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Exact Run Length Evaluation on Extended EWMA Control Chart for Autoregressive Process

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Abstract: Extended Exponentially Weighted Moving Average (Extended EWMA or EEWMA) control chart is one of the control charts which can quickly detect a small shift. The average run length (*ARL*) measures the performance of control chart. Due to the derivation of the explicit formulas for *ARL* on the EEWMA control chart for the autoregressive AR(p) process has not previously been reported. The aim of the article is to derive explicit formulas of *ARL* using a Fredholm integral equation of the second kind on EEWMA control chart for Autoregressive process, as AR(2) and AR(3) processes with exponential white noise. The accuracy of the solution obtained with the EEWMA control chart was compared to the numerical integral equation (NIE) method and extended to compare performance with Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) control charts. The results show that the *ARL* obtained by the explicit formula and the NIE method are hardly different but *ARL* of explicit formula is less the computational (CPU) time than *ARL* of NIE method. The performance of EEWMA control chart is better than the CUSUM and EWMA control charts for all situations except when the large shift sizes the EEWMA control chart performed as well as the EWMA control chart for AR(2) and AR(3) processes. And then, the EEWMA control chart is also extended to compare efficiency of EEWMA control chart with various λ. An exponential smoothing parameter of 0.05 is recommended. In addition, the simulation study, and efficacy illustration with real data on new COVID-19 cases in Thailand and Vietnam provided similar results.

Keywords: Extended EWMA control chart; autoregressive process; average run length; explicit formula

1 Introduction

Currently, the statistical process control (SPC) is very important in the manufacturing industry for monitoring, controlling, and improving processes. Control charts are one of the efficient tools of SPC and have been applied in many fields such as finance [1], health [2], and medicine [3]. The Shewhart control chart was the first to be reported and is widely used for detecting large changes in a process mean [4]. Subsequently, the Cumulative Sum (CUSUM) chart [5] and the Exponentially Weighted Moving Average

(EWMA) chart [6] have been widely employed to monitor a process mean due to their excellent performance in detecting small to moderate mean shifts. In addition, Patel et al. [7]-proposed the Modified Exponentially Weighted Moving Average (Modified EWMA) chart that is effective at detecting small size shift quickly for observations both autocorrelation and independently normally distribution. Later, Neveed et al. [8] proposed the Extended Exponentially Weighted Moving Average (EEWMA) chart that performed better than other control charts for detecting small shifts in the mean of a monitored process.

The performance of the chart is measured by Average Run Length (*ARL*). The ARL_0 denote the average number of observations before an in-control process is taken to signal to be out of control and should be large whereas the ARL_1 denote the average number of observations taken from out of control and should be as small as possible.

Many methods for evaluating *ARL* for control charts have been studied. For example, Monte Carlo simulations (MC), Markov Chain approach (MCA), Martingale approach (MA) and Numerical Integral Equation approach (NIE) and explicit formulas. Mastrangelo et al. [9] evaluated *ARL* of the traditional EWMA chart for serially correlated processes by using the Monte Carlo simulation method. Zhang et al. [10] proposed the *ARL* of the multivariate exponentially weighted moving average (MEWMA) chart and the combined control chart were evaluated with Monte Carlo simulation. Sukparungsee [11] approximated the *ARL* with optimal parameters of one and two-sided EWMA control chart using by Martingale approach. Chananet et al. [12] evaluated the *ARL* of EWMA and CUSUM control charts with Markov Chain approach based on the zero-inflated negative binomial (ZINB) model.

Many literatures for evaluating the *ARL* using NIE method and explicit formula have been studied. Areepong et al. [13] proposed the *ARL* using the numerical integral equation approach of the EWMA chart and compared the results with the Monte Carlo simulation method. Khoo et al. [14] presented a Markov chain approach for computing the *ARL* of EWMA charts. Moreover, Phanyaem et al. [15] derived the *ARL* for ARMA processes via explicit formula and numerical integral equation (NIE) method of EWMA chart. Petcharat et al. [16] investigated the derivation of the *ARL* for moving average order q process with exponential white noise by explicit formula. After that, Peerajit et al. [17] studied the NIE method of *ARL* on CUSUM chart. Supharakonsakun et al. [18] evaluated the *ARL* by NIE method on modified EWMA and compared efficiency with EWMA control chart. Sunthornwat et al. [19] derived explicit formulas of *ARL* on CUSUM chart for seasonal and non-seasonal moving average processes with exogenous variables and evaluated against the NIE method. Later, Anwar et al. [20] proposed modified-mxEWMA chart that performs very well for the monitoring of small to moderate shifts in the process and show the implementation of the wood industry. Saghir et al. [21] proposed modified EWMA chart and the performance is evaluated by *ARL*. Aslam et al. [22] proposed new Bayesian Modified-EWMA chart and its applications in mechanical and sport industry. Karoon et al. [23] developed the numerical integral equation (NIE) methods for evaluating the *ARL* on Extended EWMA chart for AR(p) process. Supharakonsakun et al. [24] presented the exact average run length based on explicit formula the observations are from moving average process with exponential white noise for modified EWMA chart. Phanthuna et al. [25] proposed the explicit formula for evaluating the *ARL* on a two-sided modified EWMA chart under the observations of AR(1) process. Recently, Phanthuna et al. [26] presented explicit formula of *ARL* for modified EWMA chart with autoregressive model involving exponential white noise.

However, the derivation of the explicit formulas for *ARL* on the EEWMA chart for autoregressive AR(p) process has not previously been reported. Therefore, the aim of this study is to derive explicit formulas of the *ARL* on the EEWMA control chart for AR(p) process, as AR(2) and AR(3) processes with exponential white noise. The explicit formulas for *ARL* were compared with the (NIE) method as the benchmark. Besides, the performance of the explicit formulas for deriving the *ARL* on the EEWMA chart was compared with those on the CUSUM and EWMA charts for both simulated data and real-world data reported.

2 Materials and Methods

2.1 Cumulative Sum (CUSUM) Control Chart

The CUSUM control chart was originally introduced by Page [5] in quality control to detect small changes in process mean, as an extension of Shewhart control chart. The CUSUM control chart can be expressed by the recursive equation below.

$$C_t = \max(0,\ C_{t-1} + X_t - a),\ t = 1,\ 2,\ \ldots \tag{1}$$

where a is non-zero constant, C_0 is the initial value of CUSUM statistics with $u \in [0, b]$, $C_0 = u$.

The stopping time of the CUSUM control chart is given by

$$\tau_b = \inf\{t > 0;\ C_t > b\},\ b > u \tag{2}$$

where τ_b is the stopping time, b is upper control limit (UCL).

2.2 Exponentially Weighted Moving Average (EWMA) Control Chart

The EWMA control chart was initially proposed by Robert [6]. It is usually used to monitor and detect small changes in process mean. The EWMA control chart can be expressed by the recursive equation below.

$$Z_t = (1 - \lambda)Z_{t-1} + \lambda X_t,\ t = 1,\ 2,\ \ldots \tag{3}$$

where X_t is a process with mean, λ is an exponential smoothing parameter with $0 < \lambda < 1$ and Z_0 is the initial value of EWMA statistics, $Z_0 = u$. The upper control limit (UCL) and Lower control limit (LCL) of EWMA control charts are given by

$$UCL = \mu_0 + Q\sigma\sqrt{\frac{\lambda}{2 - \lambda}}, \tag{4}$$

$$LCL = \mu_0 - Q\sigma\sqrt{\frac{\lambda}{2 - \lambda}}, \tag{5}$$

where μ_0 is the target mean, σ is the process standard deviation, and Q is suitable control limit width.

The stopping time of the EWMA control chart is given by

$$\tau_h = \inf\{t \geq 0 : Z_t > h\},\ h > u \tag{6}$$

where τ_h is the stopping time, h is UCL.

2.3 Extended Exponentially Weighted Moving Average (Extended EWMA or EEWMA) Control Chart

The EEWMA control chart was proposed by Neveed et al. [8]. It is developed from the EWMA control chart. This is effective to monitored and detected small changes in process mean. The EWMA control chart can be expressed by the recursive equation below.

$$E_t = \lambda_1 X_t - \lambda_2 X_{t-1} + (1 - \lambda_1 + \lambda_2)E_{t-1},\ t = 1,\ 2,\ \ldots, \tag{7}$$

where λ_1 and λ_2 are exponential smoothing parameters with $(0 < \lambda_1 \leq 1)$ and $(0 \leq \lambda_2 < \lambda_1)$ and the initial value is a constant, $E_0 = u$. The upper control limit (UCL) and Lower control limit (LCL) of the EEWMA control charts are given by

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2(1-\lambda_1+\lambda_2)}{2(\lambda_1-\lambda_2)-(\lambda_1-\lambda_2)^2}}, \tag{8}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2(1-\lambda_1+\lambda_2)}{2(\lambda_1-\lambda_2)-(\lambda_1-\lambda_2)^2}}, \tag{9}$$

where μ_0 is the target mean, σ is the process standard deviation, and L is suitable control limit width.

The stopping time of the EEWMA control chart is given by

$$\tau_{h'} = \inf\{t \geq 0 : E_t > h'\},\ h' > u \tag{10}$$

where $\tau_{h'}$ is the stopping time, $h^{'}$ is UCL.

3 Explicit Formulas of *ARL* on the EEWMA Control Chart for AR(p) Processes

Let $L(u)$ denote the ARL for the autoregressive process, to define function $L(u)$ as

$$ARL = L(u) = E_\theta(\tau_b) \geq T \tag{11}$$

where $E_\theta(\cdot)$ is the expectation under the assumption that the change point occurs at time θ and θ is the change point time.

The equation of observations for autoregressive (AR(p)) process in the case of an exponential while noise denoted can be described by

$$X_t = \eta + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \varepsilon_t \tag{12}$$

where X_t ($t = 1, 2, 3, \ldots$) is a sequence of random variables, η is a suitable constant, ϕ is an autoregressive coefficient ($-1 \leq \phi \leq 1$), and ε_t is white noise sequence of exponential ($\varepsilon_t \sim Exp(\alpha)$). The probability density function of ε_t is given by $f(x) = \frac{1}{\alpha}e^{-\frac{x}{\alpha}}$ where $x \geq 0$.

Let $L(u)$ denote ARL for AR(p) process, the EEWMA statistics E_t can be written as:

$$E_t = (1-\lambda_1+\lambda_2)Z_{t-1} + (\lambda_1\phi_1 - \lambda_2)X_{t-1} + \lambda_1\phi_2 X_{t-2} + \lambda_1\phi_3 X_{t-3} + \ldots + \lambda_1\phi_p X_{t-p} + \lambda_1\eta + \lambda_1\varepsilon_t$$

where $(0 < \lambda_1 \leq 1)$, $(0 \leq \lambda_2 < \lambda_1)$ and the initial value $E_0 = u$, and $X_{t-1}, X_{t-2}, \ldots, X_{t-p}$.

Consequently, the EEWMA statistics E_t can be written as

$$E_t = (1-\lambda_1+\lambda_2)u + (\lambda_1\phi_1 - \lambda_2)X_{t-1} + \lambda_1\phi_2 X_{t-2} + \lambda_1\phi_3 X_{t-3} + \ldots + \lambda_1\phi_p X_{t-p} + \lambda_1\eta + \lambda_1\varepsilon_t$$

If $\varepsilon_t = 0$ $LCL = 0$ and $UCL = h^{'}$, respectively. Then

$$0 \leq E_t \leq h'$$

$$0 \leq (1-\lambda_1+\lambda_2)u + (\lambda_1\phi_1 - \lambda_2)X_{t-1} + \lambda_1\phi_2 X_{t-2} + \lambda_1\phi_3 X_{t-3} + \ldots + \lambda_1\phi_p X_{t-p} + \lambda_1\eta + \lambda_1\varepsilon_t \leq h'$$

$$\frac{0 - (1-\lambda_1+\lambda_2)u - (\lambda_1\phi_1-\lambda_2)X_{t-1} - \lambda_1\phi_2 X_{t-2} - \lambda_1\phi_p X_{t-p}}{\lambda_1}$$

$$-\eta \leq \varepsilon_t \leq \frac{h' - (1-\lambda_1+\lambda_2)u - (\lambda_1\phi_1-\lambda_2)X_{t-1} - \lambda_1\phi_2 X_{t-2} - \lambda_1\phi_p X_{t-p}}{\lambda_1} - \eta$$

Let $L(u)$ denote the ARL on the EEWMA control chart. The function $L(u)$ can be derived by Fredholm integral equation of the second kind, $L(u)$ is defined as follows:

$$L(u) = 1 + \int L(E_1)f(\varepsilon_1)d\varepsilon_1 \tag{13}$$

Therefore, the function $L(u)$ is obtained as follows:

$$L(u) = 1 + \int_{\frac{0-(1-\lambda_1+\lambda_2)u-(\lambda_1\phi_1-\lambda_2)X_{t-1}-\lambda_1\phi_2X_{t-2}-\lambda_1\phi_pX_{t-p}}{\lambda_1}-\eta}^{\frac{h'-(1-\lambda_1+\lambda_2)u-(\lambda_1\phi_1-\lambda_2)X_{t-1}-\lambda_1\phi_2X_{t-2}-\lambda_1\phi_pX_{t-p}}{\lambda_1}-\eta}$$

$$L((1-\lambda_1+\lambda_2)u + (\lambda_1\phi_1-\lambda_2)X_{t-1} + \lambda_1\phi_2X_{t-2} + \ldots + \lambda_1\phi_pX_{t-p} + \lambda_1\eta + \lambda_1 y)f(y)dy$$

If $k = (1-\lambda_1+\lambda_2)u + (\lambda_1\phi_1-\lambda_2)X_{t-1} + \lambda_1\phi_2X_{t-2} + \ldots + \lambda_1\phi_pX_{t-p} + \lambda_1\eta + \lambda_1 y$ is defined for changing the integration variable, the function $L(u)$ is given by

$$L(u) = 1 + \frac{1}{\lambda_1}\int_0^{h'} L(k)f\left(\frac{k-(1-\lambda_1+\lambda_2)u-(\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1} - \phi_2X_{t-2} - \ldots - \phi_pX_{t-p} - \eta\right)dk \tag{14}$$

The $L(u)$ is Fredholm integral equation of the second kind. If $\varepsilon_t \sim Exp(\alpha)$, then

$$L(u) = 1 + \frac{1}{\lambda_1\alpha}\int_0^{h'} L(k)e^{-\frac{k}{\lambda_1\alpha}}e^{\frac{(1-\lambda_1+\lambda_2)u+(\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha}+\frac{\phi_2X_{t-2}+\ldots+\phi_pX_{t-p}+\eta}{\alpha}}dk$$

$$L(u) = 1 + \frac{e^{\frac{(1-\lambda_1+\lambda_2)u+(\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha}+\frac{\phi_2X_{t-2}+\ldots+\phi_pX_{t-p}+\eta}{\alpha}}}{\lambda_1\alpha}\int_0^{h'} L(k)e^{-\frac{k}{\lambda_1\alpha}}dk \tag{15}$$

When $G(u) = e^{\frac{(1-\lambda_1+\lambda_2)u+(\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha}+\frac{\phi_2X_{t-2}+\ldots+\phi_pX_{t-p}+\eta}{\alpha}}$, $F = \int_0^{h'} L(k)e^{-\frac{k}{\lambda_1\alpha}}dk$,

Consequently, $L(u) = 1 + \dfrac{G(u)}{\lambda_1\alpha}F.$ (16)

Consider the constant F and take turn $L(k)$ with Eq. (16), then

$$F = \int_0^{h'} L(k)e^{-\frac{k}{\lambda_1\alpha}}dk$$

$$= 1 + \int_0^{h'}\left[1 + \frac{G(k)}{\lambda_1\alpha}F\right]\cdot e^{-\frac{k}{\lambda_1\alpha}}dk$$

$$= \int_0^{h'} e^{-\frac{k}{\lambda_1\alpha}}dk + \int_0^{h'}\frac{G(k)}{\lambda_1\alpha}F\cdot e^{-\frac{k}{\lambda_1\alpha}}dk$$

$$= -\lambda_1\alpha(e^{-\frac{h'}{\lambda_1\alpha}} - 1) - \frac{F}{\lambda_1 - \lambda_2} \cdot e^{\frac{(\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha} + \frac{\phi_2X_{t-2}+\ldots+\phi_pX_{t-p}+\eta}{\alpha}} \cdot (e^{-\frac{(\lambda_1-\lambda_2)h'}{\lambda_1\alpha}} - 1)$$

$$F = \frac{-\lambda_1\alpha(e^{-\frac{h'}{\lambda_1\alpha}} - 1)}{1 + \frac{1}{\lambda_1 - \lambda_2} \cdot e^{\frac{(\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha} + \frac{\phi_2X_{t-2}+\ldots+\phi_pX_{t-p}+\eta}{\alpha}} \cdot (e^{-\frac{(\lambda_1-\lambda_2)h'}{\lambda_1\alpha}} - 1)} \tag{17}$$

Finally, substituting constant F form Eq. (17) into Eq. (16), then $L(u)$ can be written as

$$L(u) = 1 - \frac{(\lambda_1 - \lambda_2)e^{\frac{(1-\lambda_1+\lambda_2)u}{\lambda_1\alpha}} \cdot (e^{-\frac{h'}{\lambda_1\alpha}} - 1)}{(\lambda_1 - \lambda_2)e^{-\left\{\frac{(\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha} + \frac{\phi_2X_{t-2}+\ldots+\phi_pX_{t-p}+\eta}{\alpha}\right\}} + (e^{-\frac{(\lambda_1-\lambda_2)h'}{\lambda_1\alpha}} - 1)}. \tag{18}$$

The process is "in-control" with the exponential parameter $\alpha = \alpha_0$, the explicit formula of the ARL_0 for AR(p) process on the EEWMA control chart can be written as follows:

$$ARL_0 = 1 - \frac{(\lambda_1 - \lambda_2)e^{\frac{(1-\lambda_1+\lambda_2)u}{\lambda_1\alpha_0}} \cdot (e^{-\frac{h'}{\lambda_1\alpha_0}} - 1)}{(\lambda_1 - \lambda_2)e^{-\left\{\frac{(\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha_0} + \frac{\phi_2X_{t-2}+\ldots+\phi_pX_{t-p}+\eta}{\alpha_0}\right\}} + (e^{-\frac{(\lambda_1-\lambda_2)h'}{\lambda_1\alpha_0}} - 1)}. \tag{19}$$

Meanwhile, the process is "out-of-control" with the exponential parameter $\alpha = \alpha_1$ and then $\alpha_1 = (1 + \delta)\alpha_0$, where $\alpha_1 > \alpha_0$ and δ is the shift size, the explicit formula of ARL_1 for AR(p) process on the EEWMA control chart can be written as follows:

$$ARL_1 = 1 - \frac{(\lambda_1 - \lambda_2)e^{\frac{(1-\lambda_1+\lambda_2)u}{\lambda_1\alpha_1}} \cdot (e^{-\frac{h'}{\lambda_1\alpha_1}} - 1)}{(\lambda_1 - \lambda_2)e^{-\left\{\frac{(\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha_1} + \frac{\phi_2X_{t-2}+\ldots+\phi_pX_{t-p}+\eta}{\alpha_1}\right\}} + (e^{-\frac{(\lambda_1-\lambda_2)h'}{\lambda_1\alpha_1}} - 1)}. \tag{20}$$

while $(-1 \le \phi \le 1)$ is the autoregressive coefficient, $(0 < \lambda_1 \le 1)$, $(0 \le \lambda_2 < \lambda_1)$ are the smoothing parameters, the initial value $E_0 = u$, and $X_{t-1}, X_{t-2}, \ldots, X_{t-p}$ and h' is the upper control limit.

4 Numerical Integral Equation Method of *ARL* on the EEWMA Control Chart for AR(p) Processes

The NIE method is used to solve the *ARL* for the AR(p) process on the EEWMA control chart in Eq. (14). The *ARL* solution or $\tilde{L}(u)$ is approximated with the m linear equation systems over the interval $[0, h']$. A quadrature rule is used to approximate the integral by a finite sum of areas of rectangles with base h'/m and heights chosen as the values of $f(a_j)$ at the midpoints of intervals of length beginning at zero with a set of constant weights $w_j = \frac{h'}{m}$; $j = 1, 2, \ldots, m$ and $a_j = \frac{h'}{m}\left(j - \frac{1}{2}\right)$(see [24]).

Therefore, the approximating NIE method for the *ARL* on the EEWMA control chart is evaluated as follows:

$$\int_0^{h'} L(k)f(k)dk \approx \sum_{j=1}^{m} w_j f(a_j) \tag{21}$$

The system of m linear equation is showed as:

$L_{m\times1} = 1_{m\times1} + R_{m\times m}L_{m\times1}$ or $(I_m - R_{m\times m})L_{m\times1} = 1_{m\times1}$ or $L_{m\times1} = (I_m - R_{m\times m})^{-1}1_{m\times1}$

$L_{m\times1} = (I_m - R_{m\times m})^{-1}1_{m\times1}$ where $L_{m\times1} = [\tilde{L}(a_1), \tilde{L}(a_2), \ldots, \tilde{L}(a_m)]^T$, $I_m = diag(1, 1, \ldots, 1)$ and $1_{m\times1} = [1, 1, \ldots, 1]^T$.

Let $R_{m\times m}$ be a matrix, the definition of the m to m^{th} element of the matrix R is given by

$$[R_{ij}] \approx \frac{1}{\lambda_1} w_j f\left(\frac{a_j - (1-\lambda_1+\lambda_2)a_i - (\lambda_1\phi_1 - \lambda_2)X_{t-1}}{\lambda_1} - \phi_2 X_{t-2} - \ldots - \phi_p X_{t-p} - \eta\right)$$

Finally, the numerical approximation for the function $\tilde{L}(u)$ is as follows:

$$\tilde{L}(u) = 1 + \frac{1}{\lambda_1}\sum_{j=1}^{m} w_j L(a_j) f\left(\frac{a_j - (1-\lambda_1+\lambda_2)u - (\lambda_1\phi_1 - \lambda_2)X_{t-1}}{\lambda_1} - \phi_2 X_{t-2} - \ldots - \phi_p X_{t-p} - \eta\right) \quad (22)$$

5 Existence and Uniqueness of *ARL*

The solution of *ARL* shows that there uniquely exists the integral equation for explicit formulas by the Banach's Fixed-point Theorem. In this study, let T be an operation in the class of all continuous functions defined by

$$T(L(u)) = 1 + \frac{1}{\lambda_1}\int_0^{h'} L(k) f\left(\frac{k - (1-\lambda_1+\lambda_2)u - (\lambda_1\phi_1 - \lambda_2)X_{t-1}}{\lambda_1} - \phi_2 X_{t-2} - \ldots - \phi_p X_{t-p} - \eta\right) dk \quad (23)$$

According to Banach's Fixed-point Theorem, if an operator T is a contraction, and then the fixed-point equation $T(L(u)) = L(u)$ has a unique solution. To show that Eq. (23) exists and has a unique solution, theorem can be used as follows below.

Theorem 1 Banach's Fixed-point Theorem: Let (X, d) be a complete metric space and $T: X \to X$ be a contraction mapping with contraction constant $0 \le r < 1$ such that $\|T(L_1) - T(L_2)\| \le r\|L_1 - L_2\|, \forall L_1, L_2 \in X$. Then there exists a unique $L(\cdot) \in X$ such that $T(L(u)) = L(u)$, i.e., a unique fixed-point in X.

Proof: Let T defined in Eq. (23) is a contraction mapping for $L_1, L_2 \in G[0, h']$, such that $\|T(L_1) - T(L_2)\| \le r\|L_1 - L_2\|, \forall L_1, L_2 \in G[0, h']$ with $0 \le r < 1$ under the norm $\|L\|_\infty = \sup_{u\in[0,h']} |L(u)|$, so

$$\|T(L_1) - T(L_2)\|_\infty = \sup_{u\in[0,h']}\left|\frac{1}{\lambda_1\alpha} e^{\frac{(1-\lambda_1+\lambda_2)u + (\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha} + \frac{\phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \eta}{\alpha}} \int_0^{h'} (L_1(k) - L_2(k)) e^{-\frac{h'}{\lambda_1\alpha}} dk\right|$$

$$\le \sup_{u\in[0,h']}\left| \|L_1 - L_2\| \frac{1}{\lambda_1\alpha} e^{\frac{(1-\lambda_1+\lambda_2)u + (\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha} + \frac{\phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \eta}{\alpha}} \cdot (-\lambda_1\alpha)(e^{-\frac{h'}{\lambda_1\alpha}} - 1)\right|$$

$$= \|L_1 - L_2\|_\infty \sup_{u\in[0,h']}\left| e^{\frac{(1-\lambda_1+\lambda_2)u + (\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha} + \frac{\phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \eta}{\alpha}}\right| \left|1 - e^{-\frac{h'}{\lambda_1\alpha}}\right|$$

$$\le r\|L_1 - L_2\|_\infty$$

where $r = \sup_{u\in[0,h']}\left| e^{\frac{(1-\lambda_1+\lambda_2)u + (\lambda_1\phi_1-\lambda_2)X_{t-1}}{\lambda_1\alpha} + \frac{\phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \eta}{\alpha}}\right| \left|1 - e^{-\frac{h'}{\lambda_1\alpha}}\right|; \; 0 \le r < 1.$

6 Numerical Results

The absolute percentage relative error (APRE) to measure the accuracy of the *ARL* is defined as

$$\mathrm{APRE}(\%) = \frac{|L(u) - \tilde{L}(u)|}{L(u)} \times 100 \tag{24}$$

where $L(u)$ is the explicit formulas of the *ARL* on the EEWMA control chart for AR(p) process shows that Eq. (18) , which ARL_0 and ARL_1 are Eqs. (19) and (20), respectively, and $\tilde{L}(u)$ in Eq. (22) is the NIE method of the *ARL* using the Gauss-Legendre quadrature rule on the EEWMA control chart for AR(p) with the number of division points $m = 500$ nodes. The numerical results were computed by MATHEMATICA. The initial parameter values are studied at ARL_0 = 370 on the EEWMA control chart for AR(p) process, referred to as AR(2) and AR(3) processes with exponential white noise and given $\lambda_1 = 0.05$, 0.10, $\lambda_2 = 0.01$, 0.02, 0.03. The 'in-control' process had parameter value as $\alpha = \alpha_0$ with shift size ($\delta = 0$). On the other hand, the 'out-of-control' process was presented with parameter values as $\alpha_1 = (1 + \delta)\alpha_0$ with shift sizes (δ) equals 0.001, 0.003, 0.005, 0.010, 0.030, 0.050, 0.100, 0.500 and 1.000 were determined. Furthermore, the coefficient parameters of the process $\phi_1 = 0.2$, $\phi_2 = 0.2, -0.2$ were used for the AR(2) process, and $\phi_1 = \phi_2 = 0.2$, $\phi_3 = 0.2, -0.2$ were used for the AR(3) process. In addition, the speed test results were computed by the CPU time (PC System: windows10, 64-bit, Intel® Core™ i5-8250U 1.60 GHz 1.80 GHz, RAM 4 GB) in seconds.

In Tabs. 1 and 2, the *ARL* results by using the explicit formula (Eqs. (19) and (20)) and the NIE method (Eq. (22) then the absolute percentage relative error named APRE(%) show that Eq. (24). It showed that the *ARL* values derived from the explicit formulas give results close to those from the NIE method both AR(2) and AR(3) process. AR(2) process, Tab. 1 showed that the analytical results agree with NIE approximations with APRE(%) less than 0.000239% and CPU time of approximately 2.7–3.5 s whereas the CPU time of the explicit formulas is not much. AR(3) process in Tab. 2 showed that the analytical results agree with NIE approximations with APRE(%) less than 0.000216% and CPU time of approximately 2.8–3.5 s whereas the CPU time of the explicit formulas is not much. The entries inside the parentheses are the CPU time in seconds.

Table 1: Comparing *ARL* values on the EEWMA control chart for the AR(2) process using explicit formulas against the NIE method given $\lambda_1 = 0.05$, 0.10, $\lambda_2 = 0.01$, $\eta = 0$ for ARL_0 = 370

λ_1	Shift size (δ)	$\phi_1 = \phi_2 = 0.2$*			$\phi_1 = 0.2$, $\phi_2 = -0.2$**		
		Explicit	NIE (CPU time)	APRE (%)	Explicit	NIE (CPU time)	APRE (%)
0.05	0.000	370.321304	370.321192 (2.891)	0.000030	370.388734	370.388600 (3.266)	0.000036
	0.001	234.777706	234.777647 (3.047)	0.000025	239.110276	239.110204 (3.001)	0.000030
	0.003	135.885390	135.885362 (3.406)	0.000021	140.253416	140.253381 (3.079)	0.000025
	0.005	95.8318800	95.8318615 (2.906)	0.000019	99.4478549	99.4478320 (3.032)	0.000023
	0.010	55.4896135	55.4896039 (3.047)	0.000017	57.8903579	57.8903459 (2.875)	0.000021
	0.030	21.2886283	21.2886251 (3.125)	0.000015	22.2934984	22.2934945 (2.907)	0.000018
	0.050	13.5344720	13.5344701 (2.921)	0.000014	14.1755969	14.1755945 (3.187)	0.000016
	0.100	7.48364935	7.48364848 (3.172)	0.000012	7.82854173	7.82854066 (3.078)	0.000014
	0.500	2.45093356	2.45093345 (2.999)	0.000004	2.53934226	2.53934213 (2.719)	0.000005
	1.000	1.77981293	1.77981290 (3.016)	0.000002	1.83156186	1.83156182 (3.016)	0.000002

(Continued)

Table 1 (continued)

λ_1	Shift size (δ)	$\phi_1 = \phi_2 = 0.2$*			$\phi_1 = 0.2$, $\phi_2 = -0.2$**		
		Explicit	NIE (CPU time)	APRE (%)	Explicit	NIE (CPU time)	APRE (%)
0.10	0.000	370.042850	370.042130 (2.812)	0.000195	370.069839	370.068954 (3.235)	0.000239
	0.001	258.305964	258.305590 (2.844)	0.000145	265.398874	265.398393 (3.157)	0.000181
	0.003	161.332490	161.332326 (3.093)	0.000101	169.772828	169.772610 (2.967)	0.000128
	0.005	117.496757	117.496661 (2.890)	0.000082	124.992900	124.992771 (3.203)	0.000103
	0.010	70.2760299	70.2759877 (3.062)	0.000060	75.6205876	75.6205304 (3.063)	0.000076
	0.030	27.5853972	27.5853864 (3.203)	0.000039	29.9372209	29.9372064 (3.204)	0.000048
	0.050	17.5425526	17.5425468 (3.000)	0.000033	19.0425702	19.0425625 (2.891)	0.000041
	0.100	9.60903193	9.60902943 (2.985)	0.000026	10.3968364	10.3968331 (2.875)	0.000032
	0.500	2.94823486	2.94823458 (3.172)	0.000009	3.11870875	3.11870839 (3.171)	0.000012
	1.000	2.05578182	2.05578173 (3.109)	0.000004	2.14628399	2.14628388 (3.156)	0.000005

Notes: *$h' = 0.0488991$ for $\lambda_1 = 0.05$ and $h' = 0.1376787$ for $\lambda_1 = 0.10$. **$h' = 0.0530625$ for $\lambda_1 = 0.05$ and $h' = 0.1499641$ for $\lambda_1 = 0.10$.

Table 2: Comparing *ARL* values on the EEWMA control chart for the AR(3) process using explicit formulas against the NIE method given $\lambda_1 = 0.05, 0.10$, $\lambda_2 = 0.01$, $\eta = 0$ for $ARL_0 = 370$

λ_1	Shift size (δ)	$\phi_1 = \phi_2 = \phi_3 = 0.2$*			$\phi_1 = \phi_2 = 0.2$, $\phi_3 = -0.2$**		
		Explicit	NIE (CPU time)	APRE (%)	Explicit	NIE (CPU time)	APRE (%)
0.05	0.000	370.152690	370.152587 (3.250)	0.000028	370.369025	370.368902 (3.187)	0.000033
	0.001	232.684141	232.684088 (3.063)	0.000023	236.904912	236.904847 (3.298)	0.000027
	0.003	133.850860	133.850835 (2.953)	0.000019	138.011528	138.011496 (3.219)	0.000023
	0.005	94.1665824	94.1665658 (3.095)	0.000018	97.5864564	97.5864358 (3.126)	0.000021
	0.010	54.3952320	54.3952233 (3.063)	0.000016	56.6510616	56.6510508 (3.079)	0.000019
	0.030	20.8336361	20.8336333 (3.125)	0.000014	21.7738180	21.7738144 (3.015)	0.000016
	0.050	13.2442206	13.2442189 (2.907)	0.000013	13.8440442	13.8440422 (3.203)	0.000015
	0.100	7.32712291	7.32712213 (3.030)	0.000011	7.65034375	7.65034278 (2.827)	0.000013
	0.500	2.41014802	2.41014793 (3.125)	0.000004	2.49392222	2.49392210 (3.234)	0.000005
	1.000	1.75574743	1.75574740 (3.093)	0.000002	1.80505114	1.80505111 (3.484)	0.000002
0.10	0.000	370.144195	370.143544 (3.265)	0.000176	370.111076	370.110277 (3.015)	0.000216
	0.001	255.145929	255.145598 (3.172)	0.000130	261.759999	261.759576 (3.093)	0.000162
	0.003	157.658228	157.658085 (3.140)	0.000090	165.370557	165.370369 (3.015)	0.000114
	0.005	114.276687	114.276603 (3.171)	0.000073	121.058937	121.058826 (3.328)	0.000092
	0.010	68.0136145	68.0135780 (3.265)	0.000054	72.7984472	72.7983982 (3.141)	0.000067
	0.030	26.6006234	26.6006140 (3.281)	0.000035	28.6897880	28.6897755 (3.063)	0.000044
	0.050	16.9144058	16.9144007 (3.141)	0.000030	18.2468150	18.2468084 (3.140)	0.000037
	0.100	9.27732544	9.27732323 (3.312)	0.000024	9.97959567	9.97959280 (3.218)	0.000029
	0.500	370.144195	370.143544 (3.265)	0.000176	370.111076	370.110277 (3.015)	0.000216
	1.000	255.145929	255.145598 (3.172)	0.000130	261.759999	261.759576 (3.093)	0.000162

Notes: *$h' = 0.0469439$ for $\lambda_1 = 0.05$ and $h' = 0.1319420$ for $\lambda_1 = 0.10$. **$h' = 0.0509374$ for $\lambda_1 = 0.05$ and $h' = 0.1436815$ for $\lambda_1 = 0.10$.

7 Performance Comparing the ARL Results

For Tabs. 3 and 4, the EEWMA control chart is compared for various $\lambda_1 = 0.05$, 0.10 and $\lambda_2 = 0.01$, 0.02, 0.03 at $ARL_0 = 370$, $\eta = 0$, $\phi_1 = \phi_2 = 0.2$ (as an AR(2) process) and $\phi_1 = \phi_2 = \phi_3 = 0.2$ (as an AR(3) process). The ARL values are indicated that the ARL_1 on the EEWMA ($\lambda_2 = 0.03$) or EEWMA_03 control chart was reduced more sensitively than on the EEWMA with either $\lambda_2 = 0.01$ (EEWMA_01) or $\lambda_2 = 0.02$ (EEWMA_02) for all magnitudes of changes both AR(2) and AR(3) processes. Moreover, the ARL_1 on the EEWMA control chart with $\lambda_1 = 0.05$ was reduced more sensitively than on the EEWMA control chart with $\lambda_1 = 0.10$ for all situations running AR(2) and AR(3) processes. The exponential smoothing parameter 0.05 is recommended. Tab. 5 showed that the comparison of ARL values for the AR(2) process on CUSUM, EWMA and EEWMA control charts, the results presented that the ARL_1 on the EEWMA control chart with $\lambda_2 = 0.03$ was reduced the ARL_1 more than the CUSUM, EWMA, EEWMA with either $\lambda_2 = 0.01$ or $\lambda_2 = 0.02$ control charts for all shift sizes and all exponential smoothing parameter values. Similarly, the ARL results of AR(3) process in Tab. 6, the results presented that the ARL_1 on the EEWMA control chart with $\lambda_2 = 0.03$ was reduced the ARL_1 more than the CUSUM, EWMA, EEWMA with either $\lambda_2 = 0.01$ or $\lambda_2 = 0.02$ control charts for all shift sizes and all exponential smoothing parameter values as same as the ARL results of AR(2) process. Therefore, the performance of the EEWMA control chart with $\lambda_2 = 0.03$ is more efficient than the performance of the CUSUM, EWMA, EEWMA with either $\lambda_2 = 0.01$ or $\lambda_2 = 0.02$ control charts for all situations except when the large shift sizes ($\delta \geq 0.5$), the EEWMA control chart with $\lambda_2 = 0.03$ was reduced as well as the EWMA, EEWMA with either $\lambda_2 = 0.01$ or $\lambda_2 = 0.02$ control charts.

Table 3: Comparing ARL on the EEWMA control chart for AR(2) process with various λ when given $\eta = 0$, $\phi_1 = \phi_2 = 0.2$ for $ARL_0 = 370$

Shift size (δ)	$\lambda_1 = 0.05$			$\lambda_1 = 0.10$		
	$\lambda_2 = 0.01$ $h' = 0.0488991$	$\lambda_2 = 0.02$ $h' = 0.0265188$	$\lambda_2 = 0.03$ $h' = 0.0144770$	$\lambda_2 = 0.01$ $h' = 0.1376787$	$\lambda_2 = 0.02$ $h' = 0.0997870$	$\lambda_2 = 0.03$ $h' = 0.0728639$
0.000	370.3213	370.6220	370.1861	370.0429	370.1648	370.2258
0.001	234.7777	209.5365	190.2497	258.3060	237.4070	222.4548
0.003	135.8854	112.4374	96.80430	161.3325	138.5722	124.0426
0.005	95.83188	77.04483	65.11853	117.4968	98.05656	86.21334
0.010	55.48961	43.40275	36.07534	70.27603	56.96144	49.21775
0.030	21.28863	16.33608	13.43369	27.58540	21.89414	18.68865
0.050	13.53447	10.37563	8.532907	17.54255	13.91433	11.87089
0.100	7.483649	5.770465	4.769831	9.609032	7.680046	6.578097
0.500	2.450934	1.989591	1.716927	2.948235	2.491119	2.203938
1.000	1.779813	1.504181	1.342614	2.055782	1.799900	1.630912

Table 4: Comparing ARL on the EEWMA control chart for AR(3) process with various λ when given $\eta = 0$, $\phi_1 = \phi_2 = \phi_3 = 0.2$ for $ARL_0 = 370$

Shift size (δ)	$\lambda_1 = 0.05$			$\lambda_1 = 0.10$		
	$\lambda_2 = 0.01$ $h' = 0.0469439$	$\lambda_2 = 0.02$ $h' = 0.0254704$	$\lambda_2 = 0.03$ $h' = 0.0139076$	$\lambda_2 = 0.01$ $h' = 0.1319420$	$\lambda_2 = 0.02$ $h' = 0.0957177$	$\lambda_2 = 0.03$ $h' = 0.0699340$
0.000	370.1527	370.0058	370.2755	370.1442	370.0809	370.0634
0.001	232.6841	207.9606	189.1197	255.1459	235.2129	220.7210

(Continued)

Table 4 (continued)

Shift size (δ)	$\lambda_1 = 0.05$			$\lambda_1 = 0.10$		
	$\lambda_2 = 0.01$ $h' = 0.0469439$	$\lambda_2 = 0.02$ $h' = 0.0254704$	$\lambda_2 = 0.03$ $h' = 0.0139076$	$\lambda_2 = 0.01$ $h' = 0.1319420$	$\lambda_2 = 0.02$ $h' = 0.0957177$	$\lambda_2 = 0.03$ $h' = 0.0699340$
0.003	133.8509	111.2001	95.92305	157.6582	136.3804	122.4781
0.005	94.16658	76.10072	64.45710	114.2767	96.24664	84.96887
0.010	54.39523	42.82084	35.67538	68.01361	55.76243	48.42211
0.030	20.83364	16.10460	13.27734	26.60062	21.39311	18.36408
0.050	13.24422	10.22846	8.433957	16.91441	13.59477	11.66399
0.100	7.327123	5.690395	4.716288	9.277325	7.508189	6.465857
0.500	2.410148	1.967488	1.702598	2.873554	2.447154	2.173623
1.000	1.755747	1.490908	1.334281	2.015192	1.774239	1.612790

Table 5: Comparing *ARL* values for the AR(2) process on CUSUM, EWMA and EEWMA control charts when given $\lambda_1 = 0.05$, $\lambda_2 = 0.01, 0.02, 0.03$, $\eta = 0$ for $ARL_0 = 370$

ϕ_1	ϕ_2	Shift size (δ)	CUSUM ($a = 2$)	EWMA ($\lambda_2 = 0$)	EEWMA		
					EEWMA_01 ($\lambda_2 = 0.01$)	EEWMA_02 ($\lambda_2 = 0.02$)	EEWMA_03 ($\lambda_2 = 0.03$)
0.2	0.2		$b = 3.579$	$h = 0.0914794$	$h' = 0.0488991$	$h' = 0.0265188$	$h' = 0.0144770$
		0.000	370.1644	370.0520	370.3213	370.6220	370.1861
		0.001	367.7875	284.1654	234.7777	209.5365	190.2497
		0.003	363.0933	194.2544	135.8854	112.4374	96.80430
		0.005	358.4773	147.7005	95.83188	77.04483	65.11853
		0.010	347.2692	92.59445	55.48961	43.40275	36.07534
		0.030	306.7755	37.68188	21.28863	16.33608	13.43369
		0.050	272.2869	23.98603	13.53447	10.37563	8.532907
		0.100	205.9789	12.95413	7.483649	5.770465	4.769831
		0.500	43.83369	3.616823	2.450934	1.989591	1.716927
		1.000	15.80483	2.397395	1.779813	1.504181	1.342614
	−0.2		$b = 3.471$	$h = 0.0994968$	$h' = 0.0530625$	$h' = 0.0287478$	$h' = 0.0156870$
		0.000	370.2999	370.2746	370.3887	370.5370	370.6751
		0.001	367.9426	296.0073	239.1103	212.3404	192.7199
		0.003	363.2867	211.3611	140.2534	114.8813	98.65484
		0.005	358.7077	164.4419	99.44785	78.95453	66.49796
		0.010	347.5872	105.9012	57.89036	44.60220	36.90566
		0.030	307.3817	44.05912	22.29350	16.81948	13.75746
		0.050	273.0998	28.06060	14.17560	10.68373	8.737752
		0.100	207.0696	15.01612	7.828542	5.938363	4.880706
		0.500	44.50081	3.955546	2.539342	2.036011	1.746727
		1.000	16.06022	2.550454	1.831562	1.532108	1.360015

Table 6: Comparing *ARL* values for the AR(3) process on CUSUM, EWMA and EEWMA control charts when given $\lambda_1 = 0.05$, $\lambda_2 = 0.01, 0.02, 0.03$, $\eta = 0$ for $ARL_0 = 370$

$\phi_1 = \phi_2$	ϕ_3	Shift size (δ)	CUSUM ($a = 2$)	EWMA ($\lambda_2 = 0$)	EEWMA		
					EEWMA_01 ($\lambda_2 = 0.01$)	EEWMA_02 ($\lambda_2 = 0.02$)	EEWMA_03 ($\lambda_2 = 0.03$)
0.2	0.2		$b = 3.635$	$h = 0.0877273$	$h' = 0.0469439$	$h' = 0.0254704$	$h' = 0.0139076$
		0.000	370.1955	370.1327	370.1527	370.0058	370.2755
		0.001	367.8070	279.1179	232.6841	207.9606	189.1197
		0.003	363.0902	187.3070	133.8509	111.2001	95.92305
		0.005	358.4522	141.0997	94.16658	76.10072	64.45710
		0.010	347.1919	87.53147	54.39523	42.82084	35.67538
		0.030	306.5263	35.32701	20.83364	16.10460	13.27734
		0.050	271.9131	22.48317	13.24422	10.22846	8.433957
		0.100	205.4330	12.18450	7.327123	5.690395	4.716288
		0.500	43.48171	3.478614	2.410148	1.967488	1.702598
		1.000	15.67118	2.331644	1.755747	1.490908	1.334281
0.2	−0.2		$b = 3.524$	$h = 0.0953998$	$h' = 0.0509374$	$h' = 0.0276106$	$h' = 0.0150698$
		0.000	370.0741	370.0517	370.3690	370.4431	370.2123
		0.001	367.7086	289.7383	236.9049	210.8827	191.4205
		0.003	363.0365	202.1826	138.0115	113.6314	97.70710
		0.005	358.4420	155.3759	97.58646	77.98044	65.79521
		0.010	347.2848	98.61321	56.65106	43.99142	36.48428
		0.030	306.9602	40.53274	21.77382	16.57357	13.59355
		0.050	272.5951	25.80669	13.84404	10.52704	8.634091
		0.100	206.4616	13.87987	7.650344	5.852996	4.824608
		0.500	44.16430	3.774240	2.493922	2.012414	1.731631
		1.000	15.93231	2.469918	1.805051	1.517905	1.351188

8 Application to Real Data

The performance of the *ARL* constructed using explicit formulas for the EEWMA control chart for $\lambda_1 = 0.05$ and various $\lambda_2 = 0.01, 0.02, 0.03$ was compared with those of CUSUM and EWMA ($\lambda_2 = 0$) control charts using data on new COVID-19 cases in Thailand and in Vietnam from March 30th to July 7th, 2021. Lately, Areepong et al. [27] investigated monitoring COVID-19 outbreaks in Thailand, Singapore, Vietnam, and Hong Kong using by the EWMA control chart. There are 100 observations of daily. This data is a stationary time series. By looking at the autocorrelation function (ACF) and partial autocorrelation function (PACF). Both countries are the worst affected in Southeast Asia. For $\lambda_1 = 0.05$ and $\lambda_2 = 0.01, 0.02$, or 0.03, the settings for the Thailand dataset are that it is an AR(2) process with ARL_0= 370; the significance of the mean and standard deviation are 2.774663 and 1.663941, respectively; process coefficients $\phi_1 = 0.343110$, $\phi_2 = 0.527991$; the error is exponential white noise ($\alpha_0 = 0.665927$) whereas the settings for the Vietnam dataset are it is an AR(3) process with ARL_0 = 370; the significance of the mean and standard deviation are 0.214103 and 0.259871, respectively; process coefficients $\phi_1 = 0.269717$, $\phi_2 = 0.572229$, $\phi_3 = 0.219039$; the error is exponential white noise ($\alpha_0 = 0.129397$).

The results for the *ARL* of CUSUM, EWMA and EEWMA with various $\lambda_2 = 0.01$, 0.02, 0.03 control charts on AR(2) process for the Thailand dataset in Tab. 7 are agreement to the simulation results in Tab. 5. Similarly, These control charts on AR(3) process for the Vietnam dataset in Tab. 8 are agreement to the simulation results in Tab. 6. ARL_1 on an EEWMA control chart with $\lambda_2 = 0.03$ was reduced more sensitively than CUSUM, EWMA, EEWMA with $\lambda_2 = 0.01$ and EEWMA with $\lambda_2 = 0.02$ control charts for all magnitudes of changes except when the large shift sizes ($\delta \geq 0.5$), the EEWMA control chart with $\lambda_2 = 0.03$ was reduced as well as the EWMA, EEWMA with $\lambda_2 = 0.01$ and EEWMA with $\lambda_2 = 0.02$ control charts both AR(2) and AR(3) processes. The results indicate that the performances of the control charts were, in ascending order, EEWMA for $\lambda_2 = 0.03$, EEWMA for $\lambda_2 = 0.02$, EEWMA for $\lambda_2 = 0.01$, EWMA, and CUSUM, as illustrated in Figs. 1 and 2.

Table 7: Comparing *ARL* values for the AR(2) process on CUSUM, EWMA and EEWMA control charts with COVID-19 data in Thailand when given $ARL_0 = 370$, $\alpha_0 = 0.665927$, $\eta = 3.445847$, $\phi_1 = 0.343110$, $\phi_2 = 0.527991$, $\lambda_1 = 0.05$ and $\lambda_2 = 0.01$, 0.02, 0.03

Shift size (δ)	CUSUM ($a = 1.5$)	EWMA ($\lambda_2 = 0$)	EEWMA		
			EEWMA_01 ($\lambda_2 = 0.01$)	EEWMA_02 ($\lambda_2 = 0.02$)	EEWMA_03 ($\lambda_2 = 0.03$)
	$b = 2.8223$	$h = 0.0307285$	$h' = 0.00149023$	$h' = 0.0000738797$	$h' = 0.00000366589$
0.000	370.0535	370.0669	370.0996	370.1763	370.2019
0.001	367.5050	232.2021	145.5442	102.3868	78.57510
0.003	362.4756	133.3967	66.08133	42.17770	30.84456
0.005	357.5350	93.79691	42.92652	26.73477	19.36131
0.010	345.5633	54.15912	23.10964	14.16717	10.22973
0.030	302.5634	20.73229	8.508357	5.271144	3.884294
0.050	266.2953	13.17821	5.444146	3.446960	2.600501
0.100	197.6295	7.289428	3.120606	2.083703	1.655609
0.500	38.98113	2.397793	1.304992	1.094978	1.032944
1.000	14.08608	1.747635	1.116857	1.024074	1.005267

As mentioned above, the EEWMA ($\lambda_2 = 0.03$) and EWMA ($\lambda_2 = 0$) control charts are plotted by calculating E_t and Z_t for the two datasets for $\lambda_1 = 0.05$. The detecting the process with real data of the new cases COVID-19 data in Thailand (as an AR(2) process) and Vietnam (as an AR(3) process) are shown in Figs. 3 and 4, respectively. In Fig. 3, the *ARL* of the AR(2) process for the Thailand COVID-19 data on the EEWMA control chart for $\lambda_2 = 0.03$ indicates that the process was signaled as out-of-control at the 6th observation whereas on the EWMA control chart, it was detected at the 11th observation. In Fig. 4 for the EEWMA ($\lambda_2 = 0.03$) control chart, the *ARL* of AR(3) process for the Vietnam COVID-19 data on the EEWMA control chart for $\lambda_2 = 0.03$ was signaled as out-of-control process at the 9th observation whereas on the EWMA control chart, it was detected as out-of-control at the 20th observation. Therefore, the EEWMA control chart can detect shift more quickly than the EWMA control chart.

Table 8: Comparing *ARL* values for the AR(3) process on CUSUM, EWMA and EEWMA control charts with COVID-19 data in Vietnam when given ARL_0 = 370 $\alpha_0 = 0.129397$, $\eta = 0$, $\phi_1 = 0.269717$, $\phi_2 = 0.572229$, $\phi_3 = 0.219039$, $\lambda_1 = 0.05$ and $\lambda_2 = 0.01, 0.02, 0.03$

Shift size (δ)	CUSUM ($a = 0.2$)	EWMA ($\lambda_2 = 0$)	EEWMA		
			EEWMA_01 ($\lambda_2 = 0.01$)	EEWMA_02 ($\lambda_2 = 0.02$)	EEWMA_03 ($\lambda_2 = 0.03$)
	$b = 0.7101$	$h = 0.0085715$	$h' = 0.0017763$	$h' = 0.000376829$	$h' = 0.0000802665$
0.000	370.0149	370.0735	370.3624	370.1065	370.0699
0.001	366.7198	253.6347	189.3055	151.1184	123.8997
0.003	360.2121	155.7430	95.87017	69.39629	53.37970
0.005	353.8458	112.5812	64.38790	45.22358	34.19629
0.010	338.5372	66.84116	35.63176	24.40944	18.23388
0.030	284.9166	26.10418	13.25846	8.989864	6.728492
0.050	241.4673	16.60412	8.421038	5.743410	4.342770
0.100	164.3741	9.121001	4.708184	3.278241	2.545492
0.500	24.28932	2.848698	1.699288	1.341181	1.181372
1.000	10.04217	2.005967	1.332079	1.135106	1.058714

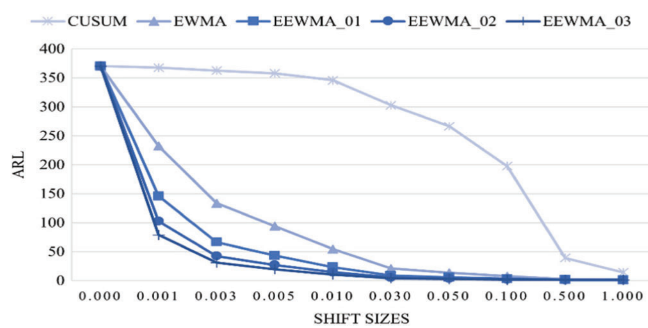

Figure 1: *ARL* values of the AR(2) process on CUSUM, EWMA and EEWMA control charts with new cases COVID-19 data in Thailand when given ARL_0 = 370

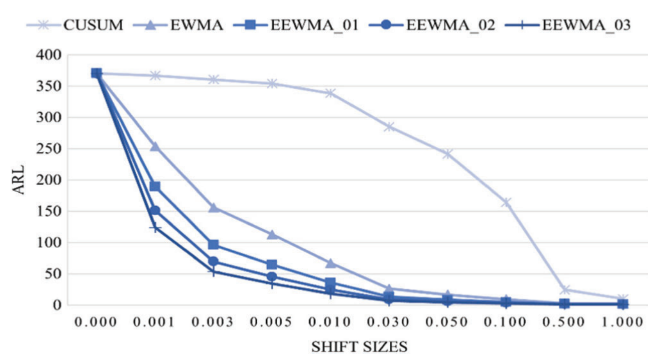

Figure 2: *ARL* values of the AR(3) process on CUSUM, EWMA and EEWMA control charts with new cases COVID-19 data in Vietnam when given ARL_0 = 370

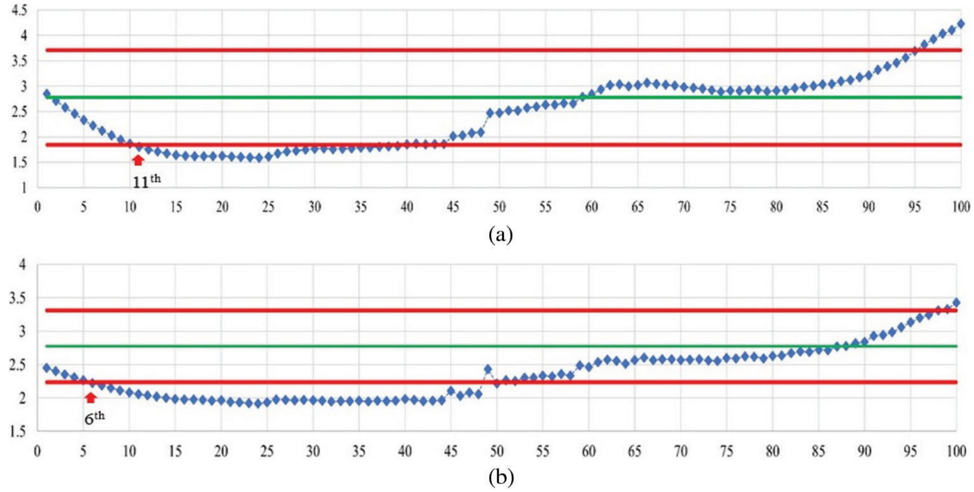

Figure 3: The detecting the AR(2) process with the Thailand COVID-19 data when given ARL_0 = 370; (a) EWMA control chart and (b) EEWMA control chart at $\lambda_2 = 0.03$

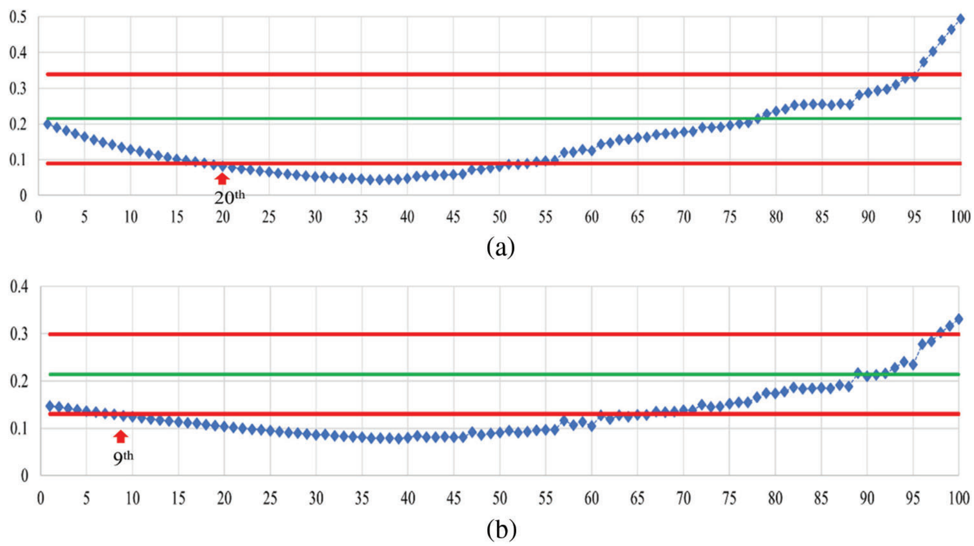

Figure 4: The detecting the AR(3) process with the Vietnam COVID-19 data when given ARL_0 = 370; (a) EWMA control chart and (b) EEWMA control chart at $\lambda_2 = 0.03$

9 Discussions and Conclusions

In the study, the performances of control charts were evaluated by using *ARL*. The explicit formulas comprise a good alternative to the NIE method for constructing the *ARL*. The analytical results agree with the NIE approximations for AR(2) and AR(3) processes with absolute percentage relative errors of less than 0.000239% and 0.000216%, respectively. The CPU time to calculate the ARL by using the NIE methods were approximately 2.7–3.5 and 2.8–3.5 s for the AR(2) and AR(3) processes, respectively, whereas they were almost instantaneous when using the explicit formulas. The performance comparison of the *ARL* using explicit formulas on the EEWMA with various λ performed better than on the CUSUM and EWMA control charts running AR(2) or AR(3) processes for most cases except for large shift sizes ($\delta \geq 0.5$) when even then, the EEWMA control chart with various λ performed as well as the EWMA control chart. The EEWMA control chart with $\lambda_2 = 0.03$ performed better than the EEWMA with either

$\lambda_2 = 0.01$ or $\lambda_2 = 0.02$, CUSUM, or EWMA control charts for most magnitudes of changes except for a large shift sizes ($\delta \geq 0.5$) when it performed at least as well as the others for AR(2) and AR(3) processes. Besides, an exponential smoothing parameter value of 0.05 is recommended. In addition, the simulation study, and the efficacy illustration with real data of new COVID-19 cases in Thailand and Vietnam provided similar results.

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