Comparative Study of the Propagation of Jet Noise in Static and Flow Environments

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Abstract: In order to analyze the effect of the background flow on the sound prediction of fine-scale turbulence noise, the sound spectra from static and flow environments are compared. It turns out that, the two methods can obtain similar predictions not only at 90 deg to the jet axis but also at mid- and high frequencies in other directions. The discrepancies of predictions from the two environments show that the effect of the jet flow on the sound propagation is related to low frequencies in the downstream and upstream directions. It is noted that there is an obvious advantage of computational efficiency for calculating in static environment, compared with that in flow environment. A good agreement is also observed to some extent between the predictions in static environment and measurements of subsonic to supersonic. It is believed that the predictions in static environment could be an effective method to study the propagation of the sound in jet flow and to predict the fine scale turbulence noise accurately in a way as well.

Keywords: Fine-scale turbulence noise; sound propagation; numerical simulation; background flow

1 Introduction

Over the last sixty years, jet mixing noise remains the dominant noise source for the aircrafts during taking off at sideline [1,2]. Consequently, it is significant to analyze the characteristics of jet mixing noise and predict the far field sound noise quickly and accurately [3,4]. The difficulty of predicting the noise of jet flow depends on the model of the sound source and the selection of the appropriate background flow to solve the sound propagator. Speaking of the latter, there are so many difficulties to solve the equations under the non-uniform flow conditions [5], and additional efforts are still to be made to analyze the properties of sound propagation of jet mixing noise.

There is now a large body of research devoted to the calculation of the sound propagation for different background flows. As early as 1952, Lighthill applied coordinate transformation in order to describe the effect of uniform flow [6,7]. Phillips considered the effect of non-uniform flow on the propagation of sound waves and replaced the time derivative operator in the original wave equation with the material derivative [8]. However, Doak pointed out that the variables selected as sound source terms by Phillips may also contain non-acoustic components in general turbulent motion [9]. Therefore, Lilley introduced the convection operator into Phillips' equation, considering the influence of background flow on sound propagation [10]. The equation proposed by Lilley is the basis of some famous jet noise computational studies [11,12]. Unfortunately, since Lilley's equation is a third-order partial differential equation, it is impossible to provide the complete solution of the equation, and only its approximate solution can be obtained. Under the condition of frequency approximation, Khavaran obtained the directivity factor of noise propagation in axisymmetric jet field [13]. Silva applied the method of geometric acoustics to improve the accuracy of the calculation results at high frequencies, but still could not get satisfactory results in the full frequency band [2]. Under the assumption of local parallel flow, Tam deduced Green function from linearized Euler equation on reciprocity principle, which could be solved without any approximation [14,15]. Therefore, good agreement between numerical results and experiment data was acquired over a wide range of frequencies and jet conditions [16,17]. In fact, in the direction of 90 deg to the jet axis, the propagation of sound is affected by the flow is so negligible that the sound spectra could be predicted as long as the reasonable sound source model is obtained. And near the 90 deg (specially at the range of 60 deg-120 deg), although it has shown that the predictions are basically consistent under the conditions of local parallel flow and the non-parallel flow by Tam [15], the effect of the local parallel flow conditions on the calculation of sound propagation is still not very clear.

It is assumed that there is no background flow, which means there is no interaction between the sound and the flow, then the sound will be free to radiate to the far field. At this time, the sound prediction only depends on the directivity of the noise source. The first step of this paper is to obtain a reasonable sound source model, based on the model of fine-scale turbulence proposed by Tam et al., and combined with the correction of turbulence characteristic scales suggested by Phillips et al. The second step is to derive the sound prediction under the hypothesis of no flow based on the reciprocity theorem. The final aim of this paper is to compare the predictions under the conditions of local parallel flow (called flow environment) and no flow (called static environment) at mid-angles, and analyze the influence of local parallel flow conditions on the sound propagation by the comparison. It believes that the comparison between the two background flows can provide references for the studies on the characteristics of the propagation of jet mixing noise.

2 Description of the Methodology

Before jet noise spectra are calculated, how to model the sound source is expected to clarify. Following the standard kinetic theory of gases, the mathematical description of the pressure fluctuation of the fine-scale turbulence structure to the surrounding environment is obtained by Tam [14], which is regarded as the sound source. It is also noted that some special turbulence scales are important for the sound source model [18,19]. The turbulence length scale model based on frequency variation proposed by Philip could agree with the turbulence scale variation well [19]. Therefore, based on Tam's sound source model and Philip's turbulence scale model, a sound source function model with good accuracy is obtained, which is used in this paper as $F(\omega)$:

$$F(\omega)=\left(\frac{\pi}{\ln 2}\right)^{\frac{3}{2}}\times\frac{l_s^3(\omega)\exp\left[l_s^2(\omega)\omega^2/\left(4\ln 2\bar{u}^2\right)\right]}{\tau_s\left[1+\omega^2\tau_s^2\left(\bar{u}\cos\Theta/\mathrm{c}_\infty-1\right)^2\right]} \quad (1)$$

ω denotes angular frequency; τ_s denotes turbulence time scale [14]; l_s denotes turbulence length scale [19]; Θ denotes observation polar angle as shown in Fig. 1; c_∞ denotes ambient sound velocity; u is the velocity components in the x -direction. And all of the mean flow variables are denoted by an overbar.

The positions of the sound source and the observation point are replaced in the reciprocal relation, where the sound source and the acoustic observation point exchange position. Based on the reciprocal theorem, the Green function of linearized Euler equation in cylindrical coordinates is deduced by Tam in the case of locally parallel flow jets [15] as:

$$
\begin{aligned}
&-\bar{\rho}\left[i\omega u_a+\bar{u}\frac{\partial u_a}{\partial x}\right]-\gamma\bar{p}\frac{\partial p_a}{\partial x}=0\\
&-\bar{\rho}\left[i\omega v_a+\bar{u}\frac{\partial v_a}{\partial x}-\frac{d\bar{u}}{dr}u_a\right]-\gamma\bar{p}\frac{\partial p_a}{\partial r}=0\\
&-\bar{\rho}\left[i\omega w_a+\bar{u}\frac{\partial w_a}{\partial x}\right]-\frac{\gamma\bar{p}}{r}\frac{\partial p_a}{\partial \phi}=0\\
&-i\omega p_a-\bar{u}\frac{\partial p_a}{\partial x}-\left[\frac{1}{r}\frac{\partial(v_a r)}{\partial r}+\frac{1}{r}\frac{\partial w_a}{\partial \phi}+\frac{\partial u_a}{\partial x}\right]=\frac{1}{2\pi}\delta(\boldsymbol{x}-\boldsymbol{x}_0)
\end{aligned}
\tag{2}
$$

v and w are r- and ϕ-directions; p and ρ is pressure and density respectively; γ is the ratio of the specific heats of the gas. A subscript a is used to denote the adjoint variables. $\boldsymbol{x}_0$ denotes the observer position of the original problem.

The sound sources are located in far field in the reciprocity relation. The incident waves are scattered and refracted in the flow environment by the time they reach the vicinity of the jet flow, which is described as the solid line in Fig. 1. The calculation of the far field spectra are expected to be conducted by a sound scattering problem with the effect of mean flow on the sound radiation, which could be referred as [15]. On the contrary, there is no scattering or refraction in the static environment as the sound radiates from the far field to the jet zone, which is the hypothetical situation used here. The incident waves just arrive at the jet zone directly, shown as the dashed line in Fig. 1.

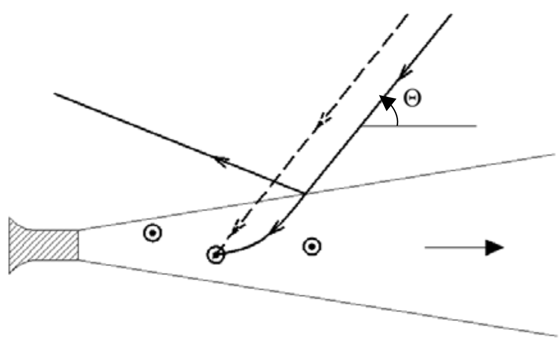


Figure 1: Sound radiation in adjoint condition

For the static environment, the variables of the adjoint Green function related to mean flow are equal to 0, and the density and the pressure in the jet zone is same as those in the far field. In other words, these variables, which should have varied with the spatial coordinates, are supposed to be substituted by $\bar{u}=\bar{v}=\bar{w}=0$, $\bar{\rho}=\bar{\rho}_\infty$ and $\bar{p}=p_\infty$. As a result, Eq. (2) could be rearranged as the following simple form:

$$\bar{\rho}_a^*=0,\qquad \boldsymbol{v}_a^*=\frac{ic_\infty^2}{\omega}\nabla p_a^* \tag{3}$$

$$\nabla^2 p_a^*+\frac{\omega^2}{c_\infty^2}p_a^*=\frac{i\omega}{2\pi c_\infty^2}\delta(\boldsymbol{x}-\boldsymbol{x}_0) \tag{4}$$

A subscript * is used to denote the variables in the static environment. It is easy to obtain the solution of Eq. (4), which satisfies the outgoing wave condition:

$$p_a^*(\boldsymbol{x},\boldsymbol{x}_0,\omega)=-\frac{i\omega}{8\pi^2 c_\infty^2}\frac{e^{i\omega|\boldsymbol{x}-\boldsymbol{x}_0|}}{|\boldsymbol{x}-\boldsymbol{x}_0|} \tag{5}$$

Therefore, after acquiring the model of the source terms and the solution of adjoint Green function, the spectrum for the observer at x_0 could be written as:

$$S^*\left(x_0,\omega\right)=2\pi\times\int_{-\infty}^{+\infty}F\left(\omega\right)\left|p_a^*\left(x,x_0,\omega\right)\right|^2 dx \qquad (6)$$

It is Eq. (5) and Eq. (6) that is the equations used to predict the far field sound pressure level of fine-scale turbulence noise in static environment. It is noted that the calculation in static environment costs 20 times less than that in flow environment due to the avoidance of the complicated two-order ordinary differential equation which is used in [15].

3 Numerical Results

The turbulence field should be simulated before the computation of the sound prediction could proceed. Fig. 2 shows the geometry and general dimensional information of the nozzle considered in this work. The computation domain chosen for the fluid computations consists of an interior nozzle domain, before body domain and jet domain which is extended 100*Dj* axially and 20*Dj* radially, described as Fig. 3. *Dj* denotes the nozzle diameter. The ICEM CFD software is used to generate the two-dimensional structured grids including about 90,000 nodes. The simulation of the fluid field is performed by density-based and axisymmetric swirl solver and standard $k-\varepsilon$ turbulence model with a second order accuracy from the commercial software, ANSYS FLUENT. The boundary condition of inlet is chosen by "Pressure-inlet" for compressible gas, where the total pressure is set as 173,190 Pa (0.9 Ma).

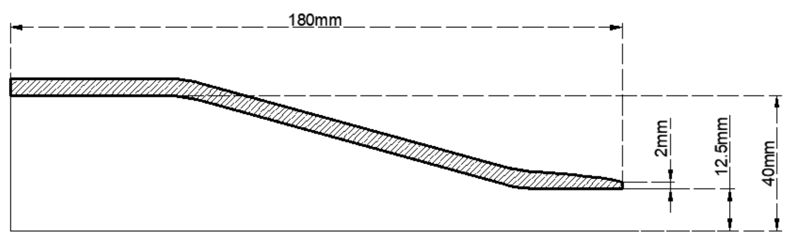


Figure 2: Nozzle geometry of axisymmetric jet flow

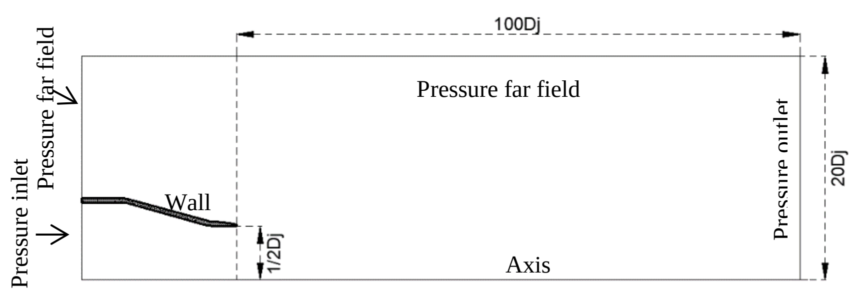


Figure 3: The schematic of computation domain

The power spectra in static environment and in flow environment are calculated by the variables acquired from turbulence field and the formulations referred as Section 2 and [14]. In this way, the sound pressure levels are obtained from different observer polar angles at the distance of 50*Dj*. The observer positions are located in the polar range of 50 deg to 150 deg, considering that the fine-scale turbulence noise is dominant among 60 deg to 120 deg [3]. The comparisons between the results in two environments for different frequencies are shown as following:

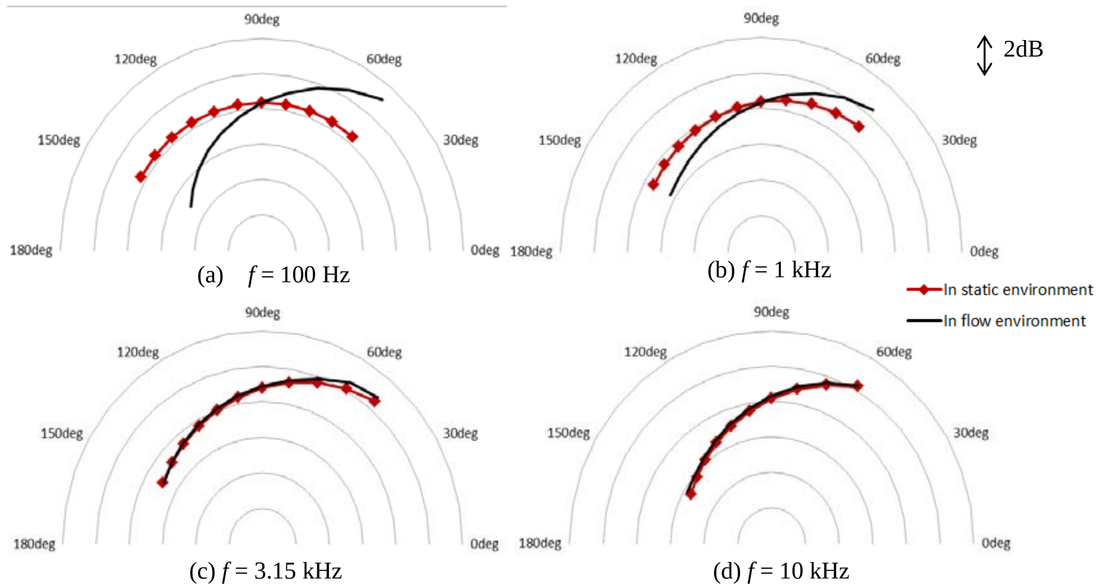
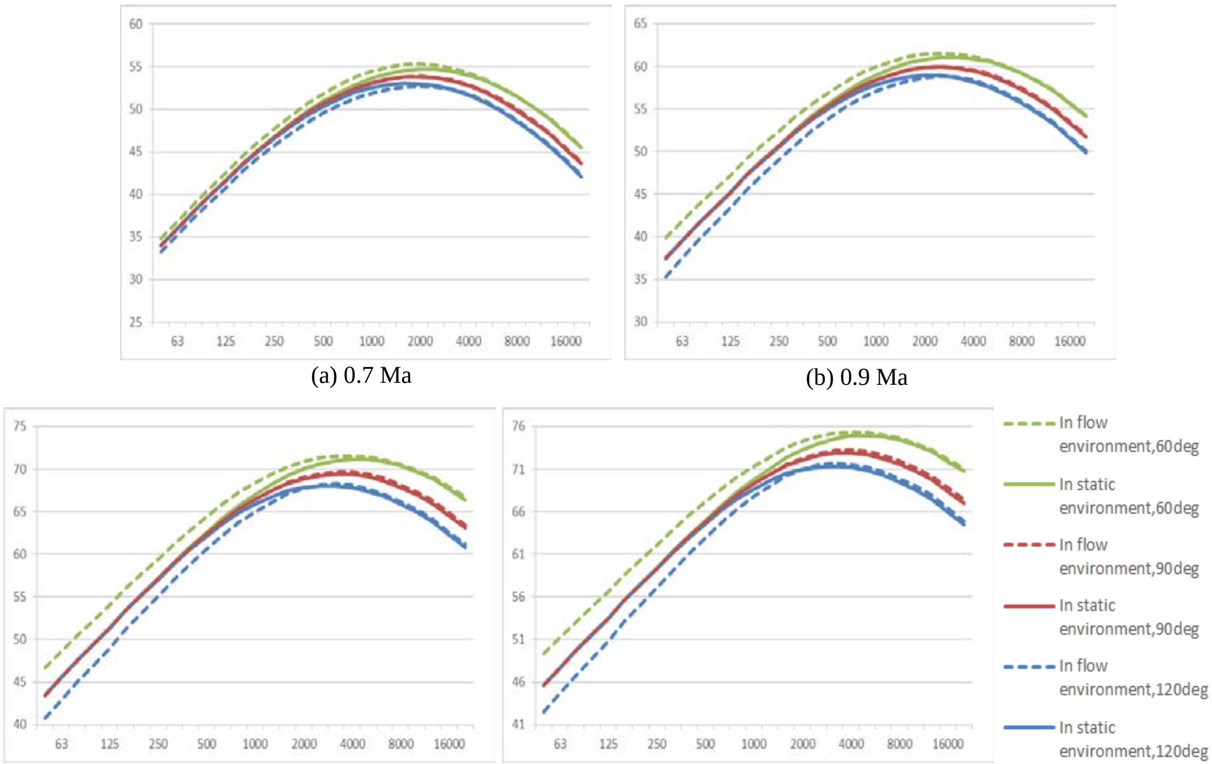


Figure 4: Comparisons between the directivity of sound source in static and flow environments

According to the calculation method provided in Section 2, the sound spectra under the hypothesis of the static environment are obtained, as shown as the red line in Fig. 4. As mentioned above, the sound prediction in the static environment depends entirely on the directivity of the sound source. Therefore, it can be seen from Fig. 4 that the noise source generated from small-scale turbulent structure is characterized as non-direction at low frequency, while the directivity shifts to the downstream with increasing frequencies. Compared the sound spectra between the two kinds of background flow, as expected, the sound spectra are basically the same in the 90 deg direction. Besides that, the discrepancies between them are related to the frequencies in other directions. Specifically, the predictions of low frequencies in the static environment are greater than those in the flow environment in the upstream direction ($\Theta > 90$ deg), and smaller in the downstream direction ($\Theta < 90$ deg). This is probably because the movement of jet flow concentrates the energy of the turbulence noise in the direction of downstream. Therefore, the directivity in the flow environment is more obvious in the downstream. On the other hand, the results of high frequencies in the two background flows are basically the same in the polar angles of 60 deg-120 deg. This suggests that within this range, the far field sound spectra of high frequencies are negligibly affected by the background flow.



(a) 0.7 Ma (b) 0.9 Ma (c) 1.3 Ma (d) 1.5 Ma

Figure 5: Comparisons between results in the two background flows in 4 Mach numbers

Several more jet conditions are expected to compare in the two environments to analyze the influence of the background flows on the jet noise prediction. The comparisons at 60 deg, 90 deg and 120 deg polar angle for 4 Mach numbers (0.7 Ma, 0.9 Ma, 1.3 Ma and 1.5 Ma) in a wide range of frequencies are showed as Fig. 5. It is clear to see that the results in the two environments have the similar performance for different Mach numbers. From 0.7 Ma to 1.5 Ma, the predictions from the flow environment and the static environment agree well with each other for a wide range of frequencies at 90 deg; at 120 deg, the results from two methods agree well for frequencies $\geq$ 1.25 kHz; and the same agreement could be observed only for frequencies $\geq$ 3.15 kHz at 60 deg. It is noted that the Mach number also plays a critical role in the influence of mean flow on sound radiation. Specifically, the greatest discrepancies are 0.9 dB and 3.9 dB for 0.7 Ma and 1.5 Ma respectively. With all of these analyses, it could be believed that the mean flow has significant influence on sound radiation for low frequencies at high Mach numbers.

4 Comparisons with Experiment Data

The calculation in static environment may be expected as a reliable method due to the efficient computation and the agreement with the prediction of flow environment to some extent. In order to verify the effective range of this method, the calculated results in static environment are compared with experiment data. Therefore, a number of experiments of unheated jets are performed subsonically through supersonically over a wide range of observation positions in the anechoic chamber. Descriptions of the anechoic facility, the jet simulator, and other details are provided in Ref. [20]. There are 8 microphones with a polar angle Θ range from 45 deg to 130 deg. The array is at a constant polar radius of $R = 50Dj$ ($Dj = 0.025$ m) from the origin of the coordinate system located on the jet centerline at the nozzle exit plane. The photograph of the microphone array and the nozzle are shown as Fig. 6 and Fig. 7.

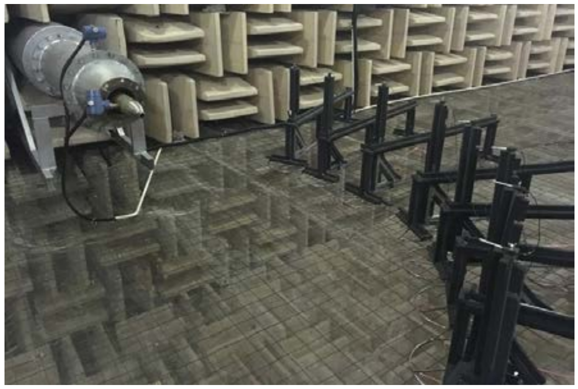


Figure 6: The photograph of the microphone array

Figure 7: The photograph of the nozzle

The unheated subsonic jet predictions in both environments are examined with experiment data, shown in Fig. 8 and Fig. 9.

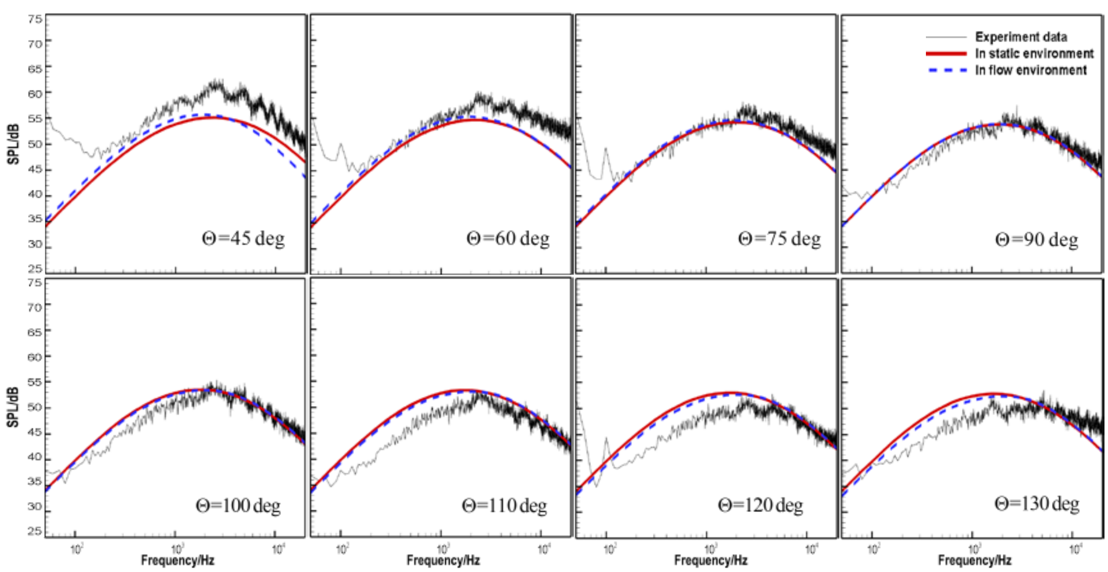


Figure 8: Predictions compared with measurement for the Mj = 0.7

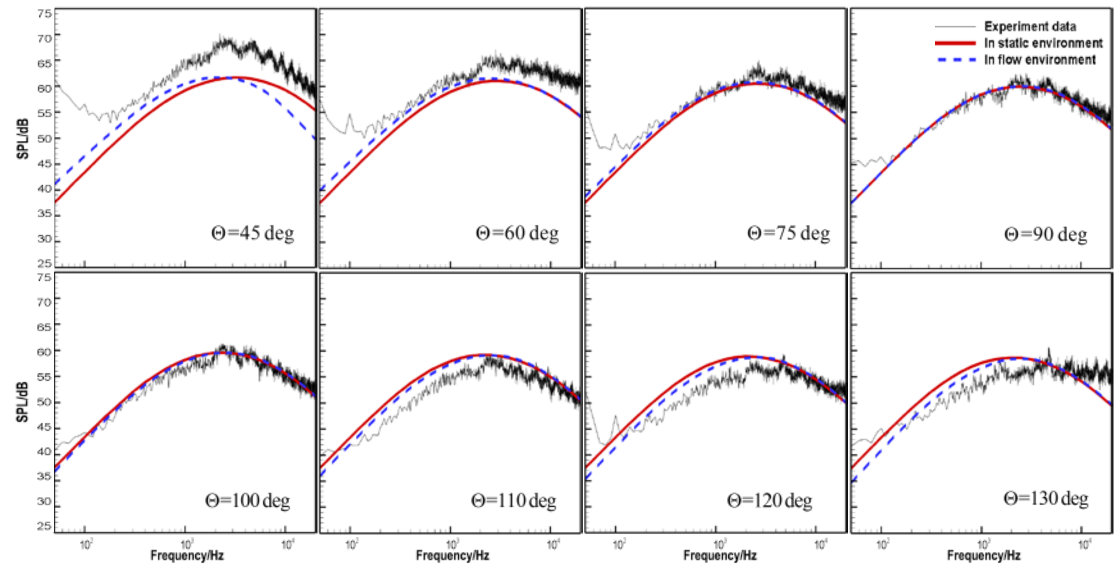


Figure 9: Predictions compared with measurement for the Mj = 0.9

From Fig. 8 and Fig. 9, the predicted peak magnitudes from both environments are less reliable at $\Theta=45\,\mathrm{deg}$ as they are close to the silence cone [14]. At the other downstream directions ($\Theta < 90\,\mathrm{deg}$), the prediction agrees very well with measurement from mid- to high frequencies but underpredicts the noise by 3dB at lower frequencies. The possible reason has already been given in Section 3. Similarly, at the upstream directions ($\Theta < 90\,\mathrm{deg}$), the prediction overpredicts the noise by 3 dB at lower frequencies.

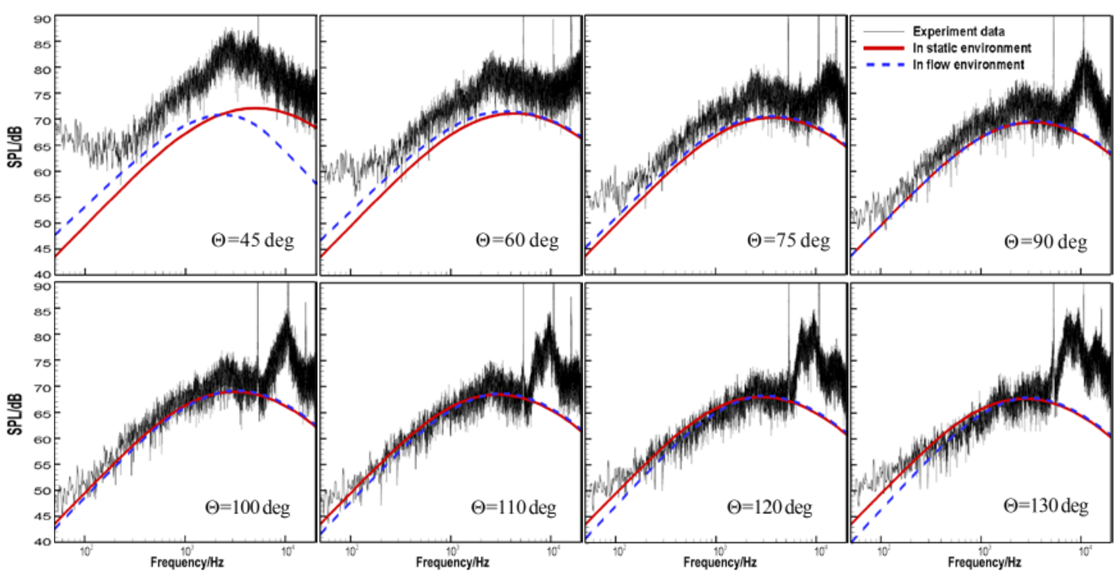


Figure 10: Predictions compared with measurement for the Mj = 1.3

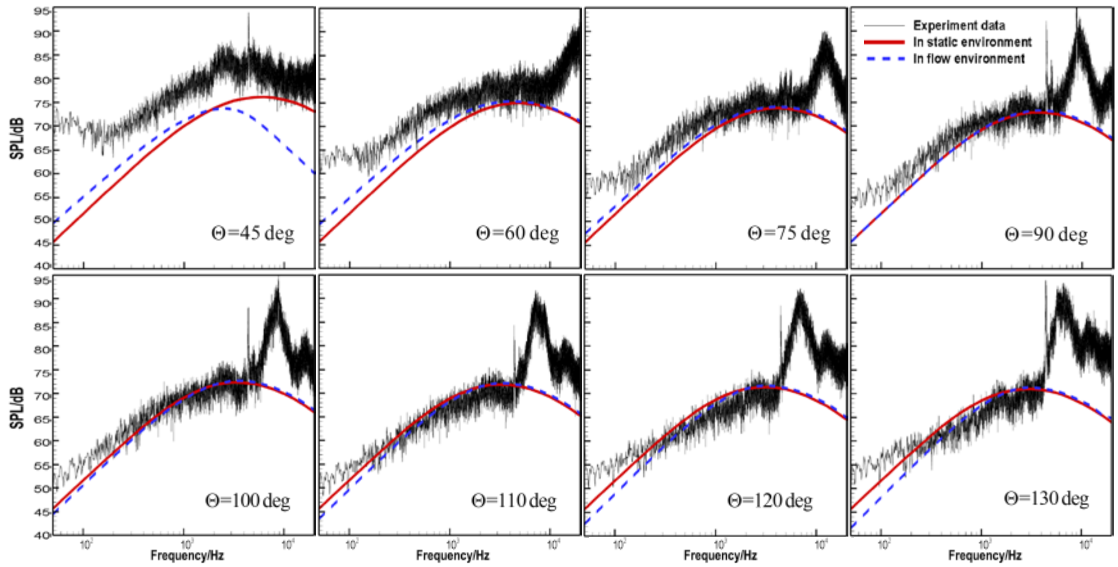


Figure 11: Predictions compared with measurement for the Mj = 1.5

The unheated supersonic jet predictions in both environments are examined with experiment data, shown in Fig. 10 and Fig. 11. Similar trends are observed relative to the subsonic cases at low to midfrequencies. The differences between the calculated and measured at high frequencies mainly due to the presence of broadband shock associated noise and screech, which should be ignored in this comparison. In general, the results of fine scale turbulence noise calculated in static environment relative to experiment data differ by ± 3 dB at the peak of the spectra for a wide range of Mach numbers, although calculating in static environment does not obey the rules of physics. Because the procedure of calculating in static environment circumvents solving complicated differential equations, shown as 15, there is an obvious advantage of this method for computational efficiency. Therefore, the method of calculating in static environment could be applied as a fast and accurate method to predict far field sound spectra of fine-scale turbulence noise to some extent.

5 Conclusions

In this paper, the mathematic derivation of the fine scale turbulence noise in static environment is presented based on reciprocity principle. Significant simplifications are made to Tam and Auriault method [15], circumventing the complex difference equation and reducing computational time prominently.

The prediction of fine scale turbulence noise in static environment shows that the turbulence sound source could be characterized as non-direction for low frequencies, and there is relatively obvious directivity near 60 deg at high frequencies. Compared the sound spectra between the static environment and the flow environment, a good agreement is observed for the full frequencies at polar angle of 90deg and only for mid- and high frequencies at the other polar angles. The discrepancies at other locations could give the reference about how the jet flow affects the sound radiation of fine scale turbulence noise. In particular, it could be concluded that due to the effect of the background flow, the jet noise prediction for low frequencies is increased in the downstream directions, and decreased in the upstream directions; and the discrepancies turns larger with increasing the Mach number.

A number of experiments are performed from subsonic to supersonic jet to verify the predictions from the static environment. It turns out that the evolution of the spectral shapes from the upstream through downstream direction shows a positive trend with respect to the results from the measurement, and the peak magnitudes are within 3 dB of the measurement. Therefore, although it is not true to calculate the turbulence sound field in static environment, there is still a good reason to apply this method to predict the fine scale turbulence noise to some extent due to the sufficient accuracy and the less computational time. Certainly, from a practical standpoint, we believe the present work should be verified by more jet conditions in the future.

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