

Memory-Type Control Charts Through the Lens of Cost Parameters

Sakthiseswari Ganasan¹, You Huay Woon^{2,*}, Zainol Mustafa¹ and Dadasaheb G. Godase³

¹Pusat Pengajian Sains Matematik, Fakulti Sains dan Teknologi, Universiti Kebangsaan Malaysia, UKM Bangi, Selangor, 43600, Malaysia

²Pusat GENIUS@Pintar Negara, Universiti Kebangsaan Malaysia, UKM Bangi, Selangor, 43600, Malaysia

³Shivaji University, Kolhapur, 416004, India

*Corresponding Author: You Huay Woon. Email: hwyu@ukm.edu.my

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Abstract: A memory-type control chart utilizes previous information for chart construction. An example of a memory-type chart is an exponentially-weighted moving average (EWMA) control chart. The EWMA control chart is well-known and widely employed by practitioners for monitoring small and moderate process mean shifts. Meanwhile, the EWMA median chart is robust against outliers. In light of this, the economic model of the EWMA and EWMA median control charts are commonly considered. This study aims to investigate the effect of cost parameters on the out-of-control average run length (ARL_1) in implementing EWMA and EWMA median control charts. The economic model was used to compute the ARL_1 parameter. The 14 input parameters were identified and the analysis was carried out based on the one-parameter-at-a-time basis. When the input parameters change based on a predetermined percentage, the ARL_1 is affected. According to the results of the EWMA chart, nine input parameters had an effect and five input parameters had no effect on the ARL_1 parameter. Further, only seven of the 14 input parameters had an effect on the ARL_1 of the EWMA median chart. However, the effect of each input parameter on the ARL_1 was different. Moreover, the ARL_1 for the EWMA median chart was smaller than the EWMA chart. This analysis is crucial to observe and determine the input parameters that have a significant impact on the ARL_1 of the EWMA and EWMA median control charts. Hence, practitioners can obtain an overview of the influence of the input parameters on the ARL_1 when implementing the EWMA and EWMA median control charts.

Keywords: Economic model; average run length; memory-type control chart; cost parameters; statistical quality control

1 Introduction

Statistical Process Control (SPC) is a collection of analytical decision making tools that are effective in achieving process reliability and enhancing capability by reducing variability. A control chart is among the



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most efficient tools in SPC [1]. The first control chart was pioneered by Dr. Walter A. Shewhart to monitor and determine whether a process is in a desired state for producing high quality goods [2].

Since then, the control chart remains to be a valuable tool that has been prominently employed in SPC. This is because it can continuously monitor a process and ultimately improve its capability. It is simple to use, yet achieves a significant impact on the enhancement of overall quality [3]. The awareness and knowledge of using control charting techniques to monitor a process and deliver high-quality production has grown exponentially [4–6]. Hence, control charts are used extensively in the manufacturing and service sectors.

The Shewhart chart has been well-known for its ease of construction and implementation. Nevertheless, it is insensitive towards small and moderate process shifts. Consequently, extensive research has been conducted to develop new control charts to improve the sensitivity of the Shewhart chart, such as the memory-type control chart [7]. A memory-type control chart is constructed based on past and current information. A good example of the memory-type control chart is the exponentially-weighted moving average (EWMA) chart.

The EWMA chart has the ability to accurately detect small-to-moderate process mean shifts under normality assumption. According to Human et al. [8], this generally does not hold when the process contains contaminated normal data. In this case, the EWMA median chart is an ideal alternative. The main benefit of this chart is that it is resistant to outliers, as it monitors the process using the sample median.

The assessment of a control chart is necessary to reveal its overall effectiveness. This is vital, as it influences the decision to use the control chart. In practice, the performance of a control chart can be accurately evaluated [9]. The common criterion to measure the performance of a control chart is the average run length (ARL). The economic designs of the memory-type control chart have been studied by Hariba et al. [10] and Serel [11], among others. The effect of cost parameters on the ARL of the EWMA chart and EWMA median chart based on the economic model is not yet available in the existing literature.

In view of this, it is essential to investigate the impact of the input parameters on the out-of-control average run length (ARL_1) of implementing the EWMA and EWMA median control charts using the economic model. From the findings, we can identify the critical input parameters that influence the performance measure of the control charts. This work presents a brief description of the EWMA and EWMA median charts. The economic model is also elaborated. Next, the sensitivity analysis of the EWMA chart and EWMA median chart was also conducted to identify the input parameters that influence the ARL_1 of control charts. Finally, concluding remarks are given.

2 The Memory-Type Control Charts

The memory-type charts considered in this study are the EWMA chart and EWMA median chart. The EWMA chart was developed by Roberts [12], and is typically used to detect small process mean shifts due to the characteristics of using previous and current data in calculation of the statistic [13].

The statistic of the EWMA chart at sampling period, u , is computed as follows:

$$z_u = \lambda x_u + (1 - \lambda)z_{u-1}, \text{ for } u = 1, 2, \dots, \quad (1)$$

where λ is a smoothing constant and $0 < \lambda \leq 1$ with $z_0 = \mu_0$. Note that μ_0 is the in-control process mean. When a z_u falls beyond the control limit, an out-of-control situation is said to have occurred. Note that the EWMA chart is converted to a Shewhart chart when λ becomes 1 [14].

The EWMA median chart uses the sample median to monitor the process, as follows:

$$\tilde{Y}_i = \begin{cases} Y_{i,((n+1)/2)} & \text{if } n \text{ is odd} \\ \frac{Y_{i,(n/2)} + Y_{i,(n/2+1)}}{2} & \text{if } n \text{ is even} \end{cases} \quad (2)$$

When calculating the sample median, it is common practice to assume that the sample size, n , is an odd number [15]. This simplifies the calculation of the sample median. The EWMA median chart's statistic is

$$Z_i = (1 - \lambda)Z_{i-1} + \lambda\tilde{Y}_i, \text{ for } i = 1, 2, \dots, \quad (3)$$

where $\lambda(0 < \lambda \leq 1)$ is a smoothing constant and $Z_0 = \mu_0$.

3 The Economic Model

The economic model obtains optimal charting parameters that minimize cost [16]. Here, the economic model involves finding the ARL_1 of the control chart. We considered the EWMA and EWMA median charts. The implementing cost of the chart was calculated based on the cost function.

When an assignable cause occurs, the process is said to become out-of-control and the process mean becomes $\mu_0 + \delta\sigma$, where δ and σ are the process shifts and process standard deviation, respectively. It is assumed that a process initiates in a state of in-control and the time for an assignable cause to occur is exponentially distributed with mean $1/\theta$. The expected cost per hour was obtained from dividing the expected cost-per-cycle by the expected cycle length. The cycle was composed of an in-control phase, followed by an out-of-control phase.

The expected cost per unit time in hours is denoted as C and can be defined as follows:

$$C = \frac{\frac{C_0}{\theta} + C_1B + \frac{b + cn}{h} \left(\frac{1}{\theta} + B \right) + \frac{sY}{ARL_0} + W}{\frac{1}{\theta} + \frac{(1 - \gamma_1)sT_0}{ARL_0} + EH}, \quad (4)$$

where

$$B = (ARL_1 - 0.5)h + F,$$

$$F = ne + \gamma_1T_1 + \gamma_2T_2,$$

$$s = \frac{1}{\lambda h} - 0.5,$$

$$EH = (ARL_1 - 0.5)h + G,$$

and

$$G = ne + T_1 + T_2.$$

The notations in Eq. (4) are defined as follows:

C_0 Expected quality cost per unit time, while the process is in-control

C_1 Expected quality cost per unit time, while the process is out-of-control

θ Process failure rate

b Fixed cost per sample

- c Cost per unit sampled
 n Sample size
 e Expected time to sample and interpret one unit
 h Sampling interval
 s Expected number of samples taken before an assignable cause occurs
 Y Cost of false alarm
 ARL_0 Average run length when the process is in-control
 ARL_1 Average run length when the process is out-of-control
 W Cost of finding and fixing an assignable cause
 $\gamma_1 = 1$ if production continues during search
 $= 0$ if production stops during search
 $\gamma_2 = 1$ if production continues during repair
 $= 0$ if production stops during repair
 T_0 Expected time to search for a false alarm
 T_1 Expected time to find the assignable cause
 T_2 Expected time to repair the process

4 The Analysis of the EWMA and EWMA Median Control Charts

Sensitivity analysis is crucial to identify the input parameter that has an impact on the performance measure, i.e., ARL_1 , of implementing the EWMA and EWMA median charts. Meanwhile, sensitivity analysis of input parameters on the EWMA and EWMA median control charts helps to determine the critical input parameters. Here, 14 input parameters ($\theta, \delta, C_0, C_1, Y, W, b, c, e, T_0, T_1, T_2, \gamma_1, \gamma_2$) related to the cost function were identified. Sensitivity analysis can be performed on a one factor at a time basis, where one of the input parameters is modified each time, while the other 13 input parameters remain unchanged. For example, the value for θ is 0.01, 0.02 and 0.04 each time, with the remainder of the input parameters remaining unchanged. In view of this, 40 different input parameter combinations ($\theta, \delta, C_0, C_1, Y, W, b, c, e, T_0, T_1, T_2, \gamma_1, \gamma_2$) were taken into account, as shown in [Tab. 1](#).

Table 1: Different combinations of the input parameters

No.	θ	δ	$C_0(\$)$	$C_1(\$)$	$Y(\$)$	$W(\$)$	$b(\$)$	$c(\$)$	e	T_0	T_1	T_2	γ_1	γ_2
1	0.01	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
2	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
3	0.04	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
4	0.02	0.43	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
5	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
6	0.02	1.72	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
7	0.02	0.86	57.12	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
8	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
9	0.02	0.86	228.48	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0

(Continued)

Table 1 (continued)

No.	θ	δ	$C_0(\$)$	$C_1(\$)$	$Y(\$)$	$W(\$)$	$b(\$)$	$c(\$)$	e	T_0	T_1	T_2	γ_1	γ_2
10	0.02	0.86	114.24	474.6	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
11	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
12	0.02	0.86	114.24	1898.4	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
13	0.02	0.86	114.24	949.2	488.7	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
14	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
15	0.02	0.86	114.24	949.2	1954.8	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
16	0.02	0.86	114.24	949.2	977.4	488.7	0	4.22	0.083	0.083	0.083	0.75	1	0
17	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
18	0.02	0.86	114.24	949.2	977.4	1954.8	0	4.22	0.083	0.083	0.083	0.75	1	0
19	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
20	0.02	0.86	114.24	949.2	977.4	977.4	5	4.22	0.083	0.083	0.083	0.75	1	0
21	0.02	0.86	114.24	949.2	977.4	977.4	10	4.22	0.083	0.083	0.083	0.75	1	0
22	0.02	0.86	114.24	949.2	977.4	977.4	0	2.11	0.083	0.083	0.083	0.75	1	0
23	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
24	0.02	0.86	114.24	949.2	977.4	977.4	0	8.44	0.083	0.083	0.083	0.75	1	0
25	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.0415	0.083	0.083	0.75	1	0
26	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
27	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.166	0.083	0.083	0.75	1	0
28	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.0415	0.083	0.75	1	0
29	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
30	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.166	0.083	0.75	1	0
31	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.0415	0.75	1	0
32	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
33	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.166	0.75	1	0
34	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.375	1	0
35	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
36	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	1.50	1	0
37	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	0	0
38	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
39	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	0	1
40	0.02	0.86	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	1

Tab. 2 demonstrates the corresponding ARL_1 values for the EWMA and EWMA median charts. Here, the input parameters are $\theta = 0.02$, $\delta = 0.86$, $C_0 = 114.24$, $C_1 = 949.2$, $Y = 977.4$, $W = 977.4$, $b = 0$, $c = 4.22$, $e = 0.083$, $T_0 = 0.083$, $T_1 = 0.083$, $T_2 = 0.75$, $\gamma_1 = 1$ and $\gamma_2 = 0$ and the corresponding ARL_1 value for the EWMA and EWMA median charts are 11.89 and 8.52, respectively. According to Tab. 2, the ARL_1

value of the EWMA chart decreases when the expected quality cost per unit time while the process is in-control, C_0 , decreases. For instance, a decrease of C_0 from 114.24 to 57.12 as in number 7 in Tab. 1 results in a decrease in the ARL_1 from 11.89 to 9.45. However, there is no change in terms of ARL_1 when C_0 increases from 114.24 to 228.48. This shows that a decrease in C_0 increases the sensitivity of the EWMA chart in terms of average run length. Meanwhile, the ARL_1 of the EWMA chart increases when the expected quality cost per unit time while the process is out-of-control, C_1 increases and vice versa. For instance, an increase of C_1 from 949.2 to 1898.4 shows an increase in the ARL_1 value from 11.89 to 11.96. The same occurs to the cost of false alarm, Y . Nevertheless, Y has a greater effect on ARL_1 than C_1 . When Y increases, ARL_1 also increases by 19.85%. When C_1 increases, ARL_1 increases by 0.59%. On the contrary, the EWMA chart's ARL_1 value decreases when the cost per unit sampled, c and fixed cost per sample, b , increase. From Tabs. 1 and 2, we can see that when c increases from 4.22 to 8.44, the ARL_1 value from 11.89 decreases to 9.45. Note that the input parameter W has no influence on the ARL_1 value.

Table 2: ARL_1 for EWMA and EWMA median control charts

No.	EWMA	EWMA Median
1	11.96 (40.38%)	8.52
2	11.89 (39.55%)	8.52
3	11.82 (38.73%)	8.52
4	24.89 (16.91%)	21.29
5	11.89 (39.55%)	8.52
6	4.37 (15.92%)	3.77
7	9.45 (10.92%)	8.52
8	11.89 (39.55%)	8.52
9	11.89 (39.55%)	8.52
10	11.82 (42.24%)	8.31
11	11.89 (39.55%)	8.52
12	11.96 (40.38%)	8.52
13	9.51 (39.65%)	6.81
14	11.89 (39.55%)	8.52
15	14.25 (42.22%)	10.02
16	11.89 (39.55%)	8.52
17	11.89 (39.55%)	8.52
18	11.89 (39.55%)	8.52
19	11.89 (39.55%)	8.52
20	10.27 (187.68%)	3.57
21	8.59 (369.40%)	1.83
22	14.25 (35.46%)	10.52
23	11.89 (39.55%)	8.52
24	9.45 (38.56%)	6.82
25	11.89 (39.55%)	8.52

(Continued)

Table 2 (continued)		
No.	EWMA	EWMA Median
26	11.89 (39.55%)	8.52
27	11.89 (39.55%)	8.52
28	11.89 (39.55%)	8.52
29	11.89 (39.55%)	8.52
30	11.89 (39.55%)	8.52
31	11.89 (39.55%)	8.52
32	11.89 (39.55%)	8.52
33	11.89 (39.55%)	8.52
34	11.89 (39.55%)	8.52
35	11.89 (39.55%)	8.52
36	11.89 (39.55%)	8.52
37	11.89 (33.60%)	8.90
38	11.89 (39.55%)	8.52
39	11.82 (33.86%)	8.83
40	11.89 (39.55%)	8.52

According to [Tab. 2](#), the ARL_1 value remains the same, regardless of the increase or decrease of the input parameter expected time to search a false alarm, T_0 , expected time to determine the assignable cause, T_1 , expected time to repair the process, T_2 and expected time to sample and interpret one unit, e . The process failure rate, θ , can be seen to have an effect on the ARL_1 value of the EWMA chart. When θ decreases by 50% (i.e., from 0.02 to 0.01), the ARL_1 value increases from 11.89 to 11.96. However, when θ rises to 0.04, the ARL_1 value decreases to 11.82. The decrease in ARL_1 occurs due to an increase in the process failure rate.

When the process shift, δ , increases by 50% from 0.86 to 1.72, the ARL_1 value is reduced by 63.25%. As the size of the process shift decreases, the ARL_1 value increases by 109.34%, from 11.89 to 24.89. A large process shift is easier to be detected than a small process shift; hence, the ARL_1 value is lower in the large process shift compared to the small process shift. The production status during search, γ_1 and production status during repair, γ_2 , show that when production stops during search and repair, the ARL_1 remains the same. The same phenomenon occurs when the production continues during search and repair. The ARL_1 decreases by 0.59% when the production stops during search and production continues during repair.

For the EWMA median chart, the input parameters, θ , C_0 , W , e , T_0 , T_1 and T_2 have no influence on the ARL_1 , i.e., the ARL_1 remains at 8.52, regardless of the increase or decrease of the input parameters. The ARL_1 value increases when the input parameters δ and c decrease and vice versa. For example, the ARL_1 increases by 23.47% when c decreases by 50%. The ARL_1 is greatly influenced by the input parameter δ . We can see that when the δ decreases or increases by 50%, the ARL_1 increases by 149.88% and decreases by 55.75%, respectively. This indicates that the smaller the δ , the higher the ARL_1 value. A different scenario occurs for the input parameter Y . The ARL_1 value decreases by 20.07% and increases by 17.61% when Y decreases and increases by 50%, respectively. Meanwhile, the ARL_1 decreases when the input parameter b increases. For instance, the ARL_1 decreases by 58.10% and 78.52% when b

increases to 5 and 10, respectively. The ARL_1 slightly decreases by 2.46% when C_1 decreases by 50%. However, the ARL_1 remains the same when C_1 increases by 50%. When $\gamma_1 = 0$ and $\gamma_2 = 0$, the ARL_1 increases by 4.46%. For the same γ_1 with $\gamma_2 = 1$, the ARL_1 increases by 3.64%.

Fig. 1 illustrates the ARL_1 profile for the EWMA and EWMA median charts based on the 40 different input parameter combinations. From Fig. 1, we notice that the influence of the input parameters on the profile of the ARL_1 for both charts is almost the same. However, the ARL_1 value is higher for the EWMA chart in comparison to the EWMA median chart. The percentage of improvement in ARL_1 when the EWMA median chart is used is denoted in parenthesis in Tab. 2. The percentage of improvement ranges from 10.92% to 369.40%. This indicates that implementing the EWMA median chart based on the economic model has better performance than the EWMA chart. As discussed, nine and seven out of 14 input parameters have an effect on the output parameter, i.e., ARL_1 value of the EWMA and EWMA median charts, respectively. The common input parameters that have an impact on the ARL_1 value for both charts are δ , C_1 , Y , b , c , γ_1 and γ_2 . This provides an overview to the practitioners on which input parameters need to be considered in measuring the performance measure, i.e., ARL_1 . However, the size of the effect of each input parameter on the ARL_1 is different. The input parameter δ has the greatest influence on the ARL_1 for the EWMA and EWMA median charts.

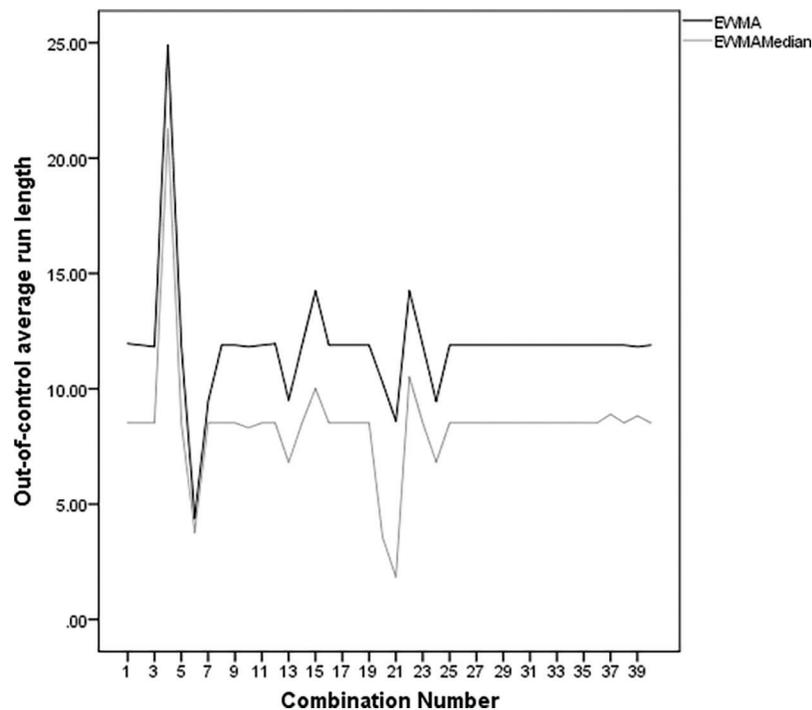


Figure 1: ARL_1 profile for the EWMA and EWMA median control charts

5 Conclusion

The EWMA control chart has been well-known for its quick detection of small and moderate process mean shifts. Meanwhile, the EWMA median chart can be used to monitor processes containing outliers. In this study, sensitivity analysis was conducted to determine the effect of the input parameters on the ARL_1 of the EWMA and EWMA median charts. The sensitivity analysis was carried out by increasing and decreasing the values of the input parameters by 50%. All input parameters, namely, θ , δ , C_0 , C_1 , Y , W , b , c , e , T_0 , T_1 , T_2 , γ_1 and γ_2 were subjected to sensitivity analysis. The findings of the study indicate

that such a variation affects the ARL_1 of the EWMA and EWMA median control charts. The input parameters that have an effect on the ARL_1 of the EWMA chart are θ , δ , C_0 , C_1 , Y , b , c , γ_1 and γ_2 . The process shift, δ , has the most impact on the ARL_1 when δ decreases by 50%. The ARL_1 of the EWMA chart increases by 109.34% when δ decreases by 50%. In contrast, input parameters e , T_0 , T_1 , T_2 and W have no effect on the ARL_1 of the EWMA control chart. For the EWMA median chart, the input parameters that have an effect on the ARL_1 are δ , C_1 , Y , b , c , γ_1 and γ_2 . On the other hand, θ , C_0 , W , e , T_0 , T_1 and T_2 , have no effect on the ARL_1 of the EWMA median chart. Similarly, the effect of δ on the ARL_1 is the highest when δ decreases by 50% and the ARL_1 increases by 149.88% for the EWMA median chart. As this study is based on the sensitivity analysis of the EWMA and EWMA median charts, future research can investigate the effect of the input parameters on the control chart with unknown process parameters.

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