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Some Thermal Modulation Effects on Directional Solidification

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Abstract: This paper deals with the investigation of thermovibrational convection induced by harmonic oscillations of the temperature boundary conditions related to the hot wall in a Bridgman-like (VB) geometrical configurations. Two different models of the VB configuration are considered (a simplified version referred to as "restricted" model and a more realistic and complete model with phase change allowed). The effects of temperature modulation are considered for both models and with regard to several possible initial (basic) states (stationary and oscillatory). In the restricted fluid cavity, we identify the existence of a critical frequency minimizing the flow intensity in the steady basic regime. In the periodic regime, the oscillation amplitude of the dynamic field can be controlled around a threshold frequency. In the full cavity, when the basic steady regime is considered, the average interface deformation is more affected under low frequency and high amplitude thermal modulation. For the quasi-periodic basic regime, the results show the existence of a modulation frequency band for which the flow is stabilized. This stability improvement depends on the thermal modulation amplitude.

keyword: Vertical Bridgman configuration, Computational Fluid dynamics, Thermal modulation, Crystal growth, Finite Volume methods.

1 Introduction

The Bridgman configuration is often used for growing semiconductor crystals. It offers simultaneously an industrial efficient process and an ideal academic configuration for fundamental studies (Adornato and Brown 1987, Brown 1988, Garandet and Alboussière 1999, Lappa 2005a). For the Vertical Bridgman configuration, the classical Rayleigh-Bénard problem gives a first approach to understand the complexity of flow development during the transition from conductive to convective regimes and its effect on the coupling with solid/liquid transition (Kaenton et al, 2002 & 2004). Stable flows are of interest in practical applications because of their beneficial impact on the redistribution of species. For example, in the crystal growth for electronic applications, convection in the liquid phase strongly affects dopant segregation and influences the interface shape. In this type of configuration, fluctuations of the interface velocity can lead to microsegregation. This phenomenon can introduce undesirable effects on the physical properties of grown crystals as was underlined for various growth techniques by Hurle and *al* (1974), Muller (1996), Lan and Yeh (2005). Several authors have studied possible strategies for the control of these phenomena in order to reduce the convection effect on the solid/liquid interface (see, e.g., Amberg and Shiomi, 2005; Lappa, 2005b and references therein).

During directional solidification, the oscillation of the phase change front can result from several factors such as fluctuations of the pulling velocity, a non stationary flow developing for strong thermal or solutal gradients or fluctuations recorded in the imposed temperature gradient. The temperature fluctuations effect on the interface velocity in the vertical Bridgman configuration was studied numerically by Stelian *et al.* (2002) for a fluid with *Pr=0.07*. The effect of the modulation period on the interface velocity has been investigated for a duration ranging between 5 and 3000*s*. As a cut-off period for the velocity amplitude was observed it was concluded that temperature oscillations with periods lower than the cut-off one have no harmful effect on the growth process.

In order to investigate flow instabilities for Bridgman configurations, under full or low gravity conditions, fluid phase models in 2D configurations have been largely used by a number of authors including Larroudé *et al.* (1994), El Ganaoui and Bontoux 1998, Semma *et al.*

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(2003), Kaenton *et al.* (2004). 3D extensions for unsteadiness and flow transition in a cavity heated from below have been provided by Accary et al (2005), Bennacer et al (2005).

It is known that both flow and thresholds of unsteadiness are strongly affected by thermal boundary conditions varying with time. Such situations are encountered for instance in periodically energized electronic components, which induce unsteady heat generation.

Many studies have appeared in the literature dealing with thermovibrational convection and similar problems (for recent results, see, e.g., Yan et al., 2005; Achour et al., 2006; Kozlov and Selin, 2006).

In particular, Antohe and Lage (1996) investigated the effects of heating amplitude and frequency on the transport phenomena considering enclosures filled with a clear fluid and a fully saturated porous medium under time periodic square wave heating in the horizontal direction for a liquid with $Pr=0.7$. It was underlined that periodic heating is very important since flow resonance appears as the heating frequency matches the natural frequency of the flow sweep inside the enclosure. The resonance frequency was shown to be independent of the heating amplitude for both the clear fluid and porous medium configurations.

Kwak and *al*. (1998) studied numerically the effects of the amplitude and frequency of the hot side sinusoidal wall oscillation on the enhancement of heat transfer in a square cavity with fixed $Ra = 10^7$ and $Pr = 0.7$ (for a configuration similar to the one used by Antohe and Lage, 1996). Once more it was observed that the maximum increase of the time-averaged heat transfer rate occurs at the resonance frequency between natural frequency of the flow and the modulation frequency (therefore by applying external proper frequency it is possible to achieve a resonant enhancement of heat transfer in the cavity). Such behavior can be useful for possible control of industrial situations, e.g. those encountered during crystal growth from the melt. In the same context, Chung and *al* (2001) studied the finite-wall effect on buoyant convection in an enclosure with pulsating exterior surface temperature. They showed that the amplitude of oscillating Nusselt number, at the central plane peaks at a particular pulsation frequency interpreted as a resonance state.

From the studies listed above, only Stelian *et al.* (2002) applied thermal modulation to a realistic situation occurring in VB configuration focusing on the oscillation of the growth velocity. This interesting study has provided information about the response of the interface velocity to hot wall thermal oscillations during growth but has been limited to the transient state.

At the best of the author's knowledge, no specific study of thermal modulation interacting with the bifurcation diagram is available.

The present issue considers the effects of the amplitude and the frequency of an imposed sinusoidal oscillation on the hot wall. Results are compared to available accurate ones without temperature modulation. For the restricted fluid phase configuration (as simplified geometrical model), it is shown the possible control of the flow motion by varying the modulation parameters. For the full configuration, external thermal modulation can allow a control strategy of the flow oscillations in order to minimize effects of instability on the solid/liquid interface.

2 Model and numerical approximations

2.1 Problem definition and model

An idealized sketch of the Vertical Bridgman (VB) growth configuration is shown on Figure 1. It consists of a vertical ampoule heated from below at temperature T'_H and cooled from the top at T'_C.The temperature profile along the lateral boundary is composed of two isothermal zones (hot temperature T'_H and cold temperature T'_C) linked by an adiabatic zone of length $L_{\Delta T}$ (for the model simplification, a prime denotes a variable with dimensions). The problem is solved in the two-dimensional Cartesian domain $D = [0, AR] \times [0, 1]$, where $AR = L/H$ is the aspect ratio of the configuration. The cavity contains a material with a low Prandtl number ($Pr=0.01$) in fluid phase and heated periodically. The sinusoidal hot temperature is characterized by its modulation amplitude ε' and frequency f' as $T'_H = \overline{T}'_H + \varepsilon' \sin(2\pi f' t')$.

The flow in the fluid phase is supposed laminar for a Newtonian and incompressible fluid. The physical properties of the fluid are assumed constant with the exception of the variations in the density (that are taken into account within the framework of the Boussinesq approximation, i.e., $\rho = \rho_0(1-\beta(T'-T'_0))$). The length, temperature and velocity are reduced using the following scaling: $x = x'/H$, $T = (T'-T'_C)/(T'_H-T'_C)$, $\boldsymbol{u}=\boldsymbol{u}'H/\alpha$ respectively. β is the thermal expansion coefficient. H is the height of the cavity and α is the thermal diffusiv-

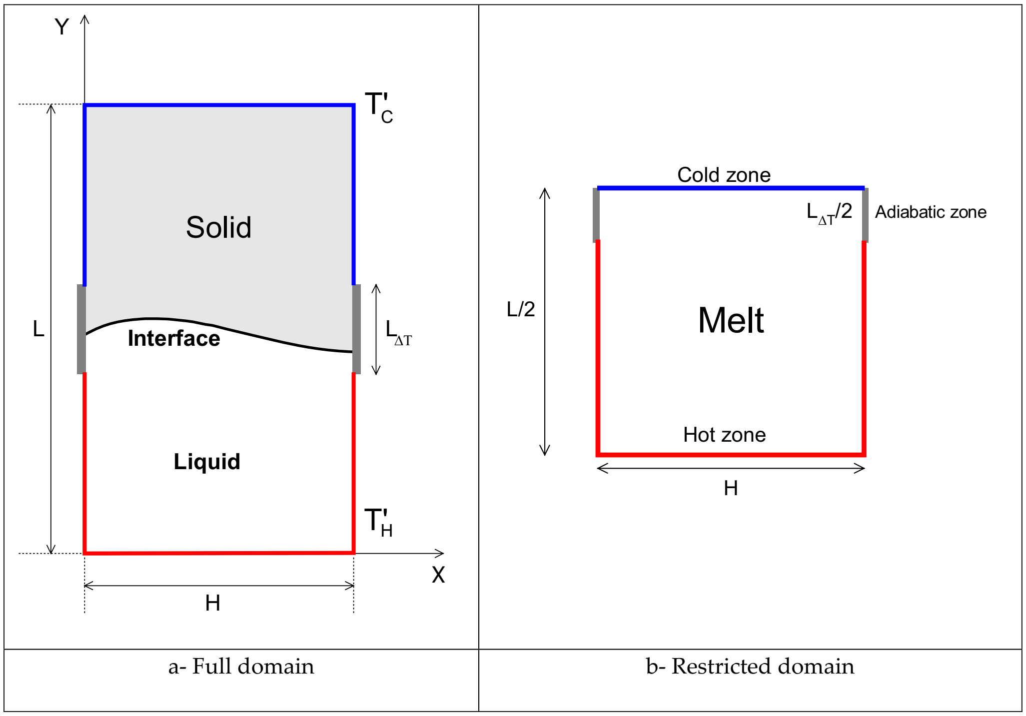


Figure 1 : Vertical Bridgman Configuration. (a) full domain, (b) restricted domain.

ity. For the full configuration, the dimensionless melting temperature is $T_m = 0.5$. The modulation amplitude and frequency in dimensionless form are given by:

$$(\varepsilon, f) = \left(\frac{\varepsilon'}{\Delta T'}, H^2 \frac{f'}{\alpha}\right) \quad (1)$$

The mathematical model is based on the enthalpy-porosity approach in which the energy equation uses a continuous enthalpy variable to avoid explicit Stefan interfacial condition and to allow the use of fixed grids (Voller and Prakash 1989, Bennon and Incropera 1987). This approach was successfully used for directional solidification involving unsteady melt interacting with front by El Ganaoui and Bontoux (1998) and Semma *et al.* (2003) .

The governing time-dependent Navier-Stokes equations, in properly non-dimensionalized form can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mathrm{Pr}\,\nabla^2 u + K^{-1}u \quad (3)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + Pr\nabla^2 v + RaPr\theta + K^{-1}v \quad (4)$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} = \nabla^2 h - \nabla.(h_l - h)u \quad (5)$$

If we suppose that the specific heats for the liquid and the solid phases are constant, the enthalpies for each phase (h_s and h_l) vary linearly with the temperature:

$$h_s = T$$

$$h_l = T + Ste^{-1} \quad (6)$$

$Ra = g\beta\Delta T H^3/\nu\alpha, \mathrm{Pr} = \nu/\alpha$ and $Ste = C_p\Delta T'/L_f$ are respectively the Rayleigh, Prandtl and Stefan numbers defining completely the physical system as control parameters.

In order to introduce the average momentum equation, we have also supposed that the two-phase zone is saturated. The Darcy source term has been adopted in the momentum equation to gradually reduce the velocity in the solidifying zone. It is defined as a function of the local liquid volume fraction as it is often used for viscous flows in porous media (Carman-Kozeny relationship, Beckermann and Viskanta 1988):

$$K = K_0\left[(1 - f_s)^3 / f_s^2\right] \quad (7)$$

K_0 depends on the solidification microstructures. The liquid volume fraction is defined as:

$$f_l = \begin{cases} 1 \text{ if } T > T_m + \tau \\ \frac{h-h_s}{h_l-h_s} \text{ if } T_m - \tau \leq T \leq T_m + \tau \\ 0 \text{ if } T < T_m - \tau \end{cases} \tag{8}$$

Where τ is a parameter characterizing the phase change size (τ is taken here equal to 10^{-2}, h_s and h_l are the sensible enthalpies corresponding to $T_m - \tau$ and $T_m + \tau$ respectively).

The associated Boundary Conditions (BCs) for the full configuration $[0,1] \times [0,2]$ are:

The cavity is dynamically submitted to impermeable conditions $u = v = 0$ and thermally to:

$$T = 0 \quad \text{at } y = 2,\ x = (0,1) \text{ and } y \in [1.25\ ,\ 2]$$

$$T = 1 + \varepsilon \sin(2\pi f) \text{ at } y = 0,\ x = (0,1) \text{ and y} \in [0\ ,\ 0.75] \tag{9}$$

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } x = 0,1 \text{ and y} \in [0.75\ ,\ 1.25]$$

Note that for the restricted cavity to the fluid phase $[0,1] \times [0,1]$, similar conditions are considered with making an exclusion of the solid part (the interface been the top wall).

In order to study the effect of the wall temperature oscillations on the heat transfer and phase change characteristics, the following quantities are introduced as proposed by Kwak *et al*. (1998):

Maximum deformation D_m of the melting interface in the vertical direction, where the interface is defined by: $I = \{(x,y)/T(x,y) = T_m\}$ and $I_y = \{y \in I\}$

$$D_m^y = \max I_y - \min I_y$$

For all study variables ϕ, we define, for periodic regimes, the relative values :

$$G(\phi) = \frac{\overline{\phi}(\varepsilon, f)}{\overline{\phi}(0,0)}$$

$$A(\phi) = \frac{\phi_{\max}(\varepsilon, f) - \phi_{\min}(\varepsilon, f)}{\overline{\phi}(0,0)} \tag{10}$$

In the above, ϕ stands for an arbitrary physical variable and $\overline{\phi}$ its average value.

The following notations for flow structure are adopted: **SS** for Stationary Symmetrical solution (two symmetrical cells); **SAS** for Stationary Asymmetrical Solution (two cells of unequal sizes and intensities). Periodic **P1** and **P2** types are also distinguished according to one or two periods exhibited.

2.2 Numerical approximations

For numerical approximations of the problem a finite volume method has been used. The approach is summarized in the following section with description of the numerical schemes chosen followed by validations.

We consider a two-dimensional convection diffusion equation for a general variable φ coupled to the continuity equation:

$$\frac{\partial \varphi}{\partial t} + \nabla . F(\varphi) = f \tag{11}$$

$$\nabla .\, u = 0 \tag{12}$$

Here $F(\varphi) = u\varphi - \gamma_\varphi \nabla \varphi$ is the advection-diffusion tensor with convective part $\boldsymbol{F}^c = u\varphi$ and diffusive part $F^d = -\gamma_\varphi \nabla \varphi$ (γ_φ is the coefficient of diffusion).

As equation (11) gives the expression for the conservation of φ in an infinitesimal domain, it is equivalent to write in any sub-domain V:

$$\int_V \varphi(x,t')dx - \int_V \varphi(x,t)dx + \int_t^{t'} \int_{\partial V} F.\tau_V(x) d\sigma(x) dx dt = \int_V \int_t^{t'} f(x,t) dx dt \tag{13}$$

where $\tau_V(\mathbf{x})$ is the normal vector to the boundary ∂Vat a point $\boldsymbol{x}$, outward to V.

In order to define a finite volume scheme, the time derivative is approximated by a finite difference scheme on an increasing sequence of time $(t_n)_{n \in IN}$with $t_0 = 0$. The discrete unknowns of φ at times $t_n = n\delta t$, are approximated as average values in the cell Varound the point $M_{i,j}$ and noted as φ_{ij}^n. Equation (13) is integrated over each cell Vusing the Gauss divergence theorem:

$$\int_V \left(\frac{\partial \varphi}{\partial t}\right)^n dx + \int_{\partial V} F^n.\tau_V d\sigma(x) dx = \int_V f(x,t_n) dx \tag{14}$$

where $(\partial\varphi/\partial t)^n$ is given by the time scheme at the time step $t_n = n\ \delta t$ in the control volume V. The next step of the method is the approximation of the convective part $\boldsymbol{F}^c.\tau_V$ and the diffusive part $\boldsymbol{F}^d.\tau_V$ of the projected flux $\boldsymbol{F}.\tau_V$ over the boundary ∂V of each control volume.

To discretize the convective fluxes various schemes can be adopted. For example, a central scheme uses a symmetric interpolation for $\varphi_{i+1/2}$, an upwind scheme utilizes a one side interpolation. Leonard (1979) introduced the QUICK scheme as a combination between the two kinds of interpolation to be more accurate for complex problems (Semma et al 2005). In the present work, the conductive terms are discretized with the central scheme and the convective terms by using the third order QUICK scheme subjected to a flux limiter ULTIMATE developed by Leonard (1991). To resolve the velocity - pressure coupling the SIMPLEC algorithm (Van Doormal and Raithby 1984) is used. The temporal discretization is done using a second order Euler scheme. Extensive validation of the performance of the present code with and without phase change has been done in Kaenton *et al*. (2002) and Semma *et al.* (2003).

3 Results and discussion

As anticipated, we will analyze the effects of thermal modulation on the dynamic and thermal behaviors for two kinds of configurations (with and without phase change). Modulation is applied on two basic states corresponding respectively to the steady and unsteady one.

In all calculations, the latent heat of the material is assumed to be very weak with respect to the overall heating energy provided to the process. For this reason it is not taken into account (see also the discussions in Semma et al 2003).

For the following results, simulations have been conducted by deploying a mesh of (*64*×*128*) in the x and y directions, respectively. Grid convergence tests were carried out by refining from *32*×*64* to *90*×*180* meshes. Convergence is assumed for a relative maximal residual value lower to 10^{-6}.

3.1 Without phase change

Without modulation, readers can found full details comparing the restricted fluid configuration and the full configuration in Keanton and *al* (2004). The present work is concentrated on the modulation effects considering basic regimes with particular significance.

Results for the restricted fluid cavity (figure 1 b) are available in Larroudé and *al*. (1994). For the low *Ra*, the solution is stationary and composed of two symmetric counter-rotating cells (called SS) with respect to the centerline ($x = 1/2$) in which the fluid rises from the vertical walls of the cavity towards the center.

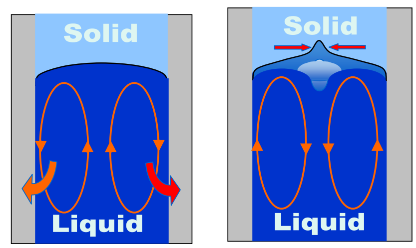


Figure 2 : Schematic illustration of hydrodynamic/solute fields interactions in a directional solidification setup.

Figure 2 illustrates interaction possibilities (from a qualitative point of view) in the case of solid/liquid coupling thermal/solutal convective fields. In the presence of thermal and solutal effects, depending on the competition between the two phenomena, convective cells can rotate in clockwise or anti-clockwise senses. In this figure, right side refers to the classical curved interface depending on the symmetric flow regime described before (inducing a symmetric concave interface). The left side shows possible solute (solvent) accumulation caused by ascending flow (Jamgotchian *et al* , 2004) .

The first transition occurs from symmetrical solution (SS) to an asymmetrical one (called SAS) for *Ra=3500* indicating the development of the convective regime. The next transition to the oscillatory flow occurs around *Ra*=17500 and leads to a periodic solution (P1) with a dimensionless internal frequency f_i =6.67. The solution is asymmetric with a flow structure dominated by a one-cell flow varying between a quasi-circular and a deformed shape because of the competition with secondary cells developing near the corners. The frequency of the oscillations increases with the convection intensity. Increasing *Ra* to 8.5×10^4, the flow becomes *P*2 type characterized by frequencies f and $f/2$ (namely f =1/3). The quasi-periodic régime (QP) which starts around *Ra* = 1.75×10^5 exhibits a complex non-periodic oscillation.

Figure 3 illustrates the transition to oscillatory flow for *Ra=27500*, it is shown that the *P*1 regime is reached af-

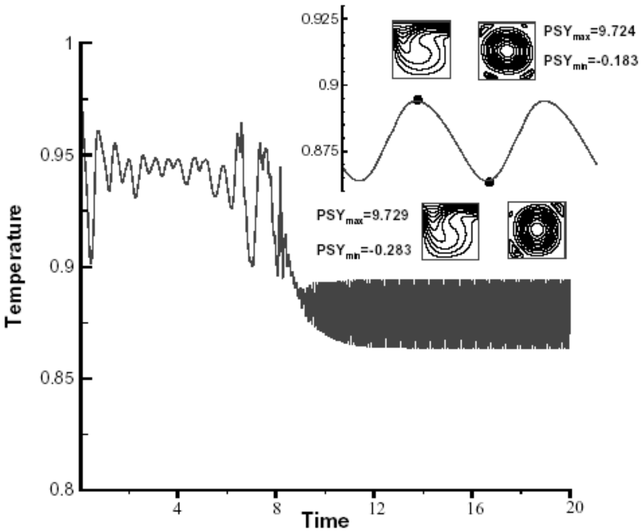


Figure 3 : Transient temperature close to hot wall with the flow structures taken in two various half-periods for *Ra=27720,* ε*=0.*

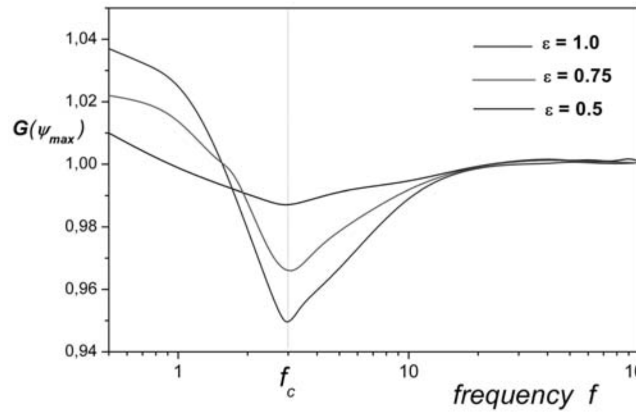


Figure 4 : Variation of *G(*ψ_{max}*)* versus frequency for Ra=10^4.

ter an interval of dimensionless time around $t = 9$ and is characterized by a competition between a main primary cell occupying the major part of the domain and four secondary cells. It is worth mentioning the important role played by 2D cavities to improve researches in such field. Presently an increasing interest is directed towards 3D extension of such studies (Bennacer et al 2006).

3.1.1 Modulated steady flow in fluid cavity

The case of oscillating hot wall temperature is considered ($\varepsilon \neq 0$) in the present section. The stationary unmodulated solution corresponding to *Ra=10*^{4} is taken as a basic state. The modulation effects on the heat transfer and fluid flow are studied by varying the nondimensional amplitudes from 0 to 1 and the nondimensional frequencies from 0 to 100.

Figure 4 shows $G(\psi_{\max})$ as a function of frequency for *Ra=10*^{4} and for ε=0.5, 0.75 and 1. For high frequencies ($f > 30$), the modulation does not exhibit significant effects on the flow intensity and heat transfer in the cavity. However, for the low frequencies, we note the presence of a critical modulation frequency around $f = 3.5$ for which $\psi_{\max}$ reaches a minimum value. This frequency, which is independent on the modulation amplitude is one of the characteristics of the regime and other simulated cases show an increase with Ra.

It should be noticed also that for the system without modulation starting initially from the steady state and increasing Ra number, the coming critical threshold will correspond to an oscillatory regime characterized by its internal frequency. It is observed that this internal frequency is the same as obtained as a response to the applied modulation when a resonance state is reached.

As a consequence, the modulation can offer a way to predict the system transition during its evolution to unsteadiness with increasing *Ra* i.e it is possible to anticipate the basic internal gravity mode by exciting the system by an adequate modulation when the resonance state is activated.

The effect of the thermal fluctuations on the heat transfer has been also analyzed. When the frequency increases, the oscillation amplitude of the Nusselt number falls exponentially. This behavior is practically independent of the considered range of Rayleigh numbers.

3.1.2 Modulated unsteady regime in fluid cavity

When the modulation is applied to the oscillatory regime, the temperature oscillation influences the fluid flow and the thermal transfer characteristics in the cavity. Heat transfer and fluid flow for ($2 \times 10^4 \leq Ra \leq 4 \times 10^4$) have been analyzed. For this range, the flow is mono-periodic with a frequency f_i and responds generally with a quasi-periodic mode to the excitation except for a characteristic frequency (f_c) exhibiting a specific behavior detailed below.

At this frequency the flow recuperates a mono-periodic regime oscillating with (f_c). Around this state, a sudden rapid change has been identified and the oscillation

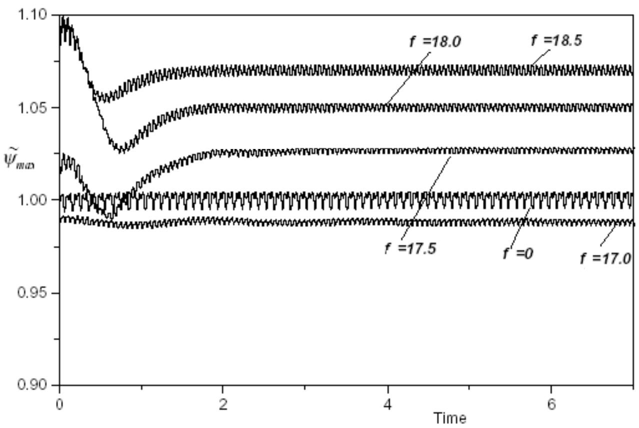


Figure 5 : Time variation of maximum relative value of stream function for various frequencies.

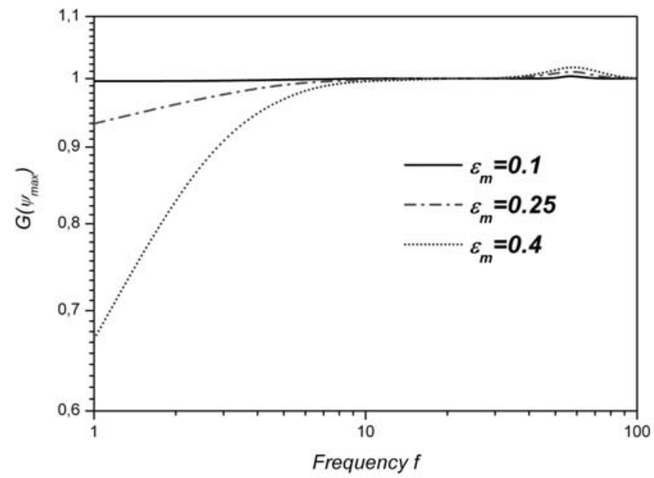


Figure 6 : Variation with frequency of average maximum flow intensity for different thermal amplitude modulation (Ra=5×10^3).

intensity of the flow is reduced to less than 50%. For all considered Ra, a correlation between the internal (i.e f_i) and the characteristic modulation frequency shows a constant ratio of $(f_c/f_i = 3/2)$. Figure 5 illustrates the behavior for Ra=4×10^4 and $\varepsilon = 1.0$, showing a strongly rapid variation of the relative value of $\psi_{\max,f}$ for various frequencies around $f = 17.75$.

Owing to the importance of this frequency and the behavior induced, we have studied the flow sensitivity to the amplitude and other frequency values around 17.75. For ε=*1*, the oscillation amplitude of the flow intensity $A(\psi_{\max})$ varies linearly with the frequency from *24 %* for $f = 17.0$ to *47 %* for $f = 18.5$. However, the oscillation amplitude of V_{max} varies in the opposite direction than $\psi_{\max}$ and decreases from 49 *% for* $f = 17.5$ to 36% *for* $f = 18.5$. This can be explained by the presence of secondary cells near the walls due to the development of local thermal gradients.

The oscillation amplitude of the Nusselt number on the cold wall as a function of the modulation amplitude exhibits a linear variation with ε:

$$A(Nu) = 3.6\times\varepsilon - 0.58 \quad \text{for } \varepsilon < 0.44 \tag{15}$$

The amplitude of the oscillation is reduced and reaches *14%* for ε=0.2. Such behavior can be very beneficial when the objective is to provide a control strategy of the flow and minimize effects of the oscillations on the solid/liquid interface with reducing at the same time unwanted striations generally encountered during crystal growth from the melt.

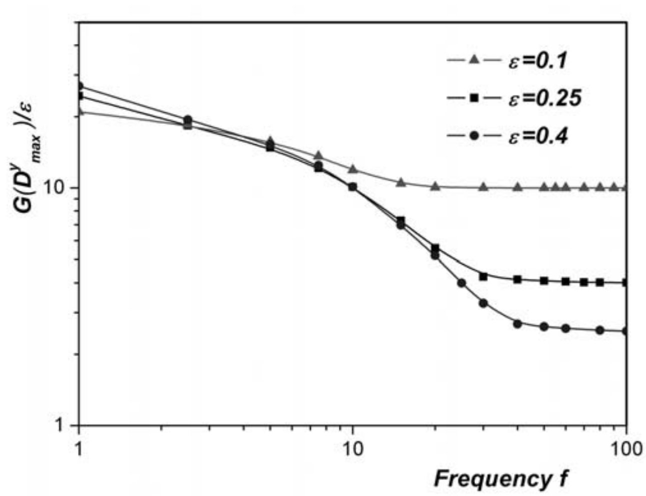


Figure 7 : Variation with frequency of average oscillation of solid/liquid interface deformation for different thermal amplitude modulation (Ra=5×10^3).

3.2 Full configuration with phase change

The full configuration is also characterized by the appearance of unstable regimes flow when Ra is increased (El Ganaoui and Bontoux 1998, Kaenton and al 2003), the steady regime corresponding relatively low values of Ra (e.g., $Ra = 5000$) and the quasi-periodic state to Ra=8×10^4.

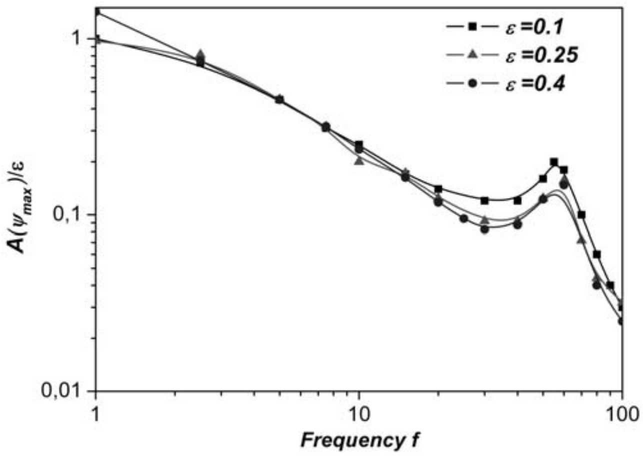


Figure 8 : Variation with frequency of amplitude oscillation of maximum stream function for different thermal modulation amplitudes (Ra=5×10^3).

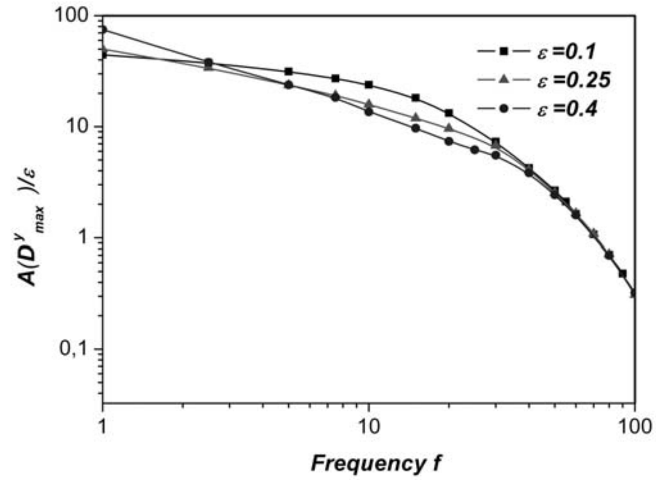


Figure 9 : Variation with frequency of amplitude oscillation of solid/liquid interface for different thermal amplitude modulation (Ra=5×10^3).

3.2.1 Modulated steady regime

We applied the thermal modulation to the stationary basic regime corresponding to Ra=5×10^3. This case is characterized by two symmetrical flow cells with the same intensity. For high frequencies ($f > 80$), the thermal fluctuations seem not to produce important effects on the average evolution of the interface deformation and flow intensity (figures 6 and 7). However, the flow intensity and the solid/liquid interface deformation are very sensitive to the variation of a low frequency. Indeed, for $f < 10$, the maximum value of $\psi_{\rm max}$decreases of more than 30% (for ε=*0.4* and $f = 0.5$) compared to the stationary value (ε=*0.0*, $f = 0$). The maximum deformation of the solid/liquid interface increases strongly in the presence of thermal modulation as showed in figure 7. The same remark is valid for the oscillation amplitudes which become very important for low frequencies (figure 7, 8 and 9) This can be due to the local thermal gradients caused by the temperature change on the hot walls.

The variation of the average and relative oscillating value of $\psi_{\rm max}$ according to the frequency (figures 6 and 8) shows the existence of a characteristic frequency (namely $f = 57$) for which the flow intensity and its amplitude oscillation reaches a local maximum value. This behavior, however, does not have an important effect on the average and amplitude oscillation of the interface deformation.

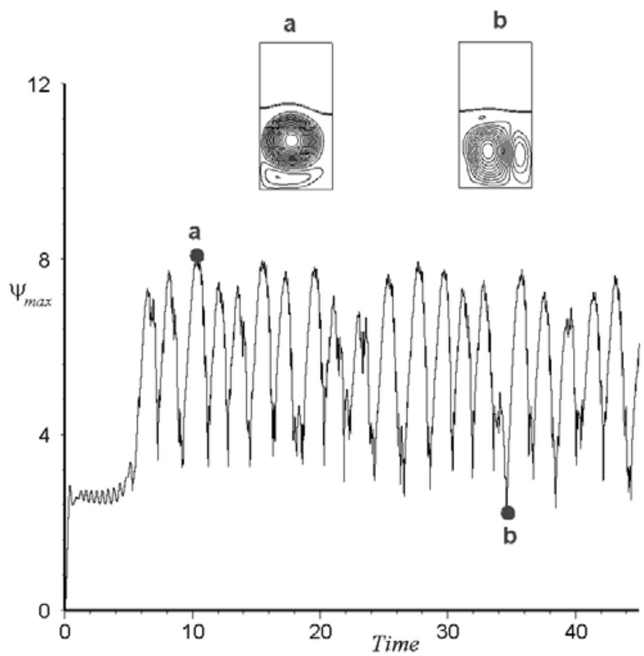
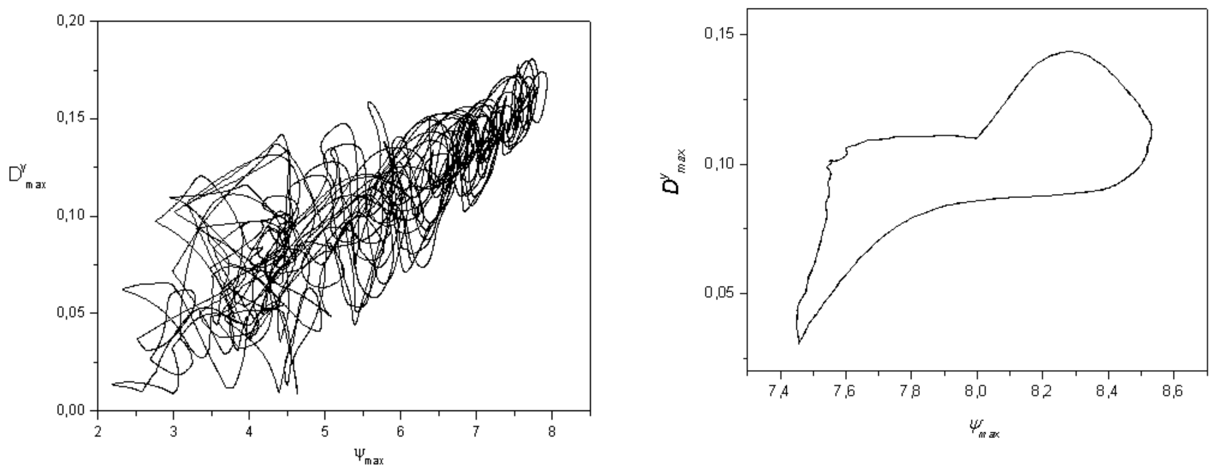


Figure 10 : Time evolution of ψ_{max} for $f = 0$, $\varepsilon = 0$, Ra=8×10^4.

3.2.2 Modulated unsteady regime

The presence of a deformable and non stationary solid/liquid interface leads to significant changes of the flow structure with respect to the purely fluid case (cavity with plane and stationary interface). Indeed, by considering only the fluid part of the configuration (figure



(a) (b)

Figure 11 : Phase trajectory $D^y_{max} = f(\psi_{max})$, for Ra=8×10^4, (a) ε=0, $f = 0$, (b) ε=0.5, $f = 4$.

1-b), we highlighted the presence of stationary monocellular flow for *Ra>3500* and there after a periodic one for *Ra>17500*. These modes of steady and periodic monocellular flow are not obtained in the full configuration because of the relaxation introduced by the solid/liquid interface.

The following part focuses on thermal modulation effects on a basic quasi-periodic regime corresponding to Ra=8×10^4. The phase trajectory $D^y_{max} = f(\psi_{max})$ for $\varepsilon = f = 0$ in figure 11-a shows clearly the complex behavior of the temporal flow regime. In figure 10, we present the time evolution of ψ_{max} for the free state (ε=0). For the two points considered, the flow structure shows a multicellular behavior caused by the relieving introduced by the deformation and the displacement of the solid/liquid interface into the zone with adiabatic lateral boundaries.

When the thermal modulation is considered, for low amplitudes (ε *<0.2*) the flow remains quasi-periodic. However for a sufficiently significant amplitude (*0.25<* ε *<0.5*), we have found the existence of a frequency band (center $f = 4$) for which the flow becomes periodic with the same excitation period (figure 11 and 12). This frequency band decreases with the oscillation amplitude of the hot temperature. The average flow intensity increases for the forced periodic regime and the flow structure is monocellular and similar to that found for the case without phase change (Semma *et al* 2005). The oscillation amplitude of the average deformation of the solid/liquid interface is reduced of more than 55%.

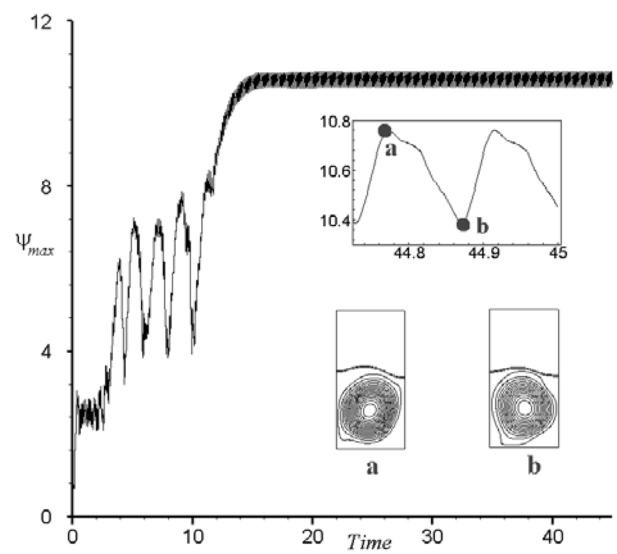


Figure 12 : Time evolution of $\psi_{max}$for f =7, ε =0.3, Ra=8×10^4.

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4 Conclusion

We have investigated thermovibrational convection in a vertical Bridgman cavity and, in particular, studied the modulation frequency dependence of the flow intensity and solid/liquid interface deformation for both steady and oscillatory basic states.

For the stationary basic regime, the solid/liquid interface deformation exhibits a strong dependence on the modulation frequency for relatively low frequency values. However, for high frequencies, the flow and interface deformation converge toward their free state values.

For the full cavity, we have showed the existence of a frequency band for which an initial quasi-periodic regime becomes perfectly periodic. The solid/liquid interface motion and flow in the melt are stabilized.

By comparing the two studies (with and without phase change) one notices that the interaction of the solid/liquid interface with the flow regime cannot be neglected and must be taken into account in a realistic study of directional solidification problems and especially when involving modulation of the heating conditions.

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Nomencalture

AR aspect ratio, W/H
H ampoule height
t time
T temperature
$\boldsymbol{u}$ velocity vector
u velocity component in x direction
v velocity component in y direction
V integration volume
p pressure
k thermal conductivity
Pr Prandtl number, $=\nu/\alpha$
Ra Rayleigh number, $= \rho g \beta \Delta T' H^3 / \alpha\mu$
Ste Stefan number, $= C_p \Delta T'/L_f$
f Dimensionless frequency
x, y Dimensionless coordinates
g gravity acceleration
$\Delta T'$ temperature difference
W ampoule width

Greek symbols

α thermal diffusivity
β_T thermal expansion coefficient
ε Modulation amplitude
μ dynamic viscosity
ν kinematics viscosity
ρ density
ψ stream function

Subscripts

H hot
C cold
i internal
max maximum
min minimum
0 reference

References

Accary G., El Ganaoui M., Raspo I. and Bennacer R. (2005), 3D finite volume approximations for stiff fluid mechanics problems. *Finite Volumes for Complex Applications IV*, F. Benkhaldoun, D. Ouazar, S. Raghay Eds., Hermès Science Publishing, 582-595

Achour K., Kaddeche S., Gharbi A., Ben Hadid H. and Henry D. (2006), On the stability of the Hadley flow under the action of an acoustic wave, *FDMP: Fluid Dynamics and Materials Processing*, 1(4):277-284.

Adornato P. M. and Brown R. A. (1987), Convection and segregation in directional solidification of dilute and non-dilute binary alloys: Effects of ampoule and furnace design. *Journal of Crystal Growth*, Volume 80, Issue 1, January, Pages 155-190.

Amberg, G. and Shiomi, **J.** (2005), Thermocapillary flow and phase change in some widespread materials processes. *FDMP: Fluid Dynamics and Materials Processing*, vol 1, pp. 81-95.

Antohe B.V., Lage J.L. (1996), Amplitude effect on convection induced by time periodic horizontal heating,. *Int. J. Heat Mass Transfer*, 39, 1121-1133.

Beckermann C., Viskanta R. (1988), Double-diffusive convection during dendritic solidification of a binary mixture, *Physico- Chemical Hydrodynamics* 2, 195-213.

Bennacer R., El Ganaoui M. and Leonardi E. (2006), Symmetry breaking of melt flow typically encountered in a Bridgman configuration heated from below *Applied Mathematical Modelling*, 30, 1249-1267.

Bennon W. D. and Incropera F. P. (1987), A continuum model for momentum, heat and species transport in binary solid-liquid phase change systems—I. Model formulation. *International Journal of Heat and Mass Transfer*, Volume 30, Issue 10, October, Pages 2161-2170.

Brown R.A. (1988), Theory of transport processes in single crystal growth from the melt, *AIChE J.* 6 , 881-911.

Chung K.H., Kwak H. S. and Hyun J. M. (2001), Finite-wall effect on buoyant convection in an enclosure with pulsating exterior surface temperature. *Int. J. Heat Mass Transfer* 44, 721-732.

El Ganaoui, M. and Bontoux P. (1998), An Homogenisation Method for Solid-Liquid Phase Change During Directional Solidification, ASME H.T.D., *Num. and Engineering Methods in Heat Transfer*, 361**,** 453-469.

Garandet, J.P., and Alboussière, T., (1999), Bridgman growth: modelling and experiments, *Prog. Crystal Growth Charact*, Vol. 38, 73-132.

Hurle, D. T. J., Jakerman, E. and Jonson, C. P. (1974), Convective Temperature Oscillations in Molten Gallium, *J. Fluid Mech*., Vol. 64, Part 3, pp. 565-576.

Jamgotchian H., Nguyen Thi H., Bergeon N. and Billia B. (2004), Double-diffusive convective modes and induced microstructure localisation during solidification of binary alloys, *International Journal of Thermal Sciences*, Volume 43, Issue 8, Pages 769-777.

Kaenton J., Semma A., Timchenko V., El Ganaoui M., Leonardi E., and de Vahl Davis G. (2004), Effects of anisotropy and solid/liquid thermal conductivity ratio on flow instabilities during inverted Bridgman growth, *Int. J. Heat and Mass Transfer*, 47, 3403-3413.

Kaenton J., Timchenko V., El Ganaoui M., de Vahl Davis G., Leonardi E. and Leong S.S. (2002), Effects of anisotropy and solid/liquid thermal conductivity ratio during inverted Bridgman growth, *Heat Transfer*, Jean Taine (ed), Elsevier, 273-278.

Kwak, H.S., Kuwahara, K. and Hyun, J.M. (1998), Resonant enhancement of natural convection heat transfer in a square enclosure. *Int. J. Heat and Mass Transfer, Vol. 41, pp.2837-2846.*

Kozlov V.G. and Selin N.V. (2006), Pendulum Thermal Vibrational Convection in a Liquid Layer with Internal Heat Generation, *FDMP: Fluid Dynamics and Materials Processing*, vol 2, No.2, pp. 107-118.

Lan, C.W. and Yeh, **B.C.** (2005), Effects of rotation on heat flow, segregation, and zone shape in a small-scale floating-zone silicon growth under axial and transversal magnetic fields. *FDMP: Fluid Dynamics and Materials Processing*, vol 1, no.1, pp. 33-43.

Lappa M. (2005a)**,** Review Thermal convection and related instabilities in models of crystal growth from the melt on earth and in microgravity: Past history and current status, *Cryst. Res. Technol. 40, No. 6, 531–549*.

Lappa M. (2005b), Review: Possible strategies for the control and stabilization of Marangoni flow in laterally heated floating zones, *FDMP: Fluid Dynamics and Materials Processing*, vol.1, no.2, pp. 171-188.

Larroudé P., Ouazzani. J., Alexander. J. I. D., and Bontoux. P. (1994), Symmetry breaking transition and oscillatory flows in a 2D directional solidification model, *Eur. J. Mech, B/Fluids*, 13, 353-381.

Leonard B. P. (1991), The ULTIMATE conservative difference scheme applied to unsteady one-dimensional advection. *Computer Methods in Applied Mechanics and engineering,* 88, 17-74.

Leonard B. P. (1979), A Stable and Accurate Convective Modelling Procedure Based on Quadratic Upstream Interpolation. *Comput. Methods Appl. Mech. Eng*., Vol. 19, pp. 59-98.

Muller C. (1996)**,** Convection and In homogeneities in Crystal Growth from the Melt, *Springer-Verlag edition, Berlin.*

Semma A, El Ganaoui M, Cheddadi A and Bontoux P., (2003), Instability of melt flow and solidification front during horizontal growth. *Comptes Rendus de l'Académie des Sciences (Mécanique)*, 331, 631-639.

Semma E., El Ganaoui M., Timchenko V. and Leonardi E. (2005), Unsteadiness and control by using thermal oundary modulation in restricted fluid domain of vertical Bridgman apparatus', *Int. J. Materials and Product Technology*, Vol. 22, N. 1, pp.122-134.

Semma A., El Ganaoui M., Cheddadi A. and Farchi A.

High order finite volume scheme for phase change problems. *Finite Volumes for Complex Applications IV, F. Benkhaldoun, D. Ouazar, S. Raghay Eds., Hermès Science Publishing*,(2005), *493-503.*

Stelian C., Duffar T., Santailler J.-L. and Nicoara, I.

(2002), Influence of temperature oscillations on the interface velocity during Bridgman crystal growth, *Journal of Crystal growth,* 237-239**,** 1701-1706.

Van Doormal J.P. and Raithby G.D. (1984), Enhancements of the SIMPLE method for predicting incompressible fluid flows, *Num. Heat Transfer*, 7, 147-163.

Voller V. R. and Prakash C. (1987), A fixed grid numerical modelling methodology for convection-diffusion mushy region phase-change problems, *International Journal of Heat and Mass Transfer*, Volume 30, Issue 8, Pages 1709-1719.

Yan, Y. Shevtsova V. and Saghir, M.Z. (2005), Numerical study of low frequency g-jitter effect on thermal diffusion, *FDMP: Fluid Dynamics and Materials Processing*, vol. 1 (4) pp 315-328.