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Article

Solid Waste Collection System Selection Based on Sine Trigonometric Spherical Hesitant Fuzzy Aggregation Information

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Abstract: Spherical fuzzy set (SFS) as one of several non-standard fuzzy sets, it introduces a number triplet (a,b,c) that satisfies the requirement $a^2 + b^2 + c^2 \leq 1$ to express membership grades. Due to the expression, SFS has a more extensive description space when describing fuzzy information, which attracts more attention in scientific research and engineering practice. Just for this reason, how to describe the fuzzy information more reasonably and perfectly is the hot that scholars pay close attention to. In view of this hot, in this paper, the notion of spherical hesitant fuzzy set is introduced as a generalization of spherical fuzzy sets. Some basic operations using sine trigonometric function are presented for spherical hesitant fuzzy sets. We define spherical hesitant fuzzy weighted average and spherical hesitant fuzzy weighted geometric aggregation operators. Based on these new aggregation operators, we propose a method for multi-criteria decision making (MCDM) in the spherical hesitant fuzzy information. Besides, a numerical real-life application about solid waste collection system selection is provided to demonstrate the validity of the proposed approaches along with relevant discussions, the merits of proposed approaches are also analyzed by validity test.

Keywords: Spherical fuzzy set; Hesitant fuzzy set; spherical hesitant fuzzy set; Sine trigonometric aggregation information; decision making

1 Introduction

The smart city idea is focused on the incorporation of information and communication technologies (ICTs) into city services to accumulate information for the allocation of assets and services, along with enhancing value of life and susceptibility. Difficulties and inefficient solutions in today's cities raise a need for the smart apps to solve existing challenges effectively. In this background, many academics, architects, urban planners, and even municipal representatives have been drawn to smart city applications. In addition, a wide variety of implementations are available for the smart city concept in areas such as town planning, waste disposal, resource management and municipal services [1–3].

In particular, the problem of solid waste collection system selection is a multi-criteria decision-making problem that should also take into account environmental, social, economic and technological aspects. Due to the labor strength of the job and the use of the large number of vehicles in these processes, sample and transport are commonly considered to be the most critical and expensive phases of the process [4]. As ever, certain issues such as quick disposal of wastes from half-full bins and thus excessive fuel use of collection and transport vehicles, higher pollution levels, inefficient usage of city assets and services are raised in the predefined scheduling [5]. In this respect, the brand-new visible light communication (VLC) technique makes it possible to communicate ultra-fastly among terminals through light bulbs and to become an essential competitor for conventional radio frequency (RF) communication like Wi-Fi [6]. Without providing some other [7] communication method, VLC can supply a room's interior lighting and data exchange at the same time.

Multiple criteria group decision making (MCGDM) method [8–14] is a significant and arising subject to depict an approach for choosing the finest alternative with group of the decision makers (DMs) and conditions. Two serious tasks are there in this procedure. The first one is to depict the atmosphere where the values of various attributes can be scrutinized successfully, while the aggregation of the depicted data is the second task. Generally, the data which depict the substances are frequently taken in the form of the deterministic or crisp in nature. Though, with the rising complications of the frameworks step by step, hardly data can be accumulated, from the records, assets and specialists, in crisp form. Therefore, to present the data more openly, a notion of fuzzy sets (FSs) [15] and its extended types for instance, Intuitionistic fuzzy set (InFSs) [16], Pythagorean fuzzy sets (PytFSs) [17], hesitant fuzzy sets (HeFSs) [18] etc., is applied by the scholars. Every element of InFS is indicated by an ordered pair, every pair is defined by a positive- membership degree (PMD) and a negative-membership grade (NMD). The totality of PMD and NMD is less than or equal to one. So, in that positions IFSs has no capacity to improve any proper outcome. To grip such state Yager [17,19] submitted the notion of Pythagorean FS (PytFS), which is the broad form of InFS. For PytFS, the square sum of PMD and NMD is less than or equal to 1. Spherical fuzzy set (SFS) [20] established by Ashraf et al. [21,22] which is the wide-ranging arrangement of the all the current approaches of FS in the fiction. SFSs can handle the vagueness more fruitfully and skillfully in decision making (DM) problems. Several DM approaches [23–27] has been established by the scholars with SF material to improve the SFS theory. Ashraf et al. [28] established the theory of the SF Dombi aggregation operators under the SF material. Chen et al. [29] established the logarithmic based (AOs) for SFNs to discuss the imprecision in DMPs. Jin et al [30] predicted the linguistic SF AOs under SF material. Rafiq et al. [31] offered the DMP established for the cosine similarity dealings according to SF material. Ashraf et al. [32] settled the spherical distance measure based on DM method in accordance with SF atmospheres. Ashraf et al. [33] adapted the SFS depiction of SF t-norms and t-conorms and described the TOPSIS based DM method in accordance with SF material. Zeng et al. [34] offered the SF rough set on the basis of TOPSIS methodology for dealing the imprecision in the process of SFSs. Torra [18] presented the idea of HeFS to stimulate the method of FS, which has a set of values without having a single value in the form of membership. HeFS is a leading tool for holding the indistinctness in DMPs.

In this paper, we offered the pioneering view of T- spherical HeFS (T-SHFS) by using the concept of SFS and HeFS to check the incredibility and imprecise figures in DMPs to reform the supreme alternative in conferring to list of criteria. A DMPs AOs acts the supreme role to aggregate the data. Since, for every aggregation procedure the rules of the operation perform a key role. It is necessary to construct fresh laws for the operation and aggregation of T-SHFNs. Consequently, the aim of this paper is to suggest some new laws for the operation of T-SHFSs. Therefore, by using the above stated proofs, we offered the MCGDM algorithm to grip the assessment material for T-SHFSs.

2 Preliminaries

This unit contains some basic definitions of FS, InFS, PytFS, SFS, HeFS and SHFS.

Definition 1. *[18]* Consider the ground set $G \neq \varphi$. A hesitant FS (HeFS) k can be described as;

$k = \{\langle g, h_k(g)\rangle | g \in G\}$,

where $h_k(g)$ be any set having some values in [0, 1].

Definition 2. *[13]* Consider the ground set $G \neq \varphi$. A spherical FS k can be described as.

$k = \{\langle g, M_k(g), I_k(g), R_k(g)\rangle | g \in G\}$,

where $M_k : g \to [0, 1]$ be positive, $L_k : g \to [0, 1]$ be neutral and $K_k : g \to [0, 1]$ be negative membership degrees with the constraint $[M_k(g)]^2 + [I_k(g)]^2 + [R_k(g)]^2 \leq 1$, for all $g \in G$.

Definition 3: Consider the ground set $G \neq \varphi$. A spherical hesitant FS k can be described as.

$k = \{\langle g, M_?(g), I_?(g), R_?(g)\rangle | g \in G\}$,

Where $M_k(g) = \{\kappa | \kappa \in [0, 1]\}$, $I_k(g) = \{\delta | \delta \in [0, 1]\}$ and $R_k(g) = \{\partial | \partial \in [0, 1]\}$, denoted the positive membership degree (PMD), neutral membership degree (NeMD) and negative membership degree (NMD) with the constraint $0 \leq (\kappa^+)^2 + \left(\delta^+\right)^2 + (\partial^+)^2 \leq 1$, for all $g \in$ R, such that

$$\kappa^+ = \bigcup_{\kappa \in M_k(g)} \max\{\kappa\},\ \delta^+ = \bigcup_{\delta \in L_k(g)} \max\{\delta\}, \text{ and } \partial^+ = \bigcup_{\partial \in K_k(g)} \max\{\partial\}.$$

Definition 4. *[13]* A T-spherical fuzzy set (T-SFS) k on G can be described as

$k = \{(g, M_k(g), I_k(g), R_k(g) | \ g \in G\}$,

where $M_k, I_k, R_k(g) : \ G \to [0, 1]$ denoted the PMD, NeMD and NMD of $g \in G$, respectively, and for each $g \in G$, it holds that $(M_k(g))^n + (I_k(g))^n + (R_k(g))^n \leq 1$ for $n \geq 1$. Analogous to its membership degrees, the degree of indeterminacy is given as $\pi_k(g) = \sqrt[n]{1 - (M_k(g))^n - (I_k(g))^n - (R_k(g))^n}$. For convenience, we call (M_k, I_k, R_k) is a T-spherical fuzzy set.

Definition 5. Suppose $k_g = \left\{M_g, I_g, R_g\right\} \in T - SpHeFS(k)(g \in G)$.The fundamental operational laws can be defined as follows;

(1) $(k_1)^c = \bigcup_{(\kappa_1, \delta_1, \partial_1) \in (M_1, L_1, K_1)} \{K_1, L_1, M_1\}$;

(2) $k_1 \cup k_2 = \bigcup_{\left(\kappa_g, \delta_g, \partial_g\right) \in (M_g, L_g, K_g)} \left\{\max\left(M_g\right), \min\left(I_g\right), \min\left(R_g\right)\right\}$;

(3) $k_1 \cap k_2 = \bigcup_{\left(\kappa_g, \delta_g, \partial_g\right) \in (M_g, L_g, K_g)} \left\{\min\left(M_g\right), \min\left(I_g\right), \max\left(R_g\right)\right\}$;

Definition 6. Let $k = \{M, I, R\}$, $k_1 = \{M_1, I_1, R_1\}$, and $k_2 = \{M_2, I_2, R_2\}$ be the three T-Spherical hesitant fuzzy numbers (T-SHFNs), $\lambda > 0$, k^c indicates the complement of k and the operations of T-SHFNs are given below:

(1) $k_1 \oplus k_2 = \bigcup_{\substack{\kappa_1 \in M_1, \delta_1 \in I_1, \partial_1 \in R_1 \\ \kappa_2 \in M_2, \delta_2 \in I_2, \partial_2 \in R_2}} \left\{\sqrt[n]{\kappa_1^n + \kappa_2^n - \kappa_1^n \kappa_1^n}, \delta_1 \delta_1, \partial_1 \partial_2\right\}$;

(2) $\lambda.k = \bigcup_{\kappa \in M, \delta \in I, \partial \in R} \left\{\sqrt{1 - (1 - \kappa^n)^\lambda}, (\delta)^\lambda, (\partial)^\lambda\right\}$;

(3) $k_1 \otimes k_2 = \bigcup_{\substack{\kappa_1 \in M_1, \delta_1 \in I_1, \partial_1 \in R_1 \\ \kappa_2 \in M_2, \delta_2 \in I_2, \partial_2 \in R_2}} \left\{ \left\{ \kappa_1 \kappa_2,\ \delta_1 \delta_1,\ \sqrt[n]{\partial_1^n + \partial_2^n - \partial_1^n \partial_2^n} \right\} \right\};$

(4) $k^\lambda = \bigcup_{\kappa \in M, \delta \in I, \partial \in R} \left\{ (\kappa)^\lambda, (\delta)^\lambda,\ \sqrt{1 - (1 - \partial^n)^\lambda} \right\}.$

3 New Sine Trigonometric Operational Laws for T-SHFS

We express a number of new operational laws for T-SHFSs in this part.

Definition 7. Consider a non-empty set G and $k = \{(g,\ M_k(g),\ I_k(g),\ R_k(g) |\ g \in G\}$ be T-SHFS, then a sine trigonometric operational laws (STOLs) of T-SHFS k can be described as in the following:

$$\sin k = \bigcup_{\kappa \in M, \delta \in I, \partial \in R} \left\{ \sin\left(\tfrac{\pi}{2}\kappa_k\right),\ \sqrt[n]{1 - \sin^n\left(\tfrac{\pi}{2}\sqrt[n]{1 - \delta_k^n}\right)}, \sqrt[n]{1 - \sin^n\left(\tfrac{\pi}{2}\sqrt[n]{1 - \partial_k^n}\right)} \right\},$$

It is obviously understood that the $\sin k$ is also T-SHFS. As it is clear from the definition of T-SHFS, $\forall g \in G$, the functions κ_k, δ_k and, ∂_k satisfy: $\kappa_k : G \to [0,\ 1],\ \delta_k : G \to [0,\ 1], \partial_k : G \to [0,\ 1]$ and $0 \le (\kappa_k(g))^n + (\delta_k(g))^n + (\partial_k(g))^n \le 1$. Further, the membership function:

$$\sin\left(\frac{\pi}{2}\kappa_k\right) : G \to [0,\ 1], \quad \forall\ g \in G \to \sin\left(\frac{\pi}{2}\kappa_k(g)\right) \in [0,\ 1],$$

the neutral function:

$$\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \delta_k^n}\right)} : G \to [0,\ 1], \quad \forall\ g \in G \to \sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \delta_k^n(g)}\right)} \in [0,\ 1],$$

and the non-membership function:

$$\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \partial_k^n}\right)} : G \to [0,\ 1], \quad \forall\ g \in G \to \sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \partial_k^n(g)}\right)} \in [0,\ 1],$$

Therefore

$$\sin k = \left\{ \sin\left(\tfrac{\pi}{2}\kappa_k\right),\ \sqrt[n]{1 - \sin^n\left(\tfrac{\pi}{2}\sqrt[n]{1 - \delta_k^n}\right)}, \sqrt[n]{1 - \sin^n\left(\tfrac{\pi}{2}\sqrt[n]{1 - \partial_k^n}\right)} \right\}$$

is a T-SHFS.

Definition 8. Let $K_i = (M,\ I,\ R)$ be T-SHFN. If

$$\sin k = \left\{ \sin\left(\tfrac{\pi}{2}\kappa_k\right),\ \sqrt[n]{1 - \sin^n\left(\tfrac{\pi}{2}\sqrt[n]{1 - \delta_k^n}\right)}, \sqrt[n]{1 - \sin^n\left(\tfrac{\pi}{2}\sqrt[n]{1 - \partial_k^n}\right)} \right\}$$

then $\sin k$ is called a sine trigonometric operator, and the value $\sin k$ is called sine trigonometric T-SHFN (ST-T-SHFN).

4 Sine Trigonometric Aggregation Operators

On the basis of STOL of T-SHFNs, we describe the below weighted averaging and geometric aggregation operators. Let ψ be the family of T-SHFNs.

Definition 9. Let $k_i = (M_i, I_i, R_i)$ be a set of n T-SHFNs, and suppose $k : \psi^n \to \psi$. If

$ST - T - SpHeFWA(k_1, k_2, \ldots, k_n) = \omega_1 \sin k_1 \oplus \omega_2 \sin k_2 \oplus \ldots \oplus \omega_n \sin k_n$

hence the mapping T-SpHeFWA is known as sine trigonometric T-SpHeF weighted averaging operator, where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight information of $\sin k_i$ i.e., $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$.

Theorem 1. Consider the set $k_i = (M_i, I_i, R_i)$ of m T-SHFNs. Then, the aggregated value by using ST-T-SpHeFWA operator is also T-SHFN, then

$$\mathrm{ST} - \mathrm{T} - \mathrm{SpHeFWA}(k_1, k_2, \ldots, k_m) = \bigcup_{\kappa_i \in M_i, \delta_i \in I_i, \partial_i \in R_i} \left(\begin{array}{c} \sqrt[n]{1 - \prod_{i=1}^{m} \left(1 - \sin^n\left(\frac{\pi}{2}\kappa_i\right)\right)^{\omega_i}}, \prod_{i=1}^{m} \left(\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \delta_i^n}\right)} \right)^{\omega_i}, \\ \prod_{i=1}^{m} \left(\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \partial_i^n}\right)} \right)^{\omega_i} \end{array} \right)$$

Proof: We prove the theorem using induction method. Since for each i, $k_i = (M_i, I_i, R_i)$ is ST-T-SpHeFWA which implies that $M_i, I_i, R_i \in [0, 1]$ and $M_i^n + I_i^n + R_i^n \le 1$.

(1) For $m = 2$, we have $ST - T - SpHeFWA(k_1, k_2) = \omega_1 \sin k_1 \oplus \omega_2 \sin k_2$

As from Definition 3.1, we can see that $\sin k_1$ and $\sin k_2$ are ST-T-SHFNs and hence $\omega_1 \sin k_1 \oplus \omega_2 \sin k_2$ is also ST-T-SHFN. Also, for k_1 and k_2, we have

$$\mathrm{ST} - \mathrm{T} - \mathrm{SpHeFWA}(k_1, k_2) = \omega_1 \sin k_1 \oplus \omega_2 \sin k_2$$

$$= \bigcup_{\kappa_1 \in M_1, \delta_1 \in I_1, \partial_1 \in R_1} \left\{ \begin{array}{c} \sqrt[n]{1 - \left(1 - \sin^n\left(\frac{\pi}{2}\kappa_1\right)\right)^{\omega_1}}, \\ \left(\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \delta_1^n}\right)} \right)^{\omega_1}, \\ \left(\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \partial_1^n}\right)} \right)^{\omega_1} \end{array} \right\} \oplus \bigcup_{\kappa_2 \in M_2, \delta_2 \in I_2, \partial_2 \in R_2} \left\{ \begin{array}{c} \sqrt[n]{1 - \left(1 - \sin^n\left(\frac{\pi}{2}\kappa_2\right)\right)^{\omega_2}}, \\ \left(\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \delta_2^n}\right)} \right)^{\omega_2}, \\ \left(\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \partial_2^n}\right)} \right)^{\omega_2} \end{array} \right\}$$

$$= \bigcup_{\kappa_i \in M_i, \delta_i \in I_i, \partial_i \in R_i} \left\{ \sqrt[n]{1 - \prod_{i=1}^{n} \left(1 - \sin^n\left(\frac{\pi}{2}\kappa_i\right)\right)^{\omega_i}}, \prod_{i=1}^{n} \left(\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \delta_i^n}\right)} \right)^{\omega_i}, \prod_{i=1}^{n} \left(\sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \partial_i^n}\right)} \right)^{\omega_i} \right\}$$

Assume Eq. (1) holds for $m = p$. Now, for $m = p + 1$, we have

$$\mathrm{T-SpHeFWA}(k_1,k_2,\ldots,k_p)\oplus\omega_{p+1}\sin k_{p+1} =$$

$$\bigcup_{\kappa_i\in M_i,\delta_i\in I_i,\partial_i\in R_i}\left\{\begin{array}{c}\sqrt[n]{1-\prod_{i=1}^{p}\left(1-\sin^n\left(\frac{\pi}{2}\kappa_i\right)\right)^{\omega_i}},\\ \prod_{i=1}^{p}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\delta_i^n}\right)}\right)^{\omega_i},\\ \prod_{i=1}^{p}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\partial_i^n}\right)}\right)^{\omega_i}\end{array}\right\}$$

$$\oplus\bigcup_{\kappa_{k+1}\in M_{k+1},\delta_{k+1}\in I_{k+1},\partial_{k+1}\in R_{k+1}}\left\{\begin{array}{c}\sqrt[n]{1-\left(1-\sin^n\left(\frac{\pi}{2}\kappa_{k+1}\right)\right)^{\omega_{k+1}}},\\ \left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\delta_{p+1}^n}\right)}\right)^{\omega_{p+1}},\\ \left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\partial_{p+1}^n}\right)}\right)^{\omega_{p+1}}\end{array}\right\}$$

$$=\bigcup_{\kappa_i\in M_i,\delta_i\in I_i,\partial_i\in R_i}\left\{\sqrt[n]{1-\prod_{i=1}^{p+1}\left(1-\sin^n\left(\frac{\pi}{2}\kappa_i\right)\right)^{\omega_i}},\prod_{i=1}^{p+1}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\delta_i^n}\right)}\right)^{\omega_i},\prod_{i=1}^{p+1}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\partial_i^n}\right)}\right)^{\omega_i}\right\}$$

Hence, Eq. (1) also valid for $m = p + 1$. So, the result is valid for all positive integer m .

Definition 10. A sine-trigonometric T-SHFN ordered weighted average (ST-T-SpHeFOWA) operator is a mapping ST-T-SpHeFOWA: $\psi^m \to \psi$, such that $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$, with $\omega_i > 0$ and $\sum_{i=1}^{m}\omega_i = 1$,

$$ST - T - SpHeFOWA(k_1, k_2, \ldots, k_m) = \omega_1 \sin k_{\rho(1)} \oplus \omega_2 \sin k_{\rho(2)} \oplus \ldots \oplus \omega_n \sin k_{\rho(m)}$$

where ρ is the permutation of $(1, 2, \ldots, m)$ such that $k_{\rho(i-1)} \geq k_{\rho(i)}$ for $i = 2, 3, \ldots, m$.

Theorem 2. For a collection of m T-SHFNs $k_i = (M_i, I_i, R_i)$, the aggregated value by using ST-T-SpHeFOWA operator is still T-SHFN and given by

$$ST - T - SpHeFOWA(k_1, k_2, \ldots, k_m)$$

$$=\bigcup_{\kappa_i\in M_i,\delta_i\in I_i,\partial_i\in R_i}\left(\begin{array}{c}\sqrt[n]{1-\prod_{i=1}^{m}\left(1-\sin^n\left(\frac{\pi}{2}\kappa_{\rho(i)}\right)\right)^{\omega_i}},\prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\delta_{\rho(i)}^n}\right)}\right)^{\omega_i},\\ \prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\partial_{\rho(i)}^n}\right)}\right)^{\omega_i}\end{array}\right)$$

.

Definition 11. A sine-trigonometric T-SHFN hybrid average (ST-T-SpHeFHA) operator is a mapping ST-T-SpHeFHA : $\psi^m \to \psi$, such that $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$, with $\omega_i > 0$ and $\sum_{i=1}^{m}\omega_i = 1$, and

$$ST-T-SpHeFHA(k_1, k_2, \ldots, k_m) = \omega_1 \sin k_{\rho(1)} \oplus \omega_2 \sin k_{\rho(2)} \oplus \ldots \oplus \omega_m \sin k_{\rho(m)}$$

where ρ is the permutation of $(1, 2, \ldots, m)$ such that $k_{\rho(i-1)} \geq k_{\rho(i)}$ for $i = 2, 3, \ldots, m$ and $k_i = m\omega_i k_i$.

Theorem 3. For a collection of m T-SHFNs $k_i = (M_i, I_i, R_i)$, the aggregated value by using ST-TSpHeFHA operator is still T-SHFN and given by

$$ST-TSHFHA(k_1, k_2, \ldots, k_m) = \bigcup_{\kappa_i \in M_i, \delta_i \in I_i, \partial_i \in R_i} \left(\begin{array}{c} \sqrt[n]{1-\prod_{i=1}^{m}\left(1-\sin^n\left(\frac{\pi}{2}\kappa_{\rho(i)}\right)\right)^{\omega_i}}, \prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\delta^n_{\rho(i)}}\right)}\right)^{\omega_i}, \\ \prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\partial^n_{\rho(i)}}\right)}\right)^{\omega_i} \end{array} \right)$$

.

Definition 12. Let $k_i = (M_i, I_i, R_i)$ is a set of m T-SHFNs and let $\mathrm{ST-T-SpHeFWG} : \psi^m \rightarrow \psi$. If

$$\mathrm{ST-T-SpHeFWG}(k_1, k_2, \ldots, k_m) = (\sin k_1)^{\omega_1} \otimes (\sin k_2)^{\omega_2} \otimes \ldots \otimes (\sin k_m)^{\omega_m}$$

then the function ST-T-SpHeFWG is called sine trigonometric T-SpHeF weighted geometric operator, where $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ is the weight vector of $\sin k_i$ with $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$.

Theorem 4. Let $k_i = (M_i, I_i, R_i)(i = 1, 2, \ldots, m)$ is a set of m T-SHFNs. Then, the aggregated value by using ST-TSpHeFWG operator is also T-SHFN and is given by

$$ST-TSpHeFWG(k_1, k_2, \ldots, k_m) = \bigcup_{\kappa_i \in M_i, \delta_i \in I_i, \partial_i \in R_i} \left(\begin{array}{c} \prod_{i=1}^{m}\left(\sin\left(\frac{\pi}{2}\kappa_i\right)\right)^{\omega_i}, \sqrt[n]{1-\prod_{i=1}^{m}\left(\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\delta_i^n}\right)\right)^{\omega_i}}, \\ \sqrt[n]{1-\prod_{i=1}^{m}\left(\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\partial_i^n}\right)\right)^{\omega_i}} \end{array} \right)$$

Definition 13. A sine trigonometric TSpHeF ordered weighted geometric (ST-TSpHeFOWG) operator is a mapping $\mathrm{ST-q-ROFOWG} : \psi^m \rightarrow \psi$, such that $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$, with $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$, and

$$ST-TSpHeFOWG(k_1, k_2, \ldots, k_m) = \left(\sin k_{\rho(1)}\right)^{\omega_1} \otimes \left(\sin k_{\rho(2)}\right)^{\omega_2} \otimes \ldots \otimes \left(\sin k_{\rho(n)}\right)^{\omega_m}$$

where ρ is the permutation of $(1, 2, \ldots, m)$ such that $k_{\rho(i-1)} \geq k_{\rho(i)}$ for $i = 2, 3, \ldots, m$.

Theorem 5. Let $k_i = (M_i, I_i, R_i)(i = 1, 2, \ldots, m)$ is a set of m T-SHFNs. Then the aggregated value by using ST-TSpHeFOWG operator is still T-SHFN and given by

$$ST-TSpHeFOWG(k_1, k_2, \ldots, k_m) = \bigcup_{\kappa_i \in M_i, \delta_i \in I_i, \partial_i \in R_i} \left\{ \begin{array}{c} \prod_{i=1}^{m}\left(\sin\left(\frac{\pi}{2}\kappa_{\rho(i)}\right)\right)^{\omega_i}, \sqrt[n]{1-\prod_{i=1}^{m}\left(\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\delta^n_{\rho(i)}}\right)\right)^{\omega_i}}, \\ \sqrt[n]{1-\prod_{i=1}^{m}\left(\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\partial^n_{\rho(i)}}\right)\right)^{\omega_i}} \end{array} \right\}$$

Definition 14. A sine trigonometric T-SpHeF hybrid weighted geometric (ST-TSpHeFHG) operator is a mapping ST-TSpHeFHG: $\phi^m \to \phi$, such that $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$, with $\omega_i > 0$ and $\sum_{i=1}^{m} \omega_i = 1$, and

$$ST - TSpHeFHG(k_1, k_2, \ldots, k_m) = \left(\sin k_{\rho(1)}\right)^{\phi_1} \otimes \left(\sin k_{\rho(2)}\right)^{\phi_2} \otimes \ldots \otimes \left(\sin k_{\rho(m)}\right)^{\phi_m}$$

where ρ is the permutation of $(1, 2, \ldots, m)$ such that $k_{\rho(i-1)} \geq k_{\rho(i)}$ for $i = 2, 3, \ldots, m$ and $k_i = k^{m\omega_i}$.

Theorem 6. Let $k_i = (M_i, I_i, R_i)(i = 1, 2, \ldots, m)$ is a set of m T-SHFNs. Then the aggregated value by using ST-TSpHeFHG operator is still T-SHFN and given by

$$ST - TSpHeFHG(k_1, k_2, \ldots, k_m) = \bigcup_{\kappa_i \in M_i, \delta_i \in I_i, \partial_i \in R_i} \left\{ \begin{array}{c} \prod_{i=1}^{m} \left(\sin\left(\frac{\pi}{2}\kappa_{\rho(i)}\right)\right)^{\omega_i}, \\ \sqrt[n]{1 - \prod_{i=1}^{m} \left(\sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \delta_{\rho(i)}^n}\right)\right)^{\omega_i}}, \\ \sqrt[n]{1 - \prod_{i=1}^{m} \left(\sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \partial_{\rho(i)}^n}\right)\right)^{\omega_i}} \end{array} \right\}$$

As similar to ST-TSHFWA operator, the ST-TSpHeFOWA, ST-TSpHeFHA, ST-TSpHeFWG, ST-TSpHeFOWG and ST-TSpHeFHG operators satisfy the properties such as boundedness, monotonicity.

Fundamental Properties of the Proposed AOs

In this subsection, we scrutinized some relations between the suggested AOs and study their various major properties as given below.

Theorem 7. For two T-SHFNs k_1 and k_2 we have, $\sin k_1 \oplus \sin k_2 \geq \sin k_1 \otimes \sin k_2$.

Proof: Let $k_1 = (M_1, I_1, R_1)$ and $k_2 = (M_2, I_2, R_2)$ be two T-SHFNs. Then, by using Definitions (i) & (ii), we get

$$\sin k_1 \oplus \sin k_2 = \left(\bigcup_{\kappa_i \in M_i, \delta_i \in I_i, \partial_i \in R_i} \left\{ \begin{array}{c} \sqrt[n]{1 - \prod_{i=1}^{2}\left(1 - \sin^n\left(\frac{\pi}{2}\kappa_i\right)\right)}, \\ \prod_{i=1}^{2} \sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \delta_i^n}\right)}, \\ \prod_{i=1}^{2} \sqrt[n]{1 - \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \partial_i^n}\right)} \end{array} \right\} \right)$$

and

$$\sin k_1 \otimes \sin k_2 = \left(\left(\begin{array}{c} \prod_{i=1}^{2} \sin\left(\frac{\pi}{2}\kappa_i\right), \\ \sqrt[n]{1 - \prod_{i=1}^{2} \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \delta_i^n}\right)}, \\ \sqrt[n]{1 - \prod_{i=1}^{2} \sin^n\left(\frac{\pi}{2}\sqrt[n]{1 - \partial_i^n}\right)} \end{array} \right) \right)$$

Since for any two non-negative real numbers c and d, their arithmetic mean is greater than or equal to their geometric mean therefore, $\frac{c+d}{2} \geq cd$ which follows that $c + d - cd \geq cd$. Thus by taking $c = \sin^n\left(\frac{pi}{2}M_1\right)$ and $d = \sin^n\left(\frac{pi}{2}M_2\right)$ we have

$$1-\left(1-\sin^n\left(\tfrac{\pi}{2}M_1\right)\right)\left(1-\sin^n\left(\tfrac{\pi}{2}M_2\right)\right)\geq\left(1-\sin^n\left(\tfrac{\pi}{2}M_1\right)\right)\left(1-\sin^n\left(\tfrac{\pi}{2}M_2\right)\right)$$

which further gives that

$$\sqrt[n]{1-\prod_{l=1}^{2}\left(1-\sin^n\left(\frac{\pi}{2}M_l\right)\right)}\geq\prod_{j=1}^{2}\sin\left(\frac{\pi}{2}M_l\right)$$

Similarly, we can obtain

$$\prod_{l=1}^{2}\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-I_l^n}\right)}\leq\sqrt[n]{1-\prod_{l=1}^{2}\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-I_l^n}\right)}$$

And

$$\prod_{l=1}^{2}\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-R_l^n}\right)}\leq\sqrt[n]{1-\prod_{l=1}^{2}\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-R_l^n}\right)}.$$

Hence, by using Definition (i), we get

$$\sin k_1\oplus\sin k_2\geq\sin k_1\otimes\sin k_2$$

Theorem 8. Let k_i, k are T-SHFNs, then

(1) $ST-TSpHeFWA(k_1\oplus k,\,k_2\oplus k,\ldots,\,k_n\oplus k)\geq ST-TSpHeFWA(k_1\otimes k,\,k_2\otimes k,\ldots,\,k_n\otimes k)$;
(2) $ST-TSpHeFWG(k_1\oplus k,\,k_2\oplus k,\ldots,\,k_{,n}\oplus k,)\geq ST-TSpHeFWG(k_1\otimes k,\,k_2\otimes k,\ldots,\,k_n\otimes k)$

Proof: We will prove part (1) only. Part (2) can be obtained in a similar way. For this, let $k_i=(M_i,\,I_i,\,R_i)$ & $k=(M,\,I,\,R)$. Since k_i and k are T-SHFNs.

$$ST-TSpHeFWA(k_1\oplus k,\,k_2\oplus k,\ldots,\,k_m\oplus k)=$$
$$\left(\bigcup_{\kappa_i\in M_i,\,\delta_i\in I_i,\partial_i\in R_i}\left(\begin{array}{c}\sqrt[n]{1-\prod_{i=1}^{m}\left(1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-(1-\kappa_i^n)(1-\kappa^n)}\right)\right)^{\omega_i}},\\ \prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\delta_i^n\delta^n}\right)}\right)^{\omega_i},\\ \prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-\partial_i^n\partial^n}\right)}\right)^{\omega_i}\end{array}\right)\right)$$

and

$$ST-TSpHeFWA(k_1\otimes k,\,k_2\otimes k,\ldots,\,k_m\otimes k)=$$
$$\left(\bigcup_{\kappa_i\in M_i,\,\delta_i\in I_i,\partial_i\in R_i}\left(\begin{array}{c}\sqrt[n]{1-\prod_{i=1}^{m}\left(1-\sin^n\left(\frac{\pi}{2}\kappa_i\kappa\right)\right)^{\omega_i}},\\ \prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{\left(1-\delta_i^n\right)\left(1-\delta^n\right)}\right)}\right)^{\omega_i},\\ \prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{\left(1-\partial_i^n\right)\left(1-\partial^n\right)}\right)}\right)^{\omega_i}\end{array}\right)\right)$$

For $M_i,\ M \in [0,\ 1]$ and by Lemma 4.15, we get $\sqrt[n]{1-(1-M_i^n)(1-M^n)} \geq M_i M$. Since sin is an increasing function, we get $\sin\left(\frac{\pi}{2}\sqrt[n]{1-(1-M_i^n)(1-M^n)}\right) \geq \sin\left(\frac{\pi}{2}M_i M\right)$ which gives that

$$\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-(1-M_i^n)(1-M^n)}\right) \geq \sin^n\left(\frac{\pi}{2}M_i M\right)$$

$$\Rightarrow 1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-(1-M_i^n)(1-M^n)}\right) \leq 1-\sin^n\left(\frac{\pi}{2}M_i M\right)$$

$$\Rightarrow \prod_{i=1}^{m}\left(1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-(1-M_i^n)(1-M^n)}\right)\right)^{\omega_i} \leq \prod_{i=1}^{m}\left(1-\sin^n\left(\frac{\pi}{2}M_j M\right)\right)^{\omega_i}$$

$$\Rightarrow \sqrt[n]{1-\prod_{i=1}^{m}\left(1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-(1-M_i^n)(1-M^n)}\right)\right)^{\omega_i}}$$

$$\geq \sqrt[n]{1-\prod_{i=1}^{m}\left(1-\sin^n\left(\frac{\pi}{2}M_i M\right)\right)^{\omega_i}}$$

Similarly, we can get

$$\prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-I_i^n I^n}\right)}\right)^{\omega_i} \leq \prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{(1-I_j^n)(1-I^n)}\right)}\right)^{\omega_i}$$

$$\prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{1-R_i^n R^n}\right)}\right)^{\omega_i} \leq \prod_{i=1}^{m}\left(\sqrt[n]{1-\sin^n\left(\frac{\pi}{2}\sqrt[n]{(1-R_i^n)(1-R^n)}\right)}\right)^{\omega_i}$$

Therefore, from above Eqs, we get

$$\mathrm{ST-TSpHeFWA}(k_1 \oplus k,\ k_2 \oplus k, \ldots,\ k_n \oplus k) \geq \mathrm{ST-TSpHeFWA}(k_1 \otimes k,\ k_2 \otimes k, \ldots,\ k_n \otimes k).$$

5 Decision Making Approach

Here, we have settled a structure for addressing improbability in decision making (DM) under TSpHeF material. Consider a DM problem with a set of m alternatives $\{G_1,\ G_2, \ldots\ldots,\ G_g\}$ and $\{V_1,\ V_2, \ldots.,\ V_h\}$ be a set of attributes with weights $(w_1,\ w_2, \ldots..,\ w_h)^T$ such that $w_t \in [0,\ 1]$, $\sum_{t=1}^{h} w_t = 1$. To assess the performance of kth alternative G_k under the tth attribute V_t, let $\left\{D_1,\ D_2, \ldots.,\ D_{\hat{j}}\right\}$ be a set of decision makers (DMs) and $(w_1,\ w_2, \ldots..,\ w_h)^T$ be DMs weights such that $w_s \in [0,\ 1]$, $\sum_{s=1}^{\hat{j}} w_s = 1$. The expert evaluation matrix is described as:

$$\begin{bmatrix} (M_{11}(g),\ I_{11}(g),\ R_{11}(g)) & (M_{12}(g),\ I_{12}(g),\ R_{12}(g)) & \cdots & (M_{1h}(g),\ I_{1h}(g),\ R_{1h}(g)) \\ (M_{21}(g),\ I_{21}(g),\ R_{21}(g)) & (M_{22}(g),\ I_{22}(g),\ R_{22}(g)) & \cdots & (M_{2h}(g),\ I_{2h}(g),\ R_{2h}(g)) \\ (M_{31}(g),\ I_{31}(g),\ R_{31}(g)) & (M_{32}(g),\ I_{32}(g),\ R_{32}(g)) & \cdots & (M_{3h}(g),\ I_{3h}(g),\ R_{3h}(g)) \\ \vdots & \vdots & \ddots & \vdots \\ (M_{g1}(g),\ I_{g1}(g),\ R_{g1}(g)) & (M_{g2}(g),\ I_{g2}(g),\ R_{g2}(g)) & \cdots & (M_{gh}(g),\ I_{gh}(g),\ R_{gh}(g)) \end{bmatrix}$$

where $(M_{gh}(g),\ I_{gh}(g),\ R_{gh}(g))$ are the three sets of some values in [0, 1], denoted the PMD, NeMD and NMD with the condition $0 \leq (\kappa^+)^n + (\delta^+)^n + (\partial^+)^n \leq 1$, for all $g \in$ R, such that

$\kappa^+ = \underset{\kappa \in M_Z(g)}{\cup} \max\{\kappa\}$, $\delta^+ = \underset{\delta \in L_Z(g)}{\cup} \max\{\delta\}$, and $\partial^+ = \underset{\partial \in K_Z(g)}{\cup} \max\{\partial\}$.

Step-1: Construct the expert evaluation matrix $(E)^{\hat{j}}$.

$$\begin{bmatrix} \left(M_{11}^{\hat{j}}(g), I_{11}^{\hat{j}}(g), R_{11}^{\hat{j}}(g)\right) & \left(M_{12}^{\hat{j}}(g), I_{12}^{\hat{j}}(g), R_{12}^{\hat{j}}(g)\right) & \cdots & \left(M_{1h}^{\hat{j}}(g), I_{1h}^{\hat{j}}(g), R_{1h}^{\hat{j}}(g)\right) \\ \left(M_{21}^{\hat{j}}(g), I_{21}^{\hat{j}}(g), R_{21}^{\hat{j}}(g)\right) & \left(M_{22}^{\hat{j}}(g), I_{22}^{\hat{j}}(g), R_{22}^{\hat{j}}(g)\right) & \cdots & \left(M_{2h}^{\hat{j}}(g), I_{2h}^{\hat{j}}(g), R_{2h}^{\hat{j}}(g)\right) \\ \left(M_{31}^{\hat{j}}(g), I_{31}^{\hat{j}}(g), R_{31}^{\hat{j}}(g)\right) & \left(M_{32}^{\hat{j}}(g), I_{32}^{\hat{j}}(g), R_{32}^{\hat{j}}(g)\right) & \cdots & \left(M_{3h}^{\hat{j}}(g), I_{3h}^{\hat{j}}(g), R_{3h}^{\hat{j}}(g)\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(M_{g1}^{\hat{j}}(g), I_{g1}^{\hat{j}}(g), R_{g1}^{\hat{j}}(g)\right) & \left(M_{g2}^{\hat{j}}(g), I_{g2}^{\hat{j}}(g), R_{g2}^{\hat{j}}(g)\right) & \cdots & \left(M_{gh}^{\hat{j}}(g), I_{gh}^{\hat{j}}(g), R_{gh}^{\hat{j}}(g)\right) \end{bmatrix}$$

where $\hat{j}$ denotes the number of experts.

Step-2: Construct the normalized decision matrix $(N)^{\hat{j}}$. Where

$$(N)^{\hat{j}} = \begin{cases} \left(M_{gh}^{\hat{j}}(g), I_{gh}^{\hat{j}}(g), R_{gh}^{\hat{j}}(g)\right) & \textit{if} \quad \text{Benefit type criteria} \\ \left(K_{gh}^{\hat{j}}(g), I_{gh}^{\hat{j}}(g), R_{gh}^{\hat{j}}(g)\right) & \textit{if} \quad \text{Cost type criteria} \end{cases}$$

Step-3: Aggregate the individual decision matrices based on the T-sphrical hesitant fuzzy aggregation operators to construct the collective matrix. Exploit the established aggregation operators to achieve the TSHFN $\mathbf{F}_t(t = 1, 2, \ldots, g)$ for the alternatives G_k, that is the established operators to obtained the collective overall preference values of $\mathbf{F}_t(t = 1, 2, \ldots, g)$ for the alternatives V_k, where $(w_1, w_2, \ldots, w_h)^T$ is the weight vector of the attributes.

Step-4: Compute the score of all the values $\mathbf{F}_t(t = 1, 2, \ldots, g)$ for the alternatives G_k.

Step-5: Rank the alternatives $G_k(k = 1, 2, \ldots, g)$ and choose the finest one having the greater value.

6 Illustrative Example

We analyze the results of the established MAGDM technique with a numerical example and compare the outcomes with the one of the existing MAGDM techniques, in this area. The aim of this research is to implement T-SpHeF data methodology in a smart city area to analyze and grade alternative waste collection systems.

Case Study; Municipal Waste Collection System Selection:

Descriptions of the Problem: For human health, aesthetics, and environmental consevation, municipal solid waste (SW) management is much more needed and essential facilities. It covers all the operations and steps necessary for waste management from selection to final disposal [1]. A critical stage of an effective waste management strategy is the identification of frequently contradictory natural, social and economic requirements and the list of alternatives. The waste disposal truck drives and stops at each building in this collection system to pick up the solid waste [2,3]. Four criteria are defined in this analysis;

Innovativeness and Aesthetics (V_1): In order to explain whether the system is innovative or not comparable to previous model, this criteria depended on the aural and physical dimensions of the system.

Maintenance Efficiency (V_2): The effectiveness of time and money invested on the maintenance of the system is taken into account by such criteria.

Sustainability (V_3): The terms of the system's environmental, financial, and sustainability practices are evaluated by such criteria.

Setup Cost Advantage (V_4): The effectiveness of the investments made during the implementation of the device is represented by such criteria.

Now following are the four alternatives concept and solution of above criteria. We suggest ideas for smart city solid waste collection (SWC) in this context, in which VLC are included. In addition, we use two new SWC ideas for smart cities in which Wi-Fi connectivity and cellular connectivity are used. Smart bins fitted with sensors, microprocessor, battery packs, compaction systems as well as solar power are taken into account in all alternatives. The bins are used both to collect waste and to collect waste data.

Wi-Fi-based SWC System (G_1): Wi-Fi software is taken into account in this alternative, and the microcontrollers at the highest point of the bins are fitted with a Wi-Fi wireless transceiver module and link with a router to relay information from the bins to the network.

Cellular Communication-based SWC System (G_2): The microcontrollers are fitted in this alternative and interact with a base station to notify the server. It is believed that all bins will connect with the same base station due to the information received by the base station. It alerts the base station about its condition when the bin becomes loaded. Then this feedback is transmitted to the server by the base station. The server processes the data and enhances the path for available garbage trucks. Next, the server notifies the trash trucks of the bins to be emptied and the routes to be driven. At last, the truck(s) will leave to clear the entire bin (s).

Li-Fi-based SWC System (G_3): We recommend a VLC-based SWC method in this alternative, which uses Li-Fi technology. The microcontrollers interact with streetlights and the streetlights send the details of the bins to the server. The microcontrollers and streetlights are fitted with light-emitting diodes (LEDs). Using their LEDs, the microcontrollers send their feedback to the streetlights, and the streetlights interact with each other and remind the server.

Waste Management Collection and Transportation with Drones (G_4): We recommend a DC-based SWC system in this alternative, which allows drones to deliver feedback. There, using Wi-Fi technology, a drone flew over the bins and interacts with them. A drone flies across all districts throughout this process and collects feedback from the garbage bins.

Application of Proposed MAGDM Method

Suppose, three experts $\mathbf{D}^{(1)}$, $\mathbf{D}^{(2)}$ and $\mathbf{D}^{(3)}$ for the analysis of the four alternative to select the best solution for the waste collection system G_1, G_2, G_3, G_4, and for importance level of the four criteria's V_1, V_2, V_3, V_4. Assume that $w = (0.2, 0.4, 0.1, 0.3)$ be the weight of the experts and their evaluation decision matrices $R^{(1)}$, $R^{(2)}$ and $R^{(3)}$ by using T-SHFNs, where $n = 2$ are shown in Tab. 1. The purpose of this numerical example is to choose the top alternative for the SWC.

Table 1: Expert evaluation information $(E)^1$

	V_1	V_2	V_3	V_4
G_1	{(.3,.2,.4)}	{(.2,.6,.5)}	{(.3,.5,.1)}	{(.1,.5,.6), (.3,.4,.5)}
G_2	{(.1,.5,.2)}	{(.2,.3,.4)}	{(.1,.1,.6), (.3,.1,.4)}	{(.1,.4,.2)}
G_3	{(.4,.1,.5)}	{(.1,.1,.6), (.3,.2,.4)}	{(.4,.2,.5)}	{(.4,.2,.5)}
G_4	{(.2,.2,.3)}	{(.1,.2,.3)}	{(.2,.4,.3), (.4,.4,.6)}	{(.2,.4,.3)}

Step-1: The expert evaluation information is in the form of *TSpHeFSs* is enclosed in Tab. 1:

Step-2: The normalized expert evaluation information in enclosed in Tab. 2:

Table 2: Normalized expert evaluation information (N)[1]

	V_1	V_2	V_3	V_4
G_1	{(.4,.2,.3)}	{(.5,.6,.2)}	{(.3,.5,.1)}	{(.6,.5,.1), (.5,.4,.3)}
G_2	{(.2,.5,.1)}	{(.4,.3,.2)}	{(.6,.1,.1), (.4,.1,.3)}	{(.2,.4,.1)}
G_3	{(.5,.1,.4)}	{(.6,.1,.1), (.4,.2,.3)}	{(.5,.2,.4)}	{(.5,.2,.4)}
G_4	{(.3,.2,.2)}	{(.3,.2,.1)}	{(.3,.4,.2), (.6,.4,.4)}	{(.3,.4,.2)}

Step-3: In this case study, we have only one expert so, we have no need to normalized *TSpHeF* information.

Step-4: In this step, we calculate the combined preference values of alternatives under criteria weight is $(0.4, 0.2, 0.3, 0.1)^T$ using proposed list of T-spherical hesitant fuzzy aggregation operators as follows:

Case 1: Using $\text{WA}_{ST-TSHF}$ **aggregation operator**

We apply ST-TSpHeFWA aggregation operator to the data provided in above matrix to find out the aggregated values. The combined preference values of each alternative using $\text{WA}_{ST-TSHF}$ aggregation operator is enclosed in Tab. 3:

Table 3: Overall preference value (ST-*TSpHeFWA*)

G_1	{.7110,.1675,.0214}, {.6689,.1456,.0416}
G_2	{.5274,.0855,.0137}, {.4803,.0855,.0171}
G_3	{.7542,.0137,.0425}, {.6657,.0240,.1031}
G_4	{.4540,.0559,.0181}, {.5191,.0559,.0209}

Case 2: Using ST-TSpHeFWG **aggregation operator**

We apply ST-TSpHeFWG aggregation operator to above matrix to find out the aggregated values. The combined preference values of each alternative using $\text{WG}_{ST-TSHF}$ aggregation operator is enclosed in Tab. 4:

Table 4: Overall preference value (ST-*TSpHeFWG*)

G_1	{.6788,.2375,.03742}, {.6520,.2205,.05477}
G_2	{.4402,.1265,.020}, {.4263,.1265,.03}
G_3	{.7463,.02,.1010}, {.6568,.02828,.1109}
G_4	{.4540,.08602,.0244}, {.4810,.08602,.0469}

Step-5: Score of collective overall preference values of each alternative is enclosed in Tab. 5:

Table 5: Score values

Operators	$S_f(G_1)$	$S_f(G_2)$	$S_f(G_3)$	$Sf(G_4)$
ST-TSpHeFWA	0.4505	0.2463	0.4983	0.2332
ST-TSpHeFWG	0.3881	0.1713	0.6803	0.2098

Step-6: Rank the alternatives $G_k(k = 1, 2, \ldots., 4)$ is enclosed in Tab. 6:

Table 6: Ranking of the alternatives

Operators	Score	Best Alternative
ST-TSpHeFWA	$S_f(G_3) > S_f(G_1) > S_f(G_2) > S_f(G_4)$	G_3
ST-TSpHeFWG	$S_f(G_3) > S_f(G_1) > S_f(G_2) > S_f(G_4)$	G_3

From the above computational process, we concluded that alternative **Li-Fi-based solid waste collection system** (G_3) is the top alternative for the waste collection system among others, and therefore it is highly recommended.

7 Reliability and Validity Test

In fact, deciding the highest suitable alternative from the decision matrices provided by the group is extremely difficult. The approach to estimate the validity and reliability of decision-making approaches was started by Ashraf et al. [21]. The steps for testing are as follows.

Test Step-1.: If we substitute the normalized element for the worse element of the alternative by presenting the appropriate alternative with no modification and also with no altering the comparable position of each decision criterion, the appropriate and effective MAGDM technique is to do so.

Test Step-2.: Through an efficient and appropriate MAGDM procedure, transitive property must be met.

Test Step-3.: When an issue with MAGDM is turned into minor issues. A combined alternative rating should be equivalent to the original rating of un-decomposed problem to ranking the alternative, we apply identical approach on minor issues used in the problem of MAGDM.

To find the best result, the MAGDM problem was transformed into a smaller one and the same proposed decision-making approach were introduced. The suitable and efficient MAGDM strategy is that the outcome would be the same as the MAGDM problem if we apply the same technique to a small problem.

Validity Test the Proposed DM Methodology

In this area [21], using the validity and reliability test mentioned above, we check the appropriation and validation of our developed methodology. The normalized spherical hesitant fuzzy information is enclosed in the Tab. 2 (given Above):

Test Step-1: We substitute the normalized element for the worse element of the alternative by presenting the appropriate alternative with no modification and also with no altering the comparable position of each decision criterion, in this step. Tab. 7 enclosed the updated decision matrix

Table 7: Updated normalized T-Spherical hesitant fuzzy information

	V_1	V_2	V_3	V_4
G_1	{(.4,.2,.3)}	{(.2,.6,.5)}	{(.3,.5,.1)}	{(.1,.5,.6), (.3,.4,.5)}
G_2	{(.2,.5,.1)}	{(.2,.3,.4)}	{(.6,.1,.1), (.4,.1,.3)}	{(.1,.4,.2)}
G_3	{(.5,.1,.4)}	{(.6,.1,.1), (.4,.2,.3)}	{(.4,.2,.5)}	{(.5,.2,.4)}
G_4	{(.2,.2,.3)}	{(.3,.2,.1)}	{(.2,.4,.3), (.4,.4,.6)}	{(.3,.4,.2)}

Now, we calculate the combined preference values of each alternative under criteria weight $(0.4, 0.2, 0.3, 0.1)^T$ using proposed list of spherical hesitant fuzzy aggregation operators as follows:

Case-1: Using ST-TSpHeFWA aggregation operator:

The collective overall preference values of each alternative using ST-TSpHeFWA aggregation operator is enclosed in Tab. 8:

Table 8: Overall preference value (*ST-TSpHeFWA*)

G_1	{.3795,.1675,.1369}, {.4419,.1456,.1217}
G_2	{.3995,.0855,.0368}, {.3228,.0855,.0459}
G_3	{.7298,.0137,.0489}, {.6296,.0240,.1187}
G_4	{.4177,.0559,.0232}, {.4480,.0559,.0269}

Case-2: Using ST-TSpHeFWG aggregation operator

The combined preference values of each alternative using **ST-TSpHeFWG** aggregation operator is enclosed in Tab. 9:

Table 9: Overall preference value (*ST-TSpHeFWG*)

G_1	{.2978,.2375,.2184}, {.4100,.2205,.1783}
G_2	{.2775,.1265,.0842}, {.2688,.1265,.0871}
G_3	{.7062,.02,.1353}, {.6214,.0282,.1428}
G_4	{.4046,.0860,.0435}, {.4315,.0860,.1063}

Now, Score of collective overall preference values of each alternative is enclosed in Tab. 10:

Table 10: Score values

Operators	$S_f(G_1)$	$S_f(G_2)$	$S_f(G_3)$	$S_f(G_4)$
ST-TSpHeFWA	0.1274	0.1214	0.4546	0.1836
ST-TSpHeFWG	0.0334	0.0555	0.4207	0.1617

Rank the alternatives $G_k(k = 1, 2, \ldots., 4)$ is enclosed in Tab. 11:

Table 11: Ranking of the alternatives

Operators	Score	Best Alternative
ST-TSpHeFWA	$S(G_3) > S(G_4) > S(G_1) > S(G_2)$	G_3
ST-TSpHeFWG	$S(G_3) > S(G_4) > S(G_2) > S(G_1)$	G_3

We get again the same alternative G_3 by using the test **Step-1**, which is also obtained by applying of our suggested method.

We are now testing the validity test **Steps-2 & 3** to demonstrate that the proposed approach is reliable and relevant. To this end, we first transformed the MAGDM problem into three smaller sub-problems such as $\{G_2, G_3, G_4\}$, $\{G_3, G_4, G_1\}$ and $\{G_3, G_1, G_2\}$. We now implement our suggested decision-making approach to the smaller problems that have been transformed and give us the ranking of alternatives as: $G_3 > G_2 > G_4$, $G_2 > G_4 > G_1$ and $G_3 > G_4 > G_1$ respectively. We analyzed that $G_3 > G_2 > G_4 > G_1$ is the same as the standard decision-making approach results when assigning a detailed ranking.

8 Conclusion

In this analysis, using the T-spherical hesitant fuzzy set decision process, alternative municipal SWC systems based on various ICTs are analyzed and graded. Alternative SWC concepts are built on the above four alternatives, taking into account the current situation and needs of a study area. The case study is performed in an area where municipal authorities embrace the smart city strategy and there are ongoing smart city initiatives.

Attributes which are described above are taken into account when implementing the suggested T-spherical hesitant fuzzy set (T-SHFS) methodology. The outcomes of the study indicated that the more effective solutions for the survey are are Li-Fi and visible light communication-based collection systems. The findings of this research illustrate that in the smart city area, these devices can be chosen and applied, especially in the sense of SWC. The use of fuzzy sets has helped us to effectively transform the uncertainty and complexity of local decision-makers and scientific experts' decisions. We developed certain robust sine-trigonometric (ST) operations laws (STOLs) for T-SHFSs and concluded new aggregation operators (AOs) to calculate T-SpHeF data which are ST weighted averaging and geometric operators.

In the future studies, the suggested T-SHFS methodology proposed here can be solve by applying TOPSIS method, q-ROFS based on real emergency and supply chain.

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