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Robust Optimal Proportional–Integral Controller for an Uncertain Unstable Delay System: Wind Process Application

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Abstract: In industrial practice, certain processes are unstable, such as different types of reactors, distillation columns, and combustion systems. To ensure greater maneuverability and improve the speed of response command, certain systems in the military and aviation fields are purposely configured to be unstable. These systems are often more difficult to control than stable systems and are of particular interest to designers and control engineers. Despite all advances in process control over the past six decades, the proportional–integral–derivative (PID) controller is still the most common. The main reasons are the simplicity, robustness, and successful applications provided by PID-based control structures. The design of proportional–integral (PI) controller for time-delay (TD) systems is a traditional and classical problem. On one hand, PI controllers are used in more than 95% of industrial processes. On the other hand, there are TD phenomena in almost all practical control systems. In this paper, we study the robustness of a PI controller in stabilizing systems containing uncertain parameters and delays. A robust controller for an unknown unstable second-order with margin TD is constructed using a generalized Kharitonov theorem for quasi-polynomials. A constructive method based on the Hermite–Biehler theorem is used to obtain all PI gains, which stabilize an uncertain and unstable second-order delay system. Genetic algorithms (GAs) and optimization methods are used to obtain optimal system and control parameters for providing the best PI control that makes the system robust and stable under uncertainty. By minimizing performance criteria such as the integral of square error, integral of absolute error, time-weighted integral of absolute error, and integral of time weighted square error, GAs and particle swarm optimization are used to optimize the PI parameters and system parameters that provide the best control under uncertainties.

Keywords: Unstable time-delay system; interval plants; Generalized Kharitonov Theorem (GKT); PI controller; Hermite–Biehler theorem; stability region; genetic algorithms; Particle swarm optimization; optimum PI controller; optimum system parameters; wind process application

1 Introduction

Time delays are found in several industrial processes and engineering systems, such as hydraulic systems, turbojet engines, chemical processes and communication networks. Delays have a considerable effect on the dynamic behavior of closed-loop control systems and can cause oscillations and even lead to instability [1]. According to Gao et al. [2], over 90% of the physical systems in process control can be modelled by first-order + time-delay (TD) and second-order + TD (about 30%) models with tolerable accuracy.

Open-loop unstable delay systems are popular in the processing industry. Compared with stable open-loop systems, open-loop unstable delay systems pose a relatively challenging problem for controller design. In an unstable delay system, the presence of an unstable pole imposes a minimum control performance limit, which could yield long settling time and excessive overshoot.

Although the proportional–integral–derivative (PID) controller is an antiquated design, numerous applications for the control of industrial process prefer and widely use it. This is because of its simple structure, satisfactory control effect, and acceptable robustness [3]. Over the past six decades, numerous techniques have been used to calculate the parameters PID controller of systems with long TD. The use of PID controllers to stabilize uncertain systems with or without delay has received a lot of attention.

A generalization of the Hermite–Biehler theorem (HBT) is one of the well-known methods for determining the stabilizing PID controller region [4]. To define all stabilizing regions of PID parameters, this approach necessitates sweeping over the proportional gain. An extended theorem, which was used to locate the PID stabilizing parameter regions, used the HBT as its basis. Farkh et al. [5] have derived the complete stabilizing set of the classical proportional–integral (PI) and PID controller parameter regions for unstable second-order TD plants.

Ma et al. [6] presented explicit lower bounds on the delay margin of second-order unstable delay systems using PID controller. In Wang et al. [7], a multiple dominant pole placement for an unstable delay system, was used to build a PID controller with a lead/lag filter. An internal model control-PID controller was proposed for an unstable second-order TD system, which shows the characteristics of inverse response [8]. PID controller tuning using genetic algorithm (GA) for stable and unstable process was presented in Ayas et al. [9]. A particle swarm optimization (PSO) algorithm was proposed to tune and retune the parameter of PID controller for a class of unstable TD systems [10].

Over the last few decades, there has been a lot of interest in the stable stability of uncertain systems. For the stability analysis of interval systems, the Kharitonov theorem (KT) is well-known. The performance and stability of plants that are exposed to uncertainties are defined as robustness.

The edge theorem in Barmish et al. [11] and the box theorem in Kwon et al. [1] were based on KT and suggested that the set of transfer functions generated by varying its perturbed coefficients in the defined ranges corresponds to a box in the parameter space and is referred to as "interval plants."

Generalized KT (GKT) states that a controller robustly stabilizes an interval system if it stabilizes a specified set of line segments in the plant parameter space [12,13].

Researchers have expanded the GKT and edge theorem to determine the robust stability of a TD system subjected to parametric uncertainty in the case of quasi-polynomials [14,15].

Relevant results relating to the stabilization of interval systems have been acquired from previous studies. In Barmish et al. [16], it was proven that a first-order controller stabilized an interval plant if and only if the controller stabilized the 16 plants of the Kharitonov plant family at the same time.

A parameter plan based on KT and the gain phase margin tester method was used in Huang et al. [17] to obtain the nonconstructive region of a PID controller, which stabilizes the entire interval plants.

The stability boundary locus can be used to find the stabilizing region of PI parameters for the regulation of a plant with unknown parameters, according to Tan et al. [18].

Patre et al. [19] suggested a two-degrees-of-freedom design technique for interval process plants to ensure robust stability and satisfactory performance.

To stabilize a delay-free interval plant family, the HBT was employed in Refs. [20,21] to formulate the proportional, PI, and PID controllers. The stabilizing problem of a PI/PID controller was studied for a first-order delay system and then used to find both PI and PID gains that stabilize an interval first-order delay system [22].

We propose in this paper to extend the work presented in Farkh et al. [23] by determining the set of all PI gains that stabilize an unstable uncertain second-order delay system with coefficients that are perturbed within prescribed ranges. Applying GAs and PSO algorithms in the robust stable region using integral performance criteria, the optimal PI controller and optimal system parameters are then calculated.

2 Problem Formulation

A tool to design a robust PI controller for an unstable second-order system with bounded uncertain parameters and bounded uncertain TD is proposed in this paper.

We consider the plant family $F(s) = \frac{Ke^{-Ls}}{s^2+a_1s+a_0}$, where $K > 0$, $L > 0$, $a_0 < 0$, and $a_1 > 0$ and $K \in [\underline{K}, \overline{K}]$, $a_1 \in [\underline{a_1}, \overline{a_1}]$, $a_0 \in [\underline{a_0}, \overline{a_0}]$ and $L \in [\underline{L}, \overline{L}]$.

By combining robust stabilization results obtained earlier [4], which are presented in Section 5, and the approach developed in Section 3, we designed a robust PI controller that stabilizes the uncertain plant family $F(s)$. In Section 4, we show that stabilizing a plant family, where $K \in [\underline{K}, \overline{K}]$, $a_1 \in [\underline{a_1}, \overline{a_1}]$, $a_0 \in [\underline{a_0}, \overline{a_0}]$, and $L = \bar{L} = L_{max}$, is enough to compute a robust stability region in the parametric space of the PI controller.

3 PI Controller Design for Unstable Second-Order Delay System

In Farkh et al. [5], all stabilizing PI controllers' computation for an unstable delay system was considered.

3.1 Theorem 1 [5]

Under the assumptions, $K > 0, L > 0, a_0 < 0$, and $a_1 > 0$, the K_p values for which there is a solution for the stabilization problem of the PI controller of unstable second-order delay system, verify

$$-\frac{a_0}{K} < K_p < \frac{1}{K}\left(a_1\frac{\alpha}{L}\sin(\alpha) - \cos(\alpha)\left(a_0 - \frac{\alpha^2}{L^2}\right)\right)$$

where α is the solution of the equation below.

$\tan(\alpha) = \frac{\alpha(2+a_1L)}{(\alpha^2 - a_1L - a_0L^2)}$ in the interval $[0,\pi]$.

The range of K_i values is given by $0 < K_i < \underset{j=1,3,5\ldots}{Min} \{a_j\}$

where $a_j = a(z_j) = \frac{z_j}{KL}\left[\sin(z_j)(a_0 - \frac{z_j^2}{L^2}) + a_1\frac{z_j}{L}\cos(z_j)\right]$

and $z_j, j = 1, 3, 5, \ldots$ are the roots, arranged in ascending order of magnitude, as follows.

$$\delta_i(z) = \frac{z}{L}\left(KK_p + \cos(z)\left(a_0 - \frac{z2}{L^2}\right) - a_1\frac{z}{L}\sin(z)\right)$$

3.2 Example

Consider the following transfer function for a second-order delay system:

$$F(s) = \frac{2e^{-0.5s}}{-0.5 + 5s + s^2}$$

To determine the values of K_p, we search for α in interval $[0, \pi]$ satisfying: $\tan(\alpha) = \frac{4.5\alpha}{(\alpha^2 - 2.625)} \Rightarrow \alpha = 1.5617$.

For K_p, the range is given by $0.25 < K_p < 7.85$.

We notice that

$$\begin{cases} 0.25 < K_p < 6.2 \ \Rightarrow K_i > 0 \\ 6.2 < K_p < 7.85 \ \Rightarrow K_i < 0 \end{cases}$$

Consequently, we choose the final range of K_p as $0.25 < Kp < 6.2$

The system stability region is obtained in (K_p, K_i); Fig. 1 presents the plane.

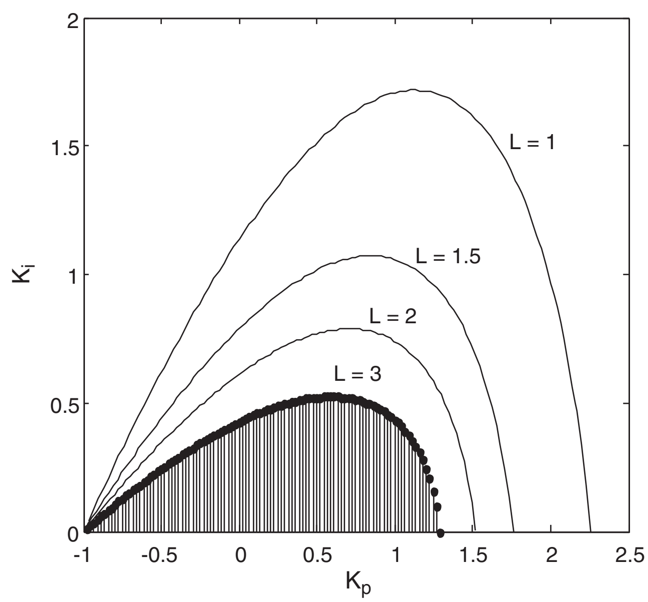

Figure 1: Sets of stabilizing PI controllers for Example 1

4 Design of a Robust Controller for an Unstable System with an Uncertain Delay

The problem of stabilizing an unstable second-order system with TD is presented in this section., where the parameters K, a_1, and a_0 are known and the TD is unknown but lies inside a known interval.

We consider the following plant family: $F_1(s) = \frac{Ke^{-Ls}}{s^2 + a_1 s + a_0}$, where $L \in [\underline{L}, \overline{L}]$, K, a_1, and a_0 are known.

4.1 Lemma 1 [4]

Consider the system with a transfer function $F_1(s)$. If a given PI controller stabilizes the delay-free system and the system $L = \bar{L} > 0$, then the same PI controller stabilizes the system $\forall L \in [0, \bar{L}]$.

4.2 Example

Consider the plant family $F_1(s) = \frac{1.9e^{-Ls}}{s^2+4s-0.4}$, where $L \in [0.1, 0.5]$ seconds. The set of all stabilizing PI controllers for different TD values can be found using the algorithm presented in Farkh et al. [5].

Fig. 2 represents these sets for $L = 1, 1.5, 2, 3$. The intersection of all these sets is the set corresponding to $L = 3$ (dashed area).

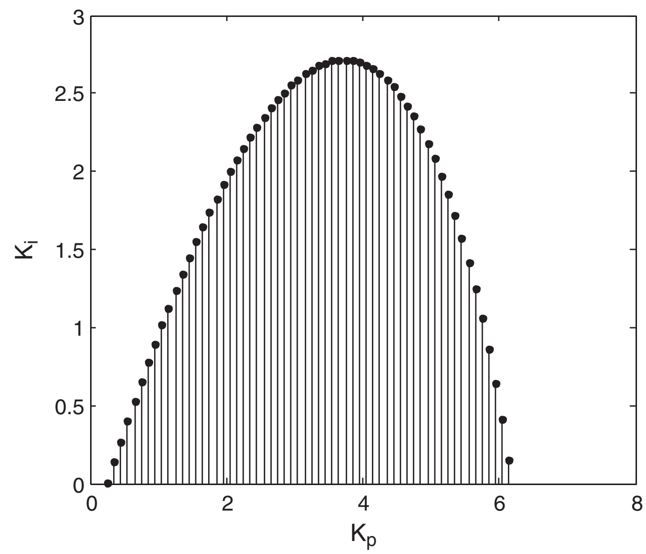

Figure 2: (K_p, K_i) Controller stability region for unstable second-order delay system

Thus, any PI controller from this set will stabilize the entire family of plants described by $F_1(s)$. In view of Lemma 1 because the closed-loop system is stable for $\bar{L}$, it is also stable for $L \in [\underline{L}, \overline{L}]$.

This result is used to simplify the problem stabilization of the family plant $F(s)$ to design a robust controller such that the following parameters verify $K \in [\underline{K}, \overline{K}]$, $a_1 \in [\underline{a_1}, \overline{a_1}]$, $a_0 \in [\underline{a_0}, \overline{a_0}]$, and $L = \bar{L}$.

The delay is set to $\bar{L}$ for the rest of this article, which is the established upper bound of the TD.

5 Robust Controller Design for Interval System with Fixed TD

The procedure for finding a robust stabilization of an unstable delay system belonging to a linear interval plant is discussed in this section, where the TD, L, is a known constant.

Consider the transfer function below:

$$F(s) = \frac{F_1(s)}{F_2(s)} e^{-Ls} \tag{1}$$

where $F_1(s)$ and $F_2(s)$ are linear interval polynomials. Our goal is to find a robust controller $C(s) = \frac{C_1(s)}{C_2(s)}$ with fixed polynomials $C_1(s)$ and $C_2(s)$ to guarantee the system's robust stability.

To compute all the stabilizing controller parameters for interval systems with TDs, we can use the GKT expanded for quasi-polynomials [13]. Before stating the GKT, we study some results from the field of robust parametric control.

Consider the following family of quasi-polynomials $\Delta(s)$:

$$\Delta(s) = F_1(s)C_1(s) + F_2(s)C_2(s) \tag{2}$$

$-\underline{F}(s) = (F_1(s), F_2(s))$ is a fixed two-tuple of real interval polynomials, where each $F_i(s)$ is a linear interval polynomial characterized by the intervals $F_{j,i}$ as follows:

$$F_{j,i} \in \left[\underline{F}_{j,i}, \overline{F}_{j,i}\right], i = 1, 2; j = 0, 1, \ldots, n_i. \tag{3}$$

$F_i(s)$ is a real independent interval polynomial described as:

$$F_i(s) = f_{0,i} + f_{1,i}s + \ldots + f_{n_i,i}s^{n_i}, i = 1, 2. \tag{4}$$

$-\ \underline{C}(s) = (C_1(s), C_2(s))$ is a fixed two-tuple of complex quasi-polynomials in the following form:

$$C_i(s) = C_i^0(s) + C_i^1(s)e^{-sL_i^1} + C_i^2(s)e^{-sL_i^2} + \ldots \tag{5}$$

where $C_i^j(s)$ are complex polynomials satisfying the following condition:

$$\text{degree}\left[C_i^0(s)\right] > \text{degree}\left[C_i^j(s)\right], j \neq 0 \tag{6}$$

In this sudy, we use $C_i(s)$ with a single delay as: $C_i(s) = C_i^0(s) + C_i^1(s)e^{-sL_i}$.

The stability problem (2) can be solved with the GKT by constructing an extremal set of line segments $\Delta_E(s) \subset \Delta(s)$, where the stability of $\Delta_E(s)$ implies the stability of $\Delta(s)$, according to Bhattacharyya et al. [2].

$\Delta_E(s)$ will be produced by constructing an extremal subset $F_E(s)$, which is derived from the Kharitonov polynomials of $F_i(s)$.

5.1 Theorem 2 [13]

Let $\underline{C} = (C_1(s), C_2(s))$ be a two-tuple of complex quasi-polynomials verifying condition (6) above, and let $\underline{F} = (F_1(s), F_2(s))$ be a polynomial with independent real intervals : the entire family $\underline{F}(s)$ is stabilized $\underline{C}(s)$ if and only if $\underline{C}$ stabilizes every two-tuple segment in $F_E(s)$. Equivalently, $\Delta(s)$ is stable if and only if $\Delta_E(s)$ is stable.

5.2 Corollary

The linear system $\underline{F}(s)$ is stabilized by $\underline{C}(s)$, if and only if the controller stabilizes the extremal transfer function $F_E(s)$. This is discussed in detail later.

To use the GKT, we must first define the most extremal set of line segments $\Delta_E(s)$. Eight Kharitonov vertex equations are derived from the segment polynomials of $F_1(s)$ and $F_2(s)$.They are expressed by Refs. [13,15]:

$K_1^m(s), m = 1, 2, 3, 4$ for $F_1(s)$

and

$K_2^m(s), m = 1, 2, 3, 4$ for $F_2(s)$

where:

$$\begin{aligned} K_i^1(s) &= \underline{f}_{i,0} + \underline{f_{i,1}}s + \overline{f_{i,2}}s^2 + \overline{f}_{i,3}s^3 + \ldots \\ K_i^2(s) &= \underline{f}_{i,0} + \overline{f_{i,1}}s + \overline{f_{i,2}}s^2 + \underline{f}_{i,3}s^3 + \ldots \\ K_i^3(s) &= \overline{f}_{i,0} + \underline{f}_1 s + \underline{f}_{i,2}s^2 + \overline{f_{i,3}}s^3 + \ldots \\ K_i^4(s) &= f_{i,0} + \overline{f_{i,1}}s + \underline{f}_{i,2}s^2 + \underline{f}_{i,3}s^3 + \ldots \end{aligned} \tag{7}$$

The extremal subset $F_E^i(s), i = 1, 2$, consists of:

$$F_E^1(s) = \frac{\lambda K_1^l(s)+(1-\lambda)K_1^k(s)}{K_2^h(s)}$$
$$F_E^2(s) = \frac{K_1^h(s)}{\lambda K_2^l(s)+(1-\lambda)K_2^k(s)} \tag{8}$$

where $\lambda \in [0, 1]$, $h = 1, 2, 3, 4$ and [l,k] = [1,2] [1,3] [2,4] [3,4].

The number of extremal equations is $(i4^i)$, where i denotes the number of perturbed polynomials and $[l, k]$ present the connection points to make Kharitonov polytope $\lambda K_i^l(s) + (1 - \lambda)K_i^k(s)$.

Some of the subset equations may be the same; hence, the extremal subset is described as [13]:

$$F_E(s) = F_E^1(s) \cup F_E^2(s) \tag{9}$$

The extremal subset of line segments (or generalized Kharitonov segment polynomials) is [2]:

$$\Delta_E(s) = \Delta_E^1(s) \cup \Delta_E^2(s) = \{ < C(s), F(s) > \, : F(s) \in F_E(s)\} \tag{10}$$

where

$$< C(s), F(s) > \, = C_1(s)F_1(s) + C_2(s)F_2(s) + \ldots + C_m(s)F_m(s) \tag{11}$$

With the knowledge that $\Delta_E(s) \subset \Delta(s)$, If the linear interval system's polynomials are all stable, the system with perturbed parameters would be as well.

The robust parametric approach control technique is an efficient control design technique, as shown by the previous findings.

For the synthesis of controllers that simultaneously stabilize a given uncertain TD system, the following will be used.

6 Robust Stabilization for Uncertain Unstable Second-Order Delay System

The problem of characterizing all PI controllers that stabilize an unstable second-order interval plant with TD is addressed in this section.

$F(s) = \frac{Ke^{-Ls}}{a_0+a_1s+s^2}$, where $K \in [\underline{K}, \overline{K}]$, $a_0 \in [\underline{a_0}, \overline{a_0}]$, $a_1 \in [\underline{a_1}, \overline{a_1}]$ and fixed TD, L.

The controller is given by:

$$C(s) = \left(K_p + \frac{K_i}{s}\right).$$

Using the GKT for quasi-polynomials, we can obtain all PI gains that stabilize $F(s)$, we will use the new transfer function $F(s)$ as: $F(s) = \frac{F_1(s)}{F_2(s)} = \frac{Ke^{-Ls}}{a_0+a_1s+s^2}$ and the new compensator as follows:

$$C(s) = \frac{C_1(s)}{C_2(s)} = \left(K_p + \frac{K_i}{s}\right)e^{-Ls}.$$

The family of closed-loop characteristic quasi-polynomials $\Delta(s, K_p, K_i)$ becomes

$$\Delta(s, K_p, K_i) = C_1(s)F_1(s) + C_2(s)F_2(s)$$
$$= K(K_i + K_ps)e^{-Ls} + (a_0 + a_1s + s^2)s \tag{12}$$

The problem of determining all stabilizing PI controllers is to find all the values of K_p and K_i for which the entire family of closed-loop characteristic quasi-polynomials is stable.

Let $K_1^j(s)$ and $K_2^j(s), j = 1, 2, 3, 4$ be the Kharitonov polynomials corresponding to $F_1(s) = K$ and $F_2(s) = a_0 + a_1 s + s^2$, respectively.
where: $K \in [\underline{K}, \overline{K}]$, $a_0 \in [\underline{a_0}, \overline{a_0}]$, and $a_1 \in [\underline{a_1}, \overline{a_1}]$.

$$\begin{cases} K_1^1(s) = K_1^2(s) = \underline{K} \\ K_1^3(s) = K_1^4(s) = \overline{K} \end{cases} \text{ and } \begin{cases} K_2^1(s) = \underline{a_0} + \underline{a_1}s + s^2 \\ K_2^2(s) = \underline{a_0} + \overline{a_1}s + s^2 \\ K_2^3(s) = \overline{a_0} + \underline{a_1}s + s^2 \\ K_2^4(s) = \overline{a_0} + \overline{a_1}s + s^2 \end{cases}$$

Let $F_E(s, \lambda)$ denote the family of 32 plant segments:

$$F_E(s,\lambda) = \left\{ \begin{array}{l} F_{lkh}(s,\lambda)/ \\ F_{lkh}(s,\lambda) = \frac{\lambda K_1^l(s) + (1-\lambda)K_1^k(s)}{K_2^h(s)} \\ \qquad \cup \\ F_{lkh}(s,\lambda) = \frac{K_1^h(s)}{\lambda K_2^l(s) + (1-\lambda)K_2^k(s)} \\ \lambda \in [0,1]; h = 1,2,3,4; \\ [l,k] = [1,2], [1,3], [2,4], [3,4] \end{array} \right\} \tag{13}$$

Then, $F_E(s, \lambda)$ it consists of the following plant segments, where the 32 extremal plants in Eq. (13) are reduced to 20:

$$F_E(s,\lambda) = \left\{ \begin{array}{l} F_1 = \frac{\underline{K}}{\underline{a_0} + \underline{a_1}s + s^2}, F_2 = \frac{\underline{K}}{\underline{a_0} + \overline{a_1}s + s^2} \\ F_3 = \frac{\underline{K}}{\overline{a_0} + \underline{a_1}s + s^2}, F_4 = \frac{\underline{K}}{\overline{a_0} + \overline{a_1}s + s^2} \\ F_5 = \frac{\overline{K}}{\underline{a_0} + \underline{a_1}s + s^2}, F_6 = \frac{\overline{K}}{\underline{a_0} + \overline{a_1}s + s^2} \\ F_7 = \frac{\overline{K}}{\overline{a_0} + \underline{a_1}s + s^2}, F_8 = \frac{\overline{K}}{\overline{a_0} + \overline{a_1}s + s^2} \\ F_9 = \frac{\overline{K} - \lambda(\overline{K} - \underline{K})}{\underline{a_0} + \underline{a_1}s + s^2}, F_{10} = \frac{\overline{K} - \lambda(\overline{K} - \underline{K})}{\underline{a_0} + \overline{a_1}s + s^2} \\ F_{11} = \frac{\overline{K} - \lambda(\overline{K} - \underline{K})}{\overline{a_0} + \underline{a_1}s + s^2}, F_{12} = \frac{\overline{K} - \lambda(\overline{K} - \underline{K})}{\overline{a_0} + \overline{a_1}s + s^2} \\ \qquad \cup \\ F_{13} = \frac{\underline{K}}{\underline{a_0} + (\overline{a_1} - \lambda(\overline{a_1} - \underline{a_1}))s + s^2}, F_{14} = \frac{\overline{K}}{\underline{a_0} + (\overline{a_1} - \lambda(\overline{a_1} - \underline{a_1}))s + s^2} \\ F_{15} = \frac{\underline{K}}{\underline{a_0} + \lambda(\overline{a_0} - \underline{a_0}) + \underline{a_1}s + s^2}, F_{16} = \frac{\overline{K}}{\underline{a_0} + \lambda(\overline{a_0} - \underline{a_0}) + \underline{a_1}s + s^2} \\ F_{17} = \frac{\underline{K}}{\underline{a_0} + \lambda(\overline{a_0} - \underline{a_1}) + \overline{a_1}s + s^2}, F_{18} = \frac{\overline{K}}{\underline{a_0} + \lambda(\overline{a_0} - \underline{a_1}) + \overline{a_1}s + s^2}, \\ F_{19} = F_4 = \frac{\underline{K}}{\overline{a_0} + \overline{a_1}s + s^2}, F_{20} = F_8 = \frac{\overline{K}}{\overline{a_0} + \overline{a_1}s + s^2} \\ \lambda \in [0,1] \end{array} \right\} \tag{14}$$

The closed-loop characteristic quasi-polynomials for each of these 32 plant segments $F_{lkh}(s, \lambda)$ are denoted by $\delta_{lkh}(s, K_p, K_i, \lambda)$ and are defined as:

$$\delta_{lkh}(s,\lambda) = sNum(F_{lkh}(s,\lambda)) + (K_i + K_p s)den(F_{lkh}(s,\lambda)) \tag{15}$$

where:

$$\begin{cases} Num(F_{lkh}(s,\lambda)) = \lambda K_1^l(s) + (1-\lambda)K_1^k(s) \cup K_1^h(s) \\ den(F_{lkh}(s,\lambda)) = K_2^h(s) \cup \lambda K_2^l(s) + (1-\lambda)K_2^k(s) \end{cases} \tag{16}$$

We posit the following theorem on using a PI controller to stabilize a second-order interval plant with TD.

6.1 Theorem 3

Let $F(s)$ be an unstable second-order interval system with uncertain TD; a PI controller stabilizes the entire family $F(s)$ if and only if each $F_{lkh}(s,\lambda) \in F_E(s,\lambda)$ is stabilized by the same PI controller.

6.2 Proof

According to Theorem 3, the entire family $\Delta(s, K_p, K_i)$ is stable if and only if $\delta_{lkh}(s, K_p, K_i, \lambda)$are all stable. Therefore, a PI controller will stabilize the entire family $F(s)$ if and only if every element of $F_E(s,\lambda)$is simultaneously stabilized by the same PI controller.

To obtain the all PI controllers that stabilize the interval plant $F(s)$ by applying this method to each $F_{lkh}(s,\lambda)$ belonging $F_E(s,\lambda)$, we use the results in Farkh et al. [5].

6.3 Example

We consider the plant family $F(s) = \frac{Ke^{-Ls}}{a_0 + a_1 s + s^2}$ where $K \in [1.9, 2.2]$, $a_0 \in [-0.6, -0.4]$, $a_1 \in [4, 6]$ and $L \in [0.1, 0.5]$.

The entire family $F_E(s,\lambda)$ is given as follows:

$$F_E(s,\lambda) = \left\{ \begin{array}{l} F_{ij}(s,\lambda)/ \\ F(s,\lambda) = \frac{\lambda_l K_1^l(s) + (1-\lambda_l) K_1^k(s)}{K_2^h(s)} \\ \quad \cup \\ F(s,\lambda) = \frac{K_1^h(s)}{\lambda_m K_2^l(s) + (1-\lambda_m) K_2^k(s)} \\ \lambda \in [0,1]; h = 1,2,3,4; \\ [l,k] = [1,2],[1,3],[2,4],[3,4] \end{array} \right\} \quad (17)$$

According to Eq. (14), we obtain

$$F_E(s,\lambda) = \left\{ \begin{array}{l} F_1 = \dfrac{1.9}{-0.6+4s+s^2}, F_2 = \dfrac{1.9}{-0.6+6s+s^2} \\ F_3 = \dfrac{1.9}{-0.4+4s+s^2}, F_4 = \dfrac{1.9}{-0.4+6s+s^2} \\ F_5 = \dfrac{2.2}{-0.6+4s+s^2}, F_6 = \dfrac{2.2}{-0.6+6s+s^2} \\ F_7 = \dfrac{2.2}{-0.4+4s+s^2}, F_8 = \dfrac{2.2}{-0.4+6s+s^2} \\ F_9 = \dfrac{2.2-0.3\lambda}{-0.6+4s+s^2}, F_{10} = \dfrac{2.2-0.3\lambda}{-0.6+6s+s^2} \\ F_{11} = \dfrac{2.2-0.3\lambda}{-0.4+4s+s^2}, F_{12} = \dfrac{2.2-0.3\lambda}{-0.4+6s+s^2} \\ \qquad \cup \\ F_{13} = \dfrac{1.9}{-0.6+(6-2\lambda)s+s^2}, F_{14} = \dfrac{2.2}{-0.6+(6-2\lambda)s+s^2} \\ F_{15} = \dfrac{1.9}{-0.4+0.2\lambda+4s+s^2}, F_{16} = \dfrac{2.2}{-0.4+0.2\lambda+4s+s^2} \\ F_{17} = \dfrac{1.9}{-0.4+0.2\lambda+6s+s^2}, F_{18} = \dfrac{2.2}{-0.4+0.2\lambda+6s+s^2}, \\ F_{19} = G_4 = \dfrac{1.9}{-0.4+6s+s^2}, F_{20} = F_8 = \dfrac{2.2}{-0.6+6s+s^2} \\ \lambda \in [0,1] \end{array} \right\} \quad (18)$$

We remark that from F_9 to F_{18}, we have an infinity of transfer function's sets due to their dependence on λ. To reduce this problem's complexity, we set λ to 0, 0.33, 0.66, and 1 as different values of $\lambda \in [0,1]$ for F_9 to F_{12}.

We set λ to 0, 0.25, 0.5, 0.75, and 1 as different values of $\lambda \in [0, 1]$ for F_{13} to F_{18}.

Therefore, we obtain

$$\begin{cases} F_{h\_1} = F_{h1}(s, \lambda = 0) \\ F_{h\_2} = F_h(s, \lambda = 0.33) \\ F_{h\_3} = F_h(s, \lambda = 0.66) \\ F_{h\_4} = F_h(s, \lambda = 1) \end{cases} \tag{19}$$

We define $F_{k\_p} = F_h\left(s, \lambda_p\right)$, where $\lambda_p \in \{0, 0.25, 0.5, 0.75, 1\}$ for $k = 13, .., 18$ and $p = 1, \ldots, 5$.

We obtain

$$\begin{cases} F_{h\_1} = F_{h1}(s, \lambda = 0) \\ F_{h\_2} = F_h(s, \lambda = 0.25) \\ F_{h\_3} = F_h(s, \lambda = 0.5) \\ F_{h\_4} = F_h(s, \lambda = 0.75) \\ F_{h\_5} = F_h(s, \lambda = 0.75) \end{cases} \tag{20}$$

The following figure (Fig. 3) presents the stabilizing $\left(K_p, K_i\right)$ values for the entire family $F(s)$ by invoking the result presented in Farkh et al. [5], which applies to each transfer function belonging to $F_E(s, \lambda)$.

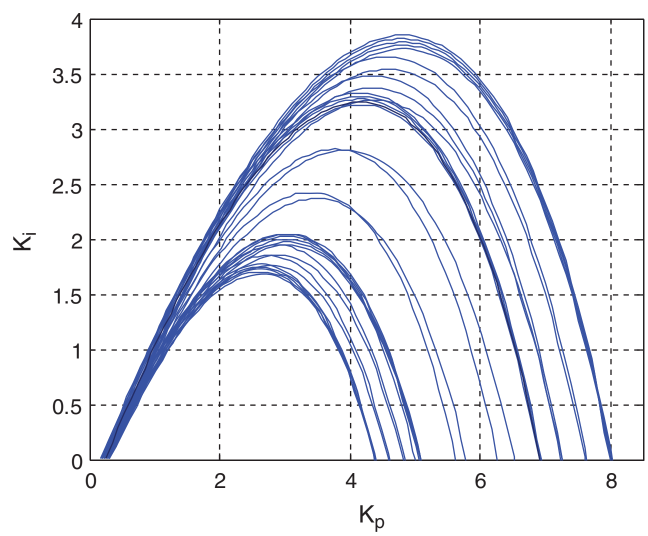

Figure 3: The stabilizing set of $\left(K_p, K_i\right)$ for *F(s)*

An overlapped area of the boundaries is the intersection of these stability regions, which constitutes the entire feasible controller sets that stabilize the entire family $F(s)$ (Fig. 4).

7 Optimization Controller Design

7.1 GAs optimization

GAs are effective stochastic search methods based on natural selection and evolutionary genetics principles.

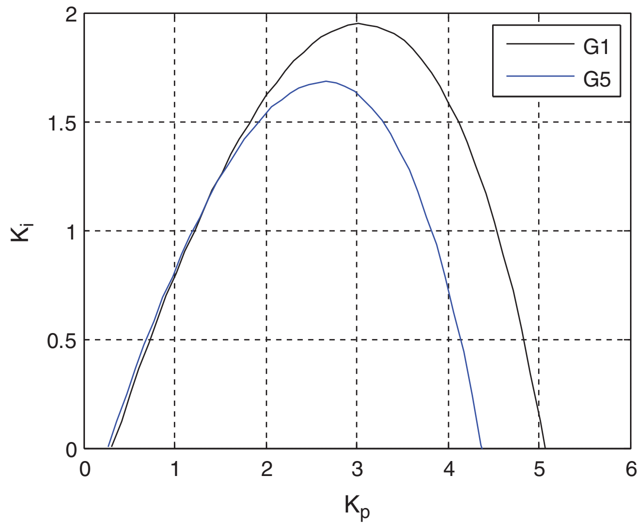

Figure 4: Final stability region in (K_p, K_i) the plan for interval plant

GAs sustain a community of individuals. Individuals are adapted to a particular setting by changing the discovery, crossover, and mutation periods. People benefit from the environment's valuable knowledge (fitness), and the selection mechanism ensures that people of higher quality are preserved.

As a result, the population's overall output increases during the development cycle, ideally contributing to an optimal solution [24]. Because of its strong capacity for global optimization, GA is used in a variety of fields.

7.2 PSO

Particle swarm optimization (PSO) [25] is an artificial intelligence-based technique for maximizing and minimizing problems.

It was inspired by the social behavior of animals, such as birds flocking and fish schooling. It starts with a random set of solutions (known as particles). Each particle has its positions (value of variables) and velocities. It improves the initial solutions by updating velocities and positions.

Fig. 5 illustrates the theory of PSO and GA optimization for control problems.

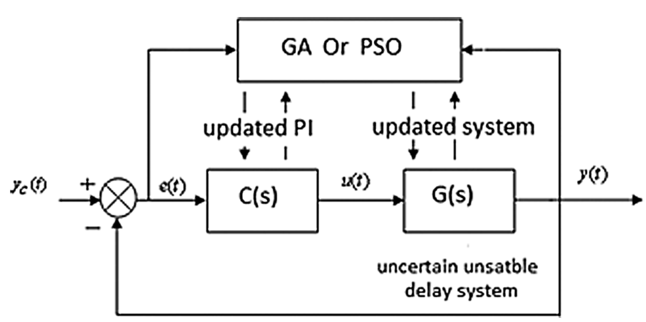

Figure 5: Optimization of controller and system parameters

In this case, it's in our best interests to find the best system and controller parameters in the robust stability area by combining one or more of the following criteria:, $ISE, IAE, ITAE$ and $ITSE$ defined by the following relationships:

$$ISE = \sum_{0}^{t_{\max}} e(t)^2, IAE = \sum_{0}^{t_{\max}} |e(t)|, ITAE = \sum_{0}^{t_{\max}} t|e(t)|, ITSE = \sum_{0}^{t_{\max}} te(t)^2$$

7.3 Example

We consider the uncertain, unstable delay system $F(s) = \frac{Ke^{-Ls}}{a_0 + a_1 s + s^2}$ where $K \in [1.9, 2.2]$, $a_0 \in [-0.6, -0.4]$, $a_1 \in [4, 6]$ and $L \in [0.1, 0.5]$.

The robust PI stability region is given in Fig. 4, from which K_p they K_i are choosing between $K_p \in [0.2727, 5.974]$ and $K_i \in [0, 1.9]$.

The following table presents the optimum PI and system parameters supplied by GA and PSO.

Using the values in Tab. 1, Figs. 6 and 7 show the closed-loop system's step responses.

The time parameter specifications are given in Tab. 2.

Table 1: Optimum PI and system parameters

	criteria	ISE	IAE	ITAE	ITSE
GA	K	2.035	2.035	2.065	1.49
	a0	−0.51	− 0.49	−0.49	−0.51
	a1	4.9	4.9	4.9	4.66
	L	0.5	0.5	0.5	0.5
	Kp	3.4439	3.51	0.99	1.49
	Ki	0.88	0.72	0.77	0.72
PSO	K	2.1588	2.1464	1.962	2.0738
	a0	−0.552	−0.471	−0.451	−0.435
	a1	4.3872	4.7618	5.994	4.327
	L	0.5	0.5	0.5	0.5
	Kp	3.6745	4.4886	3.427	3.3769
	Ki	1.4	0.48	0.877	1.4161

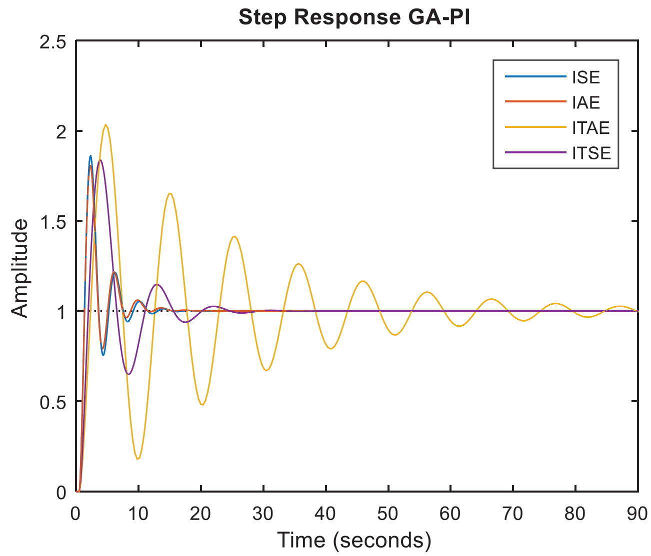

Figure 6: Step responses with GA-PI controllers

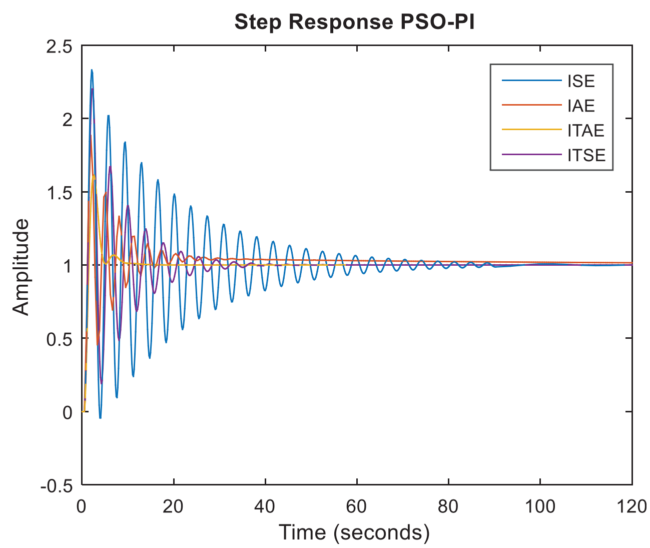

Figure 7: Step responses with PSO-PI controllers

Table 2: Time-domain specifications

		ISE	IAE	ITAE	ITSE
GA	Rise Time	0.972	1.007	0.949	0.56
	Settling Time	34.41	63.806	64.500	7.203
	Peak time	3.966	3.809	2.110	2.133
	Overshoot	58.84	98.948	111.087	71.922
PSO	Rise Time	0.485	0.5587	0.6963	0.5262
	Settling Time	75.99	93.376	8.89	31.48
	Peak time	2.11	1.9	1.6139	2.2
	Overshoot	133.1	88.375	61.3855	120.045

8 Conclusions

The HBT and GKT can be applied promptly to define the robust PI stability area for the control of uncertain and unstable second-order TD systems. The optimal system and optimal PI control parameters were computed using the integral performance criterion based on the optimization process's error.

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