

A Simplified Formulation to Estimate Influence of Gearbox Parameters on the Rattle Noise

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Abstract: Occurrence of gear rattle in transmission systems can result in severe vibration and noise, which in applications such as automobiles is an important source of user discomfort. As a result, the reduction of the rattling noise has attracted lot of concerns. The rattling noise level is affected by several gearbox parameters, an understanding of which is essential to prevent the expensive design modifications at later stages of product development. To develop such understanding at the gearbox design stage, this paper analytically evaluates the gear parameters' effect on the root mean square of the wheel gear acceleration under idling condition, which is known to be linearly correlated to the rattling noise level. Therefore, this evaluation allows for an investigation of the gear parameters' influence on the rattling noise as well. This method is then verified by comparing the analytical results with the simulation results from a dynamic model built in SIMPACK as well as previously published experimental results. Thus, the proposed analytical evaluation method can optimize the gearbox specifications at the design stage to reduce the gear rattle noise level.

Keywords: Rattle noise; gear parameter; analytical evaluation; backlash; dynamic model

Nomenclature

p, w Subscripts denoting pinion gear and wheel gear

b_{ls} Half backlash of the gear pair

d Gear diameter

k_g Gear meshing stiffness

I Inertia of the gear

ζ Dynamic transmission error

ζ Damping factor of the meshing gears

ω Rotational speed in radians per second

ω_m Nominal driving speed in radians per second

$\Delta\omega$ Amplitude of the speed fluctuation

1 Introduction

Gear rattle is a common occurrence in transmission systems, such as those in vehicles and marine vessels, under lightly loaded conditions. When gear rattle occurs, the gear teeth oscillate within the backlash while undergoing several consecutive teeth impacts that produce a high level of vibration and noise. The noise resulting from gear rattle is an importance source of passenger discomfort in automotive applications [1]. The occurrence and severity of gear rattle induced impacts and emitted noise is influenced by different components of drive-trains such as the diesel engine, the flywheel, the clutch, and the gearbox. Recent technological improvements have significantly reduced the size of engines and flywheel, as well as

mitigated the frictional torques through improvements in bearings and lubricants technology, which can increase the occurrence of gear rattle thus bringing it in focus [2-4]. Therefore, to avoid expensive modifications later, it is crucial to consider the gear rattle during the design stage of the gearbox itself.

Previous research has investigated the influence of the various transmission components on the gear rattle. The effect of the inertia of the flywheel, as well as the stiffness and hysteresis of the clutch on the gear impact has been investigated through nonlinear multi-degree of freedom (DOF) lumped parameter models of the transmission [5-7]. It was shown that by increasing the flywheel inertia, reducing the clutch rigidity and selecting adequate clutch hysteresis torque, the speed fluctuation transmitted from the diesel engine to the gearbox can be reduced, thereby reducing gear rattle severity. Gearbox parameters can also affect the rattle noise level [8-10], e.g., an increase of backlash and gear meshing stiffness can produce more rattling noise, while an increase of damping factor and the wheel gear inertia can reduce the noise level. To quantify the effect of different gear parameters on the rattling noise level, both numerical modeling and experimental based approaches have been developed. For example, a two-DOF lumped parameter model was developed for a spur gear pair to explore the effect of gear backlash and inertia in [9,11]. However, it is time-consuming to assess every gear parameters' effect on the rattle noise by individual numerical simulation that accesses the effect of each parameter for different gearboxes used in varied drive-train configurations and different investigations using different gearbox parameters do not provide an insight regarding the final expected rattle noise as a combination of all these input variables. Experimental based approaches have been carried out to investigate the effect of the variation of gear parameters on rattle noise as well [8,9]. However, an experimental investigation of gear parameters on rattle noise requires manufacturing, installation and commissioning of gears with different dimensions, which is both time-consuming and expensive. Further, the experimental results are influenced by errors in manufacturing and installation, such as gear profile error, shaft misalignment and eccentricity [1,3,12]. Therefore, an analytical approach to evaluate the important gear parameters' effect on the gear rattle noise is required. According to [9,13,14], the root mean square (RMS) value of the wheel gear acceleration is proportional to the noise level during the gear rattle and has been used as an indicator of the sound perception. Since the pure rotation motion of the gear bodies produce negligible noise, the rattle noise can be attributed to the period when gear impact occurs. Furthermore, even though several consecutive impacts can occur gear teeth face at one side before the gear teeth mesh together again, the first impact has the most dominant energy as the subsequent impacts can dampened quickly [15,16]. Thus, it can be deduced that the rattling noise level is mainly determined by the RMS value of the wheel gear acceleration during the first impact period, which is used in this work to estimate its value analytically.

In this paper the acceleration magnitude due to gear rattle is analytically evaluated to investigate the contribution of different gear parameters on the gear rattle. A two-DOF model developed for a typical drive-train configuration is simplified to a single-DOF equation of motion to describe the teeth impact. This simplified model sufficiently describes the wheel gear vibration within the backlash. The analytical solution obtained from this model is used to evaluate the RMS value of the acceleration of the wheel gear due to teeth impacts during gear rattle. This proposed formulation to access the gear rattle noise is verified using a dynamic model of a typical transmission system built in SIMPACK software environment. Previously reported experimental results in [8,9] are also shown to agree with the proposed method. Thus, the proposed evaluation method for estimating the acceleration RMS value during gear impacts can quantify the influence of gear parameters on the rattle noise, and can thus provide an easy design guideline to determine the expected severity of gear rattle at the gearbox design stage.

2 Mathematical Modeling of a Simplified Transmission

2.1 Gearbox Parameters

2.1.1 Gear Meshing Stiffness and Backlash

During operation the gear teeth can bend along the line of action due to material elasticity. The gear teeth meshing stiffness k_g can be determined using the standards mentioned in [17]. The backlash $2b_{ls}$ is

the amount of clearance between two mating pair of gear teeth that is usually kept for allowing an unobstructed operation, as shown in Fig. 1. However, torque and speed fluctuations from a prime-mover can result in the teeth to vibrate within the backlash, which results in the rattling noise.

2.1.2 Bearing Frictional Torque

Palmgren model was developed based on laboratory testing of different types and sizes of bearings to describe the bearing frictional torque due to rolling, sliding and lubricant loss effects as [18]:

$$T_{bf} = T_0 + T_1 \quad (1)$$

where, T_0 is a load independent component determined by the lubricant properties and a function of angular velocity, while T_1 is a load dependent component determined by the radial and axial load which can be assumed to be a constant for a given load. The dependence of these frictional components on bearing type, dimensions, load, rotational speed, and lubricant properties is described in [18,19].

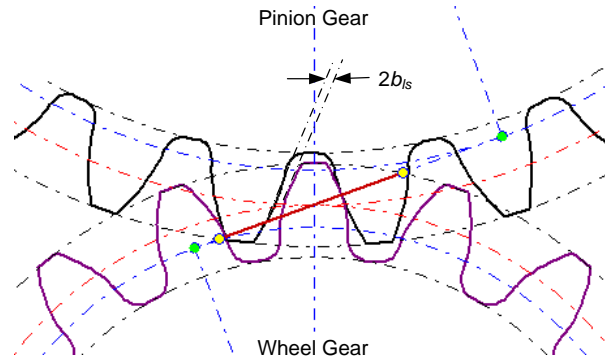


Figure 1: Gear teeth meshing for a gear pair

2.1.3 Lubricant Churning Losses

Grease lubrication is commonly used in medium-sized gearboxes, wherein all the gears are either partially or completely submerged in the lubricant. This results in a power loss component due to lubricant churning. The churning loss is a complex process for which a dynamic model was developed in [20,21], where the churning torque can be expressed as a function of the grease viscosity and immersion of the gears in the lubricant, which is dependent on dimensions of gears and the gearbox housing. The churning torque function is given as

$$T_{ch} = 0.5\rho_{lub}\omega^2r_c^3S_mT_m \quad (2)$$

where ρ_{lub} is the density of the lubricant oil, S_m is the immersion area of the gear in the lubricant, and T_m is the dimensionless drag torque. At low speeds, T_m can be obtained as

$$T_m = 1.36(h/d_c)^{0.45}(V_0/d_c^3)^{0.1}Re^{-0.21}Fr^{-0.6} \quad (3)$$

where h is the immersed depth of the gear in the lubricant, d_c is the pitch diameter, V_0 is the volume of the lubricant oil, the Reynolds number $Re = \omega r_p^2 / \nu_{lub}$, Froude number $Fr = \omega^2 r_p / \nu_{lub}$ and ν_{lub} is the dynamic viscosity of the lubricant oil.

2.2 Transmission Model under Idling Conditions

A typical five-speed manual transmission [22] is shown in Fig. 2. In this configuration, the output gears are usually rotating freely. To obtain a given gear shift configuration which corresponds to a given speed ratio, one of the three synchronizers (shown in blue color) is used to lock different gears with the output shaft. However, the 4th gear speed is obtained by directly coupling the input shaft with the output

shaft using the first synchronizer from the left. Under idling condition, all gears on the output shaft are in a free flight condition wherein the only resistance to their motion is provided by a low drag torque due to the bearing friction and lubricant churning.

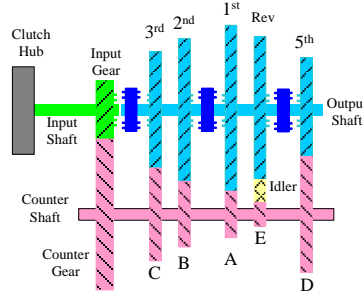


Figure 2: A typical five-speed manual transmission

To set up the dynamic model of the meshing gears, a two-DOF model consisting of the equivalent pinion gear and the corresponding wheel gear as shown in Fig. 3 is formulated. Taking the example of the 3rd gear pair, the inertias on the input and counter shafts, including gears A, B, D, E, counter gear, input gears and clutch hub, are reflected onto the pinion gear C to obtain the equivalent pinion gear inertia I_p . The governing equations of the two-DOF model thus obtained can be written as

$$I_p \ddot{\theta}_p = T_p - r_{bp} f_N - r_{bp} c_g \dot{\xi} - T_{bfp} - T_{chw} \quad (4a)$$

$$I_w \ddot{\theta}_w = r_{bw} f_N + r_{bw} c_g \dot{\xi} - T_{bfw} - T_{chw} \quad (4b)$$

where T_p is the driving torque applied on pinion gear, k_g is the gear meshing stiffness, c_g is the damping coefficient of the gear pair, and f_N is the contact force acting on the gear teeth surface. f_N can be expressed as:

$$f_N = k_g \begin{cases} \xi - b_{ls} & \xi > b_{ls} \\ 0 & |\xi| \leq b_{ls} \\ \xi + b_{ls} & \xi < -b_{ls} \end{cases} \quad (5)$$

where $\xi = r_{bp}\theta_p - r_{bw}\theta_w$ is the dynamic transmission error (DTE).

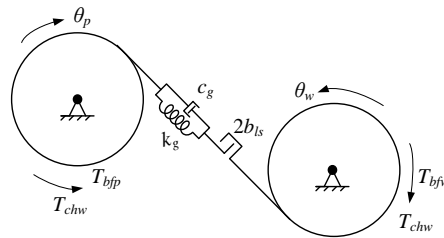


Figure 3: A two-DOF model for the transmission

3 Analytical Evaluation of the Gear Teeth Impact

3.1 Modeling of the Gear Teeth Impact

The unbalance of the rotational components in an internal combustion engine (ICE) can excite speed fluctuations in a transmission system. For example, in a six-cylinder four-stroke ICE, the input angular speed at the pinion gear as applied by the ICE can be described as [8]

$$\omega = \omega_m + \Delta\omega \cos(3\omega_m t) \quad (6)$$

where the pinion gear speed ω fluctuates at three times the input rotational frequency ω_m . As the gear impact is determined only by the speed fluctuation component, the mean gear speed can be ignored in the subsequent analysis and thus the input velocity at pinion gear is taken as:

$$\omega = \Delta\omega \cos(3\omega_m t) \quad (7)$$

This speed fluctuation can result in alternating teeth separation and impacts within the gearbox under idling conditions, as illustrated through the speed profiles of the pinion and wheel gears in Fig. 4. The loss of gear contact occurs from point B to C and then from point E to F in Fig. 4, during which the pinion gear's speed profile follows the solid line, while the wheel gear follows the dashed line. In other regions, the gears remain in contact. The area between the speed profiles of the two gears from point B to C or from point E to F is the distance travelled by gear teeth between the impacts and equals the backlash $2b_{ls}$.

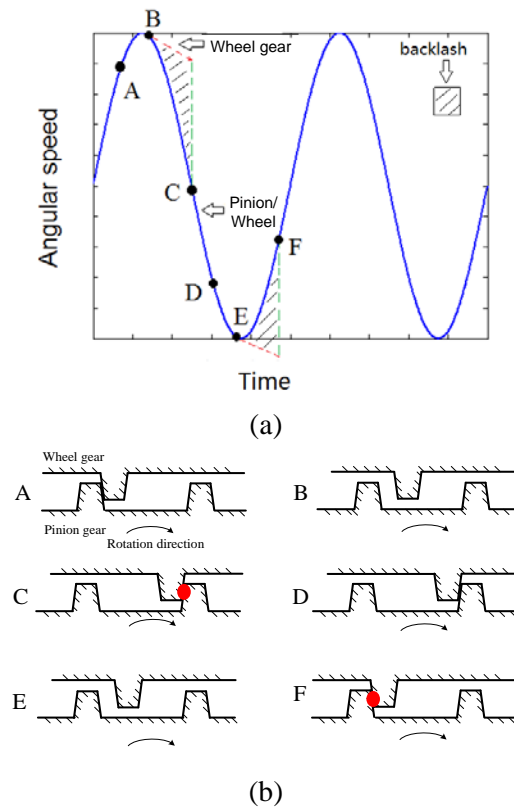


Figure 4: Gears crossing across backlash: (a) gears' speed profiles and (b) key configurations

Gear rattle occurs if the tangential deceleration at the contact point for the pinion gear is greater than that for the wheel gear. Under idling condition, only the drag torque due to bearing and lubricant acts on the wheel gear during gear rattle. Therefore, the deceleration threshold for the pinion above which gear rattle occurs is

$$\ddot{\theta}_{critical} = r_{bw} T_{drag} / (r_{bp} I_w) \quad (8)$$

where the drag torque $T_{drag} = T_{chw} + T_{bfw}$ consists of the bearing friction and the lubricant churning components. Thus, a decrease of the drag torque or an increase of the wheel gear inertia decreases the deceleration threshold limit, thereby increasing the likelihood of gear rattle.

Gear rattle noise, which is directly perceived by the end user, is correlated to the acceleration of the gear teeth. According to [9,13,14], the RMS value of the wheel gear acceleration is linear correlated with the noise level when the gear rattle occurs. And this RMS value has been used to indicate the sound perception. The noise produced by the pure rotation of the gear bodies is negligible. Therefore, the rattling noise level is mainly determined by the period when gear impact occurs. Furthermore, although several consecutive impacts might occur before the gear teeth mesh together again, the first impact has the dominant energy as the rest ones can be dampened quickly [15,16]. As a result, the rattling noise level is mainly determined by the RMS value of the wheel gear acceleration during the first impact period. This reasonable simplification enables the RMS value to be estimated analytically. Therefore, the influence of gearbox parameters' such as the meshing stiffness, damping factor, gear inertia and backlash, on the rattling noise level can be accessed by evaluating their effect on the RMS value of the wheel gear acceleration through investigation of the dynamic model of gearbox presented in Eq. (4). Taking example of a typical transmission presented in [22], the ratio of equivalent inertia of the pinion gear with respect to the largest gear on the output shaft is greater than six. Therefore, the output gear has little effect on the dynamic response of input shaft. Furthermore, the wheel gear dynamics under idling conditions can be represented by a single DOF model (similar to [8]) as shown in Fig. 5, where the wheel gear can be modeled as a rigid body vibrating along the tangential direction insides the backlash. The boundary conditions for the equation of motion for the wheel teeth vibration are time-varying due to the speed fluctuations from input at the pinion gear.

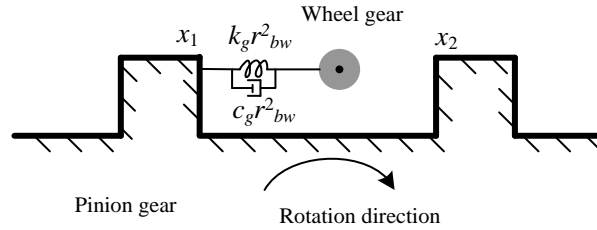


Figure 5: Single-DOF model for the gear impact

Considering an impact on the front flank side of the teeth, the equation of motion for the wheel gear from the time instant at start of collision t_0 till the time instant for separation t_1 can be obtained by substituting Eqs. (5) and (7) into the Eq. (4b) as:

$$I_w \ddot{x} + c_g r_{bw}^2 \dot{x} + k_g r_{bw}^2 x = -(T_{drag} r_{bw} + k_g r_{bw}^2 b_{ls}) + c_g r_{bp} r_{bw}^2 \Delta \omega \cos 3\omega_m t + \frac{k_g r_{bp} r_{bw}^2 \Delta \omega}{3\omega_m} \sin 3\omega_m t \quad (9)$$

where $x = r_{bw} \theta_w$ is the tangential displacement of the wheel gear. The initial conditions at $t = t_0^-$, when the gear pair are about to collide on the front flank (side x_1 in Fig. 5) are:

$$x_w(t_0^-) = -b_{ls} + \frac{r_{bw} \Delta \omega}{3\omega_m} \sin(3\omega_m t_0) \quad (10a)$$

$$\dot{x}_w(t_0^-) = v_0 = \Delta \omega \cos(3\omega_m t_0) \quad (10b)$$

where v_0 is the initial tangential speed of the pinion gear.

3.2 Analytical Solution

In this section, the equation of motion of the wheel gear is simplified and analytically solved. For a typical gear pair, the ratio k_g/c_g is greater than 1×10^4 while the idling speed of a medium sized engine ω_m is less than 200 rad/s. Compared to the third term on the right side of Eq. (9), the second term has much lower amplitude and therefore, can be neglected. Furthermore, the drag torque T_{drag} induced acceleration

component is much lower than the teeth impact force induced component during the impact period and can also be neglected. Therefore,

$$I_w \ddot{x} + c_g r_{bw}^2 \dot{x} + k_g r_{bw}^2 x \approx -k_g r_{bw}^2 b_{ls} + \frac{k_g r_{bp} r_{bw}^2 \Delta \omega}{3\omega_m} \sin 3\omega_m t \quad (11)$$

The solution of Eq. (11) can be expressed as a sum of three terms

$$x = x_h + x_{p1} + x_{p2} \quad (12)$$

where x_h is the homogeneous solution, x_{p1} is the particular solution corresponding to the constant term $-k_g r_{bw}^2 b_{ls}$, and x_{p2} is the particular solution corresponding to the sinusoidal term $\frac{k_g r_{bp} r_{bw}^2 \Delta \omega}{3\omega_m} \sin 3\omega_m t$.

These three solutions can individually be found as

$$x_h = e^{-\zeta \omega_n t} (A_1 \cos \omega_d t + B_1 \sin \omega_d t) \quad (13a)$$

$$x_{p1} = -r_{bw} b_{ls} \quad (13b)$$

$$x_{p2} = \frac{r_{bp} \Delta \omega}{3\omega_m} \cdot \frac{k_g r_{bw}^2}{\sqrt{(k_g r_{bw}^2 - 9I_w \omega_m^2)^2 + (4c_g r_{bw}^2 \omega_m)^2}} \sin 3\omega_m t \quad (13c)$$

where the natural frequency $\omega_n = \sqrt{\frac{k_g r_{bw}^2}{I_w}}$, the damped natural frequency $\omega_d = \sqrt{1 - \zeta^2} \omega_n$, and damping

factor $\zeta = \frac{c_g r_{bw}}{2\sqrt{k_g I_w}}$. For a typical gear pair, $\omega_n \gg 3\omega_m$ and $k_g \gg 4\omega_m c_g$. Therefore, Eq. (13c) can be

rewritten as:

$$x_{p2} \approx \frac{r_{bp} \Delta \omega}{3\omega_m} \sin(3\omega_m t) \quad (14)$$

Using the initial condition shown in Eq. (10), the coefficients A_1 and B_1 in x_h can be evaluated as

$$A_1 = -\frac{r_{bw} v_d \sin \omega_d t_0}{\omega_d} e^{\zeta \omega_n t_0} \quad (15a)$$

$$B_1 = \frac{r_{bw} v_d \cos \omega_d t_0}{\omega_d} e^{\zeta \omega_n t_0} \quad (15b)$$

where $v_d = v_0 - r_{bp} \Delta \omega \cos(3\omega_m t_0)$ is the tangential speed difference between the gears at time $t = t_0$ when the impact is about to occur. Thus, tangential displacement of the wheel gear (Eq. (12)) can be evaluated as

$$x = e^{-\zeta \omega_n (t-t_0)} \frac{v_d r_{bw} \sin \omega_d (t-t_0)}{\omega_d} - r_{bw} b_{ls} + \frac{r_{bp} \Delta \omega}{3\omega_m} \sin(3\omega_m t) \quad (16)$$

The wheel gear tangential acceleration can be obtained by twice differentiating the Eq. (16) with respect to time

$$\ddot{x} \approx \frac{e^{-\zeta \omega_n (t-t_0)}}{I_w} v_d \sqrt{k_g I_w r_{bw}} \cdot [(1 - 2\zeta^2) \sin \omega_d (t-t_0) + 2\zeta \cos \omega_d (t-t_0)] \quad (17)$$

It can be observed that the acceleration \ddot{x} of gear teeth excited due to their impacts is determined mainly by the homogeneous solution. The particular solution x_{p1} is constant, and it disappears in Eq. (16) during differentiation. Also, $|\ddot{x}_{p2}|$ is much lower than the homogeneous solution $|\ddot{x}_h|$. However, it must

be noted that the pinion speed fluctuation $\Delta\omega$ also affects the homogenous component of wheel gear acceleration. A larger $\Delta\omega$ produce a higher speed difference v_d , which will increase the RMS value of acceleration as shown in Eq. (17). As is shown in previous literatures [9,14], the rattling noise level is linearly correlated with the RMS value of the impact-induced wheel gear acceleration. Further, the first impact is the most dominant among the consecutive impacts that may occur on a gear face. Taking the duration of the gear's first impact to be half of the damped oscillation period for the gear teeth $t_1 - t_0 = \pi/\omega_d$, the RMS value can be obtained as

$$\text{RMS}(\ddot{x}) = \sqrt{\frac{1}{t_1 - t_0} \int_{t_0}^{t_1} \ddot{x}^2 dt} \approx v_d \sqrt{\frac{k_g (1 - e^{-2\pi\zeta}) r_{bw}}{4\pi I_w \zeta} (1 + 4\zeta^4)} \quad (18)$$

Furthermore, the shaded region in Fig. 4(a) between the wheel gear speed profile and the pinion gear speed profile can be treated as a triangle whose area equals to the backlash amount $2b_{ls}$. Therefore, the speed difference v_d and backlash $2b_{ls}$ can be related as $v_d \approx k_d \sqrt{2b_{ls}}$ where k_d is a constant coefficient for a given speed fluctuation amplitude. Thus,

$$\text{RMS}(\ddot{x}) = k_d \sqrt{\frac{2k_g (1 - e^{-2\pi\zeta}) r_{bw}}{4\pi I_w \zeta} (1 + 4\zeta^4) b_{ls}} \quad (19)$$

Based on Eq. (19), it can be seen that $\text{RMS}(\ddot{x})$ increases if the meshing stiffness k_g or the backlash $2b_{ls}$ increases. However, it will decrease if the inertia of the wheel gear I_w increases. If the damping factor ζ increases, the damping related term $\sqrt{(1 - e^{-2\pi\zeta})(1 + 4\zeta^4))/\zeta}$ decreases in the typical range of damping factor [23], $0.01 \leq \zeta \leq 0.10$, which results in a decrease of $\text{RMS}(\ddot{x})$ value.

4 Results and Discussion

The gear rattle occurring in a typical transmission, whose parameters are given in Tab. 1, is investigated in this section. Since, all the gear and rotor inertias at the input side as well as the counter shafts' inertia are reflected on the pinion gears, its equivalent inertia is much larger than that of the wheel gear. In this paper, this equivalent inertia is set to be six times of the wheel gear, which is typical in automobile transmission design [5,22,24]. The frictional characteristics associated with lubricant churning are evaluated assuming that 3/10th of the gears are immersed in the lubricant (70°C).

Table 1: Parameters of the transmission

Parameters	Value
Number of teeth (pinion/wheel gears)	25/25
Module m (mm)	4
Pressure angle α (°)	20
Equivalent inertia of the pinion gear I_p (kg.m ²)	7.0×10^{-3}
Inertia of the wheel gear I_w (kg.m ²)	1.16×10^{-3}
Gear meshing stiffness k_g (N/(m.rad))	2.4×10^8
Damping factor ζ of gear meshing	0.06
Half Backlash b_{ls} (μm)	50

4.1 Simulation Using SIMPACK

SIMPACT is a multi-body system (MBS) software that can be used for the dynamic analysis of varied mechanical or mechatronic systems. It is particularly suitable for high frequency transient analysis extending up to the acoustic range. The graphical user interface of SIMPACK can be utilized to generate and solve virtual 3D models to predict and visualize motion, coupling forces and stresses. The force element library provided in SIMPACK contains specific force elements tailored to different application areas of industry including elements for drivetrain simulations, where a gear pair module is also provided with features such as various gear types, profile modifications, dynamically changing backlash, friction models, and single or multiple teeth contact.

The typical transmission is modeled in SIMPACK (shown in Fig. 6) where a spur gear pair is connected to a large inertia to simulate the flywheel through an equivalent torsional spring-damper to simulate the clutch. The elements from SIMPACK library used to model the system are shown in Tab. 2.

Table 2: Model components represented in SIMPACK

Component	SIMPACT element
Gears	Primitive 25: Gear wheel
Flywheel	Primitive 2: Cylinder
Driving speed of the flywheel	Joint: Single axis $40 u(t)$
Stiffness and damping of clutch	Force element 52: Cardan joint
Gear meshing stiffness and damping	Force element 225: Gear pair

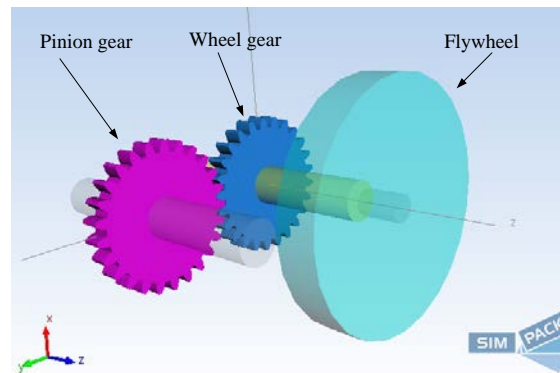


Figure 6: Dynamic model of the transmission

4.2 Result Analysis

If the gear rattle occurs, it can manifest itself as either a single-sided or double-sided impact, which depends on the magnitude of the pinion gear deceleration. For the model described in Section 4.2, letting $\omega_m = 83.78 \text{ rad/s}$ (800RPM) and $\Delta\omega = 12.57 \text{ rad/s}$ (120 RPM) the double-sided impact occurs (as seen in DTE fluctuations shown in Fig. 7) which is a typical gear rattle phenomenon [8]. Therefore, this case is analyzed to investigate the gear parameters effect on the rattle impact. From Fig. 8(a) it can be seen that the bearing frictional torque and the lubricant churning torque components are comparable under idling conditions. Both of them have a low value and have similar contributions in determining the threshold of the rattle occurrence. The teeth impacts periodically excite impulse like peaks in the gear teeth contact force profile (Fig. 8(b)), which is the root cause of the rattling noise.

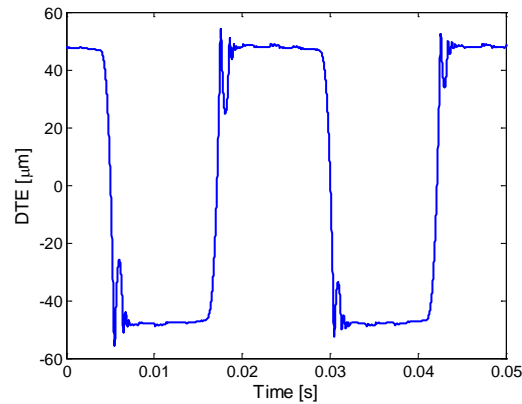


Figure 7: Dynamic transmission error under rattle

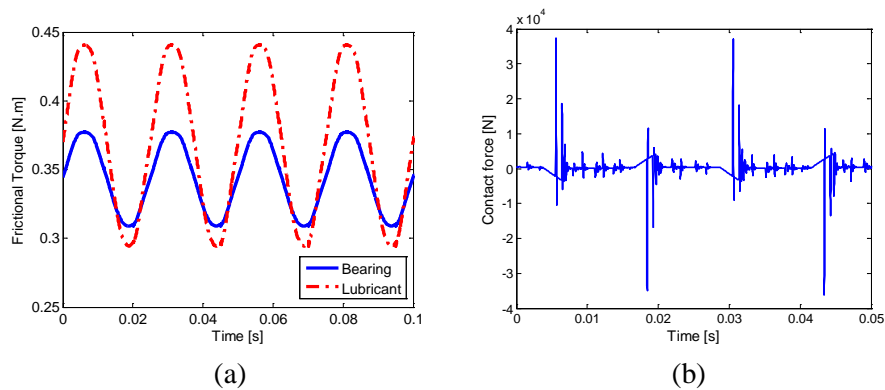


Figure 8: (a) Frictional torques and (b) gear teeth contact force at the wheel gear

The gear rattle noise level indicator presented in Eq. (19) is verified by varying the model parameters: wheel gear inertia, meshing stiffness, backlash, and meshing damping factor in SIMPACK simulations, one at a time. The wheel gear inertia, meshing stiffness and backlash are varied to $1/3$, $2/3$, 1 , $4/3$, and $5/3$ times their reference value presented in Tab. 1. The damping factor is varied from 0.02 to 0.10 using a step size of 0.02. The obtained RMS acceleration of the wheel gear is then normalized with respect to the RMS value obtained using the reference values given in Tab. 1. The result from the analytical expression and the SIMPACK dynamic model are compared in Fig. 9. It is seen that the proposed analytical expression predicts the increase of the backlash amount and the gear meshing stiffness increases the RMS acceleration of wheel, which matches closely with the results from SIMPACK simulation (Figs. 9(a) and 9(b)). A larger backlash will result in a larger gear speed difference at the time of impact, while a stiffer gear tooth will also increase the contact force magnitude at the impact. Therefore, both these factors increase the RMS acceleration as observed. A larger RMS acceleration in turn implies that the noise due to gear rattle will be higher. The backlash effect was verified in [8] through experiment, where the rattle noise level was measured for an idler gear first. Then an elastic thrust collar was fitted onto the same idler gear surface to reduce the backlash. The measurement showed that the rattling noise level can be reduced with the reduced backlash. The analytical method also predicts that an increase in inertia of the wheel gear or the damping factor reduces the RMS acceleration of the wheel gear, which matches closely with the results obtained from the SIMPACK simulations (Figs. 9(c) and 9(d)). The reduced RMS acceleration in turn indicates a lower noise level due to gear rattle. The wheel gear inertia effect was verified through the experiment in [9] where the RMS values were measured with three gears using different inertias. The results showed that high inertia resulted in a low RMS acceleration value, as well as the rattle noise. Thus,

it is shown that the proposed analytical method can be used to evaluate the gear parameters effect on the rattling noise level, and provide a gear rattle measure (Eq. (19)) to aid in the gearbox design optimization.

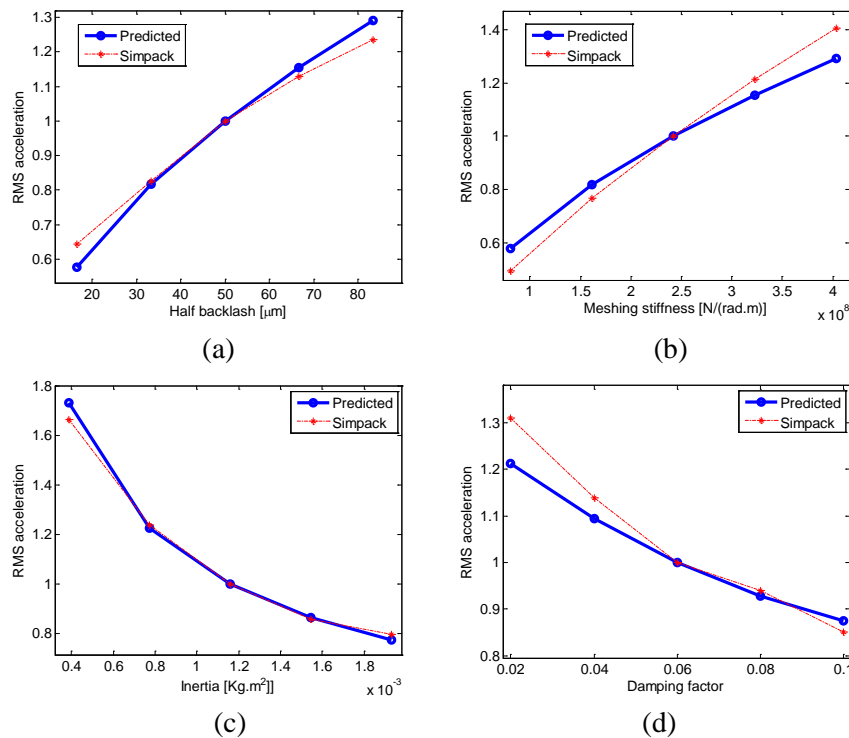


Figure 9: Normalized RMS of the wheel gear acceleration versus variation in gear parameters: (a) backlash, (b) meshing stiffness, (c) inertia of the wheel gear, and (d) damping factor

5 Conclusion

An analytical solution determining the RMS value of the rattle impact induced acceleration component for wheel gears based on a dynamic model of a typical manual transmission is presented in this paper. Since RMS acceleration value is proportional to the rattling noise level [9,14], the proposed analytical evaluation method is capable of investigating the gear parameters' effect on the rattle noise level. The evaluated RMS acceleration value for the wheel gear in the typical transmission gearbox is then used to investigate the effect backlash, meshing stiffness, inertia and meshing damping factor on the rattle noise levels. To verify the proposed analytical evaluation method, an MBS dynamic model for the typical transmission is developed in SIMPACK. The comparison of analytical and dynamic simulation analysis shows that the proposed analytical expression for RMS acceleration value can effectively predict the influence of gear parameters. Moreover the obtained analytical and simulation results also agrees with the previous experimental result in [8,9]. Thus, the proposed evaluation method provides a simple guideline that can optimize the gearbox parameters to reduce the gear rattle noise level.

Acknowledgement: This research work was supported by the funding from Shandong Natural Science Foundation (Grant No. ZR2016EEM20).

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