Tech Science Press

A Novel Method for Vibration Mitigation of Complex Mechanical Systems

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Abstract: Taking the complex mechanical systems as the research project, a theoretical multi-degree-of-freedom (MDOF) model was established. Based on the vibration characteristics analysis of this system, a novel method of vibration mitigation was proposed, which can be applied to most of the complex mechanical systems. Through this method, limited grounding stiffness was made use of and added to certain degree of freedom (DOF) discretely. Thus, the root-mean-square (RMS) of the systems amplitude can be reduced to ideal level. The MATLAB code based on this method was attached, which was tested on the theoretical model. Consider that complex mechanical systems are nonlinear and uncertain, theoretically the optimal solution of vibration mitigation is inaccessible. However, this method can always provide a relatively effective solution.

Keywords: Vibration mitigation; complex mechanical systems; grounding stiffness; MDOF; MATLAB

1 Introduction

As a common phenomenon of machines, vibration not only reduces the operating efficiency and service life of mechanical systems, but also causes a variety of safety hazards. The noise caused by vibration will make operators' working environment harsh, and even do harm to their health. For these reasons, vibration isolation has always been a prior work for engineers in the design of mechanical systems. Nowadays, industrial machinery is gradually developing towards large scale, automated and intelligent complex mechanical systems, which are normally composed of secondary mechanical systems with different functions. Ordinary vibration mitigation methods such as mass increasing and stiffness adding are usually ineffective for these systems because of the complex coupling and blending relationships between different physical process inside them. Thus, it is still a serious challenge for mechanical engineers and researchers to design effective vibration mitigation schemes for complex mechanical systems.

Advanced global optimization algorithms are constantly being proposed and improved in order to solve the optimization problem of complex mechanical systems, which include: Artificial Bee Colony, Firefly Algorithm, Cuckoo Search, Bat Algorithm, Flower Pollination Algorithm and Grey Wolf Optimizer, etc. The objective and constraint function of all these algorithms are realized by intensive calculation. Abdulbaset [1] explored and compared to study these algorithms. Qi WuChao [2] proposed a random allocation method based on the equivalent weak form of multivariate function integral, which is used to quantify and manage the uncertainty of complex mechanical systems. Vladimir Dragoş Tătaru [3] proposed a numerical method for kinematics research of complex mechanical systems. Francesco Villecco

[4] analyzed the uncertainty evaluation in the design of complex mechanical systems. In order to gain the knowledge and assembly experience of complex mechanical systems through case studies, Zhang Kai [5] presented a data mining technique which can analyze assembly data. Ol'Shanskii V.Yu [6] studied a mechanical system composed of an invariant rigid body and its subsystems, through which they obtained the general form of quadratic integration and the necessary and sufficient conditions for its existence in the absence of dynamic symmetry.

As the algorithms being proposed and improved, it is also being possible to model complex mechanical systems based on softwares like MATLAB, Solid Work, CAD and COMSOL, which gradually becoming a popular way to research these systems. This improvement greatly promoted and facilitated the study in this area. Grinchenkov D.V [7] introduced the result of applying object-oriented method into modeling complex mechanical system dynamics during the teaching process. He also provided an example of modeling the transport vehicles based on a self-created software. Gao Xu [8] proposed a model framework to analyze the reliability of repairable polymorphic complex mechanical systems. Linjie Kan [9] proposed a way of modeling complex mechanical systems, which made use of the Colored Time Hierarchical Petri Nets modeling method. In order to solve the uncertainty problem existed in the artificial immune diagnosis method, Meng Qinghua [10] proposed an immune cloud model and simulated it based on MATLAB. On these basics, more attention has been paid on the research of vibration mitigation methods for theoretical models of complex mechanical systems. Lopez Cedric [11] studied the vibration problem of mechanical systems composed of different coupling components under high-speed impact environment and proposed an anti-vibration method based on the coupling of experiment and theory. However, Consider the limited vibration mitigation rate they got in the test, this area still needs more research.

Currently, the research on vibration mitigation of complex mechanical systems is still in the preliminary status. The relationship between the assembly process and the final performance of these systems is still not clear. In the process of design and optimization, technicians usually need to use their own experience to determine various indexes, which always leads to imprecision. In this case, this paper proposed a vibration mitigation method which could be applied to most of the multi-degree-of-freedom complex mechanical systems. In the meanwhile, a theoretical model of MDOF complex mechanical systems was established based on MATLAB for the test of this method to prove its effectiveness. This method can help engineers to schedule optimization scheme in designing MDOF complex mechanical systems.

2 Method Description

While analyzing the vibration of a multiple DOF complex mechanical systems, Newton's second law of motion is still useful [12]. However, considering that the analysis of MDOF equations involves complex algebraic operations, these formulas are usually expressed in the form of matrices. Through this way, the mass matrices, damping matrices and stiffness matrices of the systems become accessible, and the natural frequencies and modes of the vibration can also be determined with the solutions of polynomial characteristic equations. Thus, the dynamic characteristics of the multiple DOF systems' free vibration and forced vibration can be obtained through MATLAB. The theoretical model in this paper will be in the state of damped forced vibration, which is more complicated.

In this paper, the Rayleigh Damping Model was adapted, which means that the motion equation of MDOF complex mechanical systems is:

$$[m]\ddot{\vec{x}}+[c]\dot{\vec{x}}+[k]\vec{x}=\vec{f}(t) \tag{2.1}$$

Assume a static complex mechanical system with N degrees of freedom, which was in the initial state since no extra grounding stiffness was added. For this state, assume the system's mass matrix was $[m]_{N\times N}$, the initial stiffness matrix was $[k_0]_{N\times N}$. Set the value of α and β, thus the initial viscous damping coefficient matrix $[c_0]_{N\times N}$ can be determined:

$$[c_0] = \alpha[m] + \beta[k_0] \tag{2.2}$$

An external excitation with P different frequencies was added to the initial state system and force it to start the damped vibration, while the external excitation vector was $[F]_{N\times 1}$with frequency vector $[f]_{1\times P}$. Calculated the system's initial amplitude of each DOF, which was matrix $[x_0]_{N\times P}$. Assume the RMS of system's initial amplitude of each DOF was $x_{i,RMS0}$ for $i \in [1, N]$, the RMS of system's initial amplitude was x_{RMS0}, while:

$$x_{i,RMS0} = \sqrt{\frac{1}{P}\sum_{j=1}^{P}[x_0]_{ij}^2} \tag{2.3}$$

$$x_{RMS0} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}x_{i,RMS0}^2} \tag{2.4}$$

The method proposed in this paper changed the system's stiffness through adding grounding stiffness to different DOF. Assume the total grounding stiffness that could be added was K_T, which could be divided into T equal grounding stiffness K_t, thus:

$$K_t = \tfrac{1}{T}K_T \tag{2.5}$$

Set the system's final stiffness matrix was $\left[k_f\right]_{N\times N}$. For DOF *i*, L_i copies of K_t were added. Thus:

$$\left[k_f\right]_{ii} = [k_0]_{ii} + L_iK_t,\ i \in [1, N] \tag{2.6}$$

$$\sum_{i=1}^{N} L_i \le T,\ i \in [1, N] \tag{2.7}$$

Set the aimed vibration mitigation rate was D_T, Which was supposed to be determined while the vibration mitigation was scheduled. Set the RMS of system's final amplitude was x_{RMSf}, while:

$$x_{RMSf} \in (0, (1 - D_T)x_{RMS0}] \tag{2.8}$$

For each K_t to be added, the least vibration mitigation rate was D_t. Thus, in order to achieve the aimed rate:

$$D_t \in \left[1 - \sqrt[T]{(1 - D_T)}\,, 1\right) \tag{2.9}$$

Among each test of vibration mitigation, D_t could be adjusted. As D_t being increased, the vibration mitigation was supposed to be more efficiency. However, it should be noticed that if D_t was too close to 1, it might also lead to the failure of vibration mitigation. The greater T provided better schedule, which would increase the amount of calculation in the meantime.

It is worth to be noticed that seeking vibration mitigation scheme for complex mechanical systems with this method on MATLAB will put forward a high demand for calculation power of the computer. For this reason, compared to normal personal computer, the workstations are the more recommended tools. Decreasing the value of T can also be used to reduce the amount of calculation, which may also reduce the effect of vibration mitigation though.

In addition to the grounding stiffness, adding coupling stiffness for complex mechanical systems is another method of vibration mitigation. However, this way brings more uncertainty and amount of calculation. In some test for relative simpler mechanical systems, coupling stiffness didn't perform better than grounding stiffness. So, in this paper, adding coupling stiffness for complex mechanical systems didn't be considered.

3 Optimization Process

During the process of vibration mitigation, the grounding stiffness was partially added. For system that had been added for q times, its RMS amplitude was supposed to be x_{RMSq}, $q \in [0, T]$. Consider that T copies of grounding stiffness could added in total, for $q = T$, $x_{RMSq} = x_{RMSf}$.

For the $(q + 1)$ time's adding, a certain DOF I with maximum vibration amplitude was supposed to be found firstly, while:

$$x_{I,RMSq} \geq x_{i,RMSq}, \ i \in [1, N] \tag{3.1}$$

Then K_t was added to DOF I and $x_{RMSq'}$ could be obtained. If $x_{RMSq'} \leq (1 - D_t)x_{RMSq}$, then this test was succeed, let $x_{RMS(q+1)} = x_{RMSq'}$ and move to the next adding. However, if $x_{RMSq'} > (1 - D_t)x_{RMSq}$, then another DOF I' with secondary maximum vibration amplitude was supposed to be found and tested, while:

$$x_{I',RMSq} \geq x_{i,RMSq}, \ i \in [1, I) \cup^{(I,N]} \tag{3.2}$$

This step was supposed to be repeated until the $(q + 1)$ time's adding was finally succeeded.

Theoretically, the RMS of System's amplitude will be decreased by the expected rate D_T after all the grounding stiffness was added. However, in practice, some exceptions might happen. For example, in a certain stage, whatever DOF the K_t was added, the RMS of system's amplitude could not be decreased by the rate that not less than D_t, which could be caused by unreasonable vibration mitigation schedule. For this case, decreasing D_t or adding plural K_t in each test would be helpful. The program code attached in Appendix A had adapted the second plan. However, it's worth to be noticed that both these two plans reduced the initial aim D_T, which might cause the decrease of the final vibration mitigation rate.

This method can also be used in system's stiffness rearrange. In this case, the grounding stiffness of each DOF are supposed to be separated a same part to compose the K_T. Then this K_T will be added back to this system through the method above. Thus, the vibration mitigation of this system will be achieved through stiffness rearrange.

4 Model Test

Established a theoretical model of a complex mechanical system with 500 DOF based on MATLAB, which meant that $N = 500$. For this initial model, any of two DOF were coupled and blended, which meant that:

$$[k_0]_{ij} \neq 0, \ i \in [1, N], \ j \in [1, N] \tag{4.1}$$

$$[c_0]_{ij} \neq 0, \ i \in [1, N], \ j \in [1, N] \tag{4.2}$$

In the initial state, the RMS of stiffness of each DOF was $k_{RMS0} \triangleq 4.3608 \times 10^9 N/m$, the RMS of mass of each DOF was $M_{RMS} \triangleq 3.1400kg$, $\alpha = 10^{-3}$ and $\beta = 10^{-6}$, from which $[c_0]$ could be determined. An external excitation was added, the RMS of which of each DOF was $F_{RMS} \triangleq 1.4039N$. Totally 1000 different frequencies of the excitation were evenly distributed between 1kHz and 2kHz, which meant that $P = 1000$. Figured out that $x_{RMS0} \triangleq 4.1239 \times 10^{-5} m$.

Set that $K_T = 10^7 N/m$, $T = 100$, which meant that $K_t = 10^5 N/m$. The average extra grounding stiffness for each DOF was only about $2 \times 10^4 N/m$, which could almost be ignored compared with k_{RMS}. Set $D_T = 30\%$ and $D_t = 99.7\%$ in the meantime.

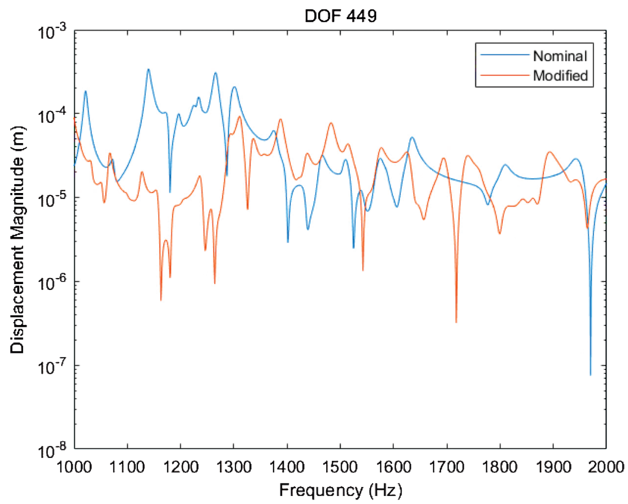

Figure 1: System's amplitude of DOF 449 in different frequencies before and after the mitigation

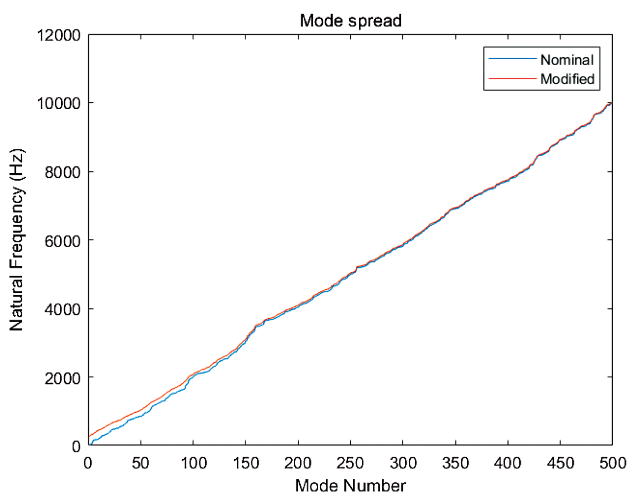

Figure 2: System's nature frequencies of each DOF before and after the mitigation

When the test was finished, all the grounding stiffness had been successfully added into the system. The system's final RMS of amplitude of each DOF was about $2.4864 \times 10^{-5}m$, the decreasing rate was about 39.708%, which was much greater than D_T. DOF 449 was the DOF with maximum decreasing rate, as shown in Fig. 1. The nature frequencies of each DOF also increased obviously, as shown in Fig. 2. Noticed that this increase was mainly concentrated in those DOF with relative lower initial nature frequencies. The vibration in frequency interval 1100 Hz~1300 Hz and interval 1500 Hz~1650 Hz was decreased obviously, but those in frequency interval 1650 Hz~1900 Hz got increased in contract, as shown in Fig. 3, which reflected the uncertainty of complex mechanical systems. In the meantime, the initial amplitude in decreasing frequency interval was relative greater, while those in increasing frequency interval was relative less, which reflected that this method could also reduce the variance of the system's amplitude in different frequency interval.

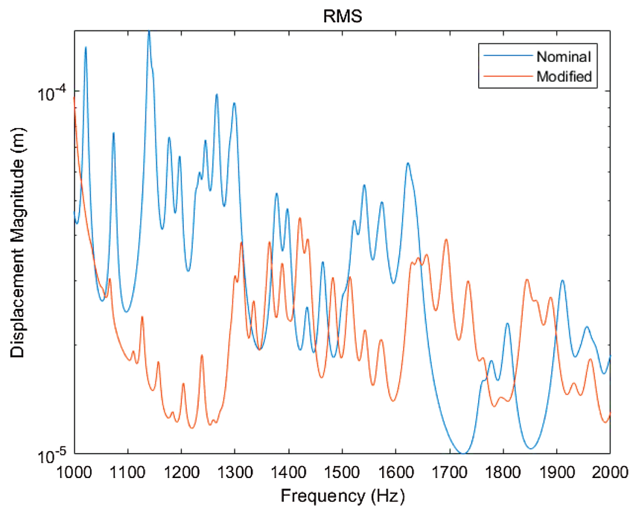

Figure 3: RMS of system's amplitude in different frequencies before and after the mitigation

5 Conclusion

Proposed a method for the vibration mitigation of MDOF complex mechanical systems and tested it on the theoretical model based on MATLAB. This method decreased the RMS of system's vibration amplitude through adding limited grounding stiffness to certain degree of freedom (DOF) discretely, which can also be used in stiffness rearrange. In Appendix A, the program code based on this method was provided for reference. The result showed that this method provided excellent vibration mitigation efficiency.

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Appendix A. The program code for model test

```
load('optimize_given.mat');
D_a = 0.997;
P = length(f);
N = size(M,1);
omega = 2*pi*f;
K_0 = K;
[V_0,D_0] = eig(K_0,M);
omega_n_0 = sqrt(diag(D_0));
f_n_0 = omega_n_0/(2*pi);
zeta_n_0 = (alpha + (omega_n_0.^2)*beta)./(2*omega_n_0);
X_0 = zeros(N, P);
for p = 1:P
    for n = 1:N
        if(f_n_0(n) < Inf)
        X_0(:,p) = X_0(:,p) + (V_0(:,n)'*F)*V_0(:,n)/(omega_n_0(n)^2 +...
        2*1i*zeta_n_0(n)*omega(p)*omega_n_0(n) - omega(p)^2);
        end
    end
end
RMS_0 = rms(rms(X_0));
K_a = 1e5;
K_add = K_a;
Num = 1;
K_mod = K_0;
RMS_mod = RMS_0;
K_g = zeros(500,2);
for i = 1:500
    K_g(i,1) = i;
end
a = 0;
X_mod = X_0;
while(a < 100)
RMS_n = rms(X_mod, 2);
RMS_f = RMS_n;
while(a < inf)
K_tes = K_mod;
X_tes = zeros(N, P);
[m, f] = max(RMS_f);
K_tes(f,f) = K_tes(f,f)+K_add;
[V_tes,D_tes] = eig(K_tes,M);
omega_n_tes = sqrt(diag(D_tes));
f_n_tes = omega_n_tes/(2*pi);
zeta_n_tes = (alpha + (omega_n_tes.^2)*beta)./(2*omega_n_tes);
for p = 1:P
    for n = 1:N
        if(f_n_tes(n) < Inf)
        X_tes(:,p) = X_tes(:,p) + (V_tes(:,n)'*F)*V_tes(:,n)/(omega_n_tes(n)^2 +...
        2*1i*zeta_n_tes(n)*omega(p)*omega_n_tes(n) - omega(p)^2);
        end
    end
end
RMS_tes = rms(rms(X_tes));
if(RMS_tes < D_a*RMS_mod)
    K_mod = K_tes;
    K_g(f,2) =  K_g(f,2)+K_add;
    X_mod = X_tes;
    RMS_mod = RMS_tes;
```

```
        a = a+Num;
        Num = 1;
    break
    else
        RMS_f(f) = min(RMS_f);
    end
    if(min(RMS_f) >= max(RMS_f))
        K_add = K_add+K_a;
        Num = Num+1;
    if(a+Num > 100)
        a = inf;
        break
    else
        break
    end
    end
    end
    end
    DEC = (RMS_0-RMS_mod)/RMS_0;
    omega_n_mod = sqrt(diag(D_mod));
    f_n_mod = omega_n_mod/(2*pi);
    DEC = (RMS_0-RMS_mod)/RMS_0;
    disp(DEC)
    save K_g
    figure
     hp = plot(1:N, f_n_0,1:N,f_n_mod);
     legend('Nominal','Modified')
     xlabel('Mode Number')
     ylabel('Natural Frequency (Hz)')
     title(('Mode spread'))
    figure
     [~,n] = max(abs(rms(abs(X_mod),2)-rms(abs(X_0),2)));
     hp1 = semilogy(f, abs(X_0(n,:)),f, abs(X_mod(n,:)));
     legend('Nominal', 'Modified')
     xlabel('Frequency (Hz)')
     ylabel('Displacement Magnitude (m)')
     title(['DOF ' num2str(n)])
    figure
     hp2 = semilogy(f, rms(abs(X_0),1),f, rms(abs(X_mod),1));
     legend('Nominal','Modified')
     xlabel('Frequency (Hz)')
     ylabel('Displacement Magnitude (m)')
     title(('RMS'))
```