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Rotational Motion of Micropolar Fluid Spheroid in Concentric Spheroidal Container

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Abstract: The slow steady rotation of a micropolar fluid spheroid whose shape deviates slightly from that of a sphere in concentric spheroidal container filled with Newtonian viscous fluid is studied analytically. The boundary conditions used are the continuity of velocity and stress components, and spin vorticity relation. The torque and wall correction factor exerted on the micropolar fluid spheroid is obtained. The dependence of wall correction factor on the micropolarity parameter, spin parameter, viscosity ratio and deformation parameter is studied numerically and its variation is presented graphically. In the limiting cases, the torque acting on solid spheroid in spheroidal container and on the solid spheroid in unbounded medium are obtained from the present analysis.

Keywords: Rotation, micropolar fluid, spin vorticity relation, torque.

1 Introduction

Studies on rotating fluid systems have received considerable attention among researchers due to its applications in engineering and science. The problem of flow of an incompressible viscous fluid contained between two concentric rotating spheres is of special interest to researchers because of its wide applications in various fields like fluid gyroscopes, colloidal science and centrifuges. Jeffery (1915) was the first who discussed the slow rotation of spheroids in an infinite fluid using curvilinear coordinates. The Stokesian flow of a viscous liquid generated by the slow steady rotation of an axisymmetric body placed in an incompressible viscous liquid was studied by Kanwal (1961). Kanwal (1961) also derived an expression for the couple experienced by the rotating body in terms of the toroidal velocity component. Munson and Joseph (1971) investigated the rotationally symmetric flow of an incompressible viscous fluid contained between two concentric spheres that rotate about a common axis with fixed angular velocities and obtained high-order analytic perturbation solution for low Reynolds number. The axisymmetric problem of viscous compressible heat-conducting fluid motion between two concentric spheres which can rotate at different angular velocities around a common axis is considered by Astafeva, Brailovskaya and Yavorskaya (1972). Munson (1974) solved the problem of motion of a viscous fluid contained between two eccentric rotating spheres using perturbation technique. Cooley (1971) studied the

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creeping flow problem of fluid motion generated by a rotating sphere. A general method for deriving exact solutions to the Stokes equations is suggested by Shankar (2009). This method can be applied to the flow problems in and around a sphere or between concentric spheres. The problem of concentric pervious spheres carrying a fluid sink at their centre and rotating slowly with different uniform angular velocities is studied by Srivastava (2013). Srivastava, Yadav and Yadav (2013) investigated the problem of rotating concentric pervious spheres with different angular velocities. The translational and rotational motion of a porous spherical shell located at the center of a spherical cavity containing incompressible Newtonian fluid was investigated analytically by Keh and Lu (2005). Saad (2010) studied the steady translation and rotation of a porous spheroid in concentric spheroidal container. The problem of rotational motion of a porous sphere situated at the centre of a spherical container containing an incompressible Newtonian viscous fluid has been tackled by Srinivasacharya and Prasad (2012). Ashmawy (2015) used a combined analytical numerical technique to study the steady rotational motion of an axially symmetric porous particle about its axis of symmetry in a viscous fluid.

The classical Navier-Stokes theory has proved to be inadequate to describe the behavior of fluids with microstructure such as animal blood, polymeric suspensions, muddy water and lubricants. In the past few years there has been increasing interest in developing theories that can accurately describe the behavior of such fluids. The theory of micropolar fluids introduced by Eringen (1966, 2001) is one of the best theories of fluids to describe the structured fluids. The micropolar fluids consist of rigid particles which can rotate with their own spins and microrotations. Micropolar fluids exhibit some microscopic effects arising from the local structure and micromotion of the fluid elements and they can sustain couple stresses. In the theory of micropolar fluids, there are two vectors describing the motion of the fluid; the classical velocity vector and the microrotation (spin) vector. The applications of these fluids are in blood flow, lubrication problem, liquid crystals, colloidal suspensions, polymeric additives, occurrence of turbulence, etc. The review article by Ariman, Turk and Sylvester (1974) and the book written by Lukaszewicz (1999) provide a useful account of the applications and theory of micropolar fluids.

In the past few years the study of non-Newtonian fluids has received special attention. Rao, Ramacharyulu and Rao (1969) studied the slow steady rotation of a sphere in a micropolar fluid. The problem of stokes flow of an axially symmetric body rotating in a micropolar fluid is analysed by Ramkisson (1977). Dennis, Ingham and Singh (1981) numerically studied the flow generated by a sphere rotating with constant angular velocity and calculated the couple for a wide range of Reynolds number. The problem of slow steady rotation of a spheroid (prolate and oblate) in an incompressible micropolar fluid is investigated by Rao and Iyengar (1981). Kamel and Fong (1993) studied the steady flow of an incompressible micropolar fluid between two rotating eccentric spheres. The slow steady rotation of an approximate sphere in an incompressible micropolar fluid is studied by Iyengar and Srinivasacharya (1995). The problem of slow steady rotation of a micropolar fluid sphere in concentric spherical container filled with viscous fluid is studied by Prasad and Gurdatta (2015). Saad (2016) studied the Stokesian flow of a spherical shaped droplet which is halfway immersed in a semi-infinite phase of a micropolar fluid. He investigated the problem in two different settings, when the movement of the droplet perpendicular to the free flat surface of the micropolar fluid and the parallel motion. Srinivas and Murthy (2016) investigated the flow of two immiscible incompressible couple stress fluids between two permeable beds flowing in axial direction under the influence of a constant pressure gradient. Recently, Prasad and Kaur (2017a,b) examined the steady, axisymmetric Stokes flow past a viscous fluid spheroid whose shape deviates slightly from that of a sphere in a micropolar fluid spheroidal cavity and the axisymmetric rotary oscillation of a micropolar fluid sphere in concentric spherical cavity filled with Newtonian viscous fluid.

The motion of a single liquid drop in another immiscible liquid, e.g., a Newtonian fluid drop in another Newtonian fluid medium, non-Newtonian fluid drop in a Newtonian liquid, etc., may well represent an idealization of various natural, industrial and biological processes, such as raindrop formation, study of blood flow, liquid-liquid extraction, prediction of atmospheric conditions and sedimentation phenomena. Many authors have studied the slow steady rotation of solid spherical or spheroidal particles in different geometries for different fluids. But very few authors have attempted to study the problem of rotation of droplets of one fluid dispersed in another immiscible fluid. There are cases where the geometry is not perfectly spherical. This motivated us to investigate the present problem.

In this paper, we study the flow generated by the slow steady rotation of a micropolar fluid spheroid in a spheroidal container containing viscous fluid by applying non zero boundary condition for microrotation vector. The boundary conditions used at the liquid-liquid interface are the continuity of velocity components, continuity of stress components and spin vorticity relation. The flow examined is axially symmetric in nature. An exact solution of the problem is obtained. The expression for torque and wall correction factor acting on the micropolar fluid spheroid is also obtained and its variation with various fluid parameters is studied.

2 Formulation of the problem

Let (r,θ,ϕ) denote a spherical polar co-ordinate system with the origin at the center of the spheroid and with $(\vec{e}_r,\vec{e}_\theta,\vec{e}_\phi)$ unit base vectors. Let the equation of the axisymmetric rotating micropolar fluid spheroid be of the form $r=a[1+f(\theta)]$. The orthogonality relations of Legendre functions $P_m(\zeta)$, $\zeta=\cos\theta$ permit us, under general circumstances, to assume the expansion $f(\zeta)=\sum_{m=1}^{\infty}\alpha_m P_m(\zeta)$. Therefore, we take the surface of the micropolar fluid spheroid to be

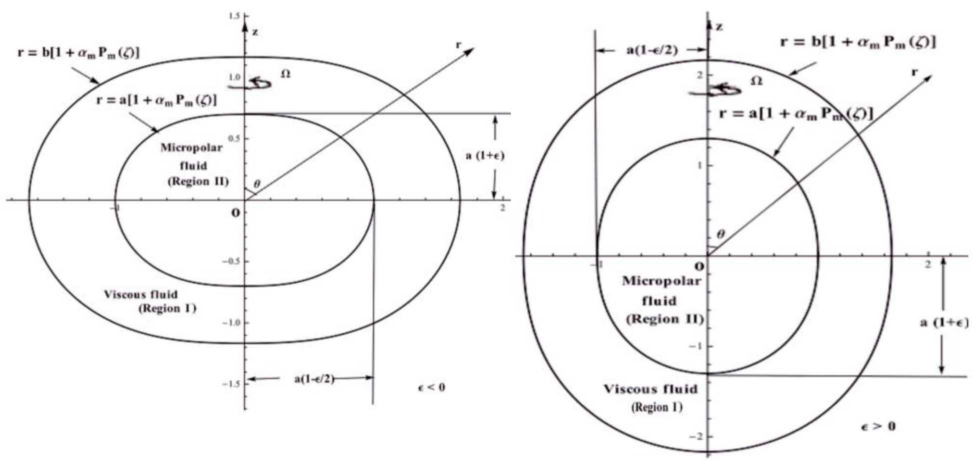
$$r=a\left[1+\alpha_m P_m(\zeta)\right]\equiv r_a, \tag{1}$$

and the surface of the Newtonian viscous fluid spheroid to be

$$r=b\left[1+\alpha_m P_m(\zeta)\right]\equiv r_b. \tag{2}$$

If all the α_m are zero, the spheroids reduces to spheres of radii a and b, respectively.

Consider the slow steady rotation of an incompressible micropolar fluid spheroid of radius r_a fixed at the center of a spheroidal cavity of radius r_b. The gap between the micopolar fluid spheroid and the cavity is filled with Newtonian viscous fluid. Assuming that the angular velocity of the micropolar fluid spheroid is Ω about the axis of symmetry $\theta = 0$ and the fluid particle is at rest. The angular velocity of the spheroidal cavity is same as that of the fluid particle in the opposite direction. The regions outside and inside the fluid spheroidal particle are denoted by regions I and II, respectively (See Figure 1).



Oblate spheroid in oblate spheroidal container ($\varepsilon < 0$)

Prolate spheroid in prolate spheroidal container ($\varepsilon > 0$)

Figure 1: The physical situation and the coordinate system.

The equations of motion for region I are

$$\nabla \cdot \vec{q}^{\,(1)} = 0, \tag{3}$$

$$\nabla p^{(1)} + \mu_1 \nabla \times \nabla \times \vec{q}^{\,(1)} = 0, \tag{4}$$

where $\vec{q}^{\,(1)}$ is the velocity vector, $p^{(1)}$ is the pressure and μ_1 is the coefficient of viscosity.

The equations of motion for the region II are the equations governing the steady flow of an incompressible micropolar fluid under Stokesian assumption with the absence of body force and body couple and are given by

$$\nabla \cdot \vec{q}^{\,(2)} = 0, \tag{5}$$

$$\nabla p^{(2)} + (\mu_2 + \kappa) \nabla \times \nabla \times \vec{q}^{\,(2)} - \kappa \nabla \times \vec{v} = 0, \tag{6}$$

$$\kappa \nabla \times \vec{q}^{\,(2)} - 2\kappa \vec{v} - \gamma_0 \nabla \times \nabla \times \vec{v} + (\alpha_0 + \beta_0 + \gamma_0) \nabla \nabla \cdot \vec{v} = 0, \tag{7}$$

where $\vec{q}^{\,(2)}$, $\vec{\nu}$ and $p^{(2)}$ are velocity vector, microrotation vector and pressure, μ_2 is the viscosity coefficient of the classical viscous fluid and κ, α_0, β_0 and γ_0 are the new viscosity coefficients for the micropolar fluids.

The equations for the stress tensor t_{ij} and the couple stress tensor m_{ij} are

$$t_{ij} = -p\delta_{ij} + \mu_2(q_{i,j} + q_{j,i}) + \kappa(q_{j,i} - \varepsilon_{ijm}\nu_m), \tag{8}$$

$$m_{ij} = \alpha_0 \nu_{m,m}\delta_{ij} + \beta_0 \nu_{i,j} + \gamma_0 \nu_{j,i}, \tag{9}$$

where the comma denotes the partial differentiation, δ_{ij} and ε_{ijm} are the Kronecker delta and the alternating tensor, respectively.

Since the rotation is assumed to be slow, the velocity $\vec{q}$ has its only component along the vector $\vec{e}_\phi$ and the microrotation vector $\vec{\nu}$ lies in the meridian plane. The flow is time independent and all the quantities are independent of ϕ. Thus, we choose the velocity and microrotation vectors as

$$\vec{q}^{\,(i)} = q_\phi^{(i)}(r,\theta)\vec{e}_\phi, i = 1,2, \tag{10}$$

$$\vec{\nu} = \nu_r(r,\theta)\vec{e}_r + \nu_\theta(r,\theta)\vec{e}_\theta. \tag{11}$$

The field equations in this case reduce to:

In the viscous fluid region $r_a \le r \le r_b$,

$$\frac{\partial p^{(1)}}{\partial r} = 0, \frac{\partial p^{(1)}}{\partial \theta} = 0, \tag{12}$$

$$Lq_\phi^{(1)} = 0, \tag{13}$$

and for the micropolar fluid region $r \le r_a$

$$L\left(L - l^2\right)q_\phi^{(2)} = 0. \tag{14}$$

Assume that $\operatorname{div}\vec{\nu} = F(r,\theta)$, $\operatorname{curl}\vec{\nu} = G(r,\theta)\vec{e}_\phi = -N^{-1}Lq_\phi^{(2)}\vec{e}_\phi$, we have

$$\left(\nabla^2 - c^2\right)F = 0, \tag{15}$$

where

$$L = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial}{\partial \theta} - \frac{1}{r^2 \sin^2\theta}, \tag{16}$$

$$l^2 = \frac{a^2\kappa(2+\chi)}{\gamma_0(1+\chi)} \quad \textit{and} \quad c^2 = \frac{2\kappa a^2}{\alpha_0 + \beta_0 + \gamma_0} \quad \chi = \frac{\kappa}{\mu_2} \quad N = \frac{\chi}{1+\chi},$$

$$\nu_r = \frac{1}{c^2}\frac{\partial F}{\partial r} - \frac{\gamma_0}{2\kappa}\frac{1}{r}\left(\frac{\partial G}{\partial \theta} + G\cot\theta\right) + \frac{1}{2r}\left(\frac{\partial q_\varphi^{(2)}}{\partial \theta} + q_\varphi^{(2)}\cot\theta\right), \tag{17}$$

$$\nu_\theta = \frac{1}{c^2}\frac{1}{r}\frac{\partial F}{\partial \theta} + \frac{\gamma_0}{2\kappa}\left(\frac{\partial G}{\partial r} + \frac{G}{r}\right) - \frac{1}{2}\left(\frac{\partial q_\phi^{(2)}}{\partial r} + \frac{q_\phi^{(2)}}{r}\right). \tag{18}$$

3 Solution of the problem

The solution of Eqs. (13)-(15) are given respectively

$$q_\phi^{(1)} = \left[a_1 r + b_1 r^{-2}\right] P_1^1(\zeta) + \sum_{n=2}^{\infty}\left[A_n r^n + B_n r^{-n-1}\right] P_n^1(\zeta), \tag{19}$$

$$q_\phi^{(2)} = \left[c_1 r + \frac{f_1}{\sqrt{r}} I_{3/2}(lr)\right] P_1^1(\zeta) + \sum_{n=2}^{\infty}\left[C_n r^n + \frac{F_n}{\sqrt{r}} I_{n+1/2}(lr)\right] P_n^1(\zeta), \tag{20}$$

$$F(r,\theta) = \frac{h_1}{\sqrt{r}} I_{3/2}(cr) P_1(\zeta) + \sum_{n=2}^{\infty}\frac{H_n}{\sqrt{r}} I_{n+1/2}(cr) P_n(\zeta). \tag{21}$$

The expression for $G(r,\theta)$ is obtained as

$$G(r,\theta) = -N^{-1}\left[\frac{f_1 l^2}{\sqrt{r}} I_{3/2}(lr) P_1^1(\zeta) + \sum_{n=2}^{\infty}\frac{F_n l^2}{\sqrt{r}} I_{n+1/2}(lr) P_n^1(\zeta)\right]. \tag{22}$$

Thus, using the expressions for F , G and $q_\phi^{(2)}$ in the equations (17) and (18), the expressions for ν_r and ν_θ are obtained as

$$\begin{aligned}\nu_r = &\left[c_1 + 2N^{-1}\frac{f_1}{r^{3/2}} I_{3/2}(lr) - \frac{1}{c^2}\frac{h_1}{r^{3/2}}(2I_{3/2}(cr) - crI_{1/2}(cr))\right] P_1(\zeta) + \\ &\sum_{n=2}^{\infty}\left[\frac{n(n+1)}{2} C_n r^{n-1} + n(n+1)N^{-1}\frac{F_n}{r^{3/2}} I_{n+1/2}(lr)\right. \\ &\left. -\frac{1}{c^2}\frac{H_n}{r^{3/2}}((n+1)I_{n+1/2}(cr) - crI_{n-1/2}(cr))\right] P_n(\zeta),\end{aligned} \tag{23}$$

$$\begin{aligned}\nu_\theta = &\left[-c_1 + N^{-1}\frac{f_1}{r^{3/2}}(I_{3/2}(lr) - lrI_{1/2}(lr)) - \frac{1}{c^2}\frac{h_1}{r^{3/2}} I_{3/2}(cr)\right] P_1^1(\zeta) + \\ &\sum_{n=2}^{\infty}\left[-\frac{(n+1)}{2} C_n r^{n-1} + N^{-1}\frac{F_n}{r^{3/2}}(nI_{n+1/2}(lr) - lrI_{n-1/2}(lr))\right.\end{aligned}$$

$$-\frac{1}{c^2}\frac{H_n}{r^{3/2}}I_{n+1/2}(cr)\Bigg]P_n^1(\zeta). \tag{24}$$

4 Boundary conditions

To determine the flow velocity components outside the fluid spheroid and velocity components and microrotation inside the fluid spheroid, we assume the continuity of velocity components. Also, we assume as in the classical case Happel and Brenner (1965), that the equilibrium theory of interfacial tension is applicable to our problem. This means that the presence of interfacial tension only produces a discontinuity in the normal stresses and does not in any way affect the tangential stresses. The latter is therefore continuous across the surface of the fluid spheroid. Hence, continuity of tangential stresses is applied at the interface. These conditions are physically realistic and mathematically consistent. Therefore, the boundary conditions on the surface $r = a[1+\alpha_m P_m(\zeta)]$ are:

$$q_\phi^{(1)} = q_\phi^{(2)}, \tag{25}$$

$$t_{r\phi}^{(1)} + \alpha_m t_{\theta\phi}^{(1)} P_m^1(\zeta) = t_{r\phi}^{(2)} + \alpha_m t_{\theta\phi}^{(2)} P_m^1(\zeta), \tag{26}$$

$$\nu_r = \frac{s}{2r}\left[\cot\theta\, q_\phi^{(1)} + \frac{\partial q_\phi^{(1)}}{\partial\theta}\right], \tag{27}$$

$$\nu_\theta = -\frac{s}{2}\left[\frac{q_\phi^{(1)}}{r} + \frac{\partial q_\phi^{(1)}}{\partial r}\right]. \tag{28}$$

On the cell surface $r = \eta^{-1}[1+\alpha_m P_m(\zeta)]$, we assume

$$q_\phi^{(1)} = -\Omega r P_1^1(\zeta). \tag{29}$$

5 Application to spheroid

As a particular example of the above analysis, we now consider the rotation of a prolate or an oblate spheroid in a spheroidal container. The surface of the spheroidal particle is represented in the Cartesian frame (x , y , z) by the equation,

$$\frac{x^2+y^2}{a^2\left(1-\frac{\varepsilon}{2}\right)^2} + \frac{z^2}{a^2(1+\varepsilon)^2} = 1. \tag{30}$$

For $\varepsilon < 0$ the spheroid is an oblate and for $0 < \varepsilon < 1$ it is a prolate. To $O(\varepsilon)$, Eq. (30) in polar form becomes $r = 1+\varepsilon P_2(\zeta)$. Here, we must take $m = 2$, $\alpha_2 = \varepsilon$. Therefore, the velocity and microrotation components are given by

$$q_\phi^{(1)} = \left[(a_1+A_1)r + (b_1+B_1)r^{-2}\right]P_1^1(\zeta) + \left[A_3 r^3 + B_3 r^{-4}\right]P_3^1(\zeta), \tag{31}$$

$$q_{\phi}^{(2)} = \left[(c_1 + C_1) r + \frac{(f_1 + F_1)}{\sqrt{r}} I_{3/2}(lr) \right] P_1^1(\zeta) + \left[C_3 r^3 + \frac{F_3}{\sqrt{r}} I_{7/2}(lr) \right] P_3^1(\zeta), \tag{32}$$

$$\nu_r = \left[(c_1 + C_1) + 2N^{-1} \frac{(f_1 + F_1)}{r^{3/2}} I_{3/2}(lr) - \frac{1}{c^2} \frac{(h_1 + H_1)}{r^{3/2}} (2I_{3/2}(cr) - crI_{1/2}(cr)) \right] P_1(\zeta) +$$

$$\left[6C_3 r^2 + 12N^{-1} \frac{F_3}{r^{3/2}} I_{7/2}(lr) - \frac{1}{c^2} \frac{H_3}{r^{3/2}} (4I_{7/2}(cr) - crI_{5/2}(cr)) \right] P_3(\zeta), \tag{33}$$

$$\nu_\theta = \left[-(c_1 + C_1) + N^{-1} \frac{(f_1 + F_1)}{r^{3/2}} (I_{3/2}(lr) - lrI_{1/2}(lr)) - \frac{1}{c^2} \frac{(h_1 + H_1)}{r^{3/2}} I_{3/2}(cr) \right] P_1^1(\zeta) +$$

$$\left[-2C_3 r^2 + N^{-1} \frac{F_3}{r^{3/2}} (3I_{7/2}(lr) - lrI_{5/2}(lr)) - \frac{1}{c^2} \frac{H_3}{r^{3/2}} I_{7/2}(cr) \right] P_3^1(\zeta). \tag{34}$$

6 Torque on the body

The torque experienced by micropolar fluid spheroid in presence of a spheroidal container is

$$T_z = \int_S \vec{r} \times (\vec{n} \cdot t^{(1)}) \cdot \vec{k} \, dS, \tag{35}$$

where $\vec{r} = a[1 + \varepsilon P_2(\zeta)] \vec{e}_r$, $\vec{n} = \vec{e}_r + \dfrac{3}{2} \varepsilon \sin 2\theta \vec{e}_\theta$, $dS = 2\pi a^2 [1 + 2\varepsilon P_2(\zeta)] \sin \theta d\theta$,

$\vec{k}$ is the unit vector in the direction of the axis of rotation and taking the integral over the surface of the boundary, we get

$$T_z = 2\pi a^3 \int_0^\pi r^3 (t_{r\phi}^{(1)} + \frac{3}{2} \varepsilon t_{\theta\phi}^{(1)} \sin 2\theta) |_{r = 1 + \varepsilon P_2(\zeta)} \sin^2 \theta d\theta, \tag{36}$$

$$T_z = -8\pi a^3 \mu_1 \Omega (b_1 + B_1), \tag{37}$$

where b_1 and B_1 are given in Appendix A.

The couple exerting on the spheroid in an unbounded medium is

$$T_z = -8\pi a^3 \mu_1 \Omega (\delta_1 + \delta_2), \tag{38}$$

where δ_1 and δ_2 are given in Appendix A.

6.1 Special cases

1. When $\varepsilon \to 0$, we get the expression for the hydrodynamic couple acting on the micropolar fluid sphere in spherical container which is given by

$$T_z = -8\pi a^3 \mu_1 \Omega b_1, \tag{39}$$

This result is previously obtained by Prasad and Gurdatta (2015).

2. If $s \to 0$ i.e., there is no microrotation at the boundary and $\sigma \to 0$ i.e., rotation of a solid spheroid in viscous fluid, we get the expression for the hydrodynamic couple acting on the solid spheroid in a cell model which is given by

$$T_z = \frac{-8\pi \mu_1 \Omega a^3}{1-\eta^3}\left(1-\frac{3\varepsilon}{5}\right).. \tag{40}$$

In the case of a perfect sphere $\varepsilon = 0$, we get the expression for the couple exerted by the fluid on a rotating sphere which was given by Happel and Brenner(1965).

3. When $\eta \to 0$ in (40), we get the torque acting on the solid spheroid

$$T_z = -8\pi \mu_1 \Omega a^3\left(1-\frac{3\varepsilon}{5}\right). \tag{41}$$

The wall correction factor W_c is defined as the ratio of the actual couple experienced by the particle in the container and the couple on a particle in an infinite expanse of fluid. With the aid of Eqs. (37) and (38) this becomes

$$W_c = \frac{T}{T_\infty} = \frac{b_1 + B_1}{\delta_1 + \delta_2}. \tag{42}$$

7 Results and discussion

The wall correction factor W_c acting on the spheroid is numerically computed for different values of spin parameter s, micropolarity parameter χ, separation parameter η, deformation parameter ε and classical ratio of viscosities between internal and external fluid σ. In all numerical computation of the wall correction factor, we assumed the value of $\dfrac{\gamma_0}{\mu_2 a^2} = 0.4$ and $\dfrac{(\alpha_0+\beta_0+\gamma_0)}{\mu_2 a^2} = 0.3$. The results are shown in Figure 2-Figure 7 and Tabel 1-Tabel 3.

Figure 2 and Figure 3 illustrate the variation of the wall correction factor W_c for the rotational motion of a micropolar fluid spheroid with the separation parameter η for various values of the micropolarity parameter χ for prolate ($\varepsilon = 0.1$) and oblate ($\varepsilon = -0.1$) spheroids, respectively. For any specified finite value of χ, the wall correction factor W_c increases monotonically with an increase in separation parameter η. It is observed that the wall correction factor of prolate and oblate spheroid increases with increasing micropolarity parameter for fixed values of s and σ. If the micropolarity parameter $\chi \to 0$, both the internal and external fluids are Newtonian and the problem reduces to the slow steady rotation of a Newtonian fluid spheroid in concentric spheroidal container filled with another Newtonian viscous fluid.

The effect of spin parameter on wall correction factor is presented in Figure 4 and Figure 5 for prolate ($\varepsilon = 0.1$) and oblate ($\varepsilon = -0.1$) spheroids, respectively. The spin parameter ranges over the interval $0 \le s \le 1$. If $s = 0$, there is no rotation of microelements near the boundary and if $s = 1$, the microrotation is equal to the fluid vorticity at the boundary. It can perceived from the figure that there is a decrease in wall correction factor compared to the case of no spin condition on microrotation vector. As expected, increasing the spin parameter decreases the wall correction factor because spin causes less motion between the fluid and the particle. This physically shows that the wall correction factor is greater in the case of zero microrotation vector than in the case of non-zero microrotation vector.

Figure 6 and Figure 7 depicts the variation of W_c with η for different values of viscosity ratio σ for prolate ($\varepsilon = 0.1$) and oblate ($\varepsilon = -0.1$) spheroids, respectively. The case $\sigma = 0$ corresponds to the rotational motion of a solid spheroid in viscous fluid. It is clear that the wall correction factor decreases with increasing values of viscosity ratio except for $\sigma = 0$. Thus, we conclude that fluid spheroid experiences less torque as compared to the solid spheroid embedded in the viscous fluid spheroid.

Tables 1- 3 show the numerical results of wall correction factor for different values of deformation parameter and separation parameter for the case of $\sigma = 0$, $\sigma = 1$, and $\sigma = 3$ keeping the values of s and χ as fixed. The numerical result shows that the wall correction factor is an increasing or a decreasing function of deformation parameter ε depending on the value of σ.

Table 1: Wall correction factor W_c for different values of separation parameter η with $\chi = 5$, $s = 0.2$ and $\sigma = 0$

	W_c				
η	$\varepsilon = -0.3$	$\varepsilon = -0.1$	$\varepsilon = 0$	$\varepsilon = 0.1$	$\varepsilon = 0.3$
0.01	1.00000	1.00000	1.00000	1.00000	1.00000
0.1	1.00080	1.00080	1.00080	1.00079	1.00079
0.3	1.02213	1.02208	1.02197	1.02185	1.02178
0.5	1.11140	1.11113	1.11055	1.10988	1.10952
0.7	1.37936	1.37824	1.37579	1.37299	1.37145
0.9	2.40678	2.39964	2.38398	2.36614	2.35625

Table 2: Wall correction factor W_c for different values of separation parameter η with $\chi = 5$, $s = 0.2$ and $\sigma = 1$

	W_c				
η	$\varepsilon = -0.3$	$\varepsilon = -0.1$	$\varepsilon = 0$	$\varepsilon = 0.1$	$\varepsilon = 0.3$
0.01	1.00000	1.00000	1.00000	1.00000	1.00000
0.1	1.00032	1.00033	1.00034	1.00035	1.00036
0.3	1.00876	1.00891	1.00922	1.00955	1.00972
0.5	1.04187	1.04261	1.04416	1.04579	1.04663
0.7	1.12389	1.12630	1.13128	1.13653	1.13926
0.9	1.30577	1.31280	1.32739	1.34274	1.35073

Table 3: Wall correction factor W_c for different values of separation parameter η with $\chi = 5$, $s = 0.2$ and $\sigma = 3$

	W_c				
η	$\varepsilon = -0.3$	$\varepsilon = -0.1$	$\varepsilon = 0$	$\varepsilon = 0.1$	$\varepsilon = 0.3$
0.01	1.00000	1.00000	1.00000	1.00000	1.00000
0.1	1.00015	1.00015	1.00016	1.00017	1.00017
0.3	1.00395	1.00405	1.00427	1.00448	1.00459
0.5	1.01855	1.01905	1.02006	1.02110	1.02162
0.7	1.05259	1.05406	1.05705	1.06009	1.06164
0.9	1.11874	1.12231	1.12956	1.13696	1.14071

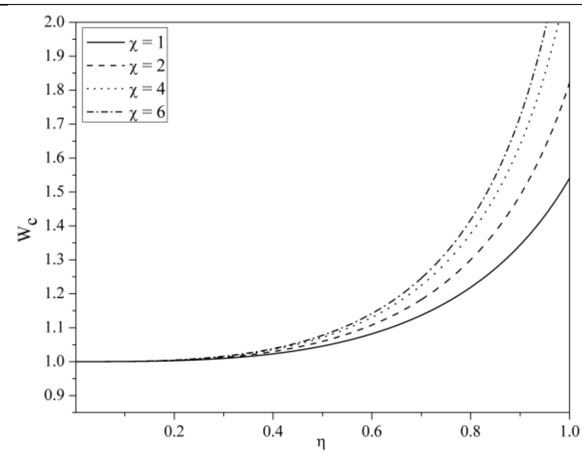


Figure 2: Variations of the Wall correction factor W_c versus η for different values of the micropolarity parameter χ with $s = 0.2$, $\sigma = 0.3$ and $\varepsilon = -0.1$.

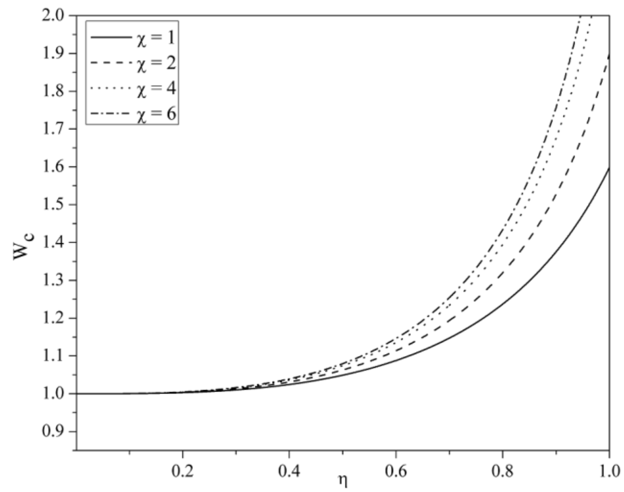


Figure 3: Variations of the Wall correction factor W_c versus η for different values of the micropolarity parameter χ with $s = 0.2$, $\sigma = 0.3$ and $\varepsilon = 0.1$.

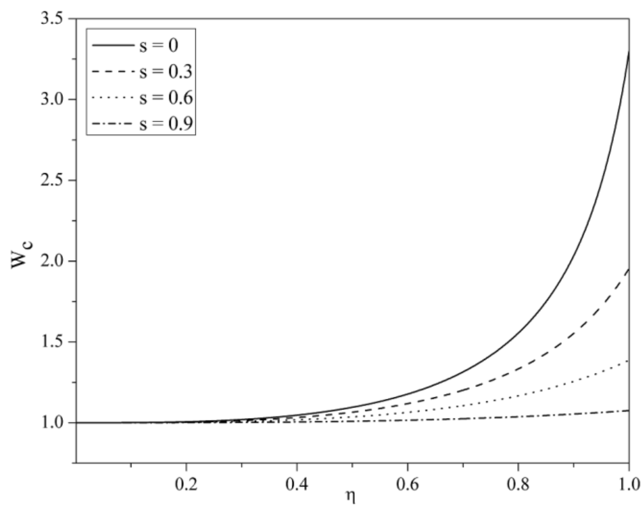


Figure 4: Variations of the Wall correction factor W_c versus η for different values of the spin parameter s with $\chi = 5$, $\sigma = 0.3$ and $\varepsilon = -0.1$.

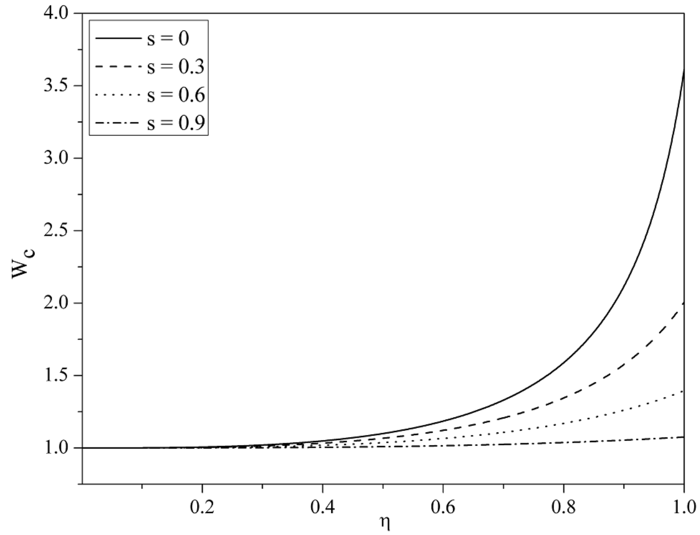


Figure 5: Variations of the Wall correction factor W_c versus η for different values of the spin parameter s with $\chi = 5$, $\sigma = 0.3$ and $\varepsilon = 0.1$.

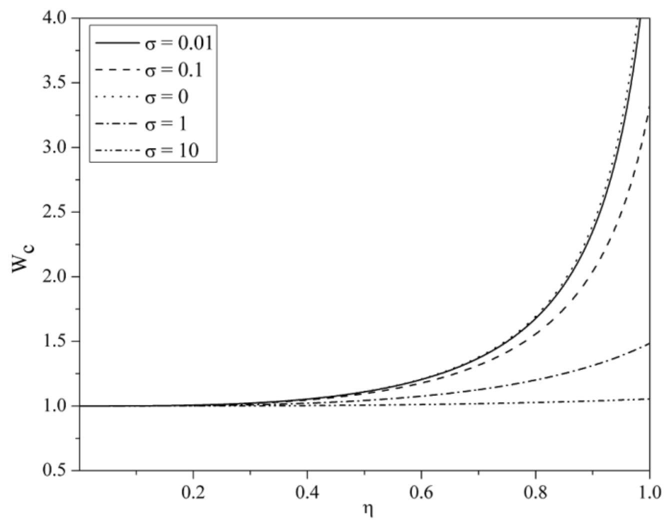


Figure 6: Variations of the Wall correction factor W_c versus η for different values of the parameter σ with $\chi = 5$, $s = 0.2$ and $\varepsilon = -0.1$.

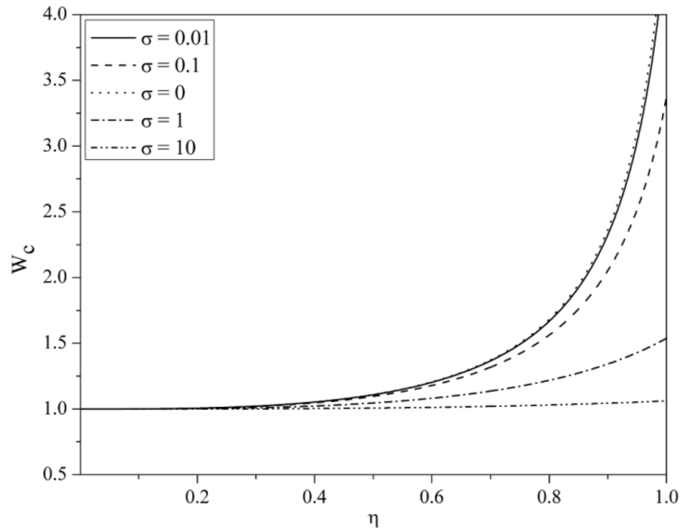


Figure 7: Variations of the Wall correction factor W_c versus η for different values of the parameter σ with $\chi = 5$, $s = 0.2$ and $\varepsilon = 0.1$.

8 Conclusion

In this paper, an analytic solution for the slow steady rotation of a micropolar fluid spheroid in a concentric spheroidal container is presented. Various useful results are obtained from the solution, particularly the closed form expression for the torque and the dependence of the wall correction factor on the various fluid parameters. It has been found that the wall correction factor is an increasing function of the separation parameter and micropolarity parameter. The effect of spin parameter on wall correction factor is also studied and it is found that the wall correction factor is a decreasing function of the spin parameter.

Acknowledgement: The authors are thankful to the reviewers for their valuable suggestions and comments that has greatly improved the presentation of the manuscript. One of the authors (Manpreet kaur) acknowledges MANF, UGC, India (F1-17.1/2015-16/MANF-2015-17-CHH-66189/(SA-III/Website)), for the financial support.

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Appendix A

Applying the boundary conditions (25)-(29), we obtain the following system of algebraic equations

$$\begin{aligned}&\left[a_1+b_1-c_1-f_1T_2\right]P_1^1(\zeta)+\alpha_m\left[a_1-2b_1-c_1+f_1\left(2T_2-lT_1\right)\right]P_1^1(\zeta)P_m(\zeta)\\&+\sum_{n=2}^{\infty}\left[A_n+B_n-C_n-F_nT_6\right]P_n^1(\zeta)=0,\end{aligned}\tag{A.1}$$

$$\begin{aligned}&\left[2f_1T_2+\frac{2N}{2-N}\frac{h_1}{c^2}T_4-3\lambda b_1\right]P_1^1(\zeta)\\&+\alpha_m\left[-2f_1\left(3T_2-lT_1\right)-\frac{2N}{2-N}\frac{h_1}{c^2}\left(3T_4-cT_3\right)+9\lambda b_1\right]P_1^1(\zeta)P_m(\zeta)\\&+\alpha_m\left[2f_1T_2-\frac{2N}{2-N}\frac{h_1}{c^2}\left(2T_4-cT_3\right)\right]P_1(\zeta)P_m^1(\zeta)\\&+\sum_{n=2}^{\infty}\left[-(n-1)C_n+2F_nT_6+\frac{2N}{2-N}\frac{H_n}{c^2}T_8+\lambda(n-1)A_n-\lambda(n+2)B_n\right]P_n^1(\zeta)=0,\end{aligned}\tag{A.2}$$

$$\begin{aligned}&\left[c_1+\frac{2}{N}f_1T_2-\frac{1}{c^2}h_1\left(2T_4-cT_3\right)-sa_1-sb_1\right]P_1(\zeta)+\\&\alpha_m\left[-\frac{2}{N}f_1\left(3T_2-lT_1\right)+\frac{1}{c^2}h_1\left((6+c^2)T_4-2cT_3\right)+3sb_1\right]P_1(\zeta)P_m(\zeta)\\&+\sum_{n=2}^{\infty}\left[\frac{n(n+1)}{2}C_n+\frac{n(n+1)}{N}F_nT_6-\frac{1}{c^2}\left((n+1)T_8-cT_7\right)H_n\right.\\&\left.-\frac{s}{2}n(n+1)A_n-\frac{s}{2}n(n+1)B_n\right]P_n(\zeta)=0,\end{aligned}\tag{A.3}$$

$$\begin{aligned}&\left[-c_1+\frac{1}{N}f_1(T_2-lT_1)-\frac{1}{c^2}h_1T_4+sa_1-\frac{s}{2}b_1\right]P_1^1(\zeta)\\&+\alpha_m\left[-\frac{1}{N}f_1\left((3+l^2)T_2-lT_1\right)+\frac{1}{c^2}h_1\left(3T_4-cT_3\right)+\frac{3}{2}sb_1\right]P_1^1(\zeta)P_m(\zeta)\\&+\sum_{n=2}^{\infty}\left[-\frac{(n+1)}{2}C_n+\frac{1}{N}F_n\left(nT_6-lT_5\right)-\frac{1}{c^2}H_nT_8\right.\\&\left.+\frac{s}{2}(n+1)A_n-\frac{s}{2}nB_n\right]P_n^1(\zeta)=0,\end{aligned}\tag{A.4}$$

$$\begin{aligned}&\left[a_1\eta^{-1}+b_1\eta^2+\eta^{-1}\right]P_1^1(\zeta)+\alpha_m\left[a_1\eta^{-1}-2b_1\eta^2+\eta^{-1}\right]P_1^1(\zeta)P_m(\zeta)\\&+\sum_{n=2}^{\infty}\left[A_n\eta^{-n}+B_n\eta^{n+1}\right]P_n^1(\zeta)=0.\end{aligned}\tag{A.5}$$

On solving the leading terms of equations (A.1)-(A.5), we will get the values of a_2, b_2, c_2, f_2 and h_2. Since the expressions are lengthy we are not presenting it here. To obtain the remaining arbitrary constants A_n, B_n, C_n, F_n and H_n, we require the following identities

$$P_1^1(\zeta)P_m(\zeta)=\frac{1}{2m+1}P_{m+1}^1(\zeta)-\frac{1}{2m+1}P_{m-1}^1(\zeta),\tag{A.6}$$

$$P_1(\zeta)P_m^1(\zeta)=\frac{m}{2m+1}P_{m+1}^1(\zeta)+\frac{m+1}{2m+1}P_{m-1}^1(\zeta),\tag{A.7}$$

$$P_1(\zeta)P_m(\zeta)=\frac{m+1}{2m+1}P_{m+1}(\zeta)-\frac{m}{2m+1}P_{m-1}(\zeta).\tag{A.8}$$

Using these in (A.1)-(A.5), and taking all the coefficient A_n, B_n, C_n, F_n and H_n are zero except at $n=m-1$ or $n=m+1$, we get

$$\xi_1\bar{c}_n+A_n+B_n-C_n-F_nT_6=0,\tag{A.9}$$

$$\xi_2\bar{c}_n+\xi_3\bar{d}_n+2F_nT_6+\frac{2N}{2-N}\frac{H_n}{c^2}T_8+\lambda(n-1)A_n-\lambda(n+2)B_n=0,\tag{A.10}$$

$$\xi_4\bar{e}_n+\frac{n(n+1)}{2}C_n+\frac{n(n+1)}{N}F_nT_6-\frac{1}{c^2}\left((n+1)T_8-cT_7\right)H_n-\frac{s}{2}n(n+1)A_n$$

$$-\frac{s}{2}n(n+1)B_n = 0, \tag{A.11}$$

$$\xi_5 \bar{c}_n - \frac{(n+1)}{2}C_n + \frac{1}{N}F_n\left(nT_6 - lT_5\right) - \frac{1}{c^2}H_n T_8 + \frac{s}{2}(n+1)A_n - \frac{s}{2}nB_n = 0, \tag{A.12}$$

$$\xi_6 \bar{c}_n + A_n \eta^{-n} + B_n \eta^{n+1} = 0, \tag{A.13}$$

where

$\xi_1 = a_1 - 2b_1 - c_1 + f_1\left(2T_2 - lT_1\right),$

$\xi_2 = -2f_1\left(3T_2 - lT_1\right) - \frac{2N}{2-N}\frac{h_1}{c^2}\left(3T_4 - cT_3\right) + 9\lambda b_1,$

$\xi_3 = 2f_1 T_2 - \frac{2N}{2-N}\frac{h_1}{c^2}\left(2T_4 - cT_3\right),$

$\xi_4 = -\frac{2}{N}f_1\left(3T_2 - lT_1\right) + \frac{1}{c^2}h_1\left((6+c^2)T_4 - 2cT_3\right) + 3sb_1,$

$\xi_5 = -\frac{1}{N}f_1\left((3+l^2)T_2 - lT_1\right) + \frac{1}{c^2}h_1\left(3T_4 - cT_3\right) + \frac{3}{2}sb_1,$

$\xi_6 = a_1\eta^{-1} - 2b_1\eta^2 + \eta^{-1},$

$T_1 = I_{1/2}(l), \quad T_2 = I_{3/2}(l), \quad T_3 = I_{1/2}(c), \quad T_4 = I_{3/2}(c),$

$T_5 = I_{n-1/2}(l), \quad T_6 = I_{n+1/2}(l), \quad T_7 = I_{n-1/2}(c), \quad T_8 = I_{n+1/2}(c),$

$\bar{c}_{m-1} = \frac{-\alpha_m}{2m+1}, \quad \bar{c}_{m+1} = \frac{\alpha_m}{2m+1},$

$\bar{d}_{m-1} = -(m+1)\bar{c}_{m-1}, \quad \bar{d}_{m+1} = m\bar{c}_{m+1},$

$\bar{e}_{m-1} = \frac{m\alpha_m}{2m+1}, \quad \bar{e}_{m+1} = \frac{(m+1)\alpha_m}{2m+1}.$

The expressions for b_1, B_1, δ_1 and δ_2 appearing in eq. (37) and eq. (38) are

$$b_1 = \left(2N(s-1)(T_4(lT_1 + 3T_2(N-3)) - cT_2T_3(N-2))\right)\delta, \tag{A.14}$$

$$\begin{aligned}
B_1 = \; & \frac{-6}{5}\varepsilon N(1-s)\Big[\Big(T_4^2\Big(6lT_1T_2N\Big(2\Big(-3+N-3s+2Ns+(N-3)(s-1)\eta^3\Big) \\
& -9\lambda(N-2)\Big)+T_1^2l^2N\Big(2-2\eta^3+s\Big(4-3N+2\eta^3\Big)+9\lambda(N-2)\Big) \\
& +3T_2^2\Big(N\Big(54-36N+6N^2-2l^2s+9Ns+l^2Ns-6N^2s \\
& +6\eta^3(N-3)^2(s-1)\Big)-3\Big(l^2(N-2)-9N\Big)(N-2)\lambda\Big)\Big) \\
& -cT_3T_4(N-2)\Big(2T_1T_2lN\Big(2+7s-2\eta^3+2s\eta^3-18\lambda\Big)-T_1^2l^2N(s-6\lambda) \\
& +T_2^2\Big(N\Big(-36+12N-9s+l^2s-12Ns+12(N-3)(s-1)\eta^3\Big) \\
& +6\Big(-l^2(N-2)+9N\Big)\lambda\Big)\Big)+c^2(N-2)\Big(2T_4^2(3T_2-T_1l)(T_2Ns \\
& -3T_2\lambda+T_1l\lambda)+T_3^2\Big(2T_1T_2lN(s-3\lambda)+T_1^2l^2N\lambda \\
& +T_2^2\Big(2N\Big(-2+N-s-Ns+(N-2)(s-1)\eta^3\Big)+\Big(-l^2(N-2)+9N\Big)\lambda\Big)\Big)\Big)\Big]\delta^2,
\end{aligned} \tag{A.15}$$

$$\begin{aligned}
\delta = \; & \Big[cT_3(N-2)\Big(T_2\Big(N\Big(2+s-2\eta^3+2s\eta^3-3\lambda\Big)-3\lambda\Big)+3lT_1\lambda\Big) \\
& +T_4\Big(-3T_2N\Big(N\Big(2+s-2\eta^3+2s\eta^3-3\lambda\Big)+6\Big(-1+\eta^3-s\eta^3+\lambda\Big)\Big) \\
& -2lT_1\Big(-6\lambda+N\Big(1-\eta^3+s\Big(\eta^3-1\Big)+3\lambda\Big)\Big)\Big)\Big]^{-1},
\end{aligned} \tag{A.16}$$

$$\delta_1 = 2\Big((lT_1+3T_2(N-3))T_4-c(N-2)T_2T_3\Big)N(s-1)\delta_3, \tag{A.17}$$

$$\begin{aligned}
\delta_2 = \; & \frac{6}{5}\varepsilon N(s-1)\Big[c(N-2)T_3T_4\Big(3NT_2^2(4N(s-1)+3(4+s-6\lambda)) \\
& -2lNT_1T_2(2+7s-18\lambda)+l^2\Big(N\Big(T_1^2-T_2^2\Big)(s-6\lambda)-12T_2^2\lambda\Big)\Big) \\
& +T_4^2\Big(6lNT_1T_2(N(2+4s-9\lambda)-6(1+s-3\lambda))+l^2\Big(2N\Big(T_1^2(1+2s-9\lambda) \\
& -3T_2^2(s-6\lambda)\Big)-3N^2\Big(T_1^2-T_2^2\Big)(s-3\lambda)-36T_2^2\lambda\Big) \\
& -9NT_2^2\Big(2N^2(s-1)+18(\lambda-1)-3N(s+3\lambda-4)\Big)\Big) \\
& +c^2(N-2)\Big(l^2\Big(2T_2^2T_3^2+N\Big(T_1^2-T_2^2\Big)T_3^2-2T_1^2T_4^2\Big)\lambda \\
& +2lT_1T_2\Big(6T_4^2\lambda+N\Big(sT_3^2-sT_4^2-3T_3^2\lambda\Big)\Big)+T_2^2\Big(-2N^2(s-1)T_3^2 \\
& -18T_4^2\lambda+N\Big(6sT_4^2+T_3^2(9\lambda-4-2s)\Big)\Big)\Big)\Big]\delta_3^2,
\end{aligned} \tag{A.18}$$

$$\begin{aligned}
\delta_3 = \; & \big[c(N-2)T_3(T_3N(2+s-3\lambda)+3(lT_1-T_2)\lambda)+T_4(-3NT_2(N(2+s-3\lambda) \\
& +6(\lambda-1))+2lT_1(N(s-3\lambda-1)+6\lambda))\big]^{-1}.
\end{aligned} \tag{A.19}$$