

# Array Shape Estimation Using Partitioned Eigenstructure Method with Sources in Unknown Localizations

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Advanced array processing approaches require accurate knowledge of the location of individual element in a sensor array. Most array shape estimation methods require the directions of sources. In this paper, an array shape estimation method based on eigen-decomposition is presented. The directions of sources do not need to be considered in advance and optimal array shape is generated through a series of iterations. To further improve the accuracy of this algorithm, a partitioned eigenstructure method is introduced. Numerical simulations using non-partitioned and partitioned method are conducted to verify the performance of the proposed technique.

**Keywords:** Array shape estimation, eigenstructure method, partitioned array.

## 1 Introduction

Nowadays, towed array has been widely used in a variety of fields from radar, oceanography, and seismology to radio astronomy. Advanced signal processing methods such as beamforming and matched-field processing and inversion are adopted in towed array applications<sup>1-3</sup>. Most array processing approaches are based on the assumption that the shape of the array remains linear or unchanged. However, due to the towing ship's speed, turning maneuvers or water currents, a towed array is hardly to maintain in linear shape<sup>4,5</sup>. So, in most cases, especially for dynamic array, localization of the sensors is a critical issue, a procedure which is named as array element estimation.

Array shape estimation methods can be broadly classified into source independent (methods that use positioning sensors such as heading sensors or depth sensors) or source dependent (methods that require calibrating sources)<sup>6</sup>. One direct approach of source independent methods, which instrument the array with depth sensors and compasses, would be to use the measurements of a depth sensor and compass located at each location to estimate its position. However, this is neither economically and mechanically feasible in practical application. Owsely<sup>7</sup> assumed the array shape could be modelled by a low order polynomial and used the limited number of sensors' information to determine the coefficients of this polynomial. Howard and Syck<sup>8</sup> introduced a spline interpolation scheme which presents better numerical stability. It is apparent that those methods are constrained to the accuracy of the measurement devices.

There are several source dependent methods which have been studied to estimate the array shape. Auxiliary sources in known locations are introduced in many literatures<sup>9-13</sup> to estimate the shape of array. Lo and Marple<sup>13</sup> proposed a calibration technique which requires calibrating sources whose directions are known. Their methods showed good performances in array shape estimation, although the locations of auxiliary sources are required. Rockah and Schultheiss<sup>14,15</sup> have shown that finding the direction of sources is possible if the location of one sensor and the direction to another sensor are known, providing the array is non-linear and the direction-of-arrival (DOAs) are distinct. Moreover, they presented an algorithm for self-calibration which is based on observing disjoint sources. However, in practice, the bearings of the sources relative to the array are changing or not accurate for moving array, especially the turning array.

Therefore, it is important to find a source dependent method which could estimate the array shape without the locations of reference sources.

This paper is specifically concerned with flexible, ocean-towed, non-linear arrays and aims to estimate the array shape with reference sources in unknown locations. The array shape is perturbed by turning maneuvers or water currents. The outline of the paper is as follows. Section 2 formulates the data model for the general problem. Section 3 presents the eigenstructure algorithm in detail and Section 4 describes partitioned method which could improve the accuracy of sensor locations. Section 5 shows numerical examples using the proposed algorithm. Some conclusions are drawn in Section 6.

## 2 Problem Formulation

Consider a hydrophone array with M sensors placed on a plane and numbered 1 through M. Set up a Cartesian coordinate system on the plane with the origin at sensor 1. The coordinates of sensor m are denoted by  $[x_m, y_m]$ . Suppose there are N narrow-band, far-field sources centered at frequency  $\omega$  with planar wavefronts ( $M > N$ ). The signals received by sensor m can be described by

$$z_m(t) = \sum_{n=1}^N s_n(t - \tau_{mn}) + v_m(t) \quad (1)$$

where  $m=1, 2, \dots, M$  and  $\{s_n(t)\}_{n=1}^N$  are the radiated signals. The delays  $\tau_{mn}$  are given by

$$\begin{aligned} \tau_{mn} &= -d_{mn} / c \\ d_{mn} &= x_m \cos \theta_n + y_m \sin \theta_n \end{aligned} \quad (2)$$

where  $\theta_n$  is the DOA of n-th source relative to the y-axis and  $d_{mn}$  is the distance from sensor 1 to sensor m in the DOA of signal n. Therefore, Equation (1) can also be expressed by

$$z_m(t) = \sum_{n=1}^N s_n e^{-j\omega\tau_{mn}} + v_m(t) \quad (3)$$

A "snapshot" taken by the sensors at time t can be described by the matrix equation,

$$\mathbf{Z}(t) = \mathbf{A}(\theta)\mathbf{S}(t) + \mathbf{V}(t) \quad (4)$$

where

$$\mathbf{Z}(t) = [z_1(t), z_2(t), \dots, z_M(t)]^T$$

$$\mathbf{V}(t) = [v_1(t), v_2(t), \dots, v_M(t)]^T$$

$$\mathbf{S}(t) = [s_1(t), s_2(t), \dots, s_N(t)]^T$$

$$\mathbf{A}(\theta) = [a(\theta_1), a(\theta_2), \dots, a(\theta_N)]$$

$$a(\theta_n) = [e^{-j\omega\tau_{1n}}, e^{-j\omega\tau_{2n}}, \dots, e^{-j\omega\tau_{mn}}]^T$$

Notice that the vector  $a(\theta_n)$  called steering vector defines the array manifold parameterized by the given array geometry defined by  $[x_m, y_m]$ .

The problem addressed here can be summarized as follows: Given the data  $Z(t)$ , estimate the sensor coordinates, as well as the uncertain directions of arrival of signals. Before the introduction of array shape estimation method, we make the following assumptions: (i) The signals  $s(t)$  and measurement signals  $z(t)$  are assumed stationary over the observation interval and ergodic. (ii) The calibrating signals are uncorrelated with the measurement noise. (iii) The number of sources is known or can be estimated.

### 3 The Eigenstructure Method

The eigenstructure method<sup>16</sup> is based on the eigen-decomposition of the sample covariance matrix of the vector of received signals. The information about sensor positions, which are contained in covariance matrix  $R$ , can be extracted through the steering vectors. The covariance matrix of the received signals are given by

$$R = E(Z(t)Z^H(t)) \quad (5)$$

Assuming  $\lambda_i$  and  $u_i$  are the eigenvalues and corresponding eigenvectors, the covariance matrix  $R$  can be expressed by,

$$R = U \Sigma U^H$$

where

$$\Sigma = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_M) \text{ and}$$

$$\lambda_1 > \lambda_2 > \dots > \lambda_M$$

$$U = [u_1, u_2, \dots, u_M]$$

Given the eigenvectors  $U$  and the number of sources  $N$ , we can construct signal subspace  $U_s$  and noise subspace

$$U_N = [u_{N+1}, u_{N+2}, \dots, u_M].$$

Schmit's dissertation<sup>17</sup> proved that each of the columns of  $A$  is orthogonal to the matrix  $U_N$ . According to this theorem, if we define

$$Q \triangleq \|U_N(\omega)^H A(\omega)\|^2 = \sum_{i=1}^Q \|U_N(\omega)^H a(\theta_i)\|^2$$

then  $Q$  should be zero. However, due to the uncertainties of DOAs and the sensor coordinates, the value of  $Q$  is also unknown. To find the optimal sensor positions and DOAs, we propose the minimization of the following cost function

$$Q \triangleq \|\hat{U}_N(\omega)^H \hat{A}(\omega)\|^2 = \sum_{i=1}^N \|\hat{U}_N(\omega)^H \hat{a}(\theta_i)\|^2 \quad (6)$$

The result of  $Q$  is defined by sensor coordinates  $[x_m, y_m]$  and DOAs. The search for the minimum of  $Q$  can be performed by many algorithms. The method proposed in the following is related to Gauss-Newton technique and iterates between two steps. The first step uses the initial sensor coordinates or the last estimated values to estimate the DOAs, which is the same as classical DOA estimation method, such as MUSIC algorithm. The second step, in turn, uses the DOAs provided by the first step to find the optimal sensor coordinates which minimize the cost function. Detail introduction about the method is in the following.

#### 3.1 DOAs Estimation

In the classical DOA estimation methods, the sensor positions are supposed to be known. To process this step, the initial sensor positions or last estimated sensor coordinates are applied to MUSIC algorithm. MUSIC algorithm is a typical DOA estimation method.

We evaluate the function defined by

$$P(\theta) = \frac{1}{\|U_N^H a(\theta)\|^2} \quad (7)$$

In practice, one selects a fairly dense set of radiated angles over the field of interest and computes the values of Equation (7). Then  $N$  peaks are chosen from those values and their corresponding angles are the estimated DOAs. Comparing Equations (6) and (7), this method of estimating the DOAs minimizes the cost function for the assumed sensor coordinates.

#### 3.2 Sensor Coordinates Optimization

This part aims to optimize the sensor coordinates which could minimize the cost function. Suppose the last estimated sensor coordinates are given by  $(x_{m0}, y_{m0})$ ,  $m=1, 2, \dots, M$ , while the sensor coordinates that minimize the cost function are given by

$$(x_m, y_m) = (x_{m0}, y_{m0}) + (\Delta x_m, \Delta y_m) \quad (8)$$

If  $(\Delta x_m, \Delta y_m)$  is small enough, the matrix  $A(\omega)$  can be expanded as follows

$$A(\omega) = A_0(\omega) + \Lambda_x A_1(\omega) + \Lambda_y A_2(\omega) \quad (9)$$

where  $A_0(\omega)$  is the matrix  $A(\omega)$  computed with the nominal sensor coordinates  $(x_{m0}, y_{m0})$  and

$$\Lambda_x \triangleq \text{diag}\{\Delta x_1, \Delta x_2, \dots, \Delta x_M\}$$

$$\Lambda_y \triangleq \text{diag}\{\Delta y_1, \Delta y_2, \dots, \Delta y_M\}$$

$$A_1(\omega) \triangleq jA_0(\omega) \frac{\omega}{c} \text{diag}\{\sin \theta_1, \sin \theta_2, \dots, \sin \theta_N\}$$

$$A_2(\omega) \triangleq jA_0(\omega) \frac{\omega}{c} \text{diag}\{\cos \theta_1, \cos \theta_2, \dots, \cos \theta_N\}$$

Then the cost function can be rewritten as

$$\begin{aligned} Q &\triangleq \|\hat{U}_N(\omega)^H \hat{A}(\omega)\|^2 \\ &= \|\hat{U}_N(\omega)^H [A_0(\omega) + \Lambda_x A_1(\omega) + \Lambda_y A_2(\omega)]\|^2 \\ &= \sum_{i=1}^Q \|\hat{U}_N(\omega)^H [a_{i0}(\omega) + \Lambda_x a_{i1}(\theta) + \Lambda_y a_{i2}(\theta)]\|^2 \\ &= \sum_{i=1}^N \|\hat{U}_N(\omega)^H [a_{i0}(\omega) + \text{diag}\{a_{i1}(\theta)\}v_x + \text{diag}\{a_{i2}(\theta)\}v_y]\|^2 \end{aligned}$$

where  $a_{i0}$ ,  $a_{i1}$  and  $a_{i2}$  are the  $i$ -th column vectors of  $A_0(\omega)$ ,  $A_1(\omega)$  and  $A_2(\omega)$  respectively, and

$$v_x = [\Delta x_1, \Delta x_2, \dots, \Delta x_M]^T, v_y = [\Delta y_1, \Delta y_2, \dots, \Delta y_M]^T$$

We construct the following vectors and matrices

$$v_{xy} = [v_x^T, v_y^T]^T$$

$$B(i) = -[\hat{U}_N(\omega)^H \text{diag}\{a_{i1}(\theta)\}, \hat{U}_N(\omega)^H \text{diag}\{a_{i2}(\theta)\}]$$

$$Z(i) = \hat{U}_N(\omega)^H a_{i0}(\theta)$$

Therefore, the cost function could be simplified as,

$$Q = \sum_{i=1}^N \|Z(i) - B(i)v_{xy}\|^2 \quad (10)$$

The real  $v_{xy}$  that minimizes the cost function is given by

$$\hat{v}_{xy} = [\text{Re}\{B^H B\}]^{-1} \text{Re}\{B^H Z\} \quad (11)$$

where

$$B \triangleq [B(1)^T, B(2)^T, \dots, B(N)^T]^T,$$

$$Z \triangleq [Z(1)^T, Z(2)^T, \dots, Z(N)^T]^T.$$

### 3.3 Iteration

The proposed procedure could be summarized as follows

- 1) Calculate the covariance matrix R and get the eigenvalues, as well as eigenvectors. Construct noise subspace  $\hat{U}_N$ .
- 2) Use the initial sensor coordinates to find the DOAs of signals.
- 3) Compute the cost function using Equation (6).
- 4) Use the estimated DOAs and last estimated sensor coordinates to construct B and Z and estimate the sensor coordinates according to Equation (11).
- 5) Use the new sensor coordinates to generate the new DOA estimates.
- 6) Compute the value of the cost function with new sensor coordinates and DOA estimates and verify the decrease of the value Q. If Q converged, then stop; otherwise, go back to Step 4.

## 4 The Partitioned Estimation Method

To further improve the accuracy of this algorithm, this section introduces the partitioned array shape estimation method. By dividing the array into several sub-arrays and applying the eigenstructure method to each sub-array, it could reduce the effect of former sensor positions errors on the latter one.

There are many different partitioning approaches and interleaved partitioning is applied in this paper. Interleaved partitioning divides an array into several disjoint interleaved subarrays. The purpose behind the interleaved partitioning scheme is to physically separate the receivers in each subarray as much as possible. By estimating the sensor coordinates in different subarrays, the cumulative positions errors on the whole array could be reduced. An example of interleaved partitioning is shown in Figure 1. In this paper, the whole array is partitioned into several segments using interleaved partitioning method.

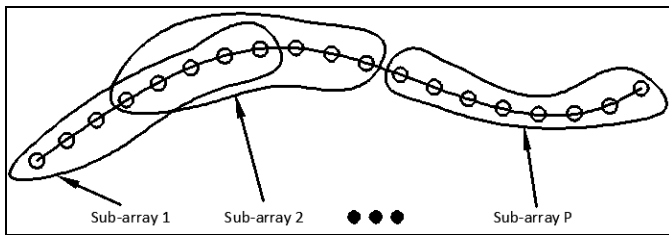


Figure 1. Interleaved partitioning array.

## 5 Numerical Studies

In this section we present several numerical simulations which illustrate the behavior of the eigenstructure method with different SNRs. Then the effect of interleaved partitioning is also shown by an example.

Consider a uniform array with 25 sensors, separated by 1 m. Two equal power narrow-band (single frequency cell) far-field P sources are

at directions  $\theta_1=30^\circ, \theta_2=60^\circ$ . The sound wave velocity is 1500 m/s and wavelength is 1 m. However, when we apply the proposed algorithm, the DOAs of signals are supposed to be unknown. The sources generate zero mean Gaussian signals. The noise is also zero mean, Gaussian, uncorrelated from sensor to sensor and uncorrelated with signals.

In this simulation, a simple curved and smooth array shape is treated as the true array shape. The original array shape (nominal shape) is very different with true array shape, which could not be used to do signal processing. The given data are the signals received by the array elements in the true array and original array element locations. The proposed method in this paper is applied to find the best array shape that is closest to the true array. To verify the stability of this method, noisy (disturbance) signals are also added to the original signals.

Figures 2 and 3 depict the estimated array shape, nominal array shape as well as the true shape with different SNRs (30 dB and 50 dB respectively). It can be seen that this method gives good results which are very close to true shape, although the nominal array shape has very big derivations with true shape. The details about the derivations are listed in Table 1.

Table 1. Sensor positions derivations using eigenstructure method.

	Maximum Derivation (m)		Average Derivation (m)	
	X	Y	X	Y
	direction	direction	direction	direction
Nominal shape	1.2280	1.9981	0.7429	1.2108
Estimated shape	30 dB	0.2990	0.7852	0.2624
50 dB	0.4581	0.5633	0.1222	0.0680

The derivations between nominal shapes and the true array are bigger than the sensor interval. The data in Table 1 indicate that the proposed algorithm significantly reduces both the maximum and average derivations in X and Y directions. In fact, most of the array elements in estimated shapes are very close to the locations of true array elements.

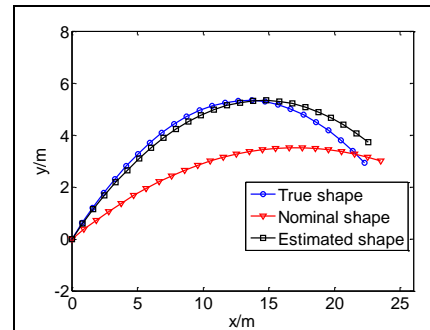


Figure 2. Array shapes (SNR=30 dB).

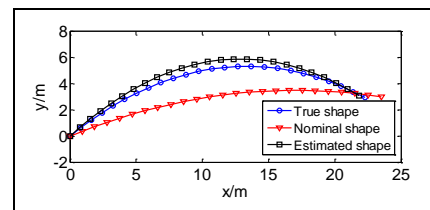


Figure 3. Array shapes (SNR=50 dB).

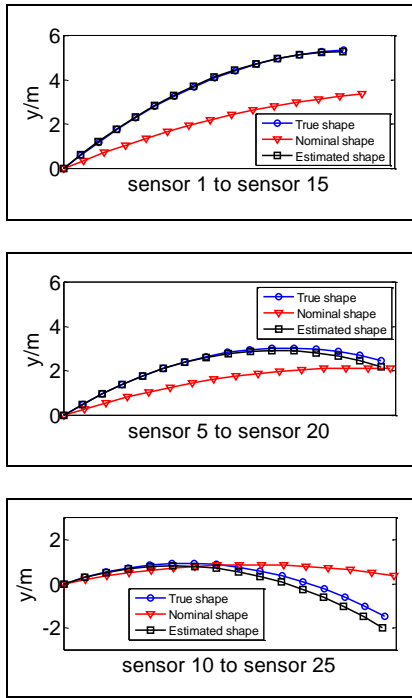


Figure 4. partitioned array shapes.

Figure 2 also implies that the derivations would increase with the increase of the index of sensor. So partitioning method is conducted to improve the accuracy of sensor locations. Figure 4 shows several figures about the estimated segmented shapes using partitioned eigenstructure method. These plots indicate that the partitioned method could provide very accurate results in the different parts of whole array. The detailed information is presented in Table 2. The partitioned eigenstructure method could prevent the cumulative error so every segment could be estimated precisely. The overall derivations are also significantly reduced.

Table 2. Sensor positions derivations using partitioned method.

	Maximum Derivation (m)		Average Derivation (m)	
	X	Y	X	Y
	direction	direction	direction	direction
Nominal shape	1.2280	1.9981	0.7429	1.2108
Estimated shape (30 dB)	0.0799	0.2922	0.0444	0.1528

## 6 Conclusion

The paper has presented an array shape estimation method with sources in uncertain localizations. Based on the eigen-decomposition method and the iteration process, the proposed algorithm leads to an optimal result for the array shape. The partitioned eigenstructure method has been subsequently introduced to reduce the derivations and get a better array shape. Numerical simulations have demonstrated that a significant improvement can be made using partitioned method. Further work would focus on the number of sources and the optimization partitioning method. Experiments are underway to verify this proposed algorithm for practical applications.

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