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Another View of Weakly Open Sets Via DNA Recombination

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Abstract: The generalized structure of deoxyribonucleic acid (DNA) is based on the rules of topological spaces. DNA recombination is one of the most important processes within DNA, as it is essential in the pharmaceutical industry as well as in gene therapy. In this paper, we are discussing the relationship between rough sets, nano topological spaces ($\Re\tau s$), nano Z open ($\Re Z0$) sets, and DNA recombination. We also created a new recombination mapping using the properties of the DNA recombination process. Further, by using the process of cutting and sticking of a sequence of genes, new topological structures are constructed and some of their properties and characterizations are investigated. Moreover, we study recombination operators in the statement "Sticky Ends". Furthermore, we use nano topological structures to prove the validity of the mathematical model of the recombination process and the extent to which the topological mathematical properties correspond to the biological properties. Finally, we use nano Z-open sets to study many topological characteristics of the neighborhood, closure, interior, limit points, frontier, border and exterior.

Keywords: DNA recombination; sticky ends; nano Z-open; operator

1 Introduction

Liellis Thivagar [1] invented the notion of nano topological spaces (short for $\Re\tau s$)), which define a subset of a universe using upper, lower approximations and a boundary region defined by an equivalence relation on it. Nano open (nano closed, nano-interior and nano-closure) sets (briefly, $\Re 0, \Re c, \Re int$ and $\Re cl$) as being defined by him. El-Maghrabi and Mubarki [2] defined Z-open sets in topological structures and investigated several of their features. The concept of nano Z-open sets plays a role in topological structures and their applications in domains such as mathematics, biology and other aspects of life. The goal of this research is to look at how mathematics is used in biological applications (DNA recombination) and prove the validity of the mathematical model of the recombination process using nano topology and $\Re Z0$. The extent to which the topological mathematical properties correspond to the biological properties. We also use nano Z-open sets to study many topological characteristics of the neighborhood, closure, interior, limit points, frontier, border and exterior.

Proposition 1.1. [3] If (U, R) is an approximation space and $X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$,
(ii) $L_R(\varphi) = U_R(\varphi) = \varphi$ and $L_R(U) = U_R(U) = U$,
(iii) $U_R\ (X \cup Y) = U_R(X) \cup U_R(Y))$,
(iv) $U_R\ (X \cap Y) \subseteq U_R(X) \cap U_R(Y))$,
(v) $L_R\ (X \cup Y) \supseteq L_R(X) \cup L_R(Y)$,
(vi) $L_R\ (X \cap Y) = L_R(X) \cap L_R(Y)$,
(vii) *If* $X \subseteq Y$ then $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$,
$(viii)$ $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$,
(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$,
(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 1.2 [1] Let $H \subseteq U$ and $(U, \tau_R(X))$ be a $\mathfrak{N}$ts. Then H is a nano regular open ($\mathfrak{N}ro$) *if* $H = \mathfrak{N}int(\mathfrak{N}cl(H))$.

Definition 1.3 [4] *If* $H \subseteq U$ *and* $(U, \tau_R(X))$ *be* a $\mathfrak{N}ts$. Then the nano θ-interior(resp. nano θ closure) of H is defined by $\mathfrak{N}\mathfrak{N}\ int_\theta(H) = \cup\{E : E \text{ is a } \mathfrak{N}\theta$ set and $\mathfrak{N}cl(E) \subseteq H\}$ *(resp.* $\mathfrak{N}\text{int}_\theta(\text{H}) = \cup\{x \in E : \mathfrak{N}\text{cl} \cap \text{H} \neq \varnothing, E \text{ is a } \mathfrak{N}\theta set\ , x \in E\}$.

Definition 1.4 [4] A subset H of U is said to be a nano θ open *(*$\mathfrak{N}\theta 0$*)(resp. nano* θ*-closed(*$\mathfrak{N}\theta$*))* set if *H= (resp.* H^c *is a nano* θ *open set).*

Definition 1.5 [5] *If* $H \subseteq U$ *and* $(U, \tau_R(X))$ *is* $\mathfrak{N}ts$, then the nano δ-interior(resp. nano δ *closure) of H is defined by* $\mathfrak{N}\text{int}_\delta(\text{H}) = \bigcup\{x \in E : \mathfrak{N}roset, E \subseteq H\}$ *(resp.*$\mathfrak{N}\text{cl}_\delta(\text{H}) = \cup\{x \in U : \mathfrak{N}\text{int}(\mathfrak{N}\text{cl}(\text{H})) \cap \text{H} \neq \varnothing, E \text{ is a } \mathfrak{N}o \text{ set}\ , x \in E\}$.

Definition 1.6 [5] A subset H of U is said to be a nano δ open *(*$\mathfrak{N}\delta o$*)(resp. nano* δ*-closed(*$\mathfrak{N}\delta c$*)) set if H=* $\mathfrak{N}\text{int}_\delta(\text{H})$ *(resp.* H^c *is a nano* δ *open set).*

Definition 1. 1 Let $K \subseteq U$ and $(U, \tau_R(X))$ be a $\mathfrak{N}$ts. Then K is said to be:

(i) nano δ-preopen [5] (briefly, $\mathfrak{N}\delta$P0)set if $K \subseteq \mathfrak{N}int(\mathfrak{N}\text{cl}_\delta(\text{K})$,
(ii) nano δ-semiopen [5] (briefly, $\mathfrak{N}\delta$S0)set if $K \subseteq \mathfrak{N}cl(\mathfrak{N}\text{int}_\delta(\text{K})$,
(iii) nano e-open [6] (briefly, $\mathfrak{N}$e0)set if $K \subseteq \mathfrak{N}int(\mathfrak{N}\text{cl}_\delta(\text{K}) \cup \mathfrak{N}cl(\mathfrak{N}\text{int}_\delta(\text{K})$,
(iv) nano θ-semiopen [6] (briefly, $\mathfrak{N}\theta SO$ set if $\mathfrak{N}cl(\mathfrak{N}\text{int}_\theta(\text{K})$,

1.1 DNA Recombination

DNA recombination is one of the most important processes within DNA, as it is essential in the pharmaceutical industry as well as in gene therapy. Bacteria can acquire new genes by incorporating environmental DNA into their genomes [7], and the recombination process may be used for gene reproduction or tissue culture, and defects in homologous recombination may lead to a gastric cancer mutation [8]. However, recombination is not accurate [9]. Since the adoption of computer programmes depends on mathematics and the description of operations before their implementation, making a simulation of any problem requires a mathematical model of the problem, so we made a mathematical model of the recombination process and verified the validity of the mathematical model using nano-topological structure and nano Z-open and we also made a conclusion. Some mathematical results on nano Z-open.

Fig. 1 depicts the steps involved in creating recombinant DNA as:

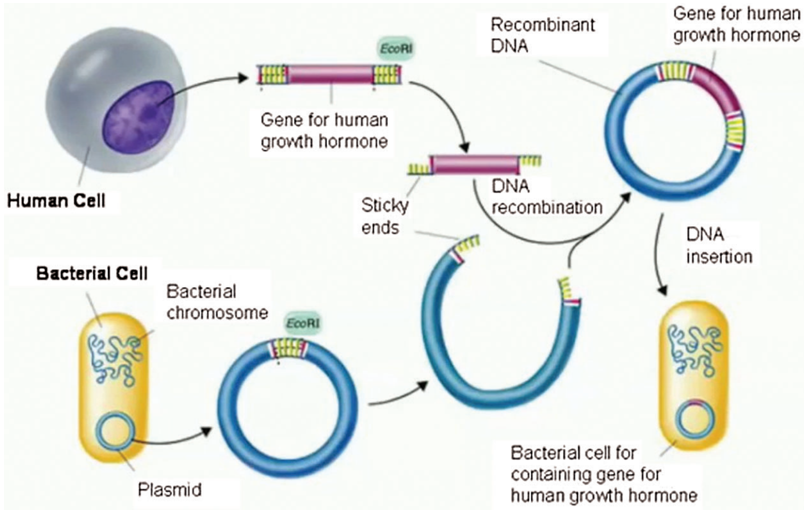

Figure 1: Recombinant DNA process

- Treat the DNA taken from both sources with the same restriction end nuclease.
- The restriction enzyme is an enzyme that cleaves DNA into fragments at or near specific recognition sites, such as sticky ends (EcoRI recognizes the sequence G ↓AATTC CTTAA↑G) and at blunt ends for example (HpaI restriction enzyme recognizes the sequence GTT↓ AACCAA↑TTG.
- Sticky ends are an overhanging segment of single-stranded DNA at the cut's ends.
- These sticky ends can base pair with any complementary sticky end-containing DNA molecule.
- When put together, complementary sticky ends can form a pair.
- A DNA ligase is used to covalently join the two strands of recombinant DNA into a molecule.
- Recombinant DNA must be copied several times before it can be used.

1.1.1 Mathematical of DNA Recombination

Mathematical modelling of biological processes is very useful in identifying the processes and designing programs that help in investigating possible solutions and avoiding errors. These models have helped in the development of other sciences and mathematics. Representing the problem of life or any process does not stop when building the mathematical model, but continues to prove the validity of the mathematical model and match the mathematical solution with the practical solution (to solve the problem of life). To prove the validity of the model, we use mathematical methods (topological, numerical analysis, differential equations, etc.). But during our work, we use topological methods more than any other branch of mathematics. Recombination is the production of offspring with combinations of hits that differ from those found in either parent. Stadler & Stadler [10,11] had defined the recombination process (most applications are inherited) and had problems with the definition, since the recombination of R(x, x) will not always give the x. Therefore, we will build on the definition of Stadler & Stadler definition of genetic re-synthesis, which is more commonly used in the manufacture of medicine and gene therapy. The DNA recombination process makes it possible to cut different strands of genotypes with a restriction enzyme (sticky ends) and join the genotypes together *via* complementary base pairing [12–14]. In this study, we consists of mathematical modelling of the recombination process. Also, we consider a method for generating nano topologies by (recombination operator deduce the equivalence classes) rough set theory *via* one of the biological applications (DNA recombination processes). We will investigate the topological properties of nano Z-open sets using nano topological structures. Finally, we explore the extent of the match between mathematical and biological results.

1.2 The DNA Recombination Operator

"A new mathematical representation is proposed for the configuration structure induced by recombination. It consists of the mapping of pairs of objects to a power set of all objects in the search space. The mapping assigns to each pair of parental (genotypes) the set of all recombinant genotypes obtainable from the parental ones."

Definition 1.3.1 [10] Let X be any set of types "strings of bites, vectors, DNA, ribonucleic acid (RNA) sequence …etc." A recombination operator on X that is defined by $F : A \times A \to P(A)$ the following condition holds $\forall h, k \in A$:

(i) $F(h,k) = F(k,h)$

(ii) $F(h,h) = \{h\}$

(iii) $\{h,k\} \subseteq F(h,k)$

(iv) $\|F(h,v)\| = \|F(h,k)\|, \forall v \in F(h,k)$.

2 Topological Spaces of DNA Recombination

By constructing a new recombination mapping based on the properties of the recombination process, we aim to use topological concepts to build flexible mathematical models in biomathematics. In addition, we investigate the topological qualities of the newly formed map as well as the topological structures of DNA that are related to it. We study the properties of recombination mapping, new topological structures, and characterizations by using the new concepts "Cut and Sticks" for sequences of genotypes. Further, we define recombination mathematically by a matrix where enzymes can be cut and the integration of two "types" introduces the meaning of the process of recombination, and as a result of improved optimization of this definition, more than once, it's a description of the recombination process is more accurate. The process of recombination consists of three elements: a gene, an enzyme, and plasma to form the mathematical model and then replace the enzyme with the slicing Boolean matrix. The functions (gene slicing and plasma slicing) and recombination composition were explained well by the way recombination between genes occurs. We have greater accuracy and better places to cut the injured part from the rest of the injured part.

Definition 2.1

Let X be a set of types "strings of bites, vectors, DNA, RNA sequence …etc." and the span of X contains all the linear combination elements of X as well as recombinant. Then the topological DNA recombination operator $R_s : X \times X \to spanX$, is defined by

R_s (x,y)= $\bigcup_{i=1}^{n} \{c_i^* x_{5'}^{3'} + c_{n-i}^* y_1, c_j^* x_{3'}^{5'} + c_{n-j}^* y_2\}$, Since, $C_i^* = \begin{pmatrix} I & \cdots & I \\ \vdots & \ddots I & \vdots \\ I & \cdots & I \end{pmatrix}$

$$C_i^* = \begin{pmatrix} 1000 & \ldots & 000 & \ldots \\ 0100 & \ldots & 000 & \ldots \\ \vdots & \vdots & \vdots & \vdots \\ 000 & \ldots 1 & 000 & \ldots \\ \vdots & \vdots & \vdots & \vdots \\ 000 & \ldots & 000 & \ldots \end{pmatrix}$$

where "$I \in \{0, 1\}$ and the matrix represents the unity of level $i \times n$, O a zero matrix, C_i^* is called the matrix slicing and $C_i^* \in M_n(f)$ [Boolean matrix] and the sense strand of DNA $X_{5'}^{3'}$, the antisense strand of DNA $X_{3'}^{5'}$."

We will conduct a new topological study on DNA recombination in which recombination between types x and y occurs, where x is the first gene and y is the plasmid.

Proposition 2.2 *Let* $R_s : X \times X \to spanX$ be the recombination operator, defined by

$R_s(x, y) = \bigcup_{i=1}^{n} \{c_i^* x_{5'}^{3'} + c_{n-i}^* y_1, c_j^* x_{3'}^{5'} + c_{n-j}^* y_2\}$, $R_s(x, y)$ consists of (offspring) can be induced by x, y and it satisfies the following:

- $\{x, y\} \in R_s(x, y)$.
- $R_s(x, y) \neq R_s(y, x)$.
- $\forall z \in R_s(x, y) \Rightarrow \|R_s(x, z)\| \leq \|R_s(x, y)\| \forall x, y, z \in X$.
- $R_s(x, x) = 2x$
- $R_s(x, y) \subseteq Span\{x, y\}$.

Proof. Obvious.

In this section, we present the definition of closure recombination space by using a recombination function.

Definition 2.3 Let A ⊆ X. We take $T_{R_s}^*(A)$ represents the closure recombination ("sticky end") operator, since $T_{R_s}^*(A) = \bigcup_{x,y} R_s(x, y)$.

We work with the topological recombination ("sticky ends") operator. The closure recombination space from the recombination operator R_s is denoted by $(X, T_{R_s}^*)$.

Theorem 2.4 The closure recombination structure $(\mathrm{X}, \mathrm{T}_{\mathrm{R_s}}^*)$ arising from recombination operator satisfies:

- $T_{R_s}^*(\varphi) = \varphi$,
- If H is a group of genes, then $H \subseteq T_{R_s}^*(H)$,
- If $H \subseteq K$, then $T_{R_s}^*(H) \subseteq T_{R_s}^*(K)$,
- $T_{R_s}^*(H) \cup T_{R_s}^*(K) = T_{R_s}^*(H \cup K)$ and $T_{R_s}^*(H) \cap T_{R_s}^*(K) \supseteq T_{R_s}^*(H \cap K)$,
- $T_{R_s}^*(T_{R_s}^*(H)) \supseteq T_{R_s}^*(K)$.

Proof. Obvious.

A space $(X, T_{R_s}^*)$, is called R_s DNA recombination structure useful for DNA recombination.

2.1 The Upper and The Lower Approximation on DNA Recombination

In the following, we study the connection between rough sets [15] concepts and DNA recombination. Also, we investigate new definitions of the class of elements depend on definitions of recombination sets that result from the definition of recombination function, which identifier as follows, $R_s : X \times X \rightarrow spanX$. Since $R_s(x,y) = \bigcup_{i=1}^{n} \{c_i^* x_{5'}^{3'} + c_{n-i}^* y_1, c_j^* x_{3'}^{5'} + c_{n-j}^* y_2\}$.

Definition 2 .1.1 Set of all recombination products can be done or (offspring can be obtained) caused by x,y, let's call it $R_s(x,y)$.

Definition 2.1.2 Let R_s *(x,y)* be a recombination set. Then the recombination class of x can be defined as $[x]_{R_s} = \{y \in U : y \in R_s(x,y)\}$ and the lower and the upper approximation of a subset X of U are defined *as:*

$\underline{R}_s(X) = \{x \in U : [x]_{R_s} \subseteq X\}$.

$\bar{R}_s(X) = \{x \in U : [x]_{R_s} \cap X \neq \varnothing\}$.

Example 2.1.3 Let the recombination process consists of three key pillars are: genes (g), plasmids(p), enzymes(e) and elements of recombination process (U) i.e. U = {g , e, p}, the recombination class of gene $[g]_R$ ={e},$[p]_R$ = {e} and $[e]_R$ = {g, p} and A any subsets of U (see, Tab. 1)

Table 1: Recombination topology

A	$\underline{R_s}(A)$	$\overline{R^s}(A)$	$b(A)$	$T^*_{R_s}(A)$
U	U	U	φ	{U,φ}
φ	φ	φ	φ	{U,φ}
{g}	φ	{e}	{e}	{U,φ,{e}}
{e}	{g, p}	{g, p}	φ	{U,φ,{g, p}}
{p}	φ	{e}	{e}	{U,φ,{e}}
{g, e}	{g, p}	U	{e}	{U,φ,{e}, {g, p}}
{g, p}	{e}	{e}	φ	{U,φ,{e}}
{e, p}	{g, p}	U	{e}	{U,φ,{g, p}, {e}}

From Tab. 1, the set of all recombination open (closed) sets consists of the indiscrete recombination space when using the general topology, and this result applies to the nano topological structure defined on the set U (enter all items of DNA recombination). But when using the nano topology, more than one topology appears. For example, when using the enzyme, only group B appears, as the effect of the enzyme is practically on the gene and the plasm. When using the gene or the plasma only, the enzyme appears, and this applies as a cofactor. These results are consistent with the biological results.

Proposition 2.1.4 let $(\mathrm{U}, \mathrm{T}^*_{\mathrm{R_s}})$ be an indiscrete recombination space, $\mathrm{H} \subseteq \mathrm{U}$.Then the limit point of H is given by

$$H\prime = \begin{cases} \varnothing & if\ H = \varnothing \\ U - \{p\} & if\ H = \{P\} \\ U & \text{if H contains more then one element matrix.} \end{cases}$$

Proof. Obviously

These results mean that:

1. That there is always an end point to the process.
2. There is an output of the process of recombination, and this is applicable to the biological concept.

Remark 2.1.5 Every subset of an indiscrete recombination space is dense. The sense that it produces a very large number of plasma carrying the gene The aim of the following proposition is to describe any item in the recombination process.

Proposition 2.1.6 Every indiscrete recombination space is a regular space. We note that from Tab. 1, the appearance of more than nano topology structures, but it appears one topological structure.

3 Nano Z-Open Sets

"Throughout this paper $(U, \tau_R(X))$ is a $\mathfrak{R}ts$ with respect to X where $X \subseteq U$ and R is an equivalence relation on U. Then U/R denotes the family of equivalence classes of U by R.' [1,3,15]

Definition 3.1 [1] If H $\subseteq$ U and $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$ is a $\mathfrak{R}$ts, then H is a nano Z -open(briefly, $\mathfrak{R}$Zo) (resp. nano Z-closed(briefly, $\mathfrak{R}$Zc)) set if H $\subseteq$ $\mathfrak{R}$int($\mathfrak{R}$cl(H)$\cup$$\mathfrak{R}$cl($\mathfrak{R}int_\delta$(H), (resp. H$\supseteq$ $\mathfrak{R}$int($\mathfrak{R}$cl$_\delta$(H)$\cap$$\mathfrak{R}$cl($\mathfrak{R}$int(H)). The family of all $\mathfrak{R}$Zo (resp. $\mathfrak{R}$Zc) sets are denoted by $\mathfrak{R}$ZO$(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X})$ (resp. $\mathfrak{R}$Z C$(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$.

Definition 3.2 If H $\subseteq$ U and $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$ is a $\mathfrak{R}$ts, then H is a nano e*-open(briefly $\mathfrak{R}$e*o) set if H $\subseteq$ $\mathfrak{R}$cl($\mathfrak{R}$int($\mathfrak{R}$cl$_\delta$(H))). The family of all $\mathfrak{R}$e*o sets are dented by $\mathfrak{R}$e*O$(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$.

Example 3.3 From Example 2.1.3.

From Tab. 2 the set of all nano Z-open sets to nano topological structure defined on the set U(enter all items of DNA recombination) is the discrete recombination space.

Table 2: Recombination and nano Z-open sets

A	$\underline{R_s}(A)$	$\overline{R^s}(A)$	$b(A)$	$T^*_{R_s}(A)$	$\mathfrak{R}ZO(U, \tau_R(A))$
U	U	U	φ	{U,φ }	P(U)
φ	φ	φ	φ	{U,φ }	P(U)
{g}	φ	{e}	{e}	{U,φ,{e}}	{U,φ,{e, g}, {e}, {e, p}}
{e}	{g,p}	{g,p}	φ	{U,φ,{g, p}}	{U,φ,{g, p}, {g}, {p}, {g, e}, {e, p}
{p}	φ	{e}	{e}	{U,φ,{e}}	{U,φ,{e}, {g,e},{e,p}}
{g, e}	{g,p}	U	{e}	{U,φ{e}, {g,p}}	P(U)
{g, p}	{e}	{e}	φ	{U,φ,{e}}	{U,φ,{e}, {g,e},{e,p}}
{e, p}	{g,p}	U	{e}	{U,φ,{g,p},{e}}	P(U)

Definition 3.4 [16] If H $\subseteq$ U and $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$ is a $\mathfrak{R}$ts, then a nano Z-interior of H is the union of all $\mathfrak{R}$Zo sets contained in H (briefly,$\mathfrak{R}$Zint(H).

Definition 3.5 [16] Let H $\subseteq$ U and $(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$ be a $\mathfrak{R}$ts. Then a nano Z-closure of H is the intersection of all $\mathfrak{R}$Zc sets containing H (briefly, $\mathfrak{R}$Zcl(H)).

Remark 3.6 Every $\mathfrak{R}$Z-open set is Nb-open (resp. e-open and e*-open).

Lemma 3.7 Let G be a subset of a space U. Then the following statement are satisfied:

(1) $\mathfrak{R}P$int$_\delta$($\mathfrak{R}Pcl(G)$)=$\mathfrak{R}Pcl(G)$)$\cap$$\mathfrak{R}int$($\mathfrak{R}cl_\delta$(G)),

(2) $\Re P\mathrm{cl}_\delta(\Re Pint(G))$=$\Re Pint(G))\cup\Re cl(\Re\mathrm{int}_\delta$(G)).

Proof. (1) Since, $\Re P\mathrm{int}_\delta(\Re Pcl(G)) = \Re Pcl(G)\cap\Re int(\Re\mathrm{cl}_\delta(\Re Pcl(G)) = \Re Pcl(G)\cap \Re int(\Re\mathrm{cl}_\delta(G\cup\Re int(\Re cl(G)) = \Re Pcl(G)\cap\Re int(\Re\mathrm{cl}_\delta(G))$

(2) It follows from (1).

Definition 3.8 [17] A subset $\mathrm{Z_x}\subseteq$ U is called a nano Z (resp. nano e^*) neighbourhood (briefly, $\Re$ZNbd (resp. $\Re\mathrm{e}^*$Nbd)) of a point x $\in$ U if there exists H $\in\Re$ZO(U, τ_R(X)) (resp. x $\in$ H $\subseteq\mathrm{Z_x}$ and a point x is called $\Re$ZNbd (resp. $\Re\mathrm{e}^*$Nbd) point of the set H. The family of all $\Re$ZNbd (resp. $\Re\mathrm{e}^*$Nbd) of a point x $\subseteq$ U is called $\Re$ZNbd (resp. $\Re\mathrm{e}^*$Nbd) system of x (briefly, $\Re$ZNbd S(x)) (resp. $\Re\mathrm{e}^*$NbdS (x)).

Theorem 3.9 An arbitrary union of $\Re$ZNbd (resp. Ne*Nbd) of a point x $\in$ U is again NZNbd (resp. $\Re\mathrm{e}^*$Nbd) of a point x $\in$ U.

Proof. Let $\{H_i : i\in I\}$ be an arbitrary collection of $\Re ZNbd$ of $x\in U$. Since $\forall i\in I$, A_i is $\Re ZNbd$ of x, $\exists L_i\in\Re\mathrm{ZO}(U,\tau_R(X))$ such that. $x\in L_i\subset H_i$, but for each $i\in I$, $H_i\subseteq\cup H_i$, therefore $x\in H_i\subset\cup A_i$ which implies that $\cup H_i$ is again $\Re ZNbd$ of x. The other cases are similar.

Lemma 3.10 The intersection of $\Re ZNbd$ (resp. $\Re\mathrm{e}^*$Nbd) of a point x $\in$ U is not a $\Re$ZNbd (resp.) of the p $\Re\mathrm{e}^*$Nbd oint x $\in$ U in general.

Example 3.11 From Example 3.3. and Tab. 2, let A = {e}. Then the $\Re$ts τ_R(X) = {U, φ, {g, p}}. In the $\Re$ts, (U, τ_R(X)), the sets {g, e} and {e, p} are $\Re$ZNbd (e) but {g,e}$\cap${e, p} = {e} is not a $\Re$ZNbd (e).

Theorem 3.12 If $(U,\tau_R(Y))$ is a $\Re$ts, then

1. Every$\Re\delta Nbd(y)$is $\Re Nbd$(y), $\forall$y $\in$ U,
2. Every$\Re\delta Nbd(y)$ is $\Re ZNbd$(y), $\forall$y $\in$ U,
3. Every$\Re\delta SNbd(y)$ is $\Re ZNbd$(y), $\forall$y $\in$ U,
4. Every$\Re PSNbd(y)$ is $\Re ZNbd$(y), $\forall$y $\in$ U,
5. Every$\Re ZNbd(y) is \Re bNbd(y),\forall y\in U$,
6. Every$\Re ZNbd(y)$ is $\Re eNbd$(y), $\forall$y $\in$ U,
7. Every$\Re ZNbd(y)$ is $\Re\mathrm{e}^*$Nbd(y), $\forall$y $\in$ U,

Proof. (3) Let A be an arbitrary $\Re\delta SNbd$ of $y\in U$. T*hen*$\exists H\in\Re\delta SO(U,\tau_R(X))$ such that $y\in H\subseteq A$. Since every $\Re\delta So$ is $\Re Zo$, then H $\Re Zo$, therefore $y\in H\subseteq A$. Then A is $\Re ZNbd$ *of* y. The other cases are similar.

Definition 3.13 Let H $\subseteq$ U and (U, τ_R(X)) be a nano topological structure. Then H is called nano Z (briefly, $\Re$Z)-dense subset if $\Re$Zcl(H) = U.

Definition 3.14 A nano topological structure is said to be a nano Z-extremally disconnected space (briefly, $\Re$Z$\mathfrak{E}DS$) if the nano Z-closure of $\Re$Zo set is $\Re$Zo set for each $\Re$Zo subset of U.

Definition 3.15 A nano topological space is called nano Z-submaximal (briefly, $\Re$Z$\mathfrak{E}\mathfrak{d}$) if each $\Re$Z-dense subset of U is $\Re$Zo set.

Remark 3.16 A nano topological space is $\Re$Z$\mathfrak{E}\mathfrak{M}$ and $\Re$Z$\mathfrak{E}\mathfrak{D}\mathfrak{S}$, then every $\Re$ZNbd is $\Re\delta$SNbd and $\Re$PNbd of x $\in$ U.

Theorem 3.17 For any point $x\in U$, $\Re$*ZNbdS(x)* satisfies

(1) $\Re$*ZNbdS(x)*$\neq\varphi$.

(2) if $H\subseteq U$ *and* $H\in\Re$*ZNbdS(x), then* $x\in H$.

(3) if $H\subseteq U$ *and* $H\in\Re$*ZNbdS(x)*, $H\subseteq C$ *then* $C\in\Re$*ZNbdS(x)*.

(4) if $H \subseteq U$ *and* $H \in \Re ZNbdS(x)$ *then* $\exists B \in \Re ZNbdS(x)S(x)$ *such that* $B \subseteq H$, $H \in \Re ZNbdS(y), \forall y \in B$.

Proof. (1) Since, $\forall x \in U$ and U is a $\Re Zo$ set, Then $x \in U$ implies that U is $\Re ZNbd$ of x. Then $U \in \Re ZNbd$S(x)$\Rightarrow \Re ZNbd$S(x)$\neq \varphi$.

(2) Let $H \in \Re ZNbd$S(x). Then H is a $\Re ZNbd$ of x implies that $\exists B \in \Re ZO(U, \tau_R(X)) \Rightarrow x \in B \subseteq H$. Thus $x \in H$.

(3) Let $H \in \Re ZNbd$S(x). Then that $\exists B \in \Re ZO(U, \tau_R(X)) \Rightarrow x \in B \subseteq H$ and $H \subseteq C \Rightarrow x \in B \subseteq H \subseteq C$. Therefore $C \in \Re ZNbd$S(x).

(4) From (3) it is obvious.

Theorem 3.18 Let $\Re$ZO$(U, \tau_R(X))$ be closed under finite intersection, H be $\Re$Zc subset of U and $x \in U - H$. Then there exists a $\Re$ZNbdA of x such that $H \cap A = \varphi$.

Proof. Let H be a $\Re$Zc set. Then $U - H$ is a $\Re Zo \Re Zo$ set. Therefore $U - H$ is $\Re ZNbd$ of each of its points. If $x \in U - H$, implies that $A \in \Re Zo$, since $x \in A \subseteq U - H$, then $H \cap A = \varphi$.

Definition 3.19 [2] A point $x \in U$ is called a nano Z- limit point of H, if for each $K \in \Re ZO(U, \tau_R(X))$ containing x satisfies $K \cap (H - x) \neq \emptyset$.

Definition 3.20 The set of all nano Z-limit points of H is a nano Z-derived set (briefly, $\Re D_Z$(H)).

Theorem 3.21 If H and K are subsets of a space U, then the following are hold.

(1) $\Re \mathfrak{D}_Z(\phi) = \phi$,

(2) if $x \in \Re D_Z(H)$, therefore $x \in \Re D_Z(H - x)$.

Proof. (1) Suppose that $x \in U$ and $G_x \in \Re ZO(U, \tau_R(X))$. Therefore $(G - x) \cap \varphi = \varphi \Rightarrow x \notin \Re D_Z(\phi)$, then $\forall x \in U, x \notin \Re D_Z(\phi)$. Thus $\Re D_Z(\phi) = \phi$.

(2) If $x \in \Re D_Z(H) \Rightarrow G \cap (H - x) \neq \varphi$ and $\forall G_x \in \Re ZO(U, \tau_R(X))$ and contains at least one point other than x of $H - x$. Thus $x \in \Re D_Z(H - x)$.

Theorem 3.22 If $H \subseteq U$ and $(U, \tau_R(X))$ is a $\Re$ts, then:

1. If $\Re$ZZC$(U, \tau_R(X))$ is closed under arbitrary union, then $H \cup \Re D_Z(H)$ is $\Re$Zc set,
2. $\Re$Zcl(H) $= H \cup \Re D_Z(H)$.

Proof. To show that$H \cup \Re D_Z(H)$ is a $\Re$Zc set, we want to prove $U - (H \cup \Re D_Z(H))$ *is a* $\Re$Zo set,we have two cases:

Case 1: Let $U - (H \cup \Re D_Z(H))$. Then the result is clear.

Case 2: Let $U - (H \cup \Re D_Z(H)) \neq \phi$. Then $x \in U - (H \cup \Re D_Z(H))$ implies that $x \notin (H \cup \Re D_Z(H))$ and hence $x \notin H$, $x \notin \Re D_Z(H) \Rightarrow G_x \in \Re ZO(U, \tau_R(X))$. Since $G \cap (H - x) = \varphi$ such that $x \notin H \Rightarrow G \cap H = \varphi$ implies $x \in G \subseteq U - H$. Thus $G \cap \Re D_Z(H) = \varphi$ implies $x \in G \subseteq U - \Re D_Z(H)$. Then $x \in G \subseteq (U - H) \cap (U - \Re D_Z(H)) = U - (H \cup \Re D_Z(H))$ implies that $x \in G \subseteq U - (H \cup \Re D_Z(H))$. Therefore $U - (H \cup \Re D_Z(H)))$ is a $\Re Zbd$ of each of its points. $U - (H \cup \Re D_Z(H))$ is a $\Re$Zo set and then $(H \cup \Re D_Z(H))$ is $\Re$Zc.

(2) By (1), if $H \cup \Re D_Z(H)$ is a $\Re$Zc set, then $H \cup \Re D_Z(H)$ is a $\Re$Zc set containing H. Therefore $\Re$Zcl(H)$\subseteq H \cup \Re D_Z(H)$ and $H \subseteq \Re$Zcl(H), implies that $\Re D_Z(H) \subseteq \Re D_Z(\Re \text{Zcl}(H) \subseteq \Re$Zcl(H) because $\Re$Zcl(H) is $\Re Zc$. Hence $H \cup \Re D_Z(H) \subseteq \Re$Zcl(H). Thus $\Re$Zcl(H)$= H \cup \Re D_Z(H)$

Theorem 3.23 If $H \subseteq U$ and $(U, \tau_R(X))$ is a Nts, then the following *holds:*

1. $\Re$Zcl(H) is the smallest $\Re Zc$ super set of H,

2. H is a $\Re Zc$ set iff $\Re Zcl(H) = H$.

Proof. (1) Let $\{F_i : i \in I\}, F_i \subseteq U, F_i$ be a $\Re Zc$ set and $H \subseteq F_i\ \forall i \in I$. Then $\Re$Zcl(H) $= \cap\{F_i : i \in I\}$, hence $\cap\{F_i : i \in I\}$ is a $\Re Zc$ set. Therefore $\Re$Zcl(H) is a $\Re Zc$ set. Also, $H \subseteq F_i$, $\forall i \in I \Rightarrow H \subseteq \cap\{F_i : i \in I\}$ = $\Re$Zcl(H). Then $\Re$Zcl(H) is a $\Re Zc$ set containing H such that $\Re$Zcl (H) = $\cap\{F_i : i \in I\}$, therefore $\Re$Zcl(H)$\subseteq F_i$, $\forall i \in I$. Consequently, $\Re$Zcl(H) is the smallest $\Re Zc$ superset of H.

(2) Let H be a $\Re Zc$ set. Then $\Re Zc$ is the superset of H and hence $\Re$Zcl(H)= H. Thus H is a $\Re Zc$ set. Then H is a $\Re Zc$ set iff $\Re$Zcl(H)= H.

Proposition 3.24 If H and K are two subsets of a space U, then the following properties hold:

(1) If $\Re Zc(U, \tau_R(X))$ is closed under finite union, then $\Re$Zcl$(H \cup K) = \Re Zcl$(H)$\cup\Re Zcl$(K) for every $H, K \in \Re Zc(U, \tau_R(X))$,
(2) If $\Re ec(U, \tau_R(X))$ is closed under finite union, then $\Re ecl(H \cup K) = \Re ecl(H) \cup \Re ecl(K)$ for every $H, K \in \Re ec(U, \tau_R(X))$.

Proof. (1) Let H and K be NZc sets in U. By hypothesis, $H \cup K$ is $\Re Zc$. Thus $\Re Zcl(H \cup K) = H \cup K = \Re Zcl(H) \cup \Re Zcl(K)$

(2) Likewise (1).

Theorem 3.25 If H and K are two subsets of a space U, Then the following are holds:

1. $\Re$Zin(H) is the largest $\Re Zo$ set contained in H.
2. H is a $\Re Zo$ set iff H = $\Re$Zin(H).
3. $\Re$Zint$(\varphi) = \varphi$ and $\Re$Zin(U) $= U$.
4. $\Re$Zint($\Re$Zin$t(H)$)) = $\Re$Zint(H).

Proof. (1) Let $B \in \Re Zo(U, \tau_R(X)), B \subseteq H$. If $x \in B$, therefore $x \in B \subseteq H$,$B \in \Re Zo(U, \tau_R(X))$, Then H is $\Re Zbd$ of x and hence $x \in B \Rightarrow x \in \Re$Zint(H). Therefore every $\Re Zo$ subset of H is contained in $\Re$Zint(H). Hence $\Re$Zint(H) is the largest $\Re Zo$ set contained in H.

(2) Let $H \in \Re ZO(U, \tau_R(X))$, $H \subseteq H$ and H be the largest $\Re Zo$ subset of H. By (1), $\Re$Zint(H) is the largest $\Re Zo$ subset of H. Hence H = $\Re$Zint(H).
(3) It is clear.
(4) By (2), H is a $\Re Zo$set iff H=$\Re$Zint(H) and by (1), $\Re$Zint(H) is the largest $\Re Zo$ set contained in H. Then $\Re$Zint($\Re$Zin$t(H)$)) = $\Re$Zint(H).

Theorem 3.26 If H and K are two subsets of a space U, then $\Re$Zint(H) $=$ H $- \Re\mathfrak{D}_Z$(U $-$ H).

Proof. Let $x \in \left(H - \Re\mathfrak{D}_3(U - H)\right) \Rightarrow x \in Handx \notin \Re\mathfrak{D}_3(U - H)$. *Then* $\exists G_x \in \Re ZO(U, \tau_R(X))$ such that $G_x \cap (U - H) = \varnothing \Rightarrow G_x \subseteq H$. Hence $x \in G \subseteq H \Rightarrow x \in \Re$Zin(H). Then $H - \Re\mathfrak{D}_3(U - H) \subseteq \Re$Zin(H).

If $x \in \Re$Zin(H) $\Rightarrow x \in H$ and $\Re$Zin(H), $x \in \Re$Zin(H) $\cap$ (U $-$ H). Therefore $x \notin \Re\mathfrak{D}_3(U - H)$). Then $x \in \left(H - \Re\mathfrak{D}_3(U - H)\right)$. Hence $\Re$Zin(H) $\subseteq H - \Re\mathfrak{D}_3(U - H)$. Then $\Re$Zin(H)=$H - \Re\mathfrak{D}_3(U - H)$.

Theorem 3.27 For H, T $\subseteq$ U, then $\Re$Zin(H $-$ T) $\subseteq \Re$Zint(H) $- \Re$Zint(T).

Proof. Let $\Re$Zin(H $-$ T) $= \Re$Zin(H $\cap$ (U $-$ T)) $\subseteq \Re$Zint$(H) \cap \Re$Zint$(U - T) \subseteq \Re$Zint(H) $\cap(U - \Re$Zint$(T)) = \Re$Zint$(H) - \Re$Zint(T).

4 Nano Z-Exterior, Eorder and Frontier Sets

Definition 4.1 For $H \subseteq U$, then a nano Z exterior (briefly, $\mathfrak{N}Z\mathfrak{E}\mathfrak{r}$) of H is defined as $\mathfrak{N}Zint(H) = \mathfrak{N}Zint(U - H)$.

Definition 4.2 [17] For $H \subseteq U$, then a nano Z border (briefly, $\mathfrak{N}Z\mathfrak{B}\mathfrak{r}$) of H is defined as $\mathfrak{N}Z\mathfrak{B}\mathfrak{r}(H) = H - \mathfrak{N}\mathrm{Zint}(H)$.

Theorem 4.3 If H and K are two subsets of a space U, then the following are satisfied:

1. $\mathfrak{N}E\eth\delta S(H) \subseteq \mathfrak{N}Z\mathfrak{E}\eth(H)$,
2. $\mathfrak{N}E\eth P(H) \subseteq \mathfrak{N}Z\mathfrak{E}\eth(H)$,
3. $\mathfrak{N}Z\mathfrak{E}\eth(H) = U - \mathfrak{N}\mathrm{Zcl}H)$,
4. $\mathfrak{N}Z\mathfrak{E}\eth(H \cup K) = \mathfrak{N}Z\mathfrak{E}\eth(H) \cap \mathfrak{N}Z\mathfrak{E}\eth(K)$,
5. $\mathfrak{N}\mathrm{Zint}(H), \mathfrak{N}Z\mathfrak{E}\eth(H)$are mutually disjoint and $U = \mathfrak{N}Z\mathfrak{E}\eth(H) \cup \mathfrak{N}\mathrm{Zint}(H)$,
6. $H \cap \mathfrak{N}Z\mathfrak{E}\eth(H) = \emptyset$,
7. $\mathfrak{N}Z\mathfrak{E}\eth(H) \subseteq U - H$
8. $\mathfrak{N}Z\mathfrak{E}\eth(H) \subseteq \mathfrak{N}\mathrm{Zint}(H^c)$,

Proof. (1) For $K \subseteq U$, $\mathfrak{N}\mathrm{S}int_\delta(K) \subseteq \mathfrak{N}\mathrm{Zint}(K)$. Put $K = U - H$, then $\mathfrak{N}\mathrm{S}int_\delta(U - H) \subseteq \mathfrak{N}\mathrm{Zint}(U - H)$. This implies $\mathfrak{N}E\eth\delta S(H) \subseteq \mathfrak{N}Z\mathfrak{E}\eth(H)$.

(2) For$K \subseteq U$, $\mathfrak{N}\mathrm{Pint}(K) \subseteq \mathfrak{N}\mathrm{Zint}(K)$ Put $K = U - H$, then $\mathfrak{N}\mathrm{Pint}(U - H) \subseteq \mathfrak{N}\mathrm{Zint}(U - H)$. This implies $\mathfrak{N}E\eth\delta S(H) \subseteq \mathfrak{N}Z\mathfrak{E}\eth(H)$.

(3) By the definition of $\mathfrak{N}Z\mathfrak{E}r(H) = \mathfrak{N}\mathrm{Zint}(U - H) = U - \mathfrak{N}\mathrm{Zcl}(H)$.

(4) Consider $\mathfrak{N}Z\mathfrak{E}r(H \cup K) = \mathfrak{N}\mathrm{Zint}(U - (H \cup K)) = \mathfrak{N}\mathrm{Zint}((U - H) \cap (U - K)) \supseteq \mathfrak{N}\mathrm{Zint}(U - H) \cap \mathfrak{N}\mathrm{Zint}(U - K) = \mathfrak{N}Z\mathfrak{E}\eth(H) \cap \mathfrak{N}Z\mathfrak{E}\eth(K)$. That is, $\mathfrak{N}Z\mathfrak{E}\eth(H \cup K) \supseteq \mathfrak{N}Z\mathfrak{E}\eth(H) \cap \mathfrak{N}Z\mathfrak{E}\eth(K)$.(1)

Also, we have $H \subseteq H \cup K$, $K \subseteq H \cup K$, then $\mathfrak{N}Z\mathfrak{E}r(H \cup K) \subseteq \mathfrak{N}Z\mathfrak{E}r(H)$ and $\mathfrak{N}Z\mathfrak{E}r(H \cup K) \subseteq \mathfrak{N}Z\mathfrak{E}r(K)$. Hence,

$$\mathfrak{N}Z\mathfrak{E}r(H \cup K) \subseteq \mathfrak{N}Z\mathfrak{E}r(H) \cap \mathfrak{N}Z\mathfrak{E}r(K). \tag{2}$$

(5) Assume that $\mathfrak{N}Z\mathfrak{E}r(H) \cap \mathfrak{N}\mathrm{Zint}(H) \neq \emptyset$. Then, $\exists x \in \mathfrak{N}Z\mathfrak{E}r(H) \cap \mathfrak{N}\mathrm{Zint}(H)$ therefore $\exists x \in \mathfrak{N}Z\mathfrak{E}r(H)$ and $x \in \mathfrak{N}\mathrm{Zint}(H) \Rightarrow x \in U - H$ and $x \in H$. contradiction, then $\mathfrak{N}Z\mathfrak{E}r(H) \cap \mathfrak{N}\mathrm{Zint}(H) = \emptyset$. Similarly, $U = \mathfrak{N}Z\mathfrak{E}\eth(H) \cup \mathfrak{N}\mathrm{Zint}(H)$.
(6) As $\mathfrak{N}Z\mathfrak{E}r(H) \cap H = H \cap \mathfrak{N}\mathrm{Zint}(U - H) \subseteq H \cap (U - H) = \emptyset$. Therefore $H \cap \mathfrak{N}Z\mathfrak{E}\eth(H) = \emptyset$.
(7) By the definition of $\mathfrak{N}Z\mathfrak{E}r(H) = \mathfrak{N}\mathrm{Zint}(U - H) \subseteq U - H$.
(8) This case is similar to (3).

Theorem 4.4 If H and K are two subsets of a space U, then the following are satisfied:

$(i)\mathfrak{N}Z\mathfrak{B}\eth(H) \subseteq \mathfrak{N}B\eth\delta S(H)$,
(ii) $\mathfrak{N}Z\mathfrak{B}\eth(H) \subseteq \mathfrak{N}B\eth\, p(H)$,
(iii) H is $\mathfrak{N}Zo$ set iff $\mathfrak{N}Z\mathfrak{B}\eth(H)=\emptyset$,
(iv) $\mathfrak{N}Z\mathfrak{B}r(H)=H-\mathfrak{N}\mathrm{Zint}(H) = H \cap \mathfrak{N}\mathrm{Zcl}(U - H)$,
(v) If $H \subseteq K$, Then $\mathfrak{N}NZ\mathfrak{B}r(K) \subseteq \mathfrak{N}Z\mathfrak{B}r(H)$,
(vi) $\mathfrak{N}Z\mathfrak{B}r(H \cup K) \subseteq \mathfrak{N}Z\mathfrak{B}r(H) \cup \mathfrak{N}Z\mathfrak{B}r(K)$,
(vii) $\mathfrak{N}Z\mathfrak{B}r(H) \cap \mathfrak{N}Z\mathfrak{B}r(K) \subseteq \mathfrak{N}Z\mathfrak{B}r(H \cap K)$,
(viii) $\mathfrak{N}Z\mathfrak{B}r(H)=\mathfrak{N}\mathfrak{D}_3(U-H)$ and $\mathfrak{N}\mathfrak{D}_3(H) = \mathfrak{N}Z\mathfrak{B}r(U-H)$.

Proof. (i) Since, $\mathfrak{N}\mathrm{S}int_\delta(H) \subseteq \mathfrak{N}\mathrm{Zint}(H) \Rightarrow U - \mathfrak{N}\mathrm{Zint}(H) \subseteq U - \mathfrak{N}\mathrm{S}int_\delta(H)$

$\Rightarrow H \cap (U - \mathfrak{N}\text{Zint}(H)) \subseteq H \cap (U - \mathfrak{N}\text{S}int_\delta(H)) \Rightarrow H - \mathfrak{N}\text{Zint}(H) \subseteq H - \mathfrak{N}\text{S}int_\delta(H)$. Therefore, $\mathfrak{N}Z\mathfrak{B}r$(H)⊆ $\mathfrak{N}Br\delta S(H)$.

(ii) Since, $\mathfrak{N}\text{Pint}(H) \subseteq \mathfrak{N}\text{Zint}(H) \Rightarrow U - \mathfrak{N}\text{Zint}(H) \subseteq U - \mathfrak{N}\text{Pint}(H) \Rightarrow H \cap (U - \mathfrak{N}\text{Zint}(H)) \subseteq H \cap (U - \mathfrak{N}\text{Pint}(H)) \Rightarrow H - \mathfrak{N}\text{Zint}(H) \subseteq H - \mathfrak{N}\text{Pint}(H)$. Then $\mathfrak{N}Z\mathfrak{B}\eth$ *(H)*⊆ $\mathfrak{N}B\eth\ p(H)$.

(iii) Let $H \subseteq U$ be a $\mathfrak{N}$Zo set iff $H = \mathfrak{N}\text{Zint}(H) \Leftrightarrow H - \mathfrak{N}\text{Zint}(H) = \emptyset \Leftrightarrow \mathfrak{N}Z\mathfrak{B}r$(H)=∅

(iv) Since, $\mathfrak{N}Z\mathfrak{B}r$(H)=$H - \mathfrak{N}\text{Zint}(H) = H \cap (U - \mathfrak{N}\text{Zint}(H)) = H \cap \mathfrak{N}\text{Zcl}(U - H)$.

$(v) If H \subseteq K, then \mathfrak{N}\text{Zint}(H) \subseteq \mathfrak{N}\text{Zint}(K) \Rightarrow U - \mathfrak{N}\text{Zint}(K) \subseteq U - \mathfrak{N}\text{Zint}(H) \Rightarrow H \cap (U - \mathfrak{N}\text{Zint}(K)) \subseteq H \cap (U - \mathfrak{N}\text{Zint}(H)) \Rightarrow K - \mathfrak{N}\text{Zint}(K) \subseteq H - \mathfrak{N}\text{Zint}(H) \Rightarrow \mathfrak{N}Z\mathfrak{B}r$(K) ⊆ $\mathfrak{N}Z\mathfrak{B}r$ (H).

(vi) Since, $H \subseteq H \cup K$, $K \subseteq H \cup K$, hence by (ix) $\mathfrak{N}Z\mathfrak{B}r(H) \supseteq \mathfrak{N}Z\mathfrak{B}r(H \cup K)$, $\mathfrak{N}Z\mathfrak{B}r(K) \supseteq \mathfrak{N}Z\mathfrak{B}r(H \cup K)$, then $\mathfrak{N}Z\mathfrak{B}r(H \cup K) \subseteq \mathfrak{N}Z\mathfrak{B}r(\text{K}) \cup \mathfrak{N}Z\mathfrak{B}r(\text{H})$.

(vii) As $H \cap K \subseteq H, H \cap K \subseteq H$ by (ix) $\mathfrak{N}Z\mathfrak{B}r(\text{H}) \subseteq \mathfrak{N}Z\mathfrak{B}r(\text{H}\cap K)$ *and*$\mathfrak{N}Z\mathfrak{B}r(\text{K}) \subseteq \mathfrak{N}Z\mathfrak{B}r(\text{H}\cap K)$, therefore) $\mathfrak{N}Z\mathfrak{B}r$(H)∩$\mathfrak{N}Z\mathfrak{B}r$(K)⊆ $\mathfrak{N}Z\mathfrak{B}r$(H∩*K*).

(viii) By the definition of $\mathfrak{N}Z\mathfrak{B}r$(H)= $H - \mathfrak{N}\text{Zint}(H) = H - \left(H - \mathfrak{N}\mathfrak{D}_3(U - H)\right) = \mathfrak{N}\mathfrak{D}_3(U - H)$ and $\mathfrak{N}\mathfrak{D}_3$*(-H*$\mathfrak{N}Z\mathfrak{B}r(U - H)$ is obtained by replacing H by $U - H$.

Definition 4.5 For H ⊆ U, then a nano Z-frontier (briefly $\mathfrak{N}Z\mathfrak{F}\eth$) of H is defined as, $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*= $\mathfrak{N}Zcl$ *(H)*-$\mathfrak{N}Zint$*(H)*.

Theorem 4.6 If H and K are two subsets of a space U, then the following are satisfied:

i) $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*⊆ $\mathfrak{N}F\eth\delta S(H)$,
ii) $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*⊆ $\mathfrak{N}F\eth\ p(H)$,
iii) $\mathfrak{N}Z\mathfrak{B}\eth$ *(H)*⊆ $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*,
iv) $\mathfrak{N}\text{Zcl}(H) = \mathfrak{N}\text{Zint}(H) \cup \mathfrak{N}Z\mathfrak{F}\eth$ *(H)*,
v) $\mathfrak{N}\text{Zint}(H) \cap \mathfrak{N}Z\mathfrak{F}\eth$ *(H)*= ∅,
vi) $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*=$\mathfrak{N}Z\mathfrak{B}\eth$ *(H)*∪$\mathfrak{N}\mathfrak{D}_3$*(H)*,
vii) H is $\mathfrak{N}$Zo set iff $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*=$\mathfrak{N}\mathfrak{D}_3$*(H)*,
viii) $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*=$\mathfrak{N}\text{Zcl}(H) \cap \mathfrak{N}\text{Zcl}(U - H)$,
ix) $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*=$\mathfrak{N}Z\mathfrak{F}\eth$ *(U-H)*.
x) $\mathfrak{N}Z\mathfrak{F}\eth$ *(H) is a* $\mathfrak{N}$Zc set ,
xi) $\mathfrak{N}\text{Zint}(H) = H - \mathfrak{N}Z\mathfrak{F}\eth$ *(H)*,
xii) $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*=∅ *iff H is both*$\mathfrak{N}$Zo set *and*$\mathfrak{N}$Zc set,
xiii) $\mathfrak{N}Z\mathfrak{F}\eth(\mathfrak{N}\text{Zint}(H)) \subseteq \mathfrak{N}Z\mathfrak{F}\eth$ *(H)*,
xiv) $U - \mathfrak{N}Z\mathfrak{F}\eth$ *(H)*= $\mathfrak{N}\text{Zint}(H) \cup \mathfrak{N}\text{Zint}(U - H)$,
xv) $\mathfrak{N}Z\mathfrak{F}\eth$ $(\mathfrak{N}\text{Zcl}(H)) \subseteq \mathfrak{N}Z\mathfrak{F}\eth$ *(H)*,
xvi) $\mathfrak{N}\text{Zcl}(H)$=*H*∪$\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*$\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*,
xvii) $\mathfrak{N}Z\mathfrak{F}\eth$ $(\mathfrak{N}Z\mathfrak{F}\eth\ (H)) \subseteq \mathfrak{N}Z\mathfrak{F}\eth$ *(H)*,
xviii) $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*∩$\mathfrak{N}\text{Z}\mathfrak{E}\eth(H) = \emptyset$,
xix) $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*∪$\mathfrak{N}\text{Z}\mathfrak{E}\eth(H)$=$\mathfrak{N}\text{Zcl}(H^c)$,
xx) $\mathfrak{N}\text{Z}\mathfrak{E}\eth(H)$, $\mathfrak{N}\text{Zint}(H)$and $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)* are forms a partition.

Proof. (i) Since, $\mathfrak{N}int_\theta(H) \subseteq \mathfrak{N}\text{Zint}(H)$ implies that $U - \mathfrak{N}\text{Zint}(H)) \subseteq U - \mathfrak{N}\text{Z}int_\theta(H)$. Also, $\mathfrak{N}\text{Zcl}(H) \subseteq \mathfrak{N}\text{S}cl_\delta(H)$. Therefore $\mathfrak{N}\text{Zcl}(H) \cap (\text{U} - \mathfrak{N}\text{Zint}(H)) \subseteq \mathfrak{N}\text{S}cl_\delta(H) \cap U - \mathfrak{N}\text{Z}int_\theta(H)$. This implies $\mathfrak{N}\text{Zcl}(H) - \mathfrak{N}\text{Zint}(H)) \subseteq \mathfrak{N}\text{S}cl_\delta(H) - \mathfrak{N}\text{S}int_\theta(H)$. Hence, $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*⊆ $\mathfrak{N}F\eth\delta S(H)$.

(ii) Since, $\mathfrak{N}\text{Pint}(H) \subseteq \mathfrak{N}\text{Zint}(H)$ implies $U - \mathfrak{N}\text{Zint}(H) \subseteq U - \mathfrak{N}\text{Pint}(H)$. Also, $\mathfrak{N}\text{Zcl}(H) \subseteq \mathfrak{N}\text{Pcl}(H)$. Therefore$\mathfrak{N}\text{Zcl}(H) \cap (U - \mathfrak{N}\text{Zint}(H)) \subseteq \mathfrak{N}\text{Pcl}(H) \cap (U - \mathfrak{N}\text{Pint}(H)$. This implies that $\mathfrak{N}\text{Zcl}(H) - \mathfrak{N}\text{Zint}(H) \subseteq \mathfrak{N}\text{Pcl}(H) - \mathfrak{N}\text{Pint}(H)$. Hence $\mathfrak{N}Z\mathfrak{F}\eth$ *(H)*⊆ $\mathfrak{N}F\eth\ p(H)$.

(iii) Since, $H \subseteq \mathfrak{R}\mathrm{Zcl}(H) \Rightarrow H \cap (U - \mathfrak{R}\mathrm{Zint}(H)) \subseteq \mathfrak{R}\mathrm{Zcl}(H) \cap (U - \mathfrak{R}\mathrm{Zint}(H))$. Therefore $H - \mathfrak{R}\mathrm{Zint}(H)) \subseteq \mathfrak{R}\mathrm{Zcl}(H) - \mathfrak{R}\mathrm{Zint}(H)$. Thus $\mathfrak{R}Z\mathfrak{B}\eth$ *(H)*$\subseteq$ $\mathfrak{R}Z\mathfrak{F}\eth$ *(H)*.

(iv) $\mathfrak{R}\mathrm{Zint}(H) \cup \mathfrak{R}Z\mathfrak{F}r(H) = \mathfrak{R}\mathrm{Zint}(H) \cup (\mathfrak{R}\mathrm{Zcl}(H) \cap (U - \mathfrak{R}\mathrm{Zint}(H)) = (\mathfrak{R}\mathrm{Zint}(H) \cup \mathfrak{R}\mathrm{Zcl}(H)) \cap (\mathfrak{R}\mathrm{Zint}(H) \cup (U - \mathfrak{R}\mathrm{Zint}(H)) = \mathfrak{R}\mathrm{Zcl}(H) \cap U = \mathfrak{R}\mathrm{Zcl}(H)$.

(v) $\mathfrak{R}\mathrm{Zint}(H) \cap \mathfrak{R}Z\mathfrak{F}r(H) = \mathfrak{R}\mathrm{Zint}(H) \cap (\mathfrak{R}\mathrm{Zcl}(H) \cap (U - \mathfrak{R}\mathrm{Zint}(H)) = (\mathfrak{R}\mathrm{Zint}(H) \cap \mathfrak{R}\mathrm{Zcl}(H)) \cap (\mathfrak{R}\mathrm{Zint}(H) \cap (U - \mathfrak{R}\mathrm{Zint}(H)) = \mathfrak{R}\mathrm{Zint}(H) \cap \varnothing = \varnothing$.

(vi) From (iv), $\mathfrak{R}\mathrm{Zcl}(H) = \mathfrak{R}\mathrm{Zint}(H) \cup \mathfrak{R}Z\mathfrak{F}\eth$ *(H)*. Then H$\cup\mathfrak{R}\mathfrak{D}_3$*(H)*=$\mathfrak{R}\mathrm{Zcl}(H) = \mathfrak{R}\mathrm{Zint}(H) \cup \mathfrak{R}Z\mathfrak{F}\eth$ *(H)*. But $H = \mathfrak{R}Z\mathfrak{E}\eth(H) \cup \mathfrak{R}\mathrm{Zint}(H)$ *by Theorem* 3.22. *Therefore* $\mathfrak{R}Z\mathfrak{E}\eth(H) \cup \mathfrak{R}\mathrm{Zint}(H) \cup \mathfrak{R}\mathfrak{D}_3$*(H)* =$\mathfrak{R}\mathrm{Zint}(H) \cup \mathfrak{R}Z\mathfrak{F}r$*(H)*. Hence $\mathfrak{R}Z\mathfrak{F}\eth$ *(H)*=$\mathfrak{R}Z\mathfrak{B}\eth$ *(H)*$\cup\mathfrak{R}\mathfrak{D}_3$*(H)*.

(vii) Let H be a $\mathfrak{R}$Zo set and by Theorem 4.4., and (iv), $\mathfrak{R}Z\mathfrak{B}\eth$ *(H)* = φ From (vi) $\mathfrak{R}Z\mathfrak{F}\eth$ *(H)*=$\mathfrak{R}Z\mathfrak{B}\eth$ *(H)*$\cup\mathfrak{R}\mathfrak{D}_3$*(H)*=$\mathfrak{R}\mathfrak{D}_3$*(H)*. Therefore if A is $\mathfrak{R}$Zo set, $\mathfrak{R}Z\mathfrak{F}\eth$ *(H)*=$\mathfrak{R}\mathfrak{D}_3$*(H)*. Conversely. Suppose $\mathfrak{R}Z\mathfrak{F}r(H)$ = $\mathfrak{R}\mathfrak{D}_3$*(H)* from (iv), $\mathfrak{R}\mathrm{Zcl}(H)$=$\mathfrak{R}\mathrm{Zint}(H) \cup \mathfrak{R}Z\mathfrak{F}\eth$*(H)*. That is $H \cup \mathfrak{R}\mathfrak{D}_3$ *(H)* $= \mathfrak{R}\mathrm{Zint}(H) \cup \mathfrak{R}Z\mathfrak{F}$ $\eth$ *(H)* by Theorem 3.22, implies $H \cup \mathfrak{R}\mathfrak{D}_3$ *(H)* = $\mathfrak{R}\mathrm{Zint}(H) \cup \mathfrak{R}\mathfrak{D}_3$ *(H)* by hypothesis. Therefore H = $\mathfrak{R}\mathrm{Zint}(H)$ and hence H is a $\mathfrak{R}$Zo set.

(viii)
$\mathfrak{R}Z\mathfrak{F}r(H) = \mathfrak{R}\mathrm{Zcl}(H) - \mathfrak{R}\mathrm{Zint}(H) = \mathfrak{R}\mathrm{Zcl}(H) \cap (U - \mathfrak{R}\mathrm{Zint}(H)) = \mathfrak{R}\mathrm{Zcl}(H) \cap \mathfrak{R}\mathrm{Zcl}(U - H)$.

(ix) $\mathfrak{R}Z\mathfrak{F}r(U - H)$ = $\mathfrak{R}\mathrm{Zcl}(U - H) - \mathfrak{R}\mathrm{Zint}$ (U-H) = $(U - \mathfrak{R}\mathrm{Zint}(H)) - (U - \mathfrak{R}\mathrm{Zcl}(H))$ =$\mathfrak{R}\mathrm{Zcl}(H) - \mathfrak{R}\mathrm{Zint}(H)$=$\mathfrak{R}Z\mathfrak{F}r(H)$

(x) *Since*, *a subset H of U is a* $\mathfrak{R}$*Zcset iff* $H = \mathfrak{R}\mathrm{Zcl}(H)$. *Consider*, $\mathfrak{R}\mathrm{Zcl}(\mathfrak{R}Z\mathfrak{F}r(H)) =$
$\mathfrak{R}\mathrm{Zcl}(\mathfrak{R}\mathrm{Zcl}(H) - \mathfrak{R}\mathrm{Zint}(H)) = \mathfrak{R}\mathrm{Zcl}(\mathfrak{R}\mathrm{Zcl}(H) \cap (U - \mathfrak{R}\mathrm{Zint}(H))) = \mathfrak{R}\mathrm{Zcl}(\mathfrak{R}\mathrm{Zcl}(H)) \cap$
$\mathfrak{R}\mathrm{Zcl}(U - \mathfrak{R}\mathrm{Zint}(H)) = \mathfrak{R}\mathrm{Zcl}(\mathfrak{R}\mathrm{Zcl}(H)) \cap \mathfrak{R}\mathrm{Zcl}(\mathfrak{R}\mathrm{Zcl}(U - H)) \subseteq \mathfrak{R}\mathrm{Zcl}(H) \cap \mathfrak{R}\mathrm{Zcl}(U - H) = \mathfrak{R}Z\mathfrak{F}r(H)$ by (iii), $\mathfrak{R}\mathrm{Zcl}(\mathfrak{R}Z\mathfrak{F}r(H)) \subseteq \mathfrak{R}Z\mathfrak{F}$ $\eth$ *(H)*. But $\mathfrak{R}Z\mathfrak{F}r(H) \subseteq$ $\mathfrak{R}\mathrm{Zcl}(\mathfrak{R}Z\mathfrak{F}r(H))$ is always true. Therefore $\mathfrak{R}\mathrm{Zcl}(\mathfrak{R}Z\mathfrak{F}r(H)) = \mathfrak{R}Z\mathfrak{F}r(H)$ and hence $\mathfrak{R}Z\mathfrak{F}$ $\eth$ *(H)* is a $\mathfrak{R}Zc$ set.

(xi) $H - \mathfrak{R}Z\mathfrak{F}r(H) = H \cap (U - \mathfrak{R}Z\mathfrak{F}r(H))$ = H $\cap(U - (\mathfrak{R}\mathrm{Zcl}(H) \cap \mathfrak{R}\mathrm{Zcl}(U - H)$ = H $\cap((U - \mathfrak{R}\mathrm{Zcl}(H)) \cup (U - \mathfrak{R}\mathrm{Zcl}(U - H)) = (H \cap (U - \mathfrak{R}\mathrm{Zcl}(H)) \cup ((H \cap (U - \mathfrak{R}\mathrm{Zcl}(\mathrm{U - H}) = \varnothing \cup$
$(H \cap \mathfrak{R}\mathrm{Zint}(H) = \mathfrak{R}\mathrm{Zint}(H)$.

(xii) If H is both $\mathfrak{R}$Zo and $\mathfrak{R}$Zc sets, then H = $\mathfrak{R}\mathrm{Zint}(H)$ and H = $\mathfrak{R}\mathrm{Zcl}(H)$. Now $\mathfrak{R}Z\mathfrak{F}\eth$ *(H)* = $\mathfrak{R}\mathrm{Zcl}(H)$ $-\mathfrak{R}\mathrm{Zint}(H) = H - H = \varphi$. Conversely, $\mathfrak{R}Z\mathfrak{F}\eth$ *(H)*=φ implies, $\mathfrak{R}\mathrm{Zcl}(H)$ $-\mathfrak{R}\mathrm{Zint}(H) = \varphi$ which implies, $\mathfrak{R}\mathrm{Zcl}(H)$ $-\mathfrak{R}\mathrm{Zint}(H) \subseteq H$. That is, $\mathfrak{R}\mathrm{Zcl}(H)$ $\subseteq H$. But, H$\subseteq$ $\mathfrak{R}\mathrm{Zcl}(H)$ is always true. Therefore H = $\mathfrak{R}\mathrm{Zcl}(H)$. Hence H is a $\mathfrak{R}$Zc set. Again $\mathfrak{R}Z\mathfrak{F}\eth$ *(H)* = φ implies $\mathfrak{R}\mathrm{Zcl}(H) - \mathfrak{R}\mathrm{Zint}(H) = \varphi$ which implies $\mathfrak{R}\mathrm{Zcl}(H) = \mathfrak{R}\mathrm{Zint}(H)$ implies $H \cup \mathfrak{R}\mathfrak{D}_3$*(H)*=$\mathfrak{R}\mathrm{Zint}(H)$ which implies H$\subseteq$ $\mathfrak{R}\mathrm{Zint}(H)$. But $\mathfrak{R}\mathrm{Zint}(H) \subseteq H$ is always true. Therefore $\mathfrak{R}\mathrm{Zint}(H)$ = H. Hence H is $\mathfrak{R}$Zo set.

(xiii) Now, $\mathfrak{R}Z\mathfrak{F}\eth$ $(\mathfrak{R}\mathrm{Zint}(H))$ = $\mathfrak{R}\mathrm{Zcl}(\mathfrak{R}\mathrm{Zint}(H))$ $-\mathfrak{R}\mathrm{Zint}(\mathfrak{R}\mathrm{Zint}(H)) \subseteq \mathfrak{R}\mathrm{Zcl}(H)$ $-\mathfrak{R}\mathrm{Zint}(H)$ as $\mathfrak{R}\mathrm{Zint}(H) \subseteq H$. This implies $\mathfrak{R}Z\mathfrak{F}\eth$ $(\mathfrak{R}\mathrm{Zint}(H)) \subseteq \mathfrak{R}Z\mathfrak{F}\eth$ *(H)*.

(xiv) Consider,
$U - \mathfrak{R}Z\mathfrak{F}r(H) = U - (\mathfrak{R}\mathrm{Zcl}(H) - \mathfrak{R}\mathrm{Zint}(H)) = (U - \mathfrak{R}\mathrm{Zcl}(H)) \cup \mathfrak{R}\mathrm{Zint}(H)) = \mathfrak{R}\mathrm{Zint}(U - H) \cup$
$\mathfrak{R}\mathrm{Zint}(H))$.

(xv) *Now* $\mathfrak{R}Z\mathfrak{F}$ $\eth$ $(\mathfrak{R}\mathrm{Zcl}(H))$ = $\mathfrak{R}\mathrm{Zcl}(\mathfrak{R}\mathrm{Zcl}(H)) - \mathfrak{R}\mathrm{Zint}(\mathfrak{R}\mathrm{Zcl}(H)) = \mathfrak{R}\mathrm{Zcl}(\mathfrak{R}\mathrm{Zcl}(H)) \cap$ $(U - \mathfrak{R}\mathrm{Zint}(\mathfrak{R}\mathrm{Zcl}(H))) = \mathfrak{R}\mathrm{Zcl}(H) \cap \mathfrak{R}\mathrm{Zcl}(\mathrm{U} - \mathfrak{R}\mathrm{Zcl}(H))$. Also, H$\subseteq$ $\mathfrak{R}\mathrm{Zcl}(H)$ $\Rightarrow$ $\mathrm{U} - \mathfrak{R}\mathrm{Zcl}(H) \subseteq U - H \Rightarrow \mathfrak{R}\mathrm{Zcl}(\mathrm{U} - \mathfrak{R}\mathrm{Zcl}(H)) \subseteq \mathfrak{R}\mathrm{Zcl}(U - H)$.$\mathfrak{R}Z\mathfrak{F}\eth(\mathfrak{R}\mathrm{Zcl}(H)) \subseteq \mathfrak{R}\mathrm{Zcl}(H)$-
$\mathfrak{R}\mathrm{Zcl}(U - H)$=$\mathfrak{R}Z\mathfrak{F}\eth(H)$. Thus $\mathfrak{R}Z\mathfrak{F}r(\mathfrak{R}\mathrm{Zcl}(H)) \subseteq \mathfrak{R}Z\mathfrak{F}\eth$ *(H)*.

(xvi) From (iv), $\mathfrak{R}\mathrm{Zcl}(H)$ = $\mathfrak{R}\mathrm{Zint}(H)) \cup \mathfrak{R}Z\mathfrak{F}\eth(H) \subseteq H \cup \mathfrak{R}Z\mathfrak{F}\eth(H)$*as* $\mathfrak{R}\mathrm{Zint}(H)) \subseteq H$. Also, from (iv), $\mathfrak{R}Z\mathfrak{F}\eth(H) \subseteq \mathfrak{R}\mathrm{Zcl}(H)$ and H$\subseteq$ $\mathfrak{R}\mathrm{Zcl}(H)$ is always true. Therefore H$\cup\mathfrak{R}Z\mathfrak{F}\eth(H) \subseteq \mathfrak{R}\mathrm{Zcl}(H)$. It follows that, H$\cup\mathfrak{R}Z\mathfrak{F}\eth(H) = \mathfrak{R}\mathrm{Zcl}(H)$.

(xviii) from (vii) $\mathfrak{N}Z\mathfrak{F}\partial(H) = \mathfrak{N}\mathrm{Zcl}(H) \cap \mathfrak{N}\mathrm{Zcl}(U - H)$ and $\mathfrak{N}Z\mathfrak{E}\partial(H) = \mathrm{U} - \mathfrak{N}\mathrm{Zcl}(H)$, then $\mathfrak{N}Z\mathfrak{F}\partial(H) \cap \mathfrak{N}Z\mathfrak{E}\partial(H) = \mathfrak{N}\mathrm{Zcl}(H) \cap \mathfrak{N}\mathrm{Zcl}(U - H) \cap (\mathrm{U} - \mathfrak{N}\mathrm{Zcl}(H)) = \varphi$.

(xix) It is clear from (xviii).

(xx) It is clear from (xix).

5 Conclusion

We consist of mathematical modelling of the recombination process and considering the method for generating nano topologies by (recombination operator deducing the equivalence classes) rough set theory via one of its biological applications (DNA recombination processes). Through the nano topological structure, we shall study the topological features of nano Z-open sets. Finally, we are exploring the extent of the match between mathematical and biological results.

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