ODS & Modal Testing Using a Transmissibility Chain

Brian Schwarz, Patrick McHargue, Mark Richardson, Vibrant Technology, Inc. Centennial, CO.

In this paper, we show how Operating Deflection Shapes (ODS's) and mode shapes can be obtained experimentally from measurements that are made using *only two sensors* and *two short wires* to connect them to a multi-channel acquisition system. This new test procedure is depicted in Figure 1. Not only is the equipment required to do a test much more cost effective, but this method can be used to test *any sized test article*, especially large ones.

The testing method introduced here involves moving a pair of sensors along together in a prescribed manor, and calculating the Transmissibility between them. The resulting chain of Transmissibility's is then post-processed to obtain a *single reference set* of cross-channel measurements, from which ODS's and mode shapes can be extracted.

A *"round trip"* example is used to show how an original set of modal parameters can be recovered by curve fitting a *single reference set* of output-only Cross spectra, and a *single reference set* of FRFs.

Key words: Fourier spectrum (DFT), Auto power spectrum (APS), Cross power spectrum (XPS), Frequency Response Function (FRF), Operating Deflection Shape (ODS), Experimental Mode Shape (EMA Mode), Operational Mode Shape (OMA Mode), Modal Assurance Criterion (MAC), Shape Difference Indicator (SDI).

Introduction

To obtain the experimental ODS's or mode shapes of a machine or structure, each degree-of-freedom (DOF) of a shape must contain the ***correct magnitude & phase relative to all other DOFs***. If all of the sensor data is ***simultaneously acquired***, each shape component will contain the correct magnitude & phase. However, simultaneous acquisition requires that all of the sensors be connected to a multi-channel acquisition system that can simultaneously acquire the data from all channels.

ODS's and mode shapes can be obtained from a set of cross-channel measurements if a ***fixed reference sensor*** is used, but the reference sensor must remain fixed throughout the test. For large test articles, the wire from the reference sensor to the acquisition system could be very long. Or, in a roving impact test, the wire from the instrumented hammer to the acquisition system could also be long.

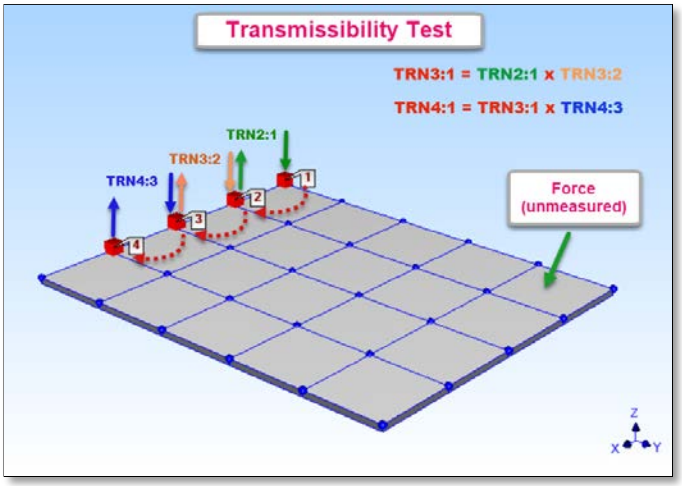
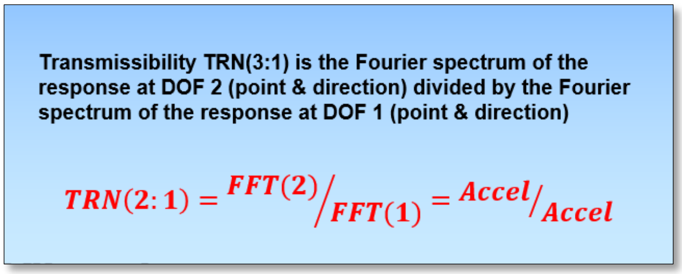


Figure 1. Transmissibility chain measurement.

Operational modal parameters (frequencies damping, mode shapes) are obtained by curve fitting a ***single reference set*** of output-only Cross spectra. An ***experimental modal model*** (a set of mode shapes containing the mass, stiffness and damping properties of the structure) is obtained by curve fitting a ***single reference set*** of calibrated FRFs.

An output-only Cross spectrum is a cross-channel frequency-based measurement that is defined as the Fourier spectrum of one response multiplied by the complex conjugate of the Fourier spectrum of the other response. An FRF is a cross-channel frequency-based measurement that is defined as the Fourier spectrum of a structural response (in displacement, velocity, or acceleration units) divided by the Fourier spectrum of an excitation force that caused the response.

A Transmissibility is also a cross-channel frequency-based measurement function. It is defined as the Fourier spectrum of one response divided by the Fourier spectrum on another response, as illustrated in Figure 2.



Transmissibility TRN(3:1) is the Fourier spectrum of the response at DOF 2 (point & direction) divided by the Fourier spectrum of the response at DOF 1 (point & direction)

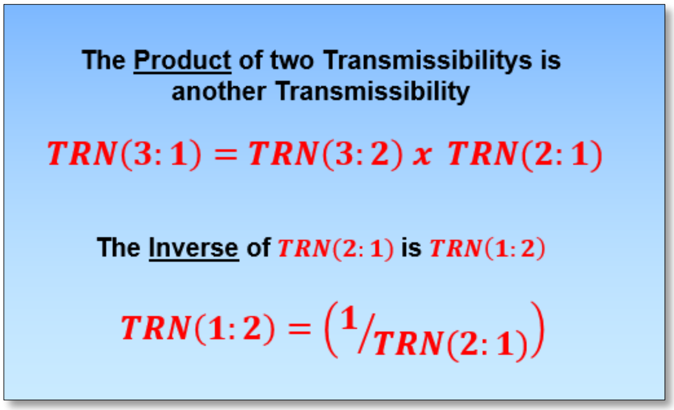
$$TRN(2:1) = {}^{FFT(2)}\!/_{FFT(1)} = {}^{Accel}\!/_{Accel}$$

Figure 2. Transmissibility.

Transmissibility properties

Transmissibility's have two unique properties that make them useful for recursive post-processing. Multiplying the Transmissibility between DOF 1 & DOF 2 by the Transmissibility between DOF 2 & DOF 3 gives the Transmissibility between DOF 1 & DOF 3. This property will be used to create a ***Transmissibility Chain***, as depicted in Figure 4.

The inverse property of Transmissibility's will also be used to the measure Transmissibility's using a simple test procedure called a ***Slinky Test***. A ***Slinky Test*** is depicted in Figure 8.



The Product of two Transmissibilitys is another Transmissibility

$$TRN(3:1) = TRN(3:2)\ x\ TRN(2:1)$$

The Inverse of $TRN(2:1)$ is $TRN(1:2)$

$$TRN(1:2) = \left({}^{1}\!/_{TRN(2:1)}\right)$$

Figure 3. Transmissibility properties.

Transmissibility chain measurement

In this new test procedure, Transmissibility's are measured in a chain fashion as depicted in Figure 1. The three measurements shown in Figure 1 are made with two sensors, (for example accelerometers), and a 2-channel acquisition system.

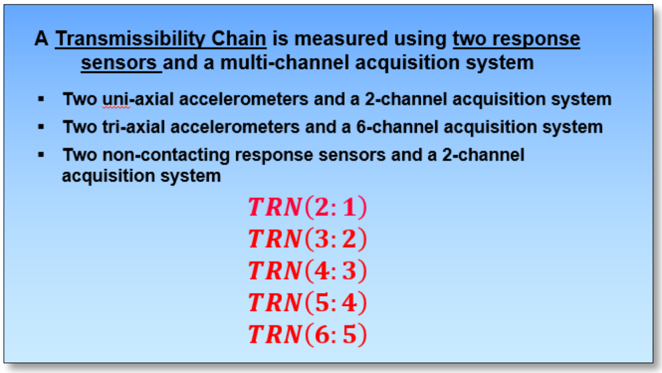
The sensor data used in the **denominator** of a Transmissibility is called the **Input**, and the sensor used in the **numerator** is called the **Output**. The test procedure is as follows;

1. Attach the sensors to points 1&2
2. Acquire vibration data with **point 1 designated as Input** and **point 2 designated as Output**
3. Calculate **TRN(2:1)**
4. Attach the sensors to points 2&3
5. Acquire vibration data with **point 2 designated as Input** and **point 3 designated as Output**
6. Calculate **TRN(3:2)**
7. Attach the sensors to points 3&4
8. Acquire vibration data with **point 3 designated as Input** and **point 4 designated as Output**
9. Calculate **TRN(4:3)**

A chain can be measured using either ***uni-axial*** or ***tri-axial*** sensors, as described in Figure 4. Tri-axial sensors will measure 3D motion of the surface at each test point. This has the advantage of yielding ODS's and mode shapes that describe the ***3D motion of the structure*** at each point.

Benefits of chain measurement

The benefits of measuring a chain of Transmissibility's are listed in Figure 5. This type of testing has its greatest advantage for testing running machines or vehicles, where the excitation forces are distributed and un-measureable. In other applications, excitation can be provided either with an impactor or with shakers, but only the responses are acquired, not the excitation forces.

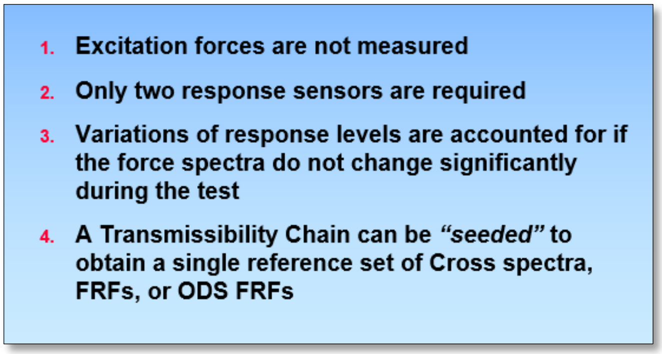


A Transmissibility Chain is measured using two response sensors and a multi-channel acquisition system

- Two uni-axial accelerometers and a 2-channel acquisition system
- Two tri-axial accelerometers and a 6-channel acquisition system
- Two non-contacting response sensors and a 2-channel acquisition system

$$TRN(2:1)$$
$$TRN(3:2)$$
$$TRN(4:3)$$
$$TRN(5:4)$$
$$TRN(6:5)$$

Figure 4. 1D or 3D chain measurement.



1. Excitation forces are not measured
2. Only two response sensors are required
3. Variations of response levels are accounted for if the force spectra do not change significantly during the test
4. A Transmissibility Chain can be *"seeded"* to obtain a single reference set of Cross spectra, FRFs, or ODS FRFs

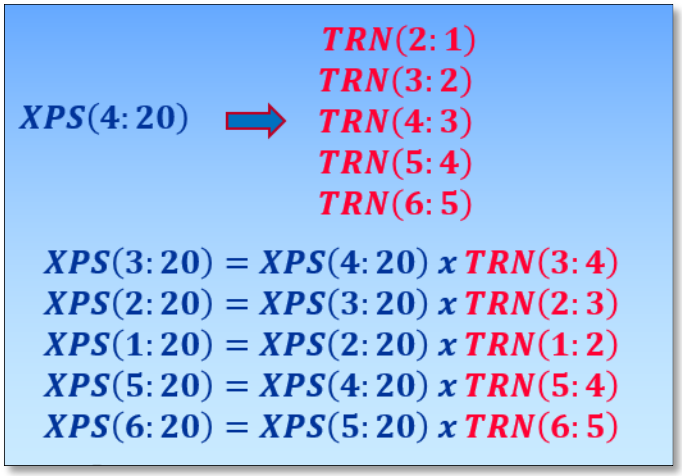
Figure 5. Benefits of chain measurement.

Seeding the chain

Once the Transmissibility Chain has been acquired for all points & directions (DOFs) on the structure, it can be **"seeded"** using either a Cross spectrum, an FRF, or an Auto spectrum.

- Seeding with a Cross spectrum (XPS) yields a ***single reference set*** of Cross spectra
- Seeding with an FRF yields a ***single reference set*** of FRFs
- Seeding with an Auto spectrum (APS) yields a ***single reference set*** of ODS FRFs
- Seeding the Chain with a calibrated measurement yields a ***single reference set*** of calibrated measurements

Notice in Figure 6 that the ***reference DOF*** of the Cross Spectrum is not used, so it can be from anywhere on the machine or structure. The only requirement for seeding a chain is that the ***Output DOF of the seed XPS matches with a DOF of one of the Transmissibility's*** in the Chain. Notice also that the Inverse property of the Transmissibility's is used during the recursive operation to ***"walk along"*** the Transmissibility Chain in both directions.

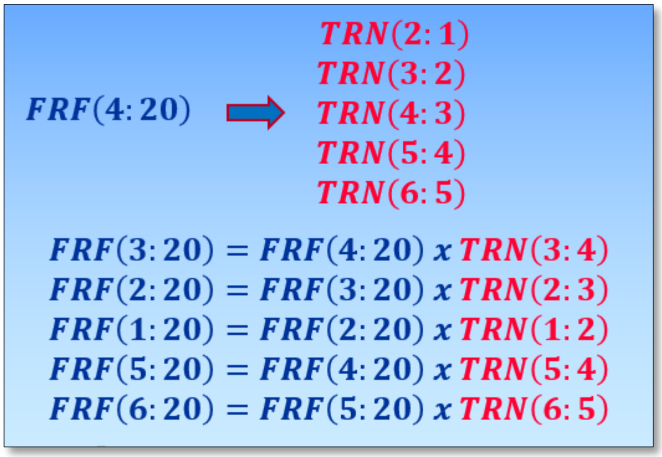


$$XPS(4:20) \Rightarrow \begin{matrix} TRN(2:1) \\ TRN(3:2) \\ TRN(4:3) \\ TRN(5:4) \\ TRN(6:5) \end{matrix}$$

$$XPS(3:20) = XPS(4:20)\ x\ TRN(3:4)$$
$$XPS(2:20) = XPS(3:20)\ x\ TRN(2:3)$$
$$XPS(1:20) = XPS(2:20)\ x\ TRN(1:2)$$
$$XPS(5:20) = XPS(4:20)\ x\ TRN(5:4)$$
$$XPS(6:20) = XPS(5:20)\ x\ TRN(6:5)$$

Figure 6. Seeding with a cross spectrum.

Seeding a Chain with FRFs is depicted in Figure 7. Again, the FRF seed can be measured from force Input anywhere on the structure. The only requirement is that the ***Output DOF of the seed FRF matches with a DOF of one of the Transmissibility's.***



$$FRF(4:20) \Rightarrow \begin{matrix} TRN(2:1) \\ TRN(3:2) \\ TRN(4:3) \\ TRN(5:4) \\ TRN(6:5) \end{matrix}$$

$$FRF(3:20) = FRF(4:20)\ x\ TRN(3:4)$$
$$FRF(2:20) = FRF(3:20)\ x\ TRN(2:3)$$
$$FRF(1:20) = FRF(2:20)\ x\ TRN(1:2)$$
$$FRF(5:20) = FRF(4:20)\ x\ TRN(5:4)$$
$$FRF(6:20) = FRF(5:20)\ x\ TRN(6:5)$$

Figure 7. Seeding with an FRF.

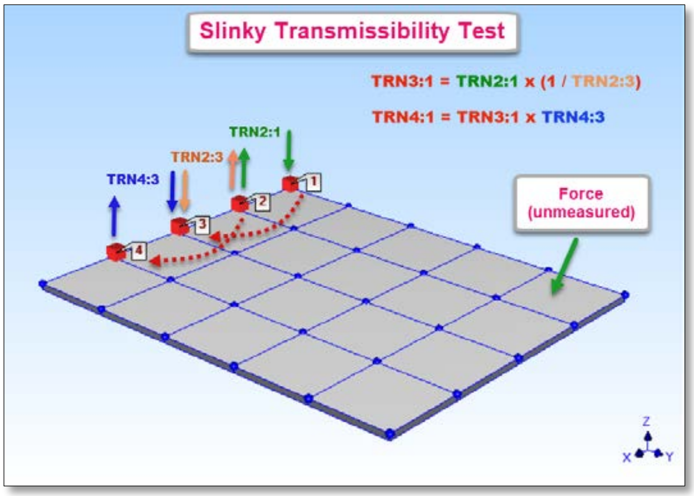


Figure 8. Slinky test.

Slinky test

There is a faster, easier way to acquire a Transmissibility Chain than the one depicted in Figure 1. We call it a ***Slinky Test,*** and it is depicted in Figure 8. Notice in Figure 8 that ***only one sensor has to be moved*** between acquisitions. Each time a sensor is moved it "***hops over"*** the other sensor to the next test point on the structure, in a manner similar to the way a ***slinky spring*** walks down a stairway.

Comparing the test in Figure 1 with the Slinky Test in Figure 8, ***six sensor moves*** were made in Figure 1 to measure three Transmissibility's, whereas ***only two sensor moves*** were required in Figure 8 to measure the same number of Transmissibility's.

Round trip simulation

In order to verify the Transmissibility Chain testing method, a ***round trip simulation*** of a test was done using a modal model of the plate structure shown in Figures 1&8. The modal model will be used to simulate random vibration of the plate. Then, Transmissibility's will be ***"acquired"*** from the responses, and seeded to calculate single reference Cross spectra and FRFs. Finally, the original mode shapes will be recovered by curve fitting the Cross spectra and the FRFs.

To obtain the modal model, a **Roving Impact Test** was performed on the aluminum plate. A uni-axial accelerometer was attached to Point 1 as a reference, the plate was impacted with an instrumented hammer at each point, ***and a calibrated FRF*** was calculated between each impact point and the reference response.

An ODS display at one of the resonance peaks in the FRFs is shown in Figure 9. Next, the FRFs were curve fit to obtain Residue mode shapes for the five modes with resonance peaks in the FRF. The mode shapes are shown in Figure 10.

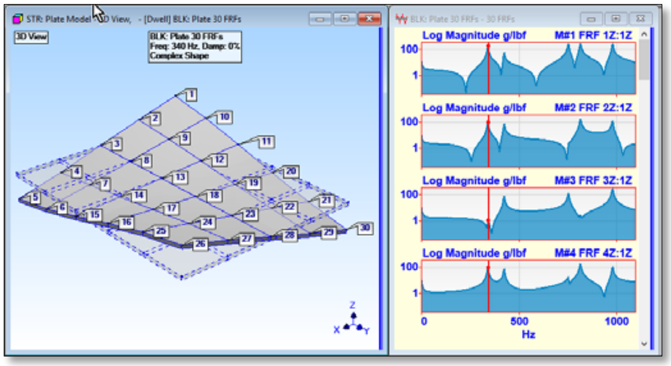


Figure 9. ODS at a resonance peak in the FRFs.

Burst random responses

The following steps were used to calculate time domain responses of the plate at all 30 points due to random excitation of the plate;

1. A sequence of ***ten burst random excitation waveforms*** was created as ***a simulated force Input*** to the plate at DOF 1Z.
2. The modal model (of calibrated mode shapes) was used to synthesize FRFs between 1Z and the 30 response DOFs of the plate.
3. The synthesized FRFs were multiplied by the Fourier spectrum of the Input force to obtain the Fourier spectrum of the Outputs, which were then transformed to the time domain waveforms shown in Figure 11.

This process is referred to as **Multi-Input Multi-Output (MIMO) Modeling & Simulation.** Any type of forcing function can be used with MIMO simulation, but a burst random force sequence was used to ***eliminate leakage*** in the response Fourier spectra.

A ***pure random Input*** would more accurately simulate real world excitation of a machine or structure. However, in this case burst random excitation was used to minimize errors so that the original mode shapes could be recovered by post-processing the Transmissibility Chain.

Some of the burst random responses are shown in Figure 11. Notice that the burst random length was chosen so that the structural responses would decay to ***"nearly zero"*** by the end of each burst sequence.

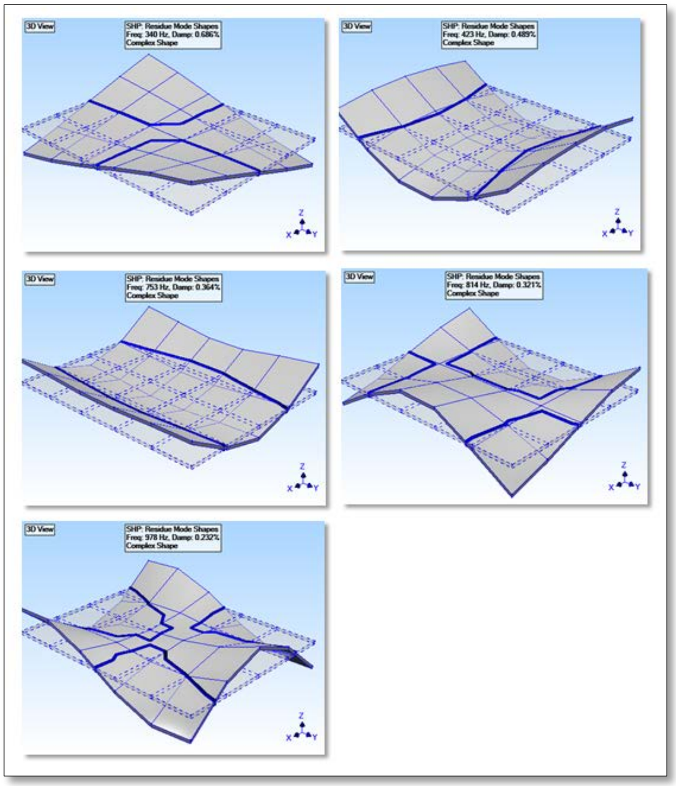


Figure 10. Mode Shapes of the plate.

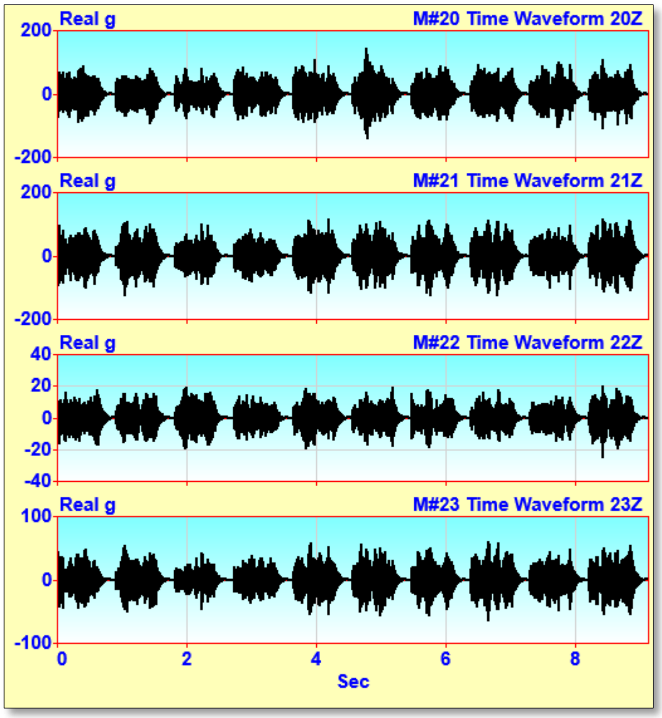


Figure 11. Burst random responses.

In real-world testing applications, the burst length will depend on the damping in the structure, which is always reflected in the damping of the resonances or modes.

Calculating a transmissibility chain

A Transmissibility Chain was calculated from time waveforms that were ***"acquired"*** from the burst random responses, shown in Figure 11. Each Transmissibility was calculated using the first response as an **Input** and the second as an **Output**, as indicated by the arrows in Figure 1.

A chain of 29 Transmissibility's was calculated between each pair of points on the aluminum plate, as depicted in Figure 4. Some of the Transmissibility's are displayed in Figure 12.

It must be emphasized that the peaks in a Transmissibility are ***not resonance peaks***. Hence, ODS's or mode shapes cannot be obtained directly from Transmissibility's. ***Transmissibility's cannot be curve fit using an FRF-based curve fitting method.*** A Transmissibility is a different complex waveform than an FRF.

Calculating single reference cross spectra

The Transmissibility Chain of 29 Transmissibility's was ***seeded***, as depicted in Figure 6, using the **Cross spectrum 1Z:1Z**. This yielded a ***single reference set of Cross spectra***, some of which are shown in Figure 13.

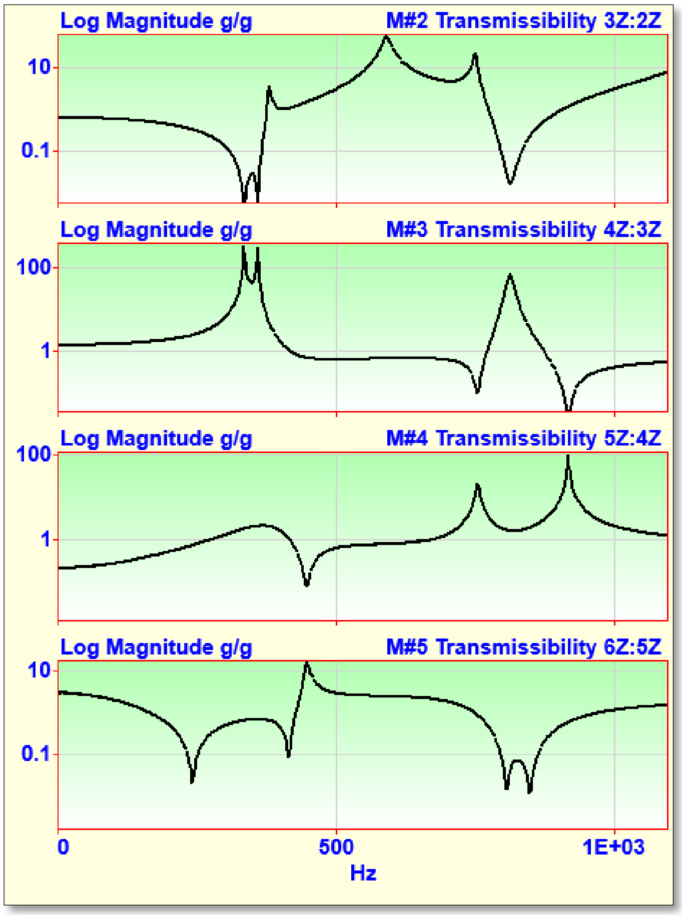


Figure 12. Transmissibility chain.

Calculating single reference frfs

The Transmissibility Chain of 29 Transmissibility's was also ***seeded***, as depicted in Figure 7, using the **driving point FRF 1Z:1Z**. This yielded a ***single reference set of FRFs***, some of which are shown in Figure 14.

Curve fitting results

The single reference set of 30 Cross spectra was curve fit using a deconvolution window followed by an FRF-based curve fitter. The resulting modal frequency & damping estimates are compared with the original modal parameters in Figure 15.

When paired with the original mode shapes, the mode shapes obtained by curve fitting the Cross spectra all had ***MAC values of 1.0.*** A MAC value ***equal to 1.0*** indicates that the two mode shapes are co-linear.

The single reference set of 30 FRFs was also curve fit. The resulting modal frequency & damping estimates are compared with the original modal parameters in Figure 16. When paired with the original mode shapes, the mode shapes obtained by curve fitting the FRFs all had ***SDI values equal to 1.0***.

The Shape Difference Indicator (SDI)[1] measures the true difference between two shapes, whereas the Modal Assurance Criterion (MAC)[2] measures the co-linearity of two shapes.

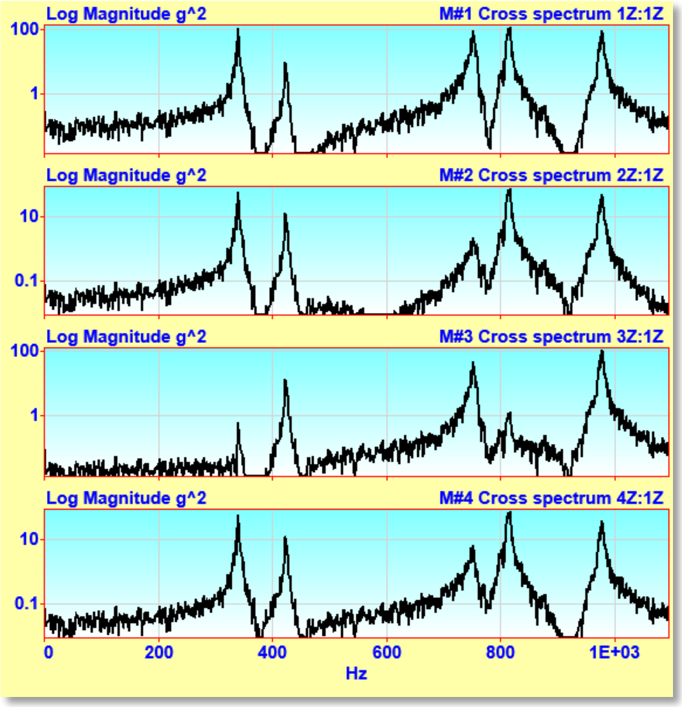


Figure 13. Single reference set of cross spectra.

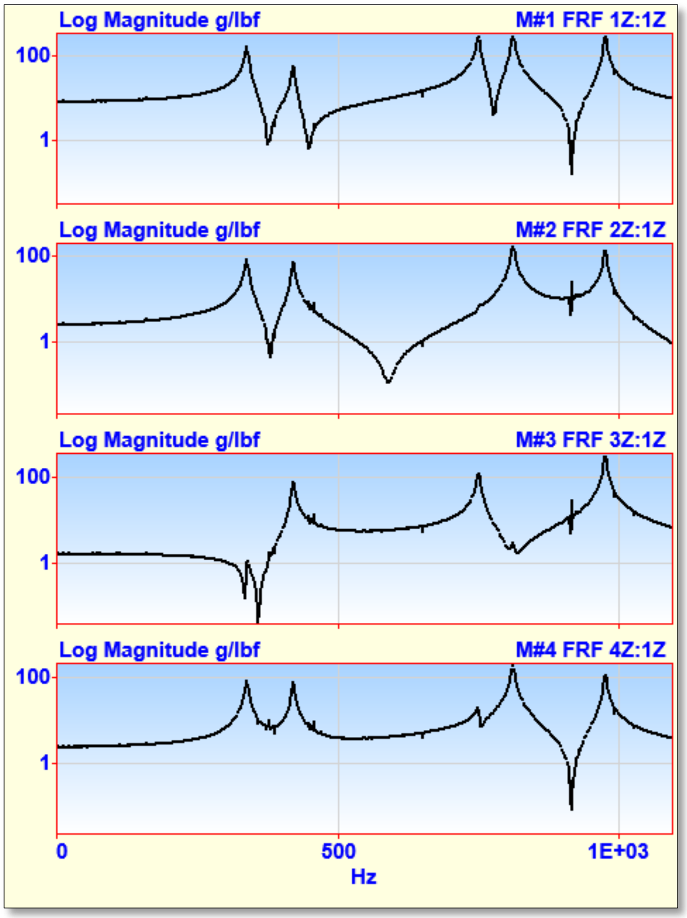


Figure 14. Single reference set of FRFs.

Curve fitting a set of Cross spectra yielded mode shapes that were ***co-linear with*** the original Residue mode shapes. Curve fitting a set of FRFs yielded mode shapes that were ***equal to*** the original mode shapes.

Table1. OMA of slinky XPS vs. Original modal parameters

Mode Number	Original Frequency (Hz)	XPS Frequency (Hz)	Original Damping (Hz)	XPS Damping (Hz)
1	340.0	339.6	2.333	1.648
2	423.0	422.8	2.070	1.501
3	752.6	752.0	2.740	3.248
4	813.8	814.2	2.610	2.623
5	978.2	977.0	2.266	2.729

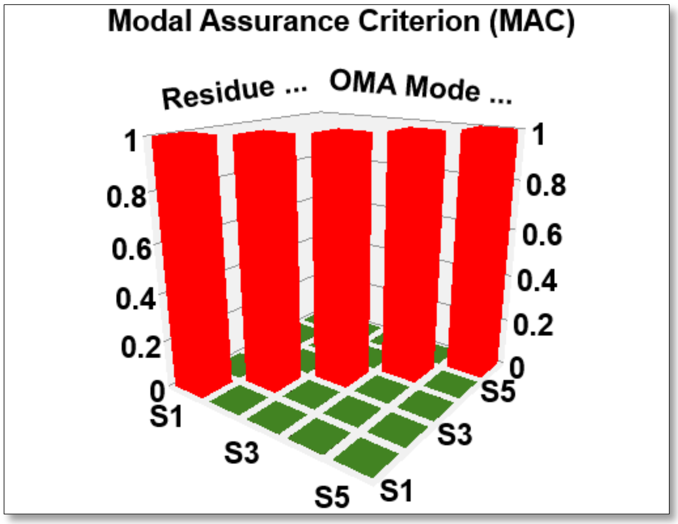


Figure 15. XPS vs. Original modal parameters.

Table 2. EMA of slinky FRFs vs. Original modal parameters

Mode Number	Original Frequency (Hz)	FRF Frequency (Hz)	Original Damping (Hz)	FRF Damping (Hz)
1	340.0	340.0	2.333	2.316
2	423.0	423.0	2.07	2.082
3	752.6	752.6	2.74	2.734
4	813.8	813.8	2.61	2.629
5	978.2	978.2	2.266	2.244

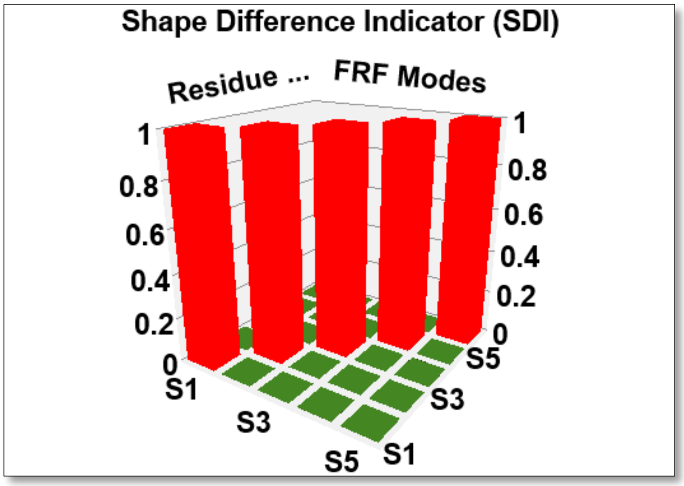


Figure 16. FRFs vs. Original modal parameters.

Conclusions

In order to obtain experimental ODS's or mode shapes, a ***single reference set*** of measurements is required. This means that ***one sensor must remain fixed*** throughout the test. To test a large structure, a long wire may be required to connect one of the sensors to the acquisition system.

In this paper, a new test procedure was introduced, which is based on the measurement of a Transmissibility Chain. A primary benefit of this testing method is that any machine or structure can be tested using two response sensors, a 2-channel acquisition system, and ***two short wires*** from the sensors to the acquisition system.

Another benefit of this method is that ***only structural responses are measured. The excitation forces do not have to be measured***. Hence, this method can be used to test running machinery, large or small, and to test large structures such as road bridges and buildings. In both of these cases, the excitation is distributed and un-measureable.

A ***Slinky Testing method*** was also introduced, which makes measurement of a Transmissibility Chain even easier. This simple method of ***hopping one sensor over the other*** between measurements is straightforward to implement.

It was shown with a ***round trip*** example how modal parameters can be recovered by curve fitting a single reference set of Cross spectra and a single reference set of FRFs. Starting with an experimental modal model of an aluminum plate, MIMO simulation was used to calculate its responses to a burst random excitation force.

The plate responses were used to calculate a Transmissibility Chain. Then, the Chain was ***seeded*** with a Cross spectrum to yield a ***single reference set of Cross spectra***. The same Chain was also seeded with an FRF to yield a ***single reference set of FRFs***. The modal parameters obtained from curve fitting both the Cross spectra and the FRFs closely matched the parameters of the original modal model.

There is one drawback to this method, however. If the seeding function, or any Transmissibility in the Chain, has errors in it, those errors will be propagated in the direction of calculation in the Chain. In Figure 17, it is shown how the noise error in the seed (**FRF 1Z:1Z**) was propagated to the other FRFs.

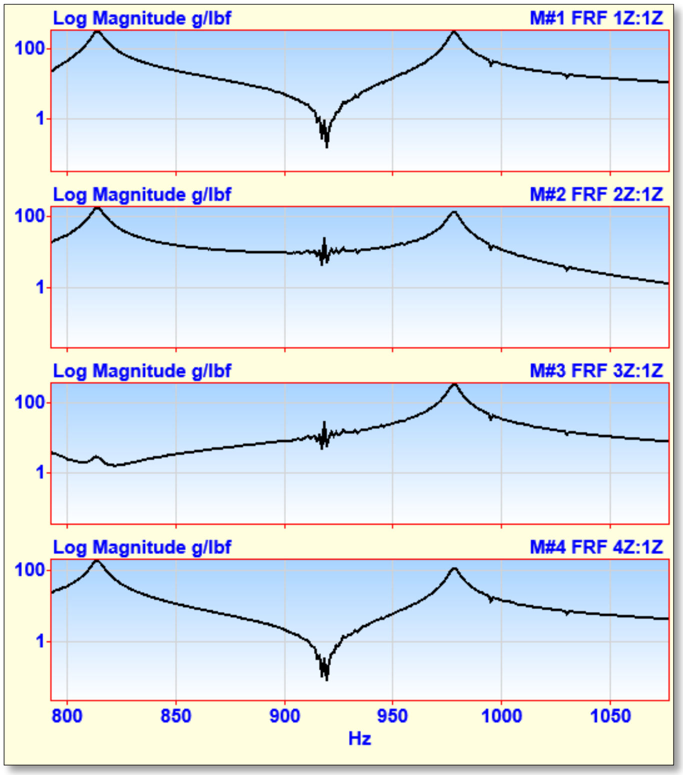


Figure 17. Noise propagation.

References

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The author can be reached at: brian.schwarz@vibetech.com