

# Weight-Minimization of Sandwich Structures by a Heuristic Topology Optimization Algorithm

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**Abstract:** A heuristic algorithm for the weight minimization of sandwich plates is presented. The method is based on a preexisting algorithm for the layerwise topology optimization of symmetric laminates under in-plane loads. The presented algorithm uses structural analyses based on finite elements and explicitly accounts for the special sandwich situation. During the optimization procedure the algorithm adds or subtracts material from the layers of the face sheets and the core of the sandwich plate in regions of high or low stresses respectively. The orientation angles of the layers of the sandwich facings are not varied in order to allow easy manufacturing. Several variants of the algorithm are discussed. In contrast to a gradient based optimization algorithm the heuristic procedure needs no sensitivity analyses, thus the computational effort is kept reasonably low. Examples lend credibility to the presented optimization method and show its efficiency in the weight reduction of sandwich plates.

**keyword:** Topology, Optimization, Sandwich structures, Algorithm, Composite materials, Finite element method

## 1 Introduction

With the increasing availability of high-performance computational and software facilities in structural engineering, the application of optimization methods concerning complicated structures becomes more and more computer-orientated as e.g. the recent works of Ibrahimbegovic and Knopf-Lenoir (2003) or Wu, Senocak, Wang, Wu and Shyy (2003) show. In the field of lightweight engineering laminate structures made of layers of fiber reinforced plastics yield a high potential for optimization with respect to the given requirements. There are multitudes of optimization procedures avail-

able. Due to the orthotropic material properties the angular orientations of the laminate plies may be design variables within an optimization procedure as well as the thicknesses of the respective layers. Thus, the overall laminate behavior may be tailored with respect to given needs of bearing capacity and / or stiffness behavior. Local variations of orientation angles and layer thicknesses may reveal some more optimization potential within a so-called parameter optimization (see e.g. Donaldson (1983), Fukunaga and Sekine (1993), Fukunaga and Vanderplaats (1991), Huang and Kröplin (1995)). However, curved fiber patterns and / or non constant ply thicknesses lead to time-consuming and expensive manufacturing processes.

Another well-established optimization strategy is the so-called topology optimization (for a detailed survey on this topic see e.g. Eschenauer and Olhoff (2001)), where the overall topology of a given structure is varied in order to achieve a topology that is well-suited to the resultant load flux. This can be accomplished by altering the given structure in a sufficiently large topology domain in such a way that material is added or removed in high- or low-stressed regions respectively (see e.g. Hinton and Sienz (1995), Xie and Steven (1993)). Considering layered structures like laminated plates or sandwich structures, it may be advantageous to apply a layerwise adaptive topology optimization in the course of an optimization procedure. Such an optimization routine starts with an initial multidirectional laminate layup with sufficient strength. The laminate layup and the ply thicknesses are not varied but the topology of the structure is allowed to be varied in a layerwise manner. This can be done in such a way that within the individual layers in regions with low stresses material is removed, whereas material is added where needed. This strategy eventually results in optimized plies with individual topologies. Thus the overall optimum topology of the given laminate is found by optimizing the topologies of the individual layers. This kind of layerwise topology optimization works in an iterative

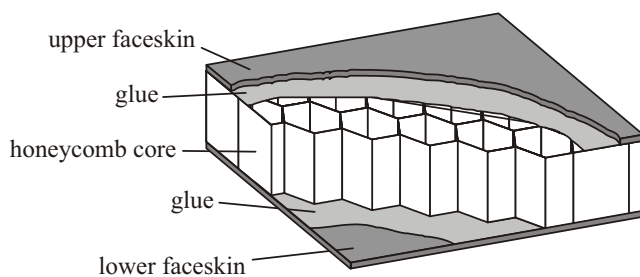
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fashion until an optimal topology of the layered structure is found with respect to the demands of minimum weight and strength criteria, and has been successfully applied by Hansel and Becker (1999 and 2000) by use of a heuristic algorithm for laminate structures under pure in-plane loading. The corresponding algorithm consists of two phases: In the first phase principal stresses and their directions are used for the layerwise detection of regions where material may be removed. In the second phase further low-stressed material, evaluated by application of an appropriate strength criterion, is removed. The algorithm employs the finite-element program code ANSYS<sup>®</sup> (Swanson Analysis Systems (1992)) which carries out the necessary structural analyses. Further optimization software is not needed.

In the present contribution an expansion of the procedure described by Hansel and Becker (1999 and 2000) with applicability to the optimization of sandwich plates with out-of-plane loads is introduced. Usually sandwich structures consist of two thin yet stiff facings made of e.g. laminated materials and a core made of a soft material. The core stabilizes the face sheets and usually is relatively thick with respect to the total sandwich thickness (see Figure 1). The most commonly used core in composite structures is the honeycomb core made of aluminum as shown in Figure 1 as an example, but there are also many other kinds of core materials available, as e.g. foam materials.



**Figure 1** : Exemplary sandwich construction with a typical honeycomb core

The sandwich facings basically carry the bending and in-plane loads whereas the core mainly bears the out-of-plane shear stresses of the sandwich and maintains the distance between the two facings. Thus with increasing core thickness the sandwich structure obtains a higher bending stiffness for the same in-plane stiffness and at

a relatively low specific weight.

For weight minimization purposes, in the present contribution a layerwise topology optimization is performed, in which the topologies of the single laminate plies and the topology of the core material are varied. To ensure a low-cost manufacturing the upper and lower faceskins consist of common prepregs with the symmetric quasi-isotropic lay-up  $[0^\circ/\pm 45^\circ/90^\circ]_S$ . During the optimization the layer orientation angles keep fixed. The ply thicknesses are allowed the discrete values 0 (no material) and  $h$  (whole ply material). The thickness of the core material is also limited to the discrete values 0 and  $H$ . The presented optimization algorithm explicitly accounts for the special structural behavior of a sandwich plate. In the final design the total sandwich structure is well adapted to the resultant local load fluxes. Several variations of the algorithm are tried for the assessment of the applicability of the presented method. Examples show that the introduced methodology yields good and plausible optimization results for sandwich plates and only a relatively low computational effort is needed.

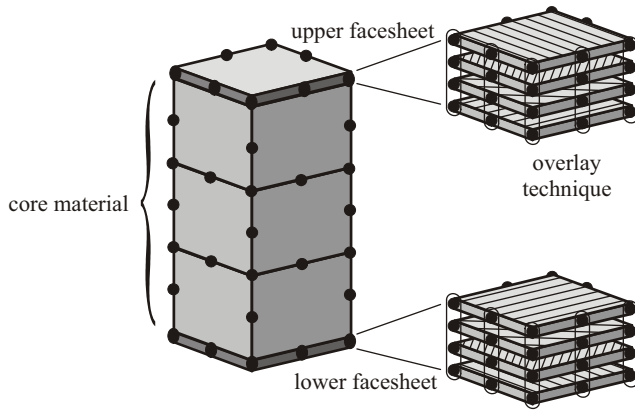
## 2 Analysis Approach

### 2.1 Finite Element Model, Material Representation

Let us consider a sandwich structure with a core to which two layered face sheets made of T300/Epoxy are glued. The facings consist of quasi-isotropic laminates with the symmetric layup  $[0^\circ/\pm 45^\circ/90^\circ]_S$ , wherein each layer is assumed with a thickness of 0.4 mm. For the structural analysis each of the two facesheets is divided into a sufficiently large number of composite finite elements. Instead of one eight-layered composite element only four single layer elements with the orientation angles  $0^\circ$ ,  $\pm 45^\circ$  and  $90^\circ$  are used. These four elements are connected with coinciding nodes so that they lie in the same plane (overlay-technique). This is feasible since we are only interested in the in-plane properties of the face sheets. For the finite element representation quadrilateral bilinear eight-node shell elements with six degrees of freedom per node are used (Figure 2).

The material properties of a typical transversely-isotropic T300/Epoxy in the linear-elastic domain are given in Table 1, where the index 1 denotes the fiber direction.

In order to keep the approach as simple as possible an isotropic linear-elastic material behavior is assumed for the core, with the effective properties given in Table



**Figure 2** : Principle of finite element representation

**Table 1** : Linear-elastic transversely-isotropic material properties of the single ply material

$E_1$	135 000 MPa	$\nu_{12}$	0.27
$E_2$	10 000 MPa	$\rho_{lam}$	1.58 g/cm <sup>3</sup>
$G_{12}$	5 000 MPa		

2. The core is discretized with three volume elements through the thickness with 20 nodes per element and three degrees of freedom for each node (see Figure 2). When each of the upper and lower faceskin plies is discretized into  $n$  single layer plate elements with the material orientation angles  $\theta_1 = 0^\circ$ ,  $\theta_2 = +45^\circ$ ,  $\theta_3 = -45^\circ$  and  $\theta_4 = 90^\circ$ , in all there are  $8n$  plate elements. The core on the other hand is discretized by  $3n$  volume elements.

**Table 2** : Assumed properties of the isotropic core material

$E$	90 MPa	$\nu$	0.27
$G$	35.43 MPa	$\rho_{core}$	0.10 g/cm <sup>3</sup>

## 2.2 Optimization model

The objective of the optimization problem is to minimize the total weight  $m$  of the sandwich construction, corre-

sponding to the following objective function  $f$ :

$$f(x) = m(x) = \underbrace{\sum_i \rho_{lam} A_i h_i}_{\text{plate elements}} + \underbrace{\sum_j \rho_{core,j} V_j}_{\text{volume elements}} \rightarrow \text{Minimum.} \quad (1)$$

Herein,  $A_i$  and  $h_i$  are the areas and thicknesses of the plate elements and  $V_j$  is the volume of the  $j$ -th element representing the core material with the density  $\rho_{core,j}$ . The topology optimization is performed by varying the thicknesses  $h_i$  of the plate elements in a discrete manner and by varying the modulus of elasticity  $E_j$  of the core material. Accordingly, the possible element thicknesses of the plate elements are

$$h_i = \begin{cases} h & \text{full thickness} \\ \varepsilon \cdot h & \text{negligible thickness} \end{cases} \quad (2)$$

Herein the small value  $\varepsilon \ll 1$  is chosen in order to conserve the positive definiteness of the stiffness matrix, but it has almost no influence on the finite element results. The stiffness  $E_j$  of the core material is defined as discrete design variables with the possible values

$$E_j = \begin{cases} E_{core} & \text{full material} \\ \varepsilon \cdot E_{core} & \text{negligible material} \end{cases} \quad (3)$$

Herein the small value  $\varepsilon$  with  $\varepsilon \ll 1$  is chosen again for numerical reasons. The distinction between different values for the core density  $\rho_{core,j}$  in Eq. (1) is necessary since in the present methodology removal of core material is simulated by applying a nearly vanishing stiffness  $E_j = \varepsilon \cdot E_{core}$  in the respective finite element. Thus, the corresponding density of the element has to be modified as well from the real density to a fictitious vanishing value.

The weight minimization by topology optimization is to be performed under strength constraints. This means that during the optimization process sufficient local strength has to be ensured everywhere within the considered sandwich structure. For the necessary strength assessment in the sandwich facings a single-ply stress failure criterion in the form of the well-known Tsai-Wu criterion (see Tsai and Wu (1971)) is applied within each layer at every location. This strength criterion introduces a failure index  $I_{TW}$  for which the expression

$$I_{TW} = \frac{\sigma_1^2}{X_t X_c} - \frac{\sigma_1 \sigma_2}{\sqrt{X_t X_c Y_t Y_c}} + \frac{\sigma_2^2}{Y_t Y_c} + \frac{\sigma_{12}^2}{S^2} + \sigma_1 \left( \frac{1}{X_t} - \frac{1}{X_c} \right) + \sigma_2 \left( \frac{1}{Y_t} - \frac{1}{Y_c} \right) \leq 1 \quad (4)$$

holds. Herein,  $S$  is the shear strength,  $X_t$  and  $X_c$  are the longitudinal strengths in tension and compression and  $Y_t$  and  $Y_c$  are the transverse strengths in tension and compression of the single ply material. The quantities  $\sigma_1$  and  $\sigma_2$  are the stresses in the principal material directions. The employed strength values of the laminate material are given in Table 3.

**Table 3** : Strength values of a typical T300/Epoxy

$X_t$	1 450 MPa	$Y_t$	55 MPa	$S$	90 MPa
$X_c$	1 400 MPa	$Y_c$	170 MPa		

The Tsai-Wu criterion is easy to apply because it does not distinguish between different failure modes, and it takes into account the interaction between the in-plane stresses in different directions. In addition to the consideration of the TSAI-WU failure criterion in both face sheets the strength of the soft core material has to be regarded. In particular here it has to be ensured that the principal shear stress  $\sigma_S$ , which can be calculated from the principal stresses  $\sigma_I$  and  $\sigma_{III}$  as follows

$$\sigma_S = \frac{1}{2}(\sigma_I - \sigma_{III}), \tag{5}$$

must be lower than the maximal allowable shear stress  $\sigma_{S,allow}$ .

For the weight minimization of the sandwich three different optimization models are considered. In the first variant only the elements in the faceskins are allowed to be modified whereas the core material is considered as unremoveable. In the second variant again the faceskins can be modified, but it has to be ensured that at least one single ply element is remaining in the upper and lower faceskins. As in the first optimization variant the core material will not be modified. The third variant offers the largest optimization potential. Material in the upper and lower faceskins and also core material can be removed. Thus, in the first and second variant the  $8n$  shell thicknesses are design variables. In the third presented variant also the elastic modulus of the volume elements is variable so that there are  $11n$  discrete design variables in all.

### 3 Heuristic Topology Optimization Algorithm for Sandwich Plates

#### 3.1 Original Heuristic Algorithm

The iterative algorithm for the topology optimization of laminate structures under in-plane loads as described by Hansel and Becker (1999 and 2000) consists of two phases. Both phases allow a specific removal of elements which are not seriously needed for the load carrying capacity of the laminate.

In the course of the first phase of the optimization algorithm the principal stress directions  $\varphi$  and  $\varphi+90^\circ$  of the respective total laminate elements are determined. The directions are calculated by the well-known relation

$$\varphi = \frac{1}{2} \arctan \left( \frac{2\tau_{xy, mean}}{\sigma_{x, mean} - \sigma_{y, mean}} \right). \tag{6}$$

Herein,  $\sigma_{x, mean}$ ,  $\sigma_{y, mean}$  and  $\tau_{xy, mean}$  are averaged laminate stresses calculated from the stresses  $\sigma_{xi}$ ,  $\sigma_{yi}$  and  $\tau_{xyi}$  of the  $i$ -th ply,

$$\sigma_{x, mean} = \frac{\sum_{i=1}^4 \sigma_{xi} d_i}{\sum_{i=1}^4 d_i}, \quad \sigma_{y, mean} = \frac{\sum_{i=1}^4 \sigma_{yi} d_i}{\sum_{i=1}^4 d_i},$$

$$\tau_{xy, mean} = \frac{\sum_{i=1}^4 \tau_{xyi} d_i}{\sum_{i=1}^4 d_i}, \tag{7}$$

where the quantity  $d_i$  is a thickness indicator and does not represent the real layer thickness. The value  $d_i=1$  denotes a present layer whereas  $d_i=0$  means a removed layer.

Of course the bearing capacity of a laminate ply is best benefited of when the orientation angle  $\theta_i$  matches as closely as possible with the principal stress flux. On the other hand, if a laminate ply exhibits an inappropriate fiber direction, i.e. there is a significant difference between the principal stress directions  $\varphi$  and  $\varphi+90^\circ$  respectively and the layer orientation angle  $\theta_i$ , it is considered as removable in the first phase of the optimization algorithm as long as there are other plies in the same location that are more suited to carry the arising stresses. For instance this means that for an angle  $\varphi=0^\circ \pm \Delta\varphi$  with a tolerance angle  $\Delta\varphi$  the  $0^\circ$  and  $90^\circ$ -plies are mainly loaded whereas the  $+45^\circ$  and  $-45^\circ$ -layers of the  $[0^\circ/\pm 45^\circ/90^\circ]_S$ -laminate are removable. Analogously,

values of  $\varphi=45^\circ \pm \Delta\varphi$  mean unnecessary  $0^\circ$  and  $90^\circ$  -plies that can be removed. The tolerance angle  $\Delta\varphi$  may be adjusted in the course of the iterative process. At the end of the first phase of the iterative optimization algorithm a laminate structure is obtained that avoids unnecessary fiber directions in the layup.

Within the second phase of the optimization algorithm the principal stresses  $\sigma_I$  and  $\sigma_{II}$  in each remaining single ply element are considered in regard of their actual magnitude:

$$\sigma_{I/II} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}. \quad (8)$$

The maximum value  $\sigma_I$  or  $\sigma_{II}$  respectively is then compared to a threshold value  $\sigma_0$ . If the maximum stress value  $\max(\sigma_I/\sigma_{II})$  is below the threshold  $\sigma_0$ , material is removed, because the element is not loaded significantly. In general, the threshold value  $\sigma_0$  will be increased successively until the ultimate load carrying capacity of the resultant laminate structure is attained with correspondingly removed material.

To gain maximum flexibility of the applied algorithm, the option to fill in again material that has been removed before is also included in the procedure. There are in general two possible criteria that allow replacement of material. First, if a removed element yields high principal stresses and a value  $I_{TW} < 1$ , material is added again. Second, if failure occurs at any laminate location, the algorithm searches for layers in the same place that may be better suited to the principal stress directions. If there are such layers they are going to be restored.

### 3.2 Extension of the Algorithm to Sandwich Plates

Considering the extension of the described algorithm to sandwich plates with laminated facings, there are at least three thinkable different strategies for an approach towards the topology optimization:

- i) Throughout the optimization procedure complete removal of material is allowed in both sandwich facings whereas the core of the sandwich remains unaltered. Thus the resultant optimal structure may have an effective connectivity different from that of the core.
- ii) At every location of the laminated facings at least one layer is supposed to remain, whereas all other respective layers are allowed to be removed. The core of the sandwich again is not varied. The resulting optimized topology is continuous and will not have any holes. If there

are any elements in the facings without any layers left after the second phase has ended, the layer with the highest principal stress is included again.

- iii) Another variant is item i) with the extension to keep core material only in regions where facings also remain. This modelling option leads to the identical topologies for the facings and the core. Within this variant the principal shear stresses in the volume elements of the sandwich core are calculated. If in any element these exceed an allowable value the corresponding elements in the facings are restored in order to reduce again the shear stresses. The removal of core material is simulated by sufficiently small values for Young's modulus of the core material. It must also be ensured that there remains core material at every location where the optimization process has left material in the facings. Figure 3 shows a flow-chart of the applied heuristic algorithm in combination with the mentioned three variants concerning the sandwich structure.

The structural analysis gives all stress components for each element. On the basis of these stresses it can be decided which individual plies are not essentially needed for the respective load transfer and thus can be removed locally. The simplest strategy to optimize a given sandwich structure without a modification of the topology would of course be a simple decrease of the layer thicknesses of the face sheets in an iterative manner until a fully stressed structure with respect to an adequate failure criterion is achieved, yet leaving the topology of the structure unaltered. This procedure can be designated as a classical dimensioning strategy (CDS). To demonstrate the advantages of the presented new methodology, it is convenient to compare the results of the heuristic algorithm with a corresponding optimized structure by the mentioned classical dimensioning strategy. We will give the results of this comparison for all presented examples.

## 4 Optimization Examples

The described heuristic algorithm has been applied successfully to several examples. In the first example the optimization procedure was tested on a quadratic sandwich plate under a torsional load by a pair of forces. The second example deals with the same geometric situation of a sandwich plate, but with only one single force. The third example considers an L-shaped sandwich plate under a combination of bending and torsional load induced by one single force.

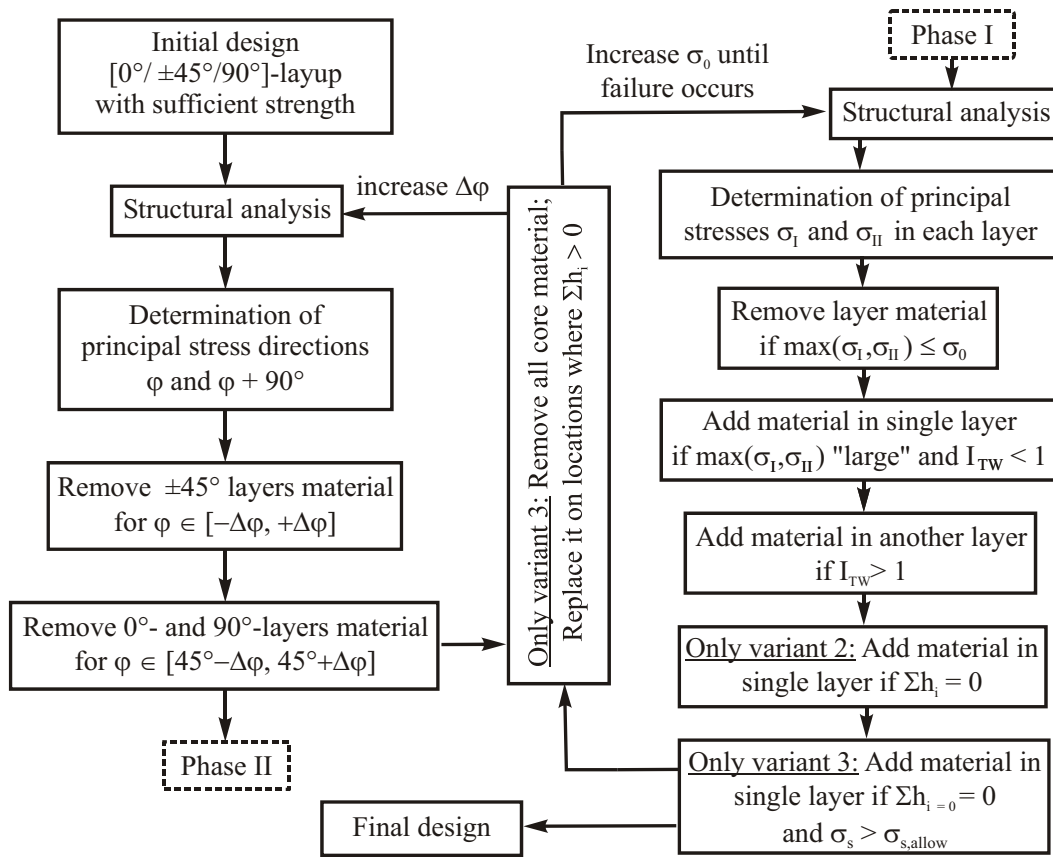


Figure 3 : Flow chart of the optimization algorithm

4.1 Sandwich Plate Under Torsional Load

Let us consider a quadratic sandwich plate with a width and length of 1800 mm and a thickness of 30 mm. The upper and lower faceskins have the quasi-isotropic laminate lay-up  $[0^\circ/\pm 45^\circ/90^\circ]_S$ . As indicated in Figure 4 the plate is clamped on one side.

At the corners of the opposite side the plate is loaded by two forces  $F$  normal to the plane that induce a torsional moment. The forces are kept constant throughout the whole optimization process, the initial designs are moderately stressed. The elements at the load introductions are treated as non-removable. For the required finite element representation the plate is discretized with a  $30 \times 30$  grid. In all there are 7200 shell elements and 2700 volume elements. The discretization is shown in Figure 4. In the initial design the sandwich structure has a total weight of 26.10 kg. This can be apportioned into 16.38 kg in the face sheets and 9.72 kg in the sandwich core, so that the predominant potential for weight reduc-

tion in a sandwich plate resides in the face sheets.

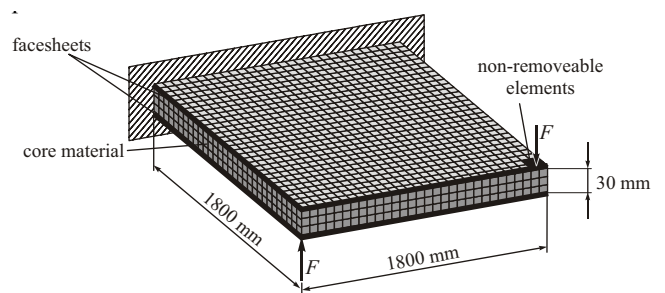
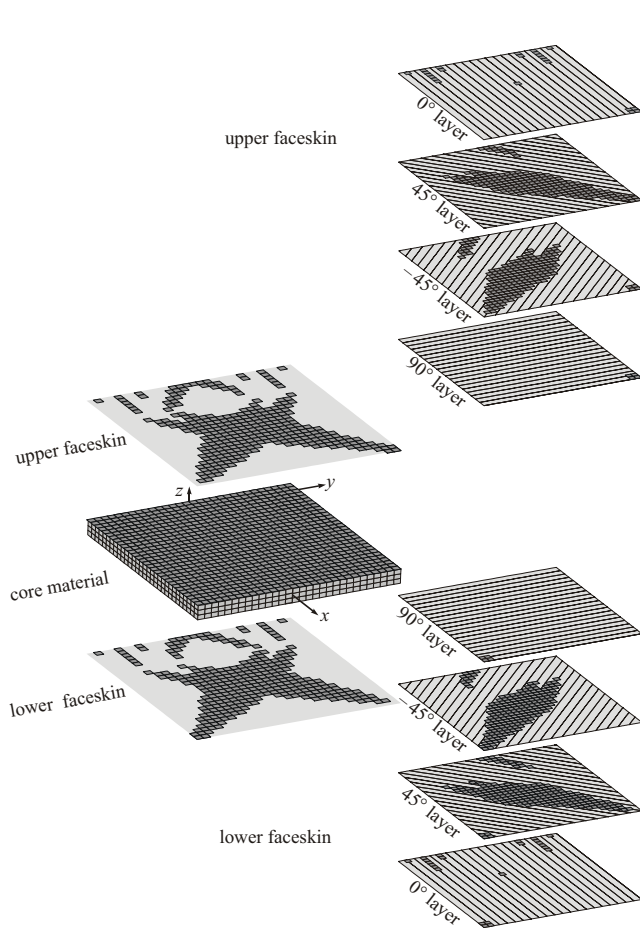


Figure 4 : Sandwich plate under torsional load by two single forces

This will also hold true for all other examples discussed in this paper. Since in the final design the maximum failure index is not exactly  $I_{TW}=1$ , the layer thicknesses will be adjusted to achieve the respective stress level in the face sheets. This virtual layer thickness will be chosen



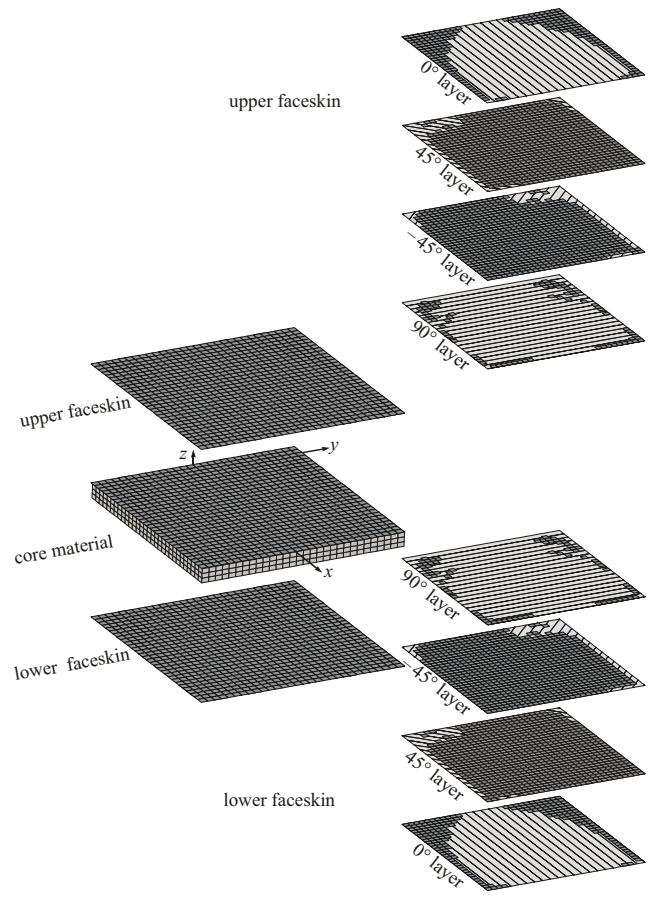
to calculate the final weight. In the first and second optimization model there are 7168 discrete design variables corresponding to all removable face sheet elements. In the third optimization model the number of design variables is increased to 9856.



**Figure 5 :** Optimal design by optimization variant 1 for the sandwich plate under torsional load

In Figure 5 the optimal design for the sandwich plate using the first optimization variant is shown. The total weight can be reduced from 26.10 kg in the initial design to 10.99 kg in the final design. Of course herein the weight of the core material is the same as in the initial design since here no material is allowed to be removed within this variant of the algorithm. The resultant optimal topology of the face sheets is such that due to the bending and torsional load flux a cross-like structure evolves which is well-adapted to the given loading conditions. In comparison, by the second optimization variant only a moderate weight reduction can be reached. The de-

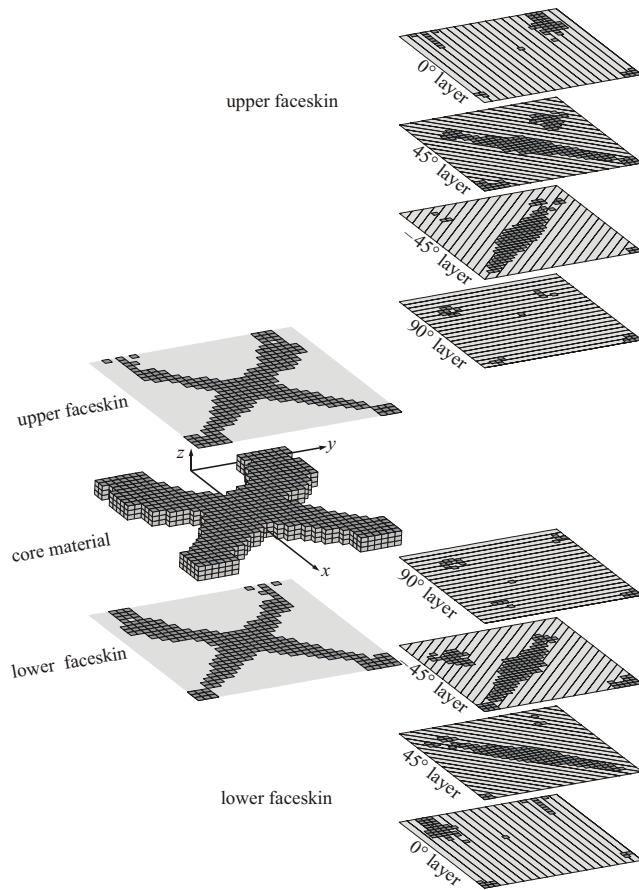
mand that at least one element must remain in the laminate skins leads to load fluxes that are less advantageous and thus the resultant layer topologies are not very convincing (Figure 6).



**Figure 6 :** Optimal design by optimization variant 2 for the sandwich plate under torsional load

Whereas the 0° - and 90° -layers are considerably thinned out, the 45° and -45° -layers remain nearly unaltered. Hence, the weight of the resultant optimal design is still 12.96 kg in all. Nevertheless, in the laminate facings more than three quarters of the initial weight could be saved. The third optimization variant leads to the best results (Figure 7).

As it was the case with the laminate facings within the first variant, a cross-like shape of the overall resultant optimal structure is found. In essence, the +45° - and -45° -plies carry the loads. The laminate weight is reduced to only 1.62 kg (≈10%) where about 50% of the core material can be removed. In all, the resultant sandwich struc-



**Figure 7 :** Optimal design by optimization variant 3 for the sandwich plate under torsional load

ture has a final weight of 6.50 kg, which is only about one fourth of the weight of the initial design. In Table 4 the final weights are listed. Optimizing the given sandwich plate by classical dimensioning while keeping the overall plate topology and dimensions constant, we achieve a final weight of 11.97 kg. Hence it is discovered that variant 1 is only slightly better than the classical method and thus yields only slight advantages for practical purposes.

Variant 2 by comparison is inferior to classical dimensioning and thus drops out of the pool of potential variants of topology optimization procedures. Even though a considerable weight saving was possible in the face sheets by variant 2, a simple reduction of layer thicknesses works better in this case. Thus variant 2 cannot be recommended for practical application. Nevertheless, optimization variant 3 yields clearly improved designs

**Table 4 :** Structural weight of initial and final designs of the sandwich plate under torsional load in comparison to classical dimensioning (\* = inferior to the classical method)

	Initial design	CDS	Optimal design variant 1	Optimal design variant 2	Optimal design variant 3
Weight of face sheets	16.38 kg	2.25 kg	1.27 kg	3.24 kg	1.62 kg
Core weight	9.72 kg	9.72 kg	9.72 kg	9.72 kg	4.88 kg
Total weight	26.10 kg	11.97 kg	10.99 kg	12.96 kg	6.50 kg
Saved weight		0 %	8.21 %	*	45.71 %

in comparison with classical sizing and delivers an optimized sandwich structure with little more than half the weight of the classically optimized plate. Therefore, in the following examples we will only apply optimization variant 3.

#### 4.2 Sandwich Plate Under a Single Load

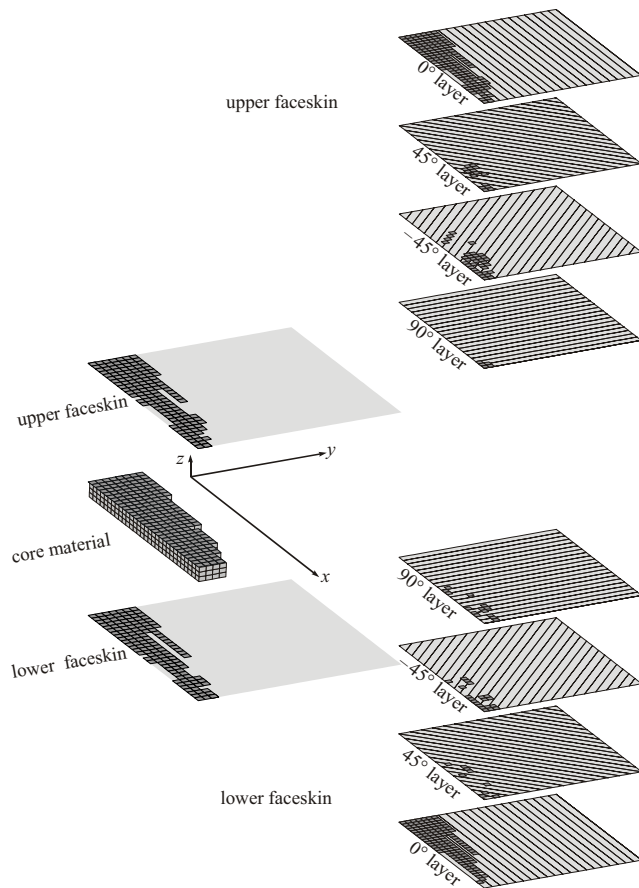
Let us now consider the geometrical situation of a square sandwich plate as aforementioned with the exception that now the plate is subject only to the left-hand single force of Figure 4. We shall only consider optimization variant 3 since this procedure will yield the best weight minimization results. The resultant optimized topology by the heuristic algorithm is displayed in Figure 8.

In Table 5 the optimization results are given. As could be expected, the load carrying capacity is mainly determined by the 0° -layers of both facings whereas the plies with the orientation angles 90°, +45° and -45° may be neglected.

**Table 5 :** Structural weight of initial and final design of the sandwich plate under a single load in comparison to classical dimensioning

	Initial design	CDS	Optimal design (variant 3)
Weight of face sheets	16.38 kg	2.78 kg	0.98 kg
Core weight	9.72 kg	9.72 kg	2.08 kg
Total weight	26.10 kg	12.50 kg	3.06 kg
Saved weight		0 %	75.93 %





**Figure 8** : Optimal design by optimization variant 3 for the sandwich plate under a single force

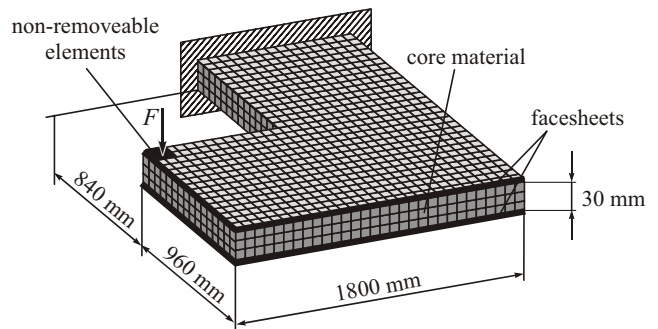
This implies that the influence of torsional effects is of minor importance in the given situation and that the single load is mainly carried by pure bending. The resultant optimized structure has a weight of 3.06 kg, which corresponds to a weight saving of more than 75% in comparison to the classically optimized sandwich plate with a final weight of 12.50 kg.

By application of the heuristic optimization algorithm it is possible to reduce the weight of the face sheets from the initial value of 16.38 kg down to 0.98 kg. This shows that the highest potential for weight reduction is again found in the optimization of the face sheets of the sandwich plate. Still, the core weight is reduced from the initial value of 9.72 kg to a final value of 2.08 kg, which is also a considerable weight reduction. It is again demonstrated that the presented heuristic optimization algorithm yields clearly improved designs with only about

one quarter of the final weight as achieved by the classical design method.

### 4.3 L-shaped Sandwich Plate under Bending Load

In the third example an L-shaped sandwich plate loaded by a single force is considered. The laminated facings have the same layups as aforementioned. The geometry of the plate is displayed in Figure 9.

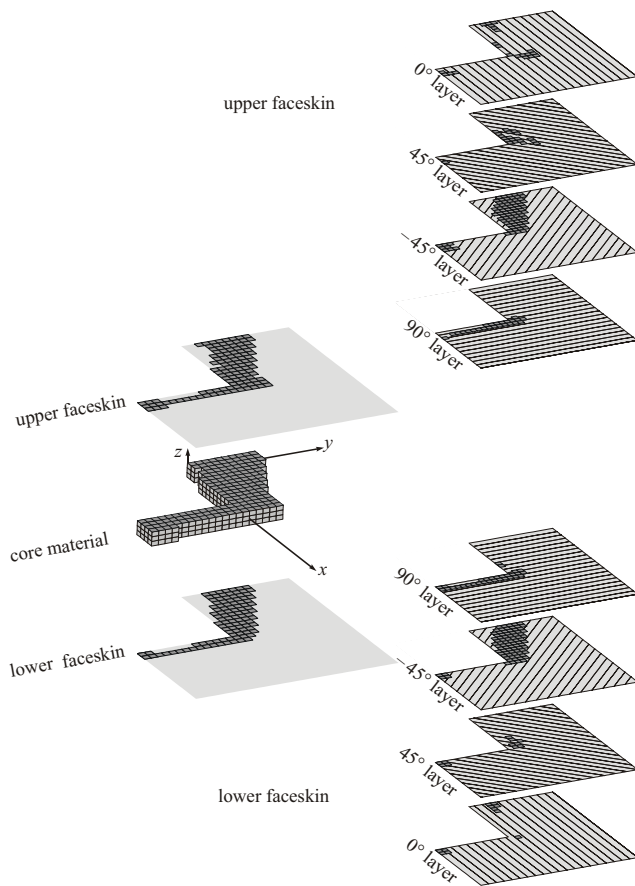


**Figure 9** : L-shaped sandwich plate under a single force

The employed finite element model consists of 5632 shell elements and 2112 volume elements. The initial weight of the structure is 20.42 kg. The optimization is performed according to optimization variant 3, which means that the thicknesses of the face sheet elements and the elastic properties of the volume elements are considered as design variables. The optimal design shown in Figure 10 has a weight of only 2.47 kg (see also Table 6, where the optimization results are given), which corresponds to a weight reduction of more than 73% in comparison with the classical method. Again, the main weight reduction was achieved in the face sheets of the sandwich plate.

**Table 6** : Structural weight of initial and final designs of the L-shaped plate in comparison to classical dimensioning

	Initial design	CDS	Optimal design (variant 3)
Weight of face sheets	12.82 kg	1.79 kg	0.61 kg
Core weight	7.60 kg	7.60 kg	1.86 kg
Total weight	20.42 kg	9.39 kg	2.47 kg
Saved weight		0 %	73.68 %



**Figure 10** : Optimal design by optimization variant 3 for the L-shaped sandwich plate

The comparison with a corresponding sandwich plate that is optimized in the classical way is also satisfactory. Such a classically optimized structure yields a total weight of 9.39 kg, hence the classical design method is clearly inferior to the applied heuristic methodology. In order to transfer the loading, mainly material in the  $90^\circ$ - and  $-45^\circ$ -layers is remaining. It is observed that the load flux is mainly carried by the  $90^\circ$ -plies in the protruding part of the L-shaped plate, which corresponds to a carrying behavior by pure bending. In the remaining part of the sandwich plate the load is mainly carried by the  $-45^\circ$ -layers which correlates to a mixed carrying capacity by torsion and bending. The determined optimal sandwich structure is thus well adapted to the local load fluxes in the pre-determined topology domain and is accurately predicted by the applied heuristic optimization algorithm.

## 5 Conclusions

In the present contribution a well-established algorithm for a layerwise adaptive topology optimization of laminated structures under pure in-plane load was extended on the topology optimization of sandwich plates under bending and / or torsional load. The proposed optimization algorithm leads to weight-minimal sandwich construction designs with a significantly reduced weight. The optimized structures are well-adapted to the given structural needs. The proposed method enables the user to design sandwich structures with only a fraction of the initial weight, thus in this regard the obtained optimization results are very convincing. The presented heuristic topology optimization algorithm is straightforward, easy to apply within a structural analysis program and in contrast to a more formal mathematical optimization the method needs no sensitivity analyses. This keeps the numerical effort reasonably low.

It should be noted, however, that even though the present methodology has proven as quite efficient, a comparison with the results of a classical dimensioning strategy (CDS) may not be most suitable in all aspects. On the one hand such a comparison shows that the presented heuristic algorithm yields clearly better results, on the other hand classical dimensioning is a method that simply decreases the individual layer thicknesses until the structure is fully stressed without altering the overall topology of the structure in any way. Hence, both methods belong to different classes of methodologies. Nevertheless, it is convenient to address CDS as a comparison since it is felt that this is the most simple engineering method that a design engineer would resort to in order to conduct an initial optimization step before applying more sophisticated procedures of optimization.

After optimization some additional “engineering post processing” is still needed to improve the final designs. The laminate facings of the optimized structures can be manufactured easily from standard prepregs with the corresponding topologies of the individual laminate layers before the actual lay-up and subsequent curing.

Finally, it must be mentioned that an optimization of sandwich plates by means of the presented heuristic algorithm can only lead to statements on optimal topologies concerning the in-plane state of stress in conjunction with adequate stress criteria. No statements are possible on higher failure modes like e.g. the wrinkling of the face

sheets or the crushing of the sandwich core as well as the three-dimensional problem of interlaminar stress transfer and stress concentrations within the irregular topologies of the face sheets. These questions, however, require further investigations which cannot be part of the present contribution. Thus, it should be emphasized that the investigation of such failure modes and stress concentration problems is vital for the reliable design of sandwich structures, nevertheless the presented heuristic optimization algorithm can provide the design engineer with a good idea on a well-adapted topological design of a sandwich structure.

In regard of the given advantages the optimization procedure outlined can really be recommended for the direct engineering application.

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