 SDHM, vol.12, no.1, pp.17-41, 2017

Structural Damage Detection in Framed Structures Using Under Foundation Settlement/ Rotation of Bases

Siddesha H[1] and Manjunath N Hegde[2]

Abstract This paper describes the damage detection in framed structures due to the vertical support settlement and rotation of footing bases. The damage detection procedure proposed by Nobahari and Seyedpoor (2013) is used to detect the damage in the members of the frame. In the present study, instead of using the flexibility matrix (referred here as original flexibility matrix) method, the generalized flexibility matrix is used in the same algorithm and the results are compared. The algorithm uses flexibility matrix and strain energy concept to detect the damage in the members. The behaviour of the frame is discussed through changes observed in flexibility in the associated degree of freedom. Finally, the results indicate that, the damage index determined by generalized flexibility matrix method is more reliable than using the original flexibility matrix based method for the problem of settlement and rotation of footing base.

Keywords: Damage detection, original flexibility matrix based method, generalized flexibility matrix based method.

1 Introduction

In the recent years, many high rise structures and underground structures like tunnels, metro stations, pipe networks are constructed due to the restriction of space above the ground surface. If proper measures are not taken during the excavation and compaction of the soil for their construction, damage can occur to the adjacent existing structure.

---

[1]Department of Civil Engineering, Siddaganga Institute of Technology, Tumakuru-572 103, Karnataka, India

[2] Department of Civil Engineering, Dr Ambedkar Institute of Technology, Bengaluru – 560 056, Karnataka, India

Here the damage is mainly due to the settlement of support and rotation of bases which is a local damage for the existing or old structure. This damage leads to the damages in the members of the structure. The extent of damage depends on the number of storeys, type of soil, depth of excavation etc. [Giardina, Hendriks and Rots (2015)] studied vulnerability of masonry buildings subjected to tunnelling-induced settlements.

Many researchers investigated the various damage detection methods to locate and quantify the local damages in the structures. These local damages are mainly due to the change in Young's modulus, reduction in cross sectional area and reduction in stiffness etc. in the certain members of the structure. The local damages present in the structure will alter its dynamic characteristics like natural frequencies and mode shapes [Lee (2009); Cawley and Adams (1979); Yan, Cheng, Wu and Yam (2007)]. It is easy to detect the presence of damage in a complex structure from changes in natural frequencies but it is difficult to determine the location of damage.

In the present study, the damage is induced to the frames through vertical settlement and rotation of footing base. The damage index is computed from the original flexibility matrix based method and generalized flexibility matrix based method and is compared for various modes. The flexibility matrix and strain energy concept proposed by Nobahari and Seyedpoor (2013) is used to detect the damage.

Large volumes of investigation have been reported involving reduction in Young's modulus, stiffness in the particular elements of the frame. But the damage induced by foundation settlement and their detection has not received much attention.

2 Flexibility based damage detection method

The flexibility based damage detection method is one of the promising methods to detect the damage in structures. This method is most popular because, using a first few natural frequencies the damage in the structures can be detected. Change in flexibility method is used to detect the damage in beams [Pandey and Biswas (1994); Pandey and Biswas (1995)]. Generalized flexibility matrix based approach and changes in natural frequencies are used to detect the damage in framed structure [Li, Wu, Zeng and Lim (2010)]. Modal flexibility based damage detection method is proposed for cantilever beam type structures [Sung, Koo and Jung (2014)]. A multi stage damage localization methodology is adopted by Grande and Imbimbo (2016) based on Dempester-shafer's theory for beam and four storey structure. Flexibility based structural damage detection algorithm without requiring unknown structural mass is proposed by Zhang, Xu, Guo and Wu (2013). Weng, Zhu, Xia and Mao (2013) proposed a new sub-structuring method for damage detection for portal frame structure and T.V Tower. The sub-structural flexibility matrix is decomposed into its eigen values and eigen vectors, which are used as indicators for damage detection. Bernal (2014) investigated a new approach to interrogate changes in experimentally extracted changes in flexibility to localize and quantify the damage. Flexibility based damage locating vector method was verified experimentally for three dimensional truss structure [Gao, Spencer and Bernal (2007)]. The DLV method was used to detect the multiple damages in a 3D frame structure [Monajemi, Razak and Ismail (2013)]. Damage diagnosis technique based on changes in stiffness and dynamically measured flexibility of the structures are adopted in Yan and Golinval (2005). Weng, Zhu, Li, Xia and Ye (2016) used orthogonal projector to extract the sub-structural modal flexibility matrices to detect the damage in beams. Best achievable flexibility change approach was used to quantify the damage in spring mass system, two storey frame and space truss [Yang, Sun (2011)]. Montazer and Seyedpoor (2014) proposed strain change flexibility index to locate the damage in truss structures. Experimental studies were conducted on steel grid model and damage assessment was made using flexibility based curvature technique [Catbas, Gul and Burkett (2008)]. Hosseinzadeh, Amiri, Razzaghi, Koo and Sung (2016) presented an effective method to detect and estimate the structural damage in shear frame and truss by introducing an objective function based on Modal Assurance Criteria and modal flexibility matrix.

3 Induction of damage

The following two damage scenarios are considered in the present work.

3.1 Damage due to vertical settlement of support

Pressure on footing exerted by the subgrade due to settlement is given by,

$$p = K_s \times \Delta \tag{1}$$

where,

K_s= modulus of subgrade reaction

Δ = unit settlement, p = Pressure on subgrade

The equivalent spring stiffness due to unit support settlement is determined by the equation

$$k = K_s \times F_A \tag{2}$$

Where k = spring stiffness, K_s is taken as 5000 kN/m^3 (loose sand), F_A = Footing area

3.2 Damage due to rotation of base of the footing

The damage due to rotation of bases [Bowles(2014)] is determined by the equation

$$\frac{M}{\tan\theta} = \frac{E_s B^2 L}{(1-\mu^2) I_\theta} \tag{3}$$

Where M = overturning moment resisted by the base dimension B, L= footing length, μ= Poisson's ratio, E_s = modulus of elasticity of soil, θ = base rotation and I_θ = Influence factor.

Here E_s = 10 MPa, μ = 0.3, B = 1.5 m, L = 1.0 m, for small angles $\tan\theta \approx \theta$ and I_θ is taken for rigid foundation.

4 Damage detection method

The methodology of damage detection as proposed in Nobahari and Seyedpoor (2013) is outlined below.

Considering a structural system of n degrees of freedom, the (global) original flexibility matrix is given by:

$$F_o = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \varphi_i \varphi_i^T \tag{4}$$

Considering a structural system of n degrees of freedom, the (global) generalized flexibility matrix [Li, Wu, Zeng and Lim (2010)] is given by:

$$F_g = \sum_{i=1}^{n} \frac{1}{\omega_i^4} \varphi_i \varphi_i^T \tag{5}$$

Where φ_i is the mass-normalized mode shape i and ω_i is the corresponding frequency.

Each columnar coefficients of the flexibility matrix (defined in Eq.(4) and Eq.(5)) denotes the nodal displacement pattern of the structure when a unit force is applied to the degree of freedom corresponding to that column. The columnar coefficients of the flexibility matrix f_{ij} $(j=1,......,nd)$ is utilized to obtain the strain energy of a structural element and is designated here as Flexibility Strain Energy (FSE) [Nobahari and Seyedpoor (2013)]. The FSE of e^{th} element for j^{th} column of the flexibility matrix is expressed as

$$fse_j^e = \frac{1}{2} {f_j^e}^T K^e f_j^e \quad , \quad j=1,.......,nd, \; e=1,....,ne \tag{6}$$

Where K^e is the stiffness matrix of e^{th} element of the structure f_j^e is the vector of corresponding nodal displacements of element *e*. Also, *ne* is the total number of structural elements and *nd* is the total number of columns in the flexibility matrix.

The normalized FSE for each element is given by,

$$nfse_j^e = \frac{fse_j^e}{fse_j}, \quad j = 1,....nd \; , e=1,...ne \tag{7}$$

Where $nfse_j^e$ is the normalized FSE for e^{th} element of j^{th} column of the flexibility matrix. fse_j is the summation of FSE of all the elements and fse_j^e is FSE of individual elements.

$$mnfse^e = \frac{\sum_{j=1}^{nd} nfse_j^e}{nd}, \quad e=1,...ne \tag{8}$$

$nfse_j^e$ is the normalized FSE for e^{th} element of j^{th} column of the flexibility matrix. The efficient parameter $mnfse^e$ is the mean of Eq.(7) for the *nd* columns.

The Flexibility Based Damage Index (FSEBI) is determined by,

$$FSEBI^e = \max \left[0, \frac{(mnfse^e)^d - (mnfse^e)^h}{(mnfse^e)^h}\right], \; e=1,....,ne \tag{9}$$

When the damage occurs in the element, the FSE and consequently the efficient parameter $mnfse^e$ increases. The efficient parameter $mnfse^e$ is evaluated twice, one for healthy structure and another for damaged structure indicated by $(mnfse^e)^h$ and $(mnfse^e)^d$ respectively. The relative change of efficient parameter is a good indicator for estimating the presence and location of damage. The element stiffness matrix of the healthy structure is used for estimating the parameter $mnfse^{e^d}$. For the damaged element the $FSEBI^e$ will be greater than zero and for healthy element $FSEBI^e$ will be equal to zero.

In the present study, $FSEBI^e$ is represented by damage index. The damage index is determined by using original flexibility matrix based method and generalized flexibility matrix based method, which is first proposed by Li, Wu, Zeng, Lim (2010). Two damage scenarios are considered for the 2D frames namely vertical settlement of support and rotation of footing bases.

5 Numerical investigations

The configurations of the RC frame for single bay and two bays are shown in Fig. 1 and Fig. 2, respectively. The width of the bay in each case is 4 m and each storey height is taken as 3 m for all the frame configurations. The material properties of these frames are as follows: modulus of elasticity E = 22.36 × 10^6 kN/m^2, mass density ρ = 2500 kg/m^3. In addition to this, the member dimensions are: width = 250 mm and depth = 450 mm. Every node has three degrees of freedom. The numbers shown (Fig.1 and Fig.2) in square box represent the element numbers and the numbers indicated without square box are the node numbers. The damages are introduced to these frames by inducing unit settlement of the base.

The damage due to vertical support settlement and rotation of bases are induced at node 2 and node 3 along vertical and rotational degrees of freedom for single and two bay frames respectively.

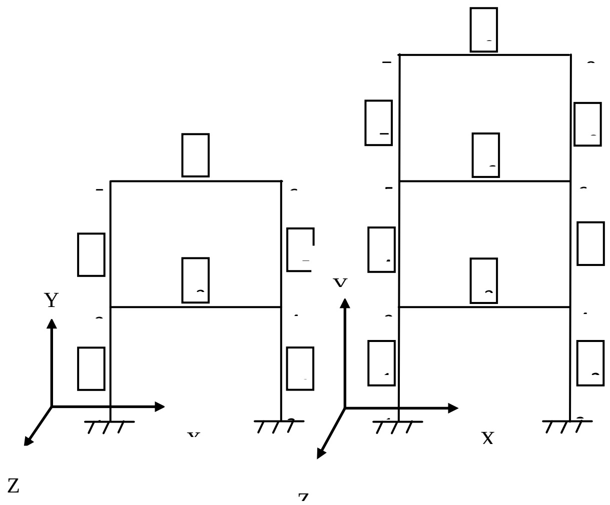


Figure 1: Single bay frames. (i) Two storey,

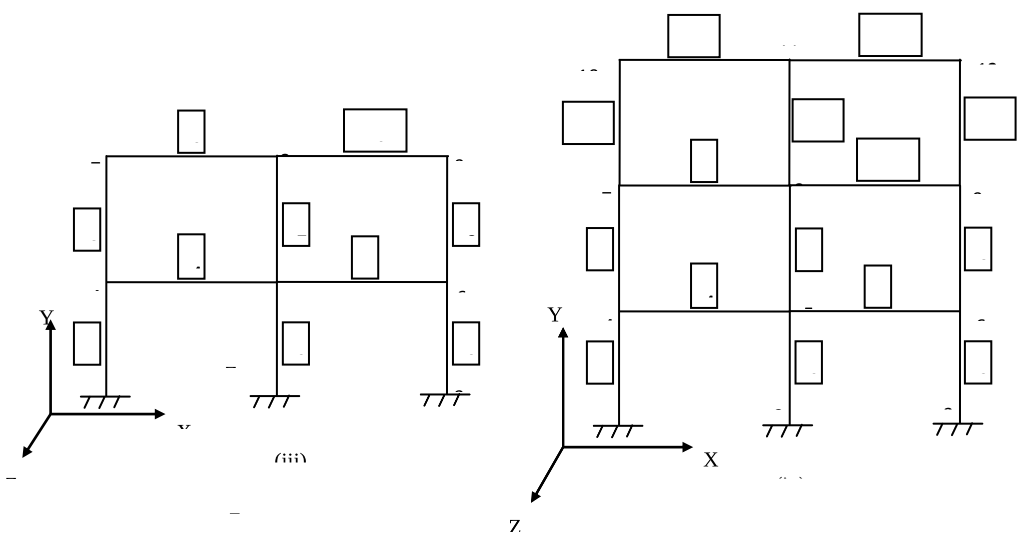


Figure 2: Two bay frames. (iii) Two storey, (iv)Three storey.

The damage in different members of the frame is detected for all the configurations of the frame by computing the damage index using the concept of flexibility matrix and strain energy. In the present study, the damage index is computed using the algorithm proposed in Nobahari and Seyedpoor (2013) by both original flexibility matrix and generalized flexibility matrix. The first three modes of vibration are considered in the flexibility based damage detection methods.

5.1 Changes in natural frequencies

Table 1 to Table 4 show the comparison between natural frequencies of the healthy and damaged frames considered for damage detection. The natural frequency decreases from 6.83% to 48.86% in single bay two storey frame due to vertical support settlement. Similarly, the natural frequency decreases from 1.64% to 7.51% due to rotation of bases. In single bay three storey frames, the natural frequency decreases from 10.02% to 42.84 % and 2.38% to 5.25% due to vertical support settlement and rotation of footing base respectively. In the case of vertical support settlement, highest percentage changes are observed in the third mode, while the first two modes indicate very little change.

The natural frequency decreases from 2.77% to 50.56% and 1% to 4.42% due to vertical support settlement and rotation of base in the cases of two bay two storey frame respectively. However, in the case of two bay three storey frames, the natural frequency decreases from 4.07% to 40.74% and 1.56% to 2.99% due to vertical support settlement and rotation of bases respectively. It can also be observed that, the natural frequency decreases with increase in number of storeys in single and two bay frames due to increased stiffness of the frames. However, the frequency method gives only the global information of the damage in frames and cannot be used directly to locate the damaged elements and quantify the elemental damage. Therefore, damage detection using flexibility method is preferable.

Table 1: Natural frequency (Hz) for single bay two storey frame

Mode No.	Single bay two storey frame				
	Healthy frame	Damaged frame			
		Vertical settlement	% Diff.	Rotation about the base	% Diff.
1	1.185	1.104	6.83	1.096	7.51
2	4.202	3.009	28.39	4.013	4.50
3	8.276	4.232	48.86	8.140	1.64

Table 2: Natural frequency (Hz) for single bay three storey frame

Mode No.	Single bay three storey frame				
	Healthy frame	Damaged frame			
		Vertical settlement	% Diff.	Rotation about the base	% Diff.
1	0.742	0.666	10.24	0.703	5.25
2	2.604	2.343	10.02	2.486	4.53
3	4.992	2.853	42.84	4.873	2.38

Table 3: Natural frequency (Hz) for two bay two storey frame

Mode No.	Two bay two storey frame				
	Healthy frame	Damaged frame			
		Vertical settlement	% Diff.	Rotation about the base	% Diff.
1	1.154	1.122	2.77	1.103	4.42
2	3.913	3.059	21.82	3.804	2.78
3	7.936	3.923	50.56	7.856	1.00

Table 4: Natural frequency (Hz) for two bay three storey frame

Mode No.	Two bay three storey frame				
	Healthy frame	Damaged frame			
		Vertical settlement	% Diff.	Rotation about the base	% Diff.
1	0.736	0.706	4.07	0.714	2.99
2	2.502	2.389	4.51	2.433	2.75
3	4.617	2.736	40.74	4.545	1.56

5.2 Member behaviour

To assess the member behaviour under the settlement of foundation and rotation of base problems, the change in flexibility in the case of original flexibility matrix based method (ΔF_o) and generalized flexibility matrix based method (ΔF_g) is computed as

$$\Delta F_o = |\ F_o^d - F_o^h\ | \tag{9}$$

$$\Delta F_g = | F_g^d - F_g^h | \tag{10}$$

Where F_o^d and F_g^d are the flexibility matrices of damaged cases in the original and generalized flexibility matrix methods, respectively. F_o^h and F_g^h are the flexibility matrices of healthy cases in the original and generalized flexibility matrix methods, respectively.

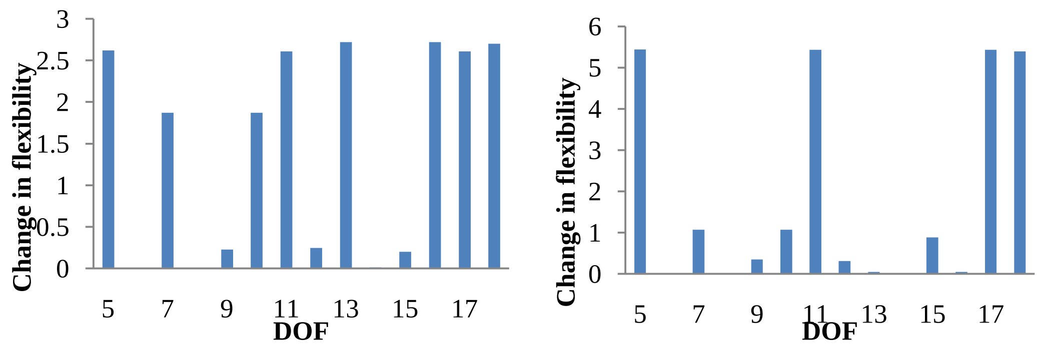


Figure 3: Change in flexibility for vertical support settlement at node 2 in single bay two storey frame in first mode (a) original flexibility matrix based method (b) generalized flexibility matrix based method.

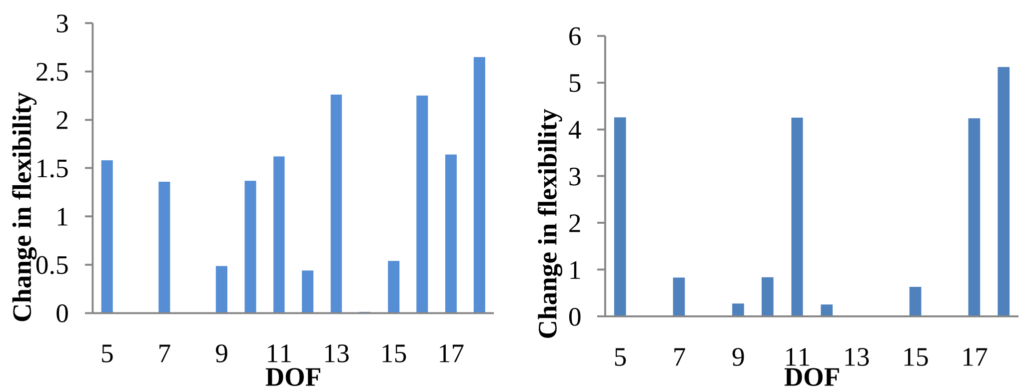


Figure 4: Change in flexibility for vertical support settlement at node 2 in single bay two storey frame for first two modes (a) original flexibility matrix based method (b) generalized flexibility matrix based method.

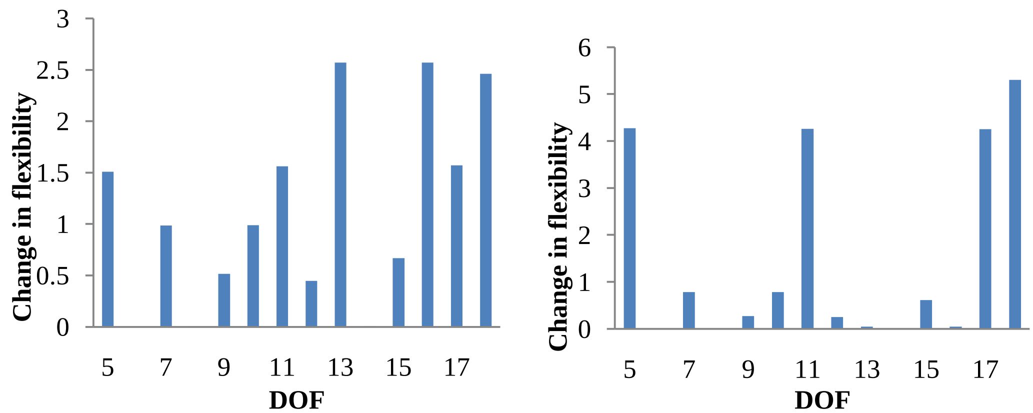


Figure 5. Change in flexibility for vertical support settlement at node 2 in single bay two storey frame for first three modes (a) original flexibility matrix based method (b) generalized flexibility matrix based method.

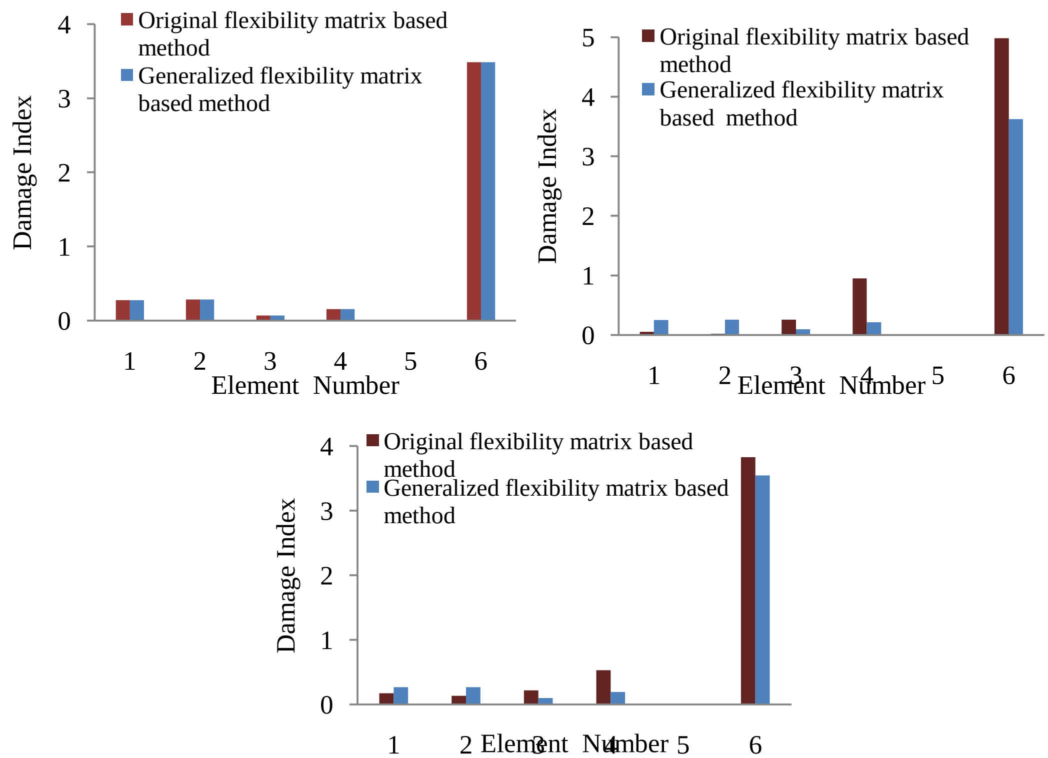


Figure 6. Damage detection results for single bay two storey frames by inducing the vertical support settlement (a) First mode (b) First two modes (c) First three modes.

Fig. 3 to Fig. 5 show the change in flexibility for single bay two storey frame under vertical support settlement in first, first two and first three modes. It is observed that, there is significant rise in the flexibility of nodes 2, 4 and 6 in the Y direction. The nodes 4 and 6 are above the damaged node 2, suggesting that the settlement at node 2 alters the flexibility of all nodes above it vertically. In Fig. 6, the most potentially damaged element is 6 in the case of single bay two storey frames. This may be due to the rise in flexibility at the node 6 in the Z direction as well as some change in flexibility at node 5 in the Z direction.

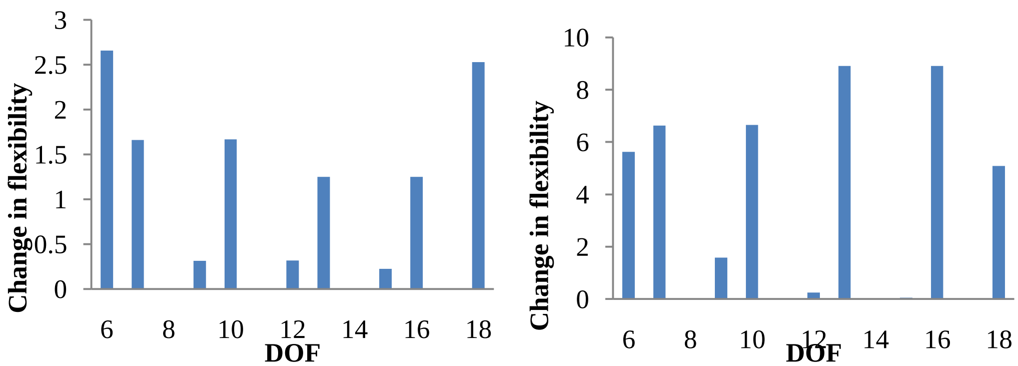


Figure 7: Change in flexibility for rotation of base at node 2 in single bay two storey frame for first mode (a) original flexibility matrix based method (b) generalized flexibility matrix based method.

Fig. 7 to Fig.9 show the change in flexibility due to rotation of base at node 2 in the case of single bay two storey frame in different modes. Induction of rotation at node 2 leads to the rise in flexibility in both X and Z directions. There is significant rise in flexibility at the nodes 5 and 6. This leads to the damage in the member 6. The damage in the member 6 is also indicated in the Fig.10.

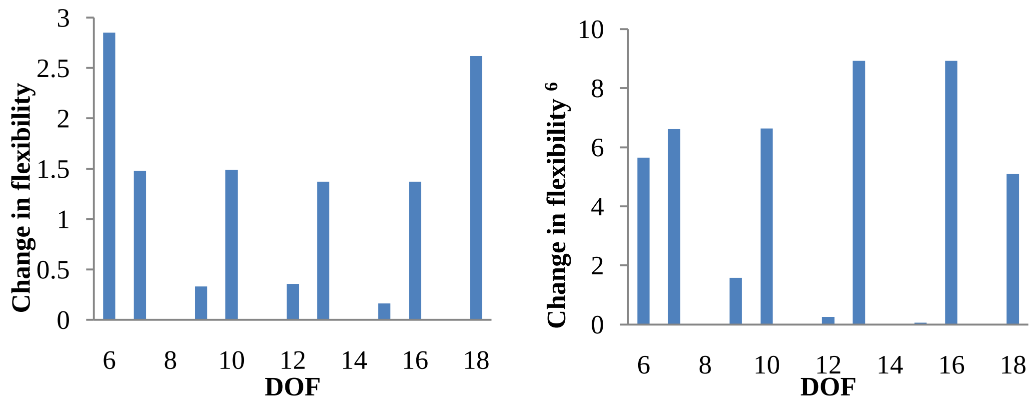


Figure 8: Change in flexibility for rotation of base at node 2 in single bay two storey frame for first two modes (a) original flexibility matrix based method (b) generalized flexibility matrix based method.

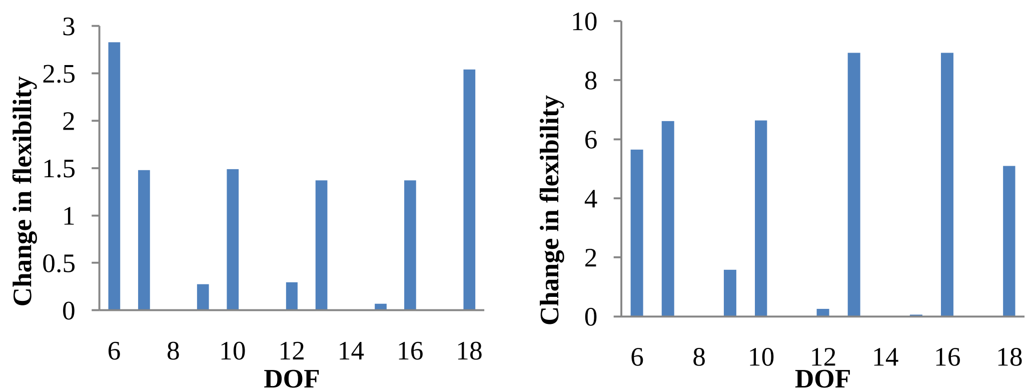


Figure 9: Change in flexibility for rotation of base at node 2 in single bay two storey frame for first three modes (a) original flexibility matrix based method (b) generalized flexibility matrix based method.

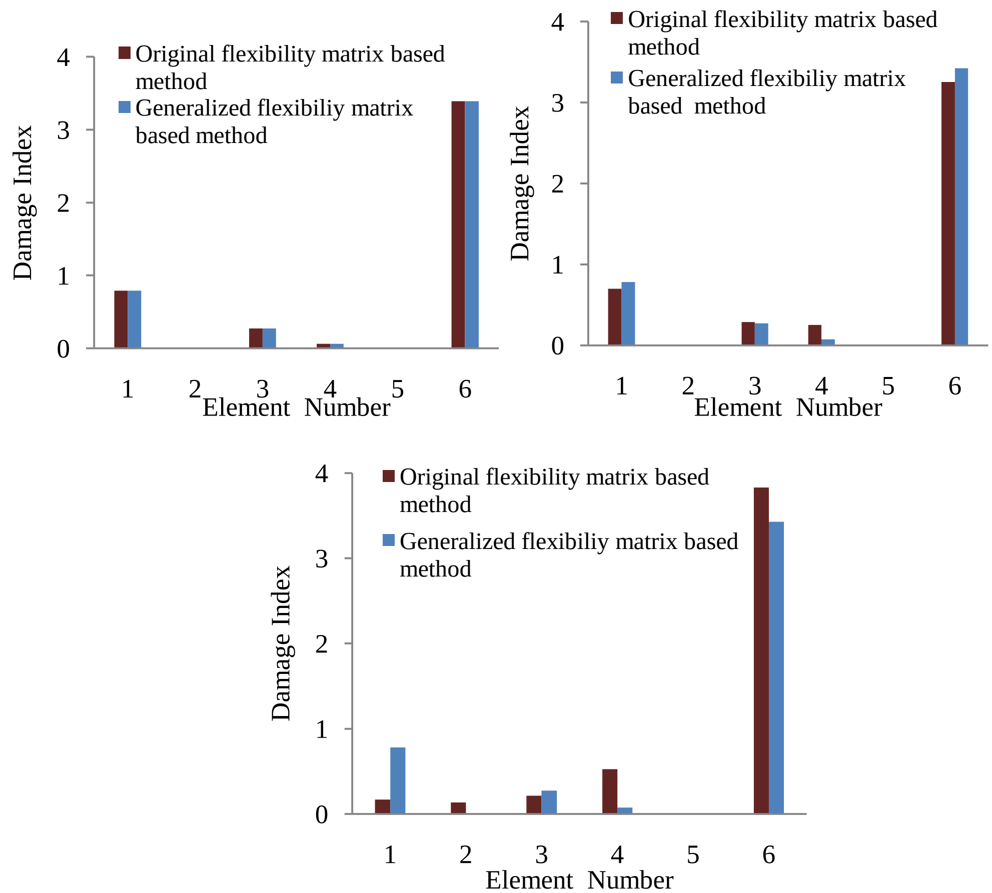


Figure 10: Damage detection results for single bay two storey frames by inducing the rotation of bases (a) First mode (b) First two modes (c) First three modes.

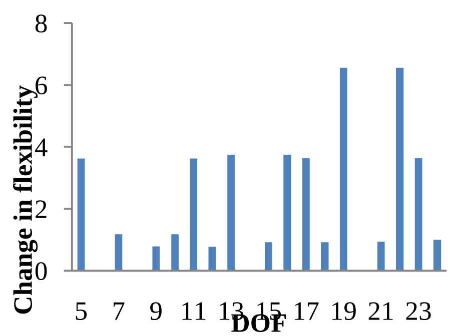


Figure 11: Change in flexibility for single bay three storey frame at node 2 in first mode using generalized flexibility method due to vertical support settlement

Fig. 11 show the change in flexibility for single bay three storey frames due to vertical support settlement. The change in flexibility is computed using generalized flexibility matrix method for three storey single bay and two bay frames. The rise in flexibility is observed in nodes 7 and 8 in the X direction. The damage could be in the member 9 due to the rise in flexibility at nodes 7 and 8. The flexibility rise is increasing from node 3 to node 8 in the X direction due to the sway action. Due to the rise in flexibility in X and Z direction, at nodes 3 and 4, there may be damage in the columns 1 and 2. The damaged members i.e 1,2 and 9 are shown in Fig.12. The original flexibility based method is unable to detect the damage in the member 9(Fig.12 (b)) in first two modes and in the members 1 and 2 (Fig. 12(c)) in first three modes of vibration.

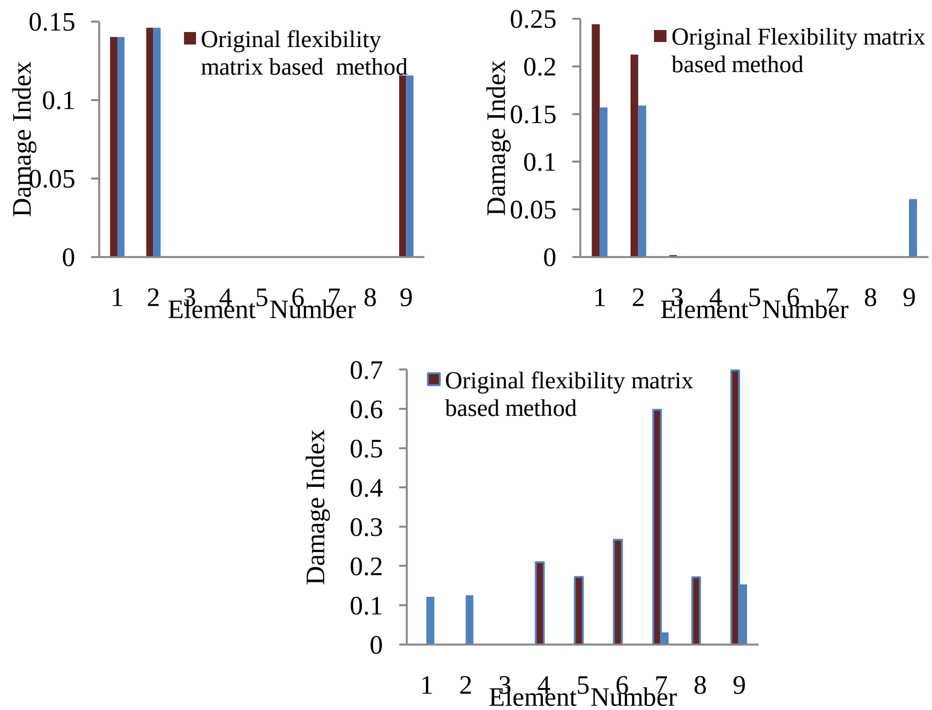


Figure 12: Damage detection results for single bay three storey frames by inducing the vertical support settlement (a) First mode (b) First two modes (c) First three modes.

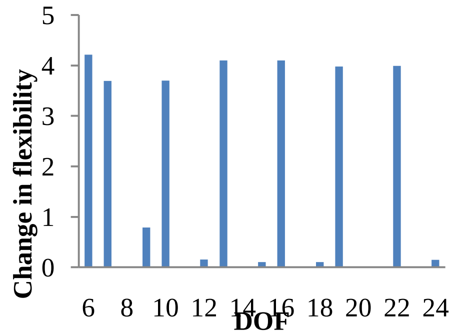
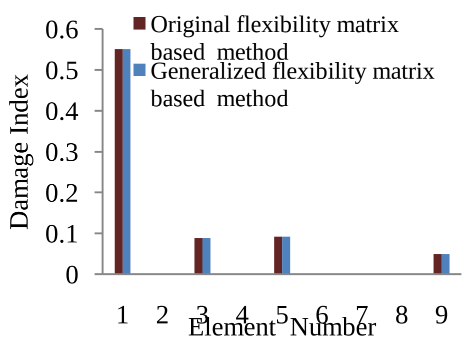
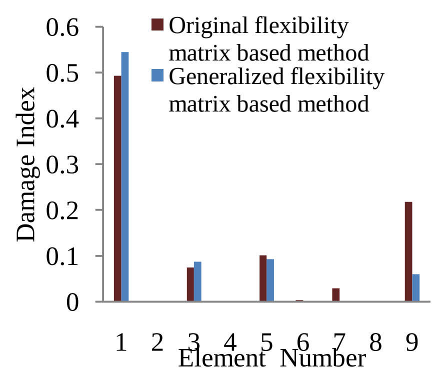


Figure 13: Change in flexibility for single bay three storey frame at node 2 in first mode using generalized flexibility method due to rotation of base.

The damaged element in single bay three storey frames due to rotation of base is 1 (Fig.14). This is due to the rise in flexibility at node 3 in X and Z directions (Fig. 13). The top floor beam suffers less damage due to the rigid body motion of the beam. The critically damaged members determined from generalized flexibility method are 4, 6 and 9 in all the modes of vibration for two bay two storey frames (Fig.16) due to vertical support settlement.





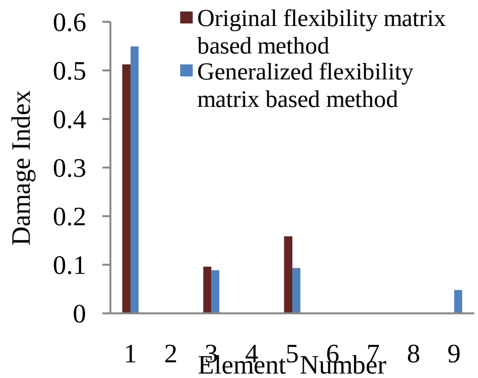


Figure 14: Damage detection results for single bay three storey frames by inducing the rotation of bases (a) First mode (b) First two modes (c) First three modes.

In two bay two storey frame, the rise in flexibility at nodes 4 and 5 in the X and Z direction can damage the member 4 (Fig.15). This beam is connected between the nodes 4 and 5. Since the column member 6 is connected at node 4, this can also undergo damage. The member 9 is also damaged, because it is connected to member 6 through the node 7. The original flexibility based method is unable to detect the damages in the elements 4, 6 and 9 in first two and first three modes of vibration (Fig. 16(b) and Fig.16(c)). Due to the addition of bay in the X direction, the rise in flexibility in the X direction is reduced.

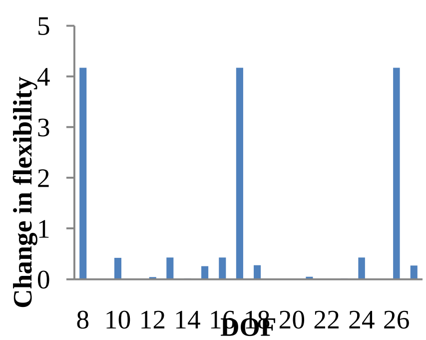


Figure 15: Change in flexibility for two bay two storey frame at node 3 in first mode using generalized flexibility method for vertical support settlement

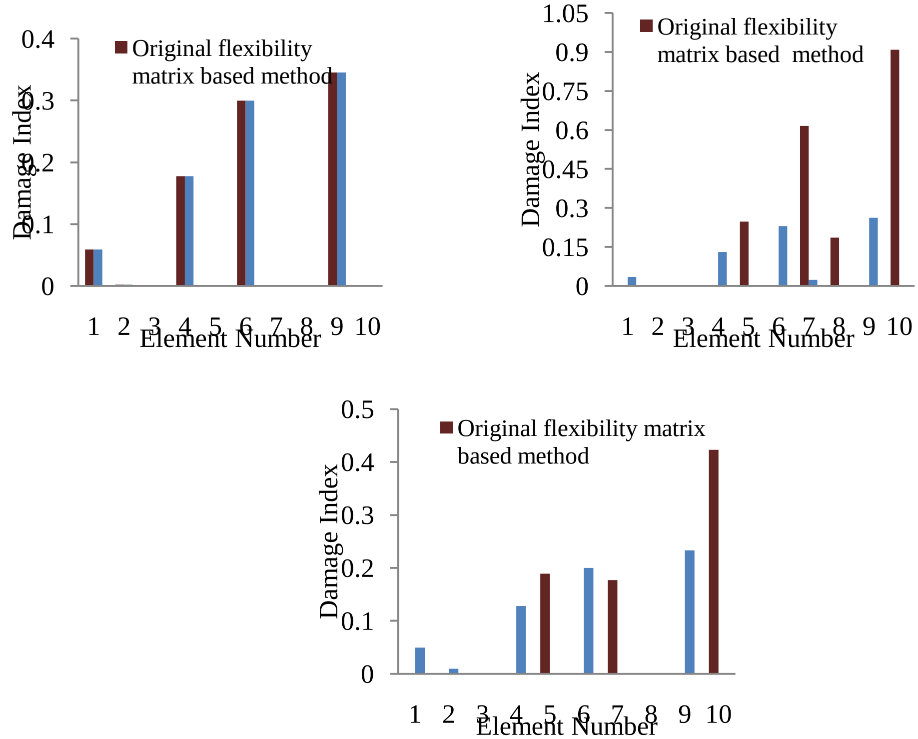
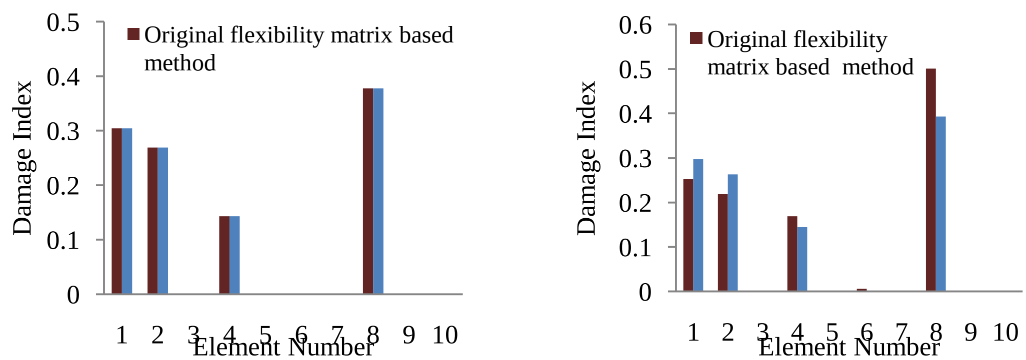


Figure 16: Damage detection results for two bay two storey frames by inducing the vertical support (a) First mode (b) First two modes (c) First three modes.

In Fig.17, the potential damaged members due to rotation of bases in two bay two storey frames are 1, 2, 4 and 8. The columns 1 and 2 are damaged due to the rise in flexibility at nodes 4 and 5 in both X and Z directions (Fig.18). Since the beam (member 4) is connected in between these nodes, it also gets damaged. The rise in flexibility at nodes 6 and 9 in the X direction causes the column member 8 to damage.



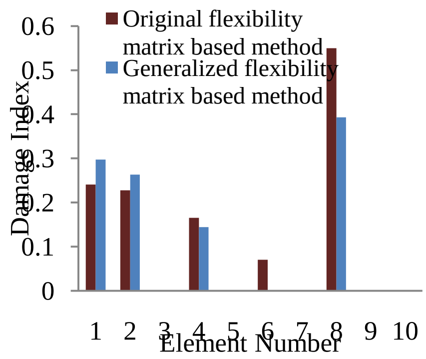


Figure 17: Damage detection results for two bay two storey frames by inducing the rotation of bases (a) First mode (b) First two modes (c) First three modes.

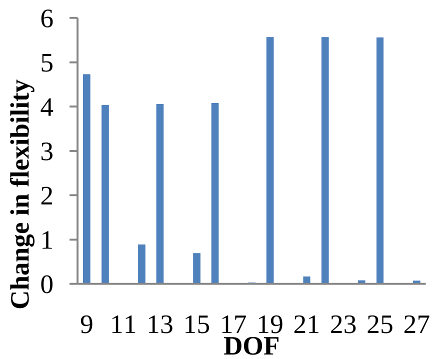


Figure 18: Change in flexibility for two bay two storey frame at node 3 in first mode using generalized flexibility method due to rotation of base.

For two bay three storey frames, the most critical members due to vertical support settlement are 4, 6, 9, 11 and 14 (Fig.19). In Fig. 19(b), the original flexibility based method is unable to detect the damage in the elements 9,11 and 14. As explained in two bay two storey frame, the similar members get damaged in two bay three storey frame under vertical support settlement (Fig. 20).

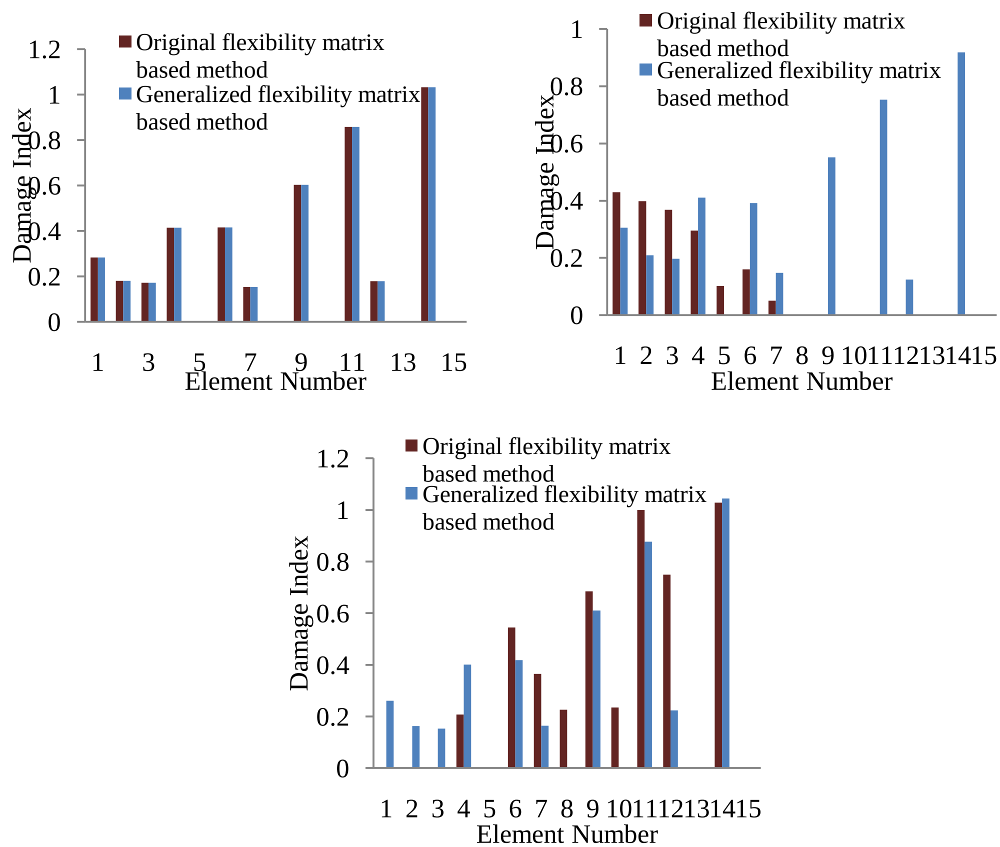


Figure 19: Damage detection results for two bay three storey frames by inducing the vertical support settlement (a) First mode (b) First two modes (c) First three modes.

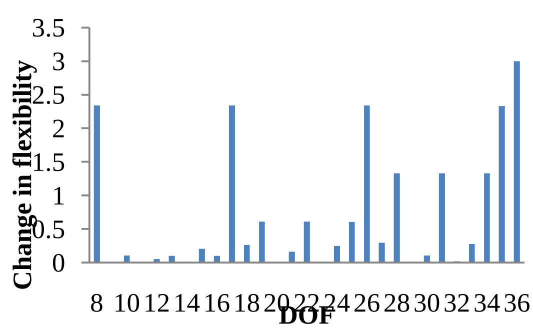


Figure 20: Change in flexibility for two bay three storey frame due to vertical support settlement at node 3 in first mode using generalized flexibility method.

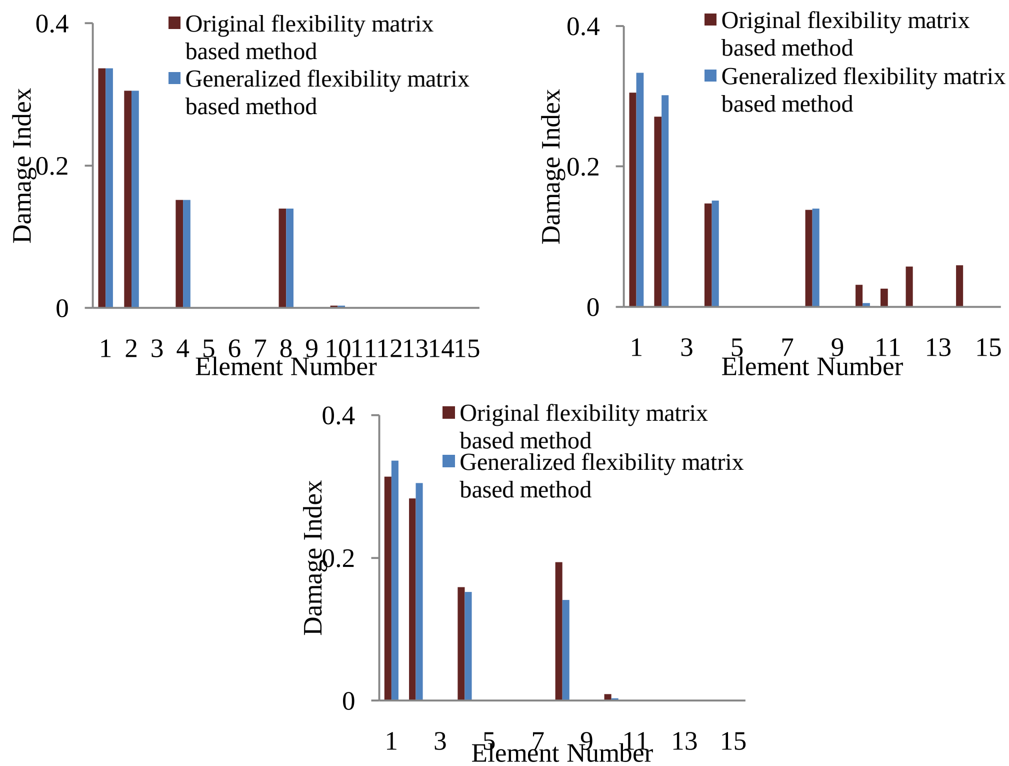


Figure 21: Damage detection results for two bay three storey frames by inducing the rotation of bases (a) First mode (b) First two modes (c) First three modes.

The potentially damaged elements in two bay three storey frames due to rotation of bases are 1, 2, 4 and 8 (Fig.21). In Fig. 22, the pattern of rise in flexibility is similar to two bay two storey frame. Among all the damaged configurations of the frame, the damage index magnitude computed in the first mode is compared well by both of the methods. In general, it is evident that the changes in flexibility of the nodes correlate well with damage location and magnitude predicted by the generalized flexibility method.

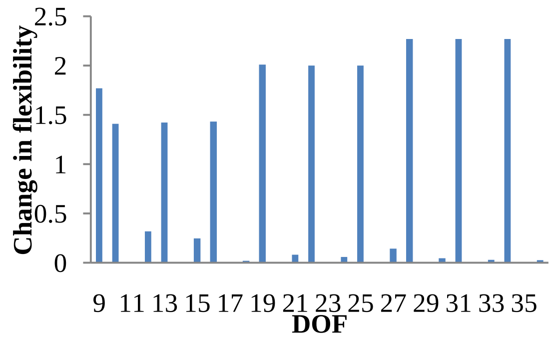


Figure 22: Change in flexibility for two bay three storey frame due to rotation of base at node 3 in first mode using generalized flexibility method.

5.3 Comparison between two damage detection methods

It is revealed that, the damage index magnitude obtained from the generalized flexibility matrix based method is nearly same in all the modes of vibration. As the number of storeys or number bays increases, original flexibility method fails to indicate the damage in certain members as the higher number of modes are considered. Whereas, the generalized flexibility method shows a consistent pattern of damaged members in all the configurations of the frame considered.

This is mainly due to the reason that, the effect of truncating higher –order modes are considerably reduced in the generalized flexibility matrix.

6 Conclusions

In this paper, flexibility matrix and the concept of strain energy is used to detect the damage due to vertical support settlement and rotation of footing bases for framed structures. The original flexibility matrix and the generalized flexibility matrix are used in the same algorithm and comparison has been made between the two methods. The behaviour of member has been discussed through the change in flexibility for various frames. The following conclusions can be drawn from the changes in flexibility matrix.

1. Rise in flexibility is observed in all nodes above the node where vertical settlement is induced in Y direction.

2. Rise in flexibility is observed in all the nodes in the X direction due to rotation of bases.

3. The addition of a bay in the lateral direction reduces the rise in flexibility in the Z direction.

The following conclusions were drawn among both the damage detection methods.

1. The effect of truncating higher –order modes can be considerably reduced in the generalized flexibility matrix based method in comparison with original flexibility based damage detection method.

2. Numerical simulations show that, the generalized flexibility matrix based method works better for determining the damage in the critical members of the frame due to the settlement problems and it shows more consistent results compared to original flexibility method.

3. In the original flexibility method, it is sufficient to consider only the first mode of vibration for predicting the damage and identifying the damage locations.

References

Aimin Yan; Jean-Claude Golinval. (2005): Structural damage localization by combining flexibility and stiffness method. *Engineering Structures*, vol. 27, pp.1752-1761.

Cawley, P.; Adams, R.D. (1979): The location of defects in structures from measurements of natural frequencies. *Journal of strain Analysis for Engineering Design*, vol. 14, pp. 49-57.

Catbas, F.N.; Gul, M.; Burkett,J.L. (2008): Damage assessment using flexibility and flexibility based curvature for structural health monitoring. *Smart Materials and Structures*, vol. 17, pp. 1-12.

Dionisio Bernal. (2014): Damage localization and quantification form the image changes in flexibility. *Journal of Engineering Mechanics*, vol. 140, pp.279-286.

Grande, E.; Imbimbo, M. (2016): A multi-stage approach for damage detection in structural systems based on flexibility. *Mechanical Systems and Signal Processing*, vol. 76-77, pp. 455-475.

Gao, Y.; Spencer, B.F.; M. ASCE.; Bernal, D. (2007): Experimental verification of the flexibility-based damage locating vector method. *Journal of Engineering Mechanics,* vol. 133, pp.1043-1049.

Giorgia Giardina; Max A.N. Hendriks; Jan G. Rots. (2015): Damage functions for the vulnerability assessment of masonry buildings subjected to tunnelling. *Journal of Structural Engineering*, vol. 141 pp.1-12.

Hooman Monajemi; Hashim Abdul Razak; Zubaidah Ismail. (2013): Damage detection in frame structures using damage locating vectors. *Measurement,* vol. 46, pp. 3541-3548.

Jing Li.; Baisheng Wu.; Zeng, Q.C.; Lim, C.W. (2010): A generalized flexibility matrix based approach for structural damage detection. *Journal of Sound and Vibration,* vol. 329, pp. 4583-4587.

Jinhee Lee. (2009): Identification of multiple cracks in a beam using natural frequencies. *Journal of Sound and Vibration*, vol. 320, pp. 482-490.

Joseph E. Bowles.(2014): Foundation analysis and design. *McGraw Hill Education (India) Private Limited*, New Delhi, fifth edition.

Mehdi Nobahari; Seyed Mohammad Seyedpoor. (2013): An efficient method for structural damage localization based on the concepts of flexibility matrix and strain energy of a structure. *Structural Engineering and Mechanics*, vol. 46, pp.231-244.

Montazer, M.; Seyedpoor, S.M. (2014): A new flexibility based damage index for damage detection of truss structures. *Shock and Vibration,* pp. 1-12.

Pandey, A.K.; Biswas, M. (1994): Damage detection in structures using changes in flexibility. *Journal of Sound and Vibration*, vol. 169, pp.3-17.

Pandey, A.K.; Biswas, M. (1995): Experimental verification of flexibility difference method for locating the damage in structures. *Journal of Sound and Vibration*, vol. 184, pp.311-328.

Sung, S.H.; Koo, K.Y.; Jung, H.J. (2014): Modal flexibility-based damage detection of cantilever beam-type structures using baseline modification. *Journal of Sound and Vibration*, vol. 333, pp. 4123-4138.

Shun Weng; Hong-Ping Zhu; Yong Xia; Ling Mao. (2013): Damage detection using the eigenparameter decomposition of substructural flexibility matrix. *Mechanical Systems and Signal Processing*, vol. 34, pp. 19-38.

Shun Weng; Hong-Ping Zhu; Peng-Hui Li; Yong Xia; Ling Ye. (2016): Construction of orthogonal projector for the damage identification by measured substructural flexibility. *Measurement*, vol. 88, pp. 441-455.

Yan, Y.J.; Cheng, L.; Wu, Z.Y.; Yam, L.H. (2007): Development of vibration-based structural damage detection technique. *Mechanical Systems and Signal Processing*, vol. 21, pp. 2198-2211.

Yang,Q.W.; Sun, B.X. (2011): Structural damage identification based on best achievable flexibility change. *Applied Mathematical Modelling*, vol. 35, pp. 5217-5224.

Zare Hosseinzadeh, A.; Ghodrati Amiri, G.; Seyed Razzaghi, S.A.; Koo, K.Y.; Sung, S.H.(2016): Structural damage detection using sparse sensor installation by optimization procedure based on the modal flexibility matrix. *Journal of Sound and Vibration*, vol. 381, pp. 65-82.

Zhang,J.; Xu, J.C.; Guo, S.L.; Wu, Z.S. (2013): Flexibility-based structural damage detection with unknown mass for IASC-ASCE benchmark studies. *Engineering Structures*, vol. 48, pp. 486-496.