_x = \frac{xq_w}{\alpha_f(T_w - T_\infty)}, \tag{10}$$

where

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad q_w = -\alpha_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{11}$$

Substituting, Eq. (11) in Eq. (10), after some manipulation the $C_f$ and $Nu_x$ are given by

$$\begin{aligned} C_f \mathrm{Re}_x^{1/2} &= C_3 f''(0) \\ Nu_x \mathrm{Re}_x^{-1/2} &= -\frac{C_4}{C_5}\theta'(0), \end{aligned} \tag{12}$$

## 4 An Analytical Solution for Momentum and Energy Equations

Let us assume the solution of the Eq. (6)

$$f(\eta) = V_C + \Lambda\eta + \frac{\lambda - \Lambda}{k(1 + l_1 k - l_2 k^2)}\{1 - \exp(-k\eta)\}, \tag{13}$$

in order to get the algebraic formula connecting with the physical parameters by equating Eqs. (13) into (6). The constraint $k$ must be positive to achieve the solution.

$$\begin{aligned} &(\lambda - \Lambda)\Big\{-(c_3 l_2 \beta V_C + \eta\Lambda\beta c_3 l_2)k^5 + (c_3 l_1 \beta V_C - c_3 l_2 + \eta\Lambda l_1 \beta c_3)k^4 \\ &+ (c_3\beta V_C + l_1 c_3 + V_C l_2 + \eta\Lambda\beta c_3 + \eta\Lambda l_2)k^3 + \left(dc_3\beta + l_2\frac{c_1}{c_2}Q\sin^2(\tau) + c_3\right. \\ &\left. -V_C l_1 + 2\Lambda l_2 - c_3\beta\Lambda - \eta\Lambda l_1\right)k^2 + \left(-l_1\frac{c_1}{c_2}Q\sin^2(\tau) - V_C - 2\Lambda l_1 - \eta\Lambda\right)k \\ &-\lambda - \frac{c_1}{c_2}Q\sin^2(\tau) - \Lambda\Big\} = 0, \end{aligned} \tag{14}$$

For limiting case $\Lambda = 0$, then Eqs. (13) and (14) reduce to

$$f(\eta) = V_C + \frac{\lambda[1 - \exp(-k\eta)]}{k(1 + l_1 k - l_2 k^2)}, \text{ provided } k > 0, \tag{15}$$

$$\begin{aligned} &\Big\{-c_3 l_2 \beta V_C k^5 + (c_3 l_1 \beta V_C - c_3 l_2)k^4 + (c_3\beta V_C + l_1 c_3 + V_C l_2)k^3 + \left(dc_3\beta + l_2\frac{c_1}{c_2}Q\sin^2(\tau).\right. \\ &\left. + c_3 - V_C l_1)k^2 + \left(-l_1\frac{c_1}{c_2}Q\sin^2(\tau) - V_C\right)k - \lambda - \frac{c_1}{c_2}Q\sin^2(\tau) - \Lambda\Big\} = 0 \end{aligned} \tag{16}$$

Further, we assuming the solution of temperature equation

$$\theta(\eta) = \exp(-k\eta), \tag{17}$$

Substituting (13) and (17), in (8), then we found the solution as

$$\lambda C_4 \,\mathrm{Pr} + k[\mathrm{Pr}\, C_4 V_C - k(C_5 + R)](1 + l_1 k - l_2 k^2) = 0, \tag{18}$$

After some manipulation on equating (16) and (18), we get the root of the equation as

$$k=\sqrt{\frac{C_2(C_5+R)-\Pr C_2C_3C_4\pm\sqrt{(\Pr C_2C_3C_4-C_2(C_5+R))^2-4\beta C_1C_2C_3C_4(C_5+R)\Pr QSin[\tau]^2}}{2\beta C_2C_3(C_5+R)}} \tag{19}$$

$$V_C=\frac{k(C_5+R)}{\Pr C_4}-\frac{\lambda}{k(1+l_1k-l_2k^2)}, \tag{20}$$

where, $k>0$ is independent of $\lambda$, $l_1$ and $l_2$, and also the Parameter $V_C$ will change with these parameters, the $C_f$ and $Nu$ are then becomes

$$-f''(0)=\frac{\lambda k}{1+l_1k-l_2k^2}, \qquad -\theta'(0)=k, \tag{21}$$

Therefore, the $C_f$ will decreases by increasing first and second order slip

Next we take particular case $\Lambda=1$, and $\lambda=1$, then Eq. (13) becomes

$$f(\eta)=V_C+\eta, \tag{22}$$

Substituting this in Eq. (7), the solution obtained as

$$\left.\theta(\eta)=\frac{Erfc\left[\frac{\sqrt{d}(V_C+\eta)}{\sqrt{2}}\right]}{Erfc\left[\frac{\sqrt{d}V_C}{\sqrt{2}}\right]}\text{where}\quad \mathrm{d}\ =\frac{\mathrm{PrC_4}}{(\mathrm{C_5}+R)}\right\}. \tag{23}$$

where $Erfc=1-Erf(t)=-\frac{2}{\sqrt{\pi}}\int_0^{\infty}e^{-t^2}dt$ means the complementary error function, hence we get the analytical expression as

$$\theta'(0)=-\left\{\frac{\exp\left[-\frac{dV_C^2}{2}\right]\sqrt{\frac{2}{\pi}}\sqrt{d}}{Erfc\left[\frac{\sqrt{d}V_C}{\sqrt{2}}\right]}\right\}. \tag{24}$$

## 5 Figures and Discussions

The Jeffrey nanofluid flow is considered in the current investigation, the impact of inclined MHD and second order slip is taken into account with this flow in the presence of *Cu-water* nanofluid. The flow is conducted due to a stretching/shrinking sheet. The governing PDEs are mapped into ODEs with the help of similarity transfromations, the momentum equation of this problem is verified exactly to get solution domain. By using direct integration technique to verify the energy equation and also the solution domain connected with heat equation through $f(\eta)$. To understand the problem tharougly skin friction and Nusselt number can be verified. The results of this investigation can be conducted with the help of graphs on varying the different parameters.

Figs. 2 and 3 demonstrate that the impact of $f(\eta)$ on similarity variable $\eta$ for stretching and shrinking case. In Fig. 2a it is observed that the stretching case condition, i.e., $\lambda = 1$, in this the flow of a fluid increases with increase of Radiation parameter $R$, and also the boundary layer thickness is away from the x-axis. Here the flow fluid increases with increase of $\beta$ value. Whereas Fig. 2b indicates the shrinking case $\lambda = -1$, in this also in this the flow of a fluid increases with increase of Radiation parameter $R$, and also the boundary layer thickness is away from the x-axis. But the flow fluid decreases with increase of $\beta$ value. Thermal radiation is much impact on fields it increases the flow velocity and heat transfer rate.

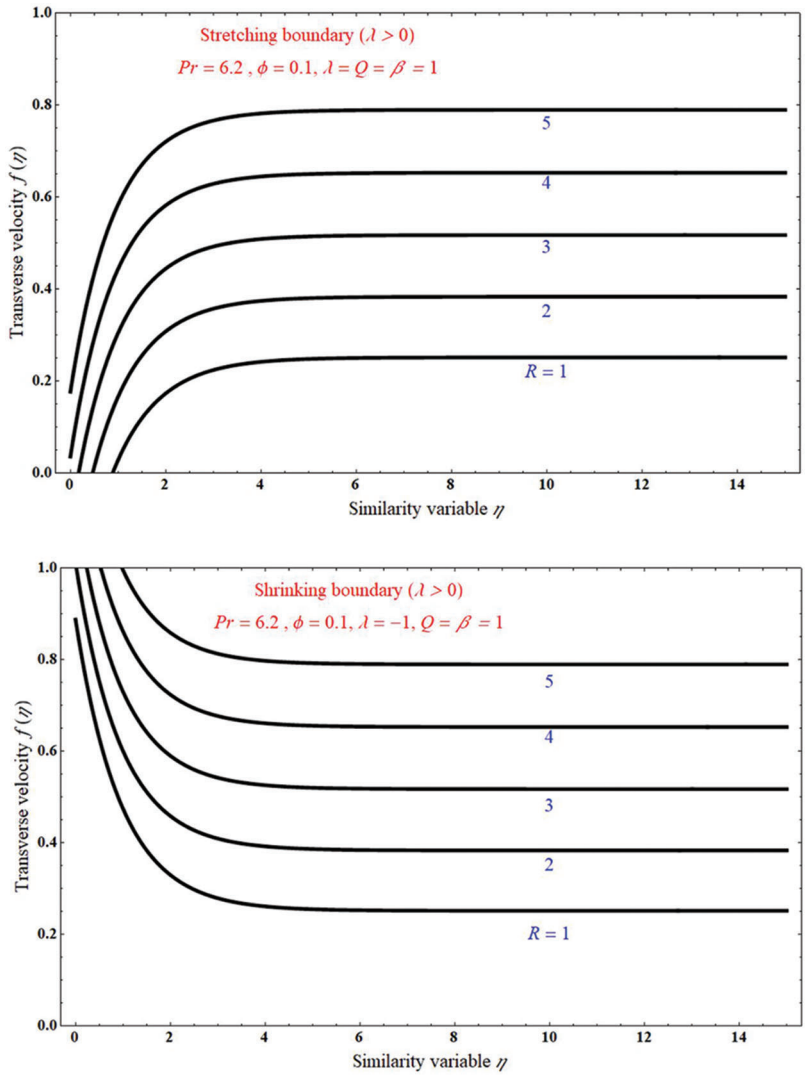


**Figure 2:** The impact of transverse velocity on similarity variable $\eta$ for unlike values of radiation parameter $R$, at (a) stretching case $\lambda = 1$, and (b) shrinking case $\lambda = -1$

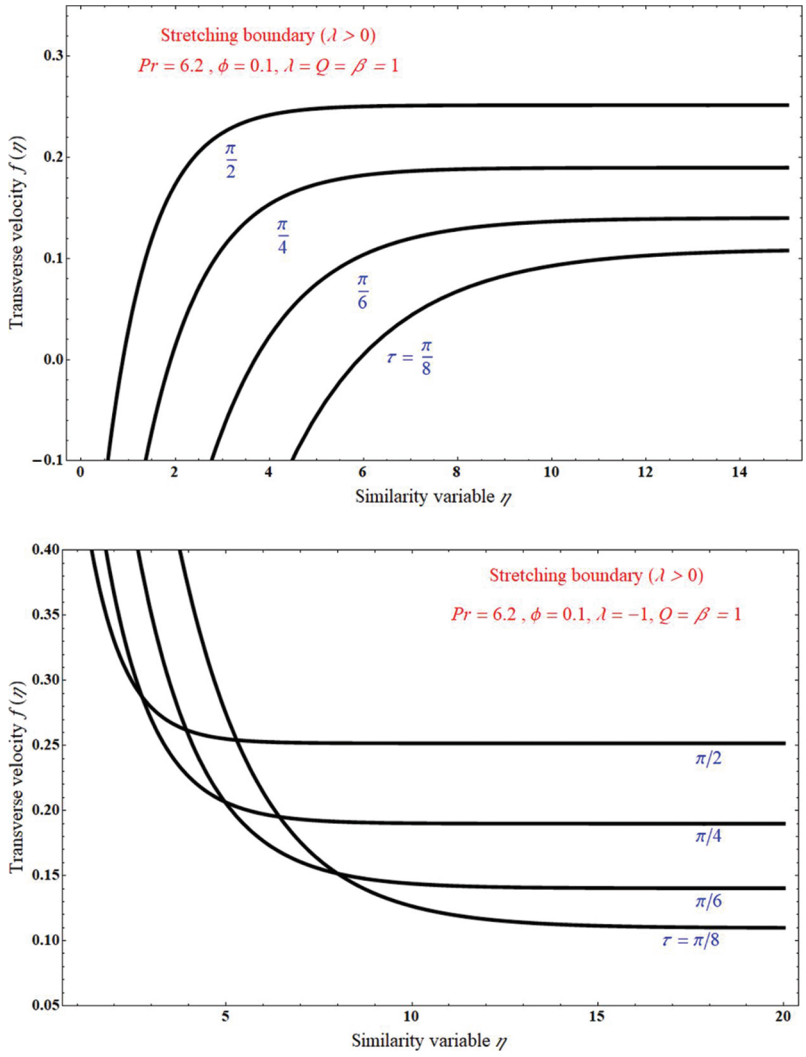


**Figure 3:** The impact of transverse velocity on similarity variable $\eta$ for unlike values of inclined angle $\tau$, at (a) stretching case $\lambda = 1$, and (b) shrinking case $\lambda = -1$

Figs. 3a and 3b analyse the flow of a fluid with unlike values of inclined angle $\tau$ for stretching and shrinking case, respectively. In both stretching and shrinking case the transverse velocity decreases with increase of inclined angle $\tau$, and boundary value thickness is towards the x-axis.

The effect of tangential velocity on similarity variable $\eta$ can be represented by Figs. 4 and 5. In Figs. 5a and 5b it is indicating that the tangential velocity decreases for stretching case and it is increased for shrinking case when we varying the Chandrasekhar's number $Q$, respectively, boundary layer thickness is away from the x-axis in stretching case, and boundary layer thickness is towards the x-axis in shrinking case. Figs. 6a and 6b represent the stretching case and shrinking case, respectively. In Fig. 6a it is observed that the tangential velocity increases with increases of inclined angle $\tau$ and this effect is reversed in Fig. 6b, also boundary layer thickness is away and towards the x-axis, respectively.

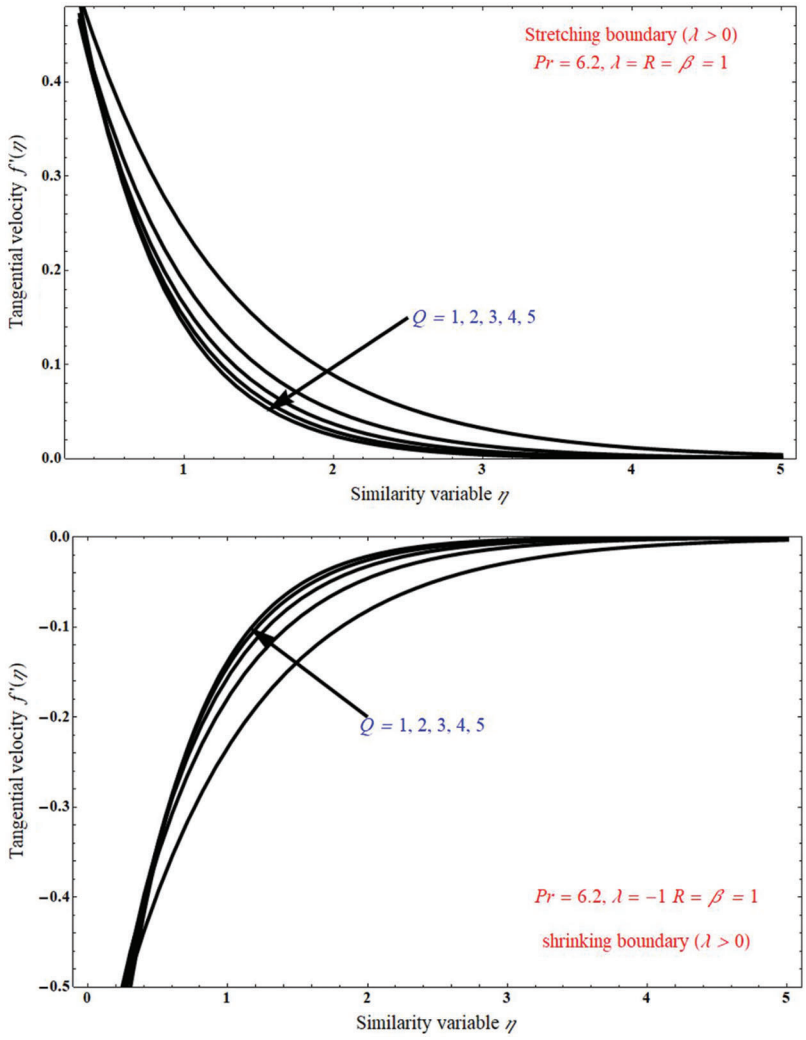


**Figure 4:** The impact of tangential velocity on similarity variable $\eta$ for unlike values of Chandrasekhar's number $Q$, at (a) stretching case $\lambda = 1$, and (b) shrinking case $\lambda = -1$

The impact of skin friction coefficient $C_f$ on the functions Chandrasekhar's number $Q$ and Prandtl number $Pr$ can be represented from the Figs. 7a and 7b, respectively. In Fig. 7a it is clear that the $C_f$ is increases with increasing the inclined angle $\tau$, and also from Fig. 7b it is conclude that the $C_f$ is decreases with increasing the Chandrasekhar's number $Q$. Furthermore, from both of the figure we observed that the $C_f$ will decreases by increasing the slip parameters $l_1$ and $l_2$. The large values of $Q$ causes the lesser skin friction and lesser heat transfer rate.

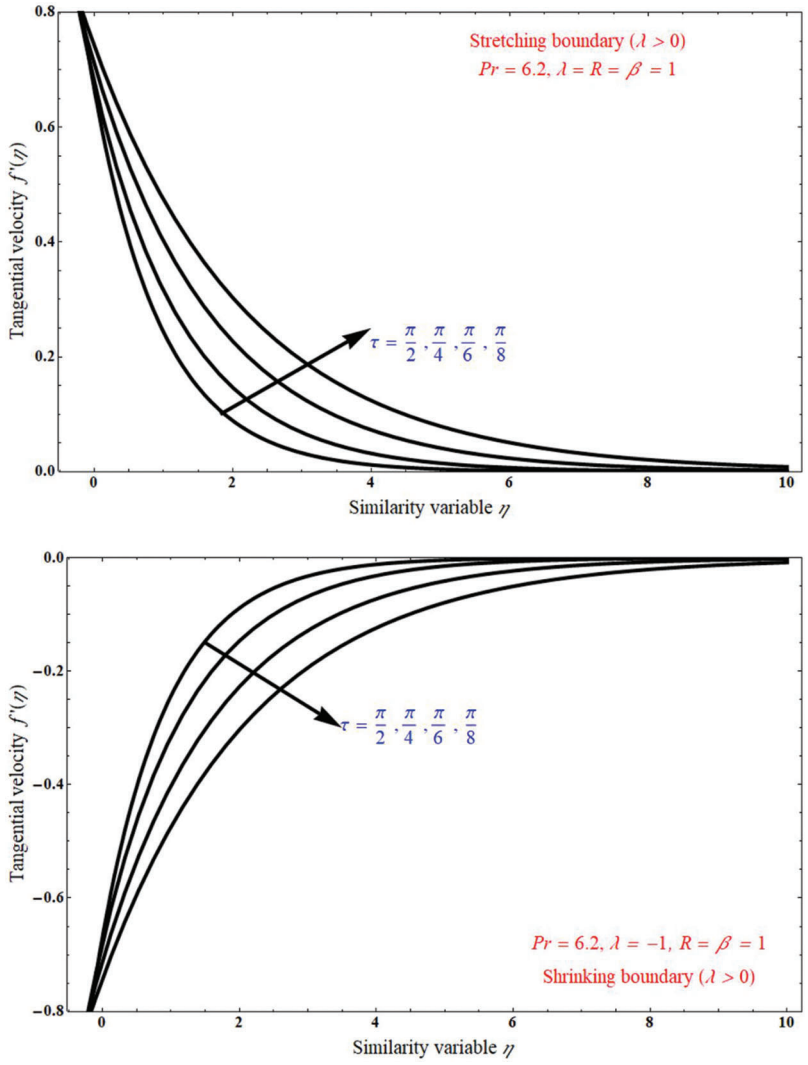


**Figure 5:** The impact of tangential velocity on similarity variable $\eta$ for unlike values of inclined angle $\tau$, at (a) stretching case $\lambda = 1$, and (b) shrinking case $\lambda = -1$

Fig. 8 represents the impact of temperature profile $\theta(\eta)$ expressed in Eq. (17), on the similarity variable $\eta$, in this it is analysed that the $\theta(\eta)$ increases with inclined angle $\tau$, and the boundary value thickness is away from the x-axis. Similarly, Figs. 8a and 8b represent the effect of temperature profile $\theta(\eta)$ derived in Eq. (23), on the similarity variable $\eta$, $\theta(\eta)$ decreases with increase of mass transpiration $V_C$ and increases with increase of Radiation parameter $R$, it is observed graphically in Figs. 8a and 8b, respectively.

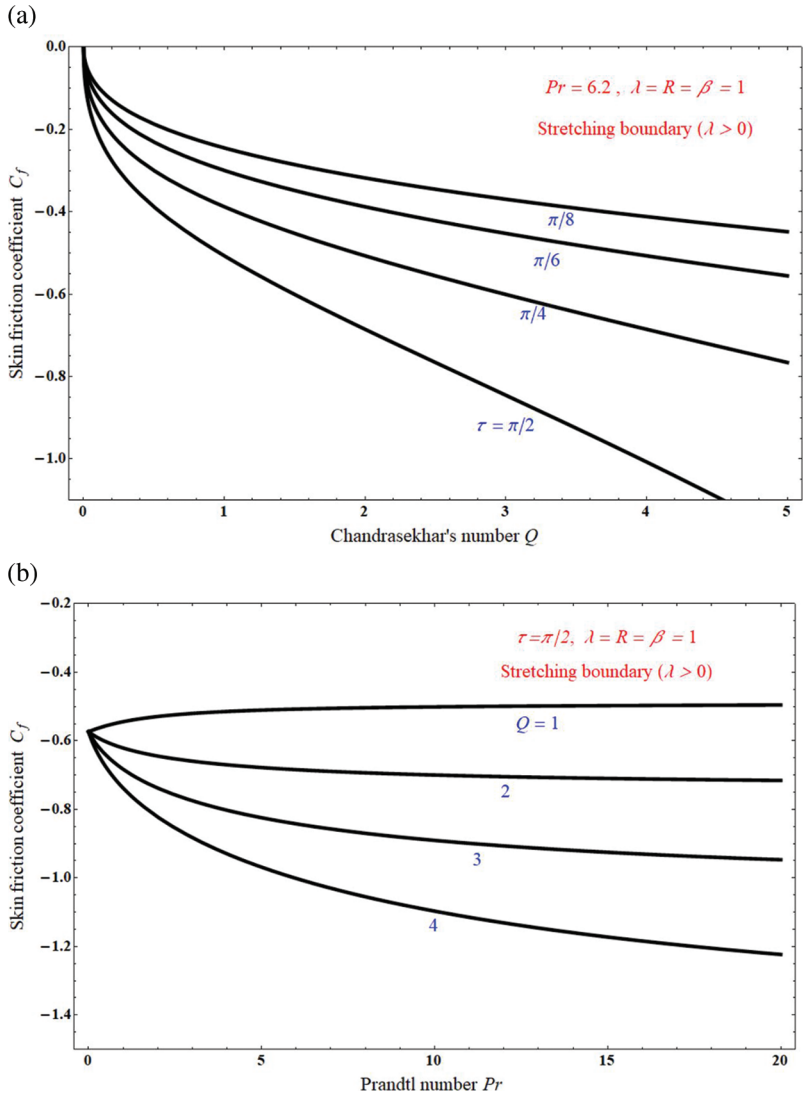


**Figure 6:** a) The effect of skin friction coefficient $C_f$ on Chandrasekhar's number $Q$ for unlike values of inclined angle $\tau$; b) The effect of skin friction coefficient $C_f$ on Prandtl number Pr for unlike values of Chandrasekhar's number $Q$

Figs. 9a and 9b demonstrated that the impact of Nusselt number $Nu$ derived from (12) and (21), on the inclined angle $\tau$ and Radiation parameter $R$ in the presence of a function Chandrasekhar's number $Q$, and Mass transpiration $V_C$. In both figures the Nusselt number $Nu$ decreases with increase of inclined angle $\tau$ and Radiation parameter $R$.

Similarly, the same investigation carried out for $Al_2O_3$-*water* nanoparticle, it also shows the approximately same effect when we compared with *Cu-water* nanoparticle. The present investigation helps to conduct new research on nanofluids with stretching sheet problems and also direct integration technique helps to solve heat transfer equation easily.

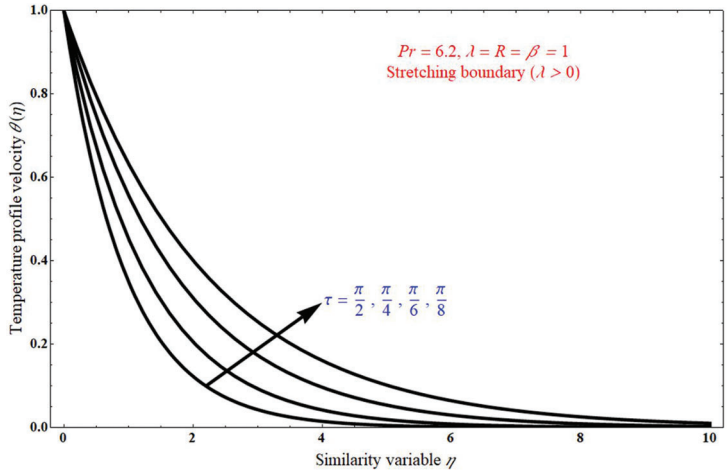


**Figure 7:** The impact of temperature profile velocity $\theta(\eta)$ on similarity variable $\eta$ for unlike values of inclined angle $\tau$

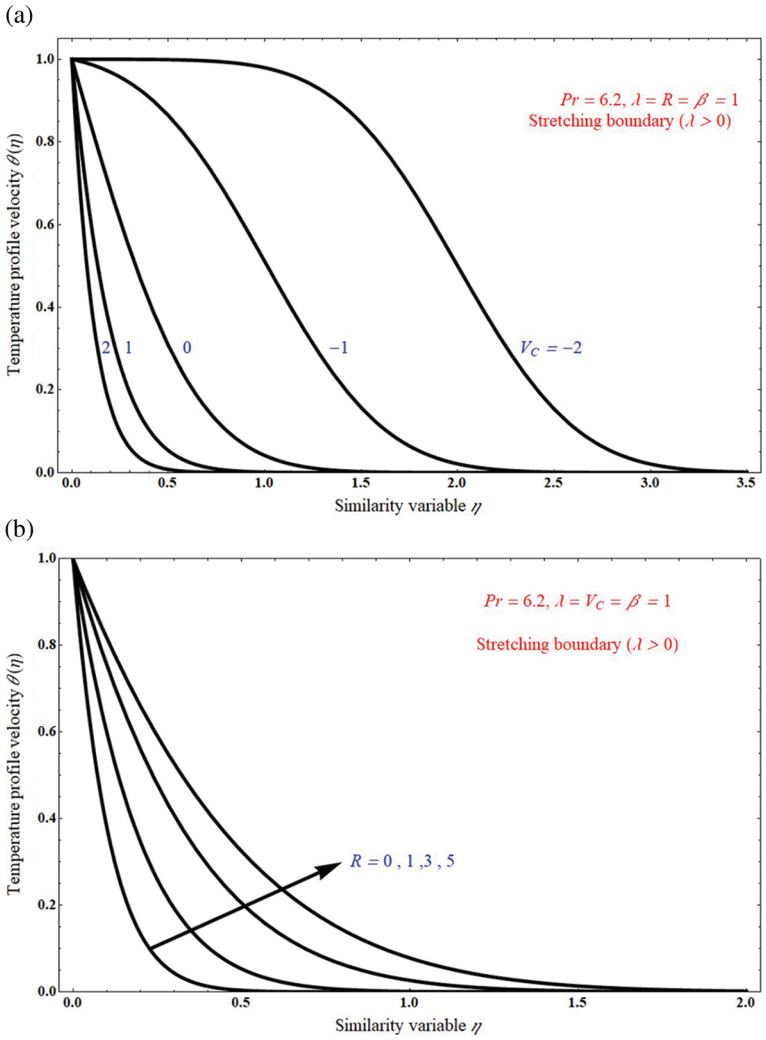


**Figure 8:** a) The impact of temperature profile velocity $\theta(\eta)$ on similarity variable $\eta$ for unlike values of mass transpiration $V_C$; b) The impact of temperature profile velocity $\theta(\eta)$ on similarity variable $\eta$ for unlike values of radiation parameter $R$

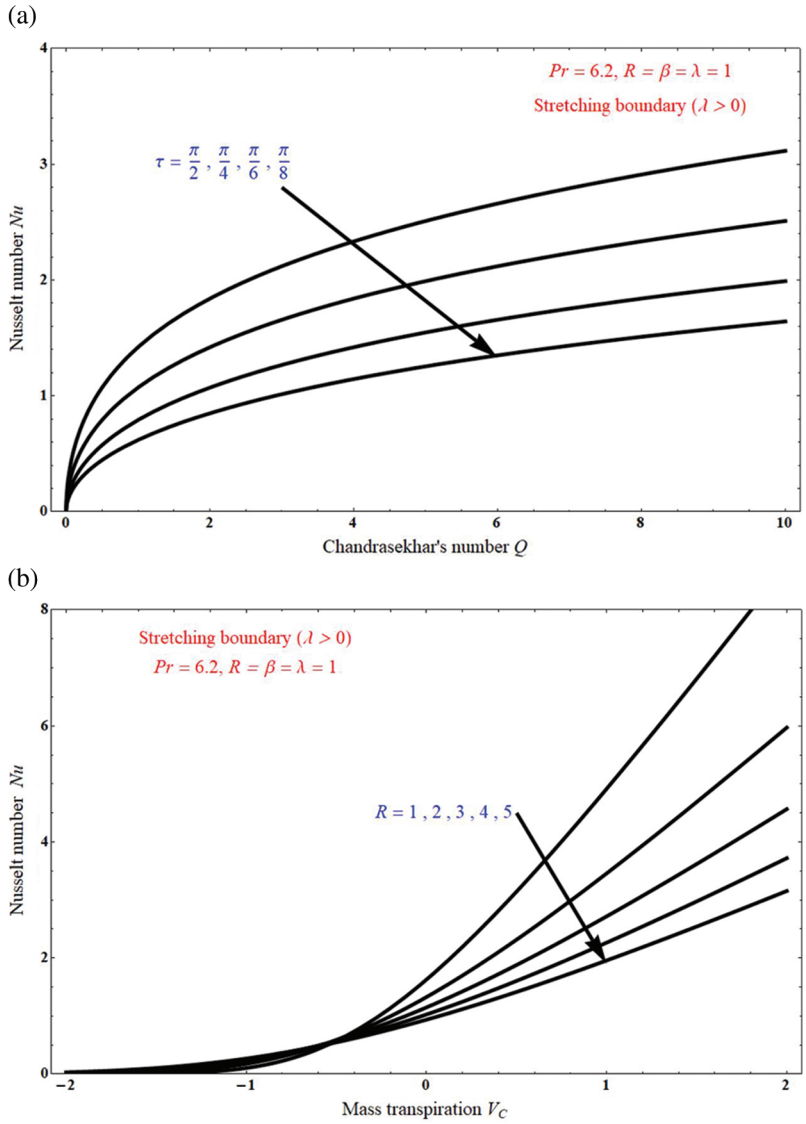


**Figure 9:** a) The effect of Nusselt number $Nu$ on Chandrasekhar's number $Q$ for unlike values of inclined angle $\tau$; b) The effect of Nusselt number $Nu$ on mass transpiration $V_C$ for unlike values of radiation parameter $R$

## 6 Concluding Remarks

The current article investigates the Jeffrey nanofluid flow in the presence of *Cu-water* nanofluid. The nonlinear PDE's are changed into ODEs with the help of suitable similarity transformations. Later the momentum equation solved analytically to get the solution domain, this domain is linked with heat equation through $f(\eta)$. Then the heat equation is solved under direct integration technique to get the solution. Additionally, inclined magnetic field and radiation term is taken into account in the whole calculation. The foremost findings of this analysis are given below:

- Flow fluid decreases with increase of Deborah number $\beta$ value.
- The skin friction coefficient $C_f$ will decreases by increasing the slip parameters $l_1$ and $l_2$.

- The rate of heat transfer increases by adding nanofluid.
- Temperature $T$ is not influenced by parameters $\lambda$, $l_1$, and $l_2$.
- The classical Crane [2] problem is obtained from Eq. (6) if $Q = \tau = \beta = \Lambda = 0$, and $C_1 = C_2 = C_3 = 1$.
- The classical Pavlov [6] problem is obtained from Eq. (6) if $\beta = \Lambda = 0$, $\tau = 90^\circ$ and $C_1 = C_2 = C_3 = Q = 1$.
- The Turkyilmazoglu [24] problem is obtained from Eqs. (6)–(8) if $R = 0$, $\tau = 90^\circ$ and $C_1 = C_2 = C_3 = 1$, $Q = M$.
- The present work is help to conduct future research on nanofluid flows, and also direct integration technique is one of the easy ways to solve the energy equation, it helps to many reasearchers.

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