

## Stability Analysis of Cyber-Physical Micro Grid Load Frequency Control System with Time-Varying Delay and Non-Linear Load Perturbations

D. Vijeswaran<sup>1, \*</sup> and V. Manikandan<sup>2</sup>

**Abstract:** In a cyber-physical micro-grid system, wherein the control functions are executed through open communication channel, stability is an important issue owing to the factors related to the time-delay encountered in the data transfer. Transfer of feedback variable as discrete data packets in communication network invariably introduces inevitable time-delays in closed loop control systems. This delay, depending upon the network traffic condition, inherits a time-varying characteristic; nevertheless, it adversely impacts the system performance and stability. The load perturbations in a micro-grid system are considerably influenced by the presence of fluctuating power generators like wind and solar power. Since these non-conventional energy sources are integrated into the power grid through power electronic interface circuits that usually works at high switching frequency, noise signals are introduced into the micro-grid system and these signals gets super-imposed to the load variations. Based on this back ground, in this paper, the delay-dependent stability issue of networked micro-grid system combined with time-varying feedback loop delay and uncertain load perturbations is investigated, and a deeper insight has been presented to infer the impact of time-delay on the variations in the system frequency. The classical Lyapunov-Krasovskii method is employed to address the problem, and using a standard benchmark micro-grid system, and the proposed stability criterion is validated.

**Keywords:** Delay-dependent stability, time-varying delay, open communication network, nonlinear perturbations, Lyapunov-Krasovskii functional, linear Matrix Inequality (LMI).

### 1 Introduction

In recent times, the micro-grid system has become an essential add-on feature of the conventional large scale power system. The micro-grid system is an independent power entity that encompasses small pockets of distributed generating units and loads as shown in Fig. 1. The distributed generation usually involves a low capacity micro-turbine unit catering to the base load, and fluctuating power generators like wind and solar power,

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<sup>1</sup> Department of Electrical and Electronics Engineering, Coimbatore Institute of Technology, Coimbatore 641014, India.

\* Corresponding Author: D. Vijeswaran. Email: vijeswaran72@gmail.com.

fuel cell with electrolyzer system, super-conducting capacitor banks and plug-in electric vehicles are also common in micro-grid system catering to the peak load of the power system [Gündüz, Sönmez and Ayasun (2017)]. The loads connected to the micro-grid system include domestic, small and medium scale industry loads. The storage units like battery banks, flywheels, super conducting magnetic storage elements are optional but desirable add-ons in a micro-grid system. The micro-grid substantially improves the operational performance of the conventional power grids; nevertheless, it enhances the system security and creditability paving the way to low cost power generation at a better power quality. The units connected to the micro-grid system have varied roles to play in enhancing the performance of a conventional grid connected system. The environment dependent non-conventional energy sources like wind and solar are basically fluctuating power generators. The power generated by these sources is intermittent in nature. To compensate for the system uncertainties which these fluctuating power generators offer, a gas based micro turbine is added to cater the base load. In the event of any unexpected real power imbalance, to maintain the frequency of the system at desired value, a fuel cell with electrolyser unit is appropriately incorporated into the micro-grid system, since frequency regulation is not possible by the micro turbine itself [Khalil, Rajab, Alfergani et al. (2017); Mahmoud, Hussain and Abido (2014)]. To achieve efficient load frequency control (LFC), local and master controllers are used in tandem in a typical micro-grid system. Under cyber physical framework, the information exchange among them is exercised through an open communication network as discrete data packets [Richard (2003)]. As a result, a time-delay is encountered in the feedback control system, and depending upon the network traffic condition, this network induced delay is time varying in nature. This time delay adversely affects the overall performance and stability of the closed loop system. If this delay exceeds a critical magnitude called stable delay margin, the closed loop system is driven to unstable operating conditions. Hence under delayed data interchange, for effective operation of a micro-grid load frequency regulation, the stability margin of the micro-grid closed loop system needs to be assessed. In this research paper this open problem is addressed pertaining to networked micro-grid system.

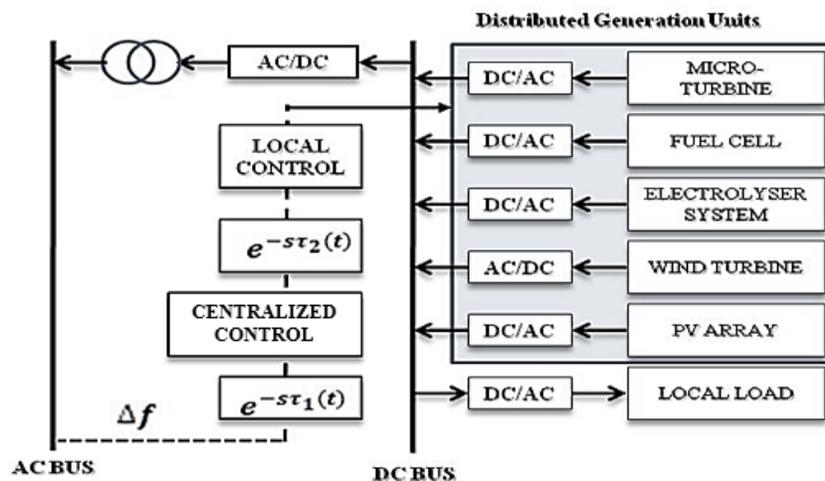


Figure 1: Micro-grid system with communication delay

### 2 Micro-grid with communication delays

In a power system, load is an uncertain entity. As the real power fed into the grid in response to the load variations, the system frequency deviates from the desired value. To regulate the frequency to the desired value, load frequency control strategy is employed [Kundur, Balu and Lauby (1994)]. In a conventional load frequency control, the incremental frequency variable  $\Delta f(t)$ , that gives a measure of imbalance between generation and demand is the error variable. The micro-grid central controller (MGCC), located in the master control facility in the micro-grid system, acts based on the error variable and sends suitable command to local control so that the distributed generation in the micro-grid is operated and co-ordinated simultaneously to achieve a stabilised operation with appropriate frequency and voltage profile. The centralised controller continuously monitors the error variable,  $\Delta f(t)$  and decides the control effort for regulating the system frequency at desired value. This, in turn, ensures that the generation always matches the demand in the system. While accomplishing this task in a networked system environment, two time-varying delays are introduced in the feedback path. The delays are sensor to controller delay  $\tau_1(t)$  and controller to plant delay  $\tau_2(t)$ . Depending upon the network traffic conditions, these delays may have dissimilar characteristics. Invariably, the delays pose a serious threat to the desirable performance of the closed loop system [Jiang, Yao, Wu et al. (2012)]. If these time delays are allowed to exceed a critical margin, the overall system loses stability. In this unstable condition, loss of synchronism of a generating unit from grid takes place paving way to high amplitude low frequency oscillations to be induced in the process [Mary and Rangarajan (2016)]. The power swing may eventually lead to catastrophic tripping of various generator units connected to the power system grid paving way to overall blackout unless islanding scheme takes over to minimise the system outage.

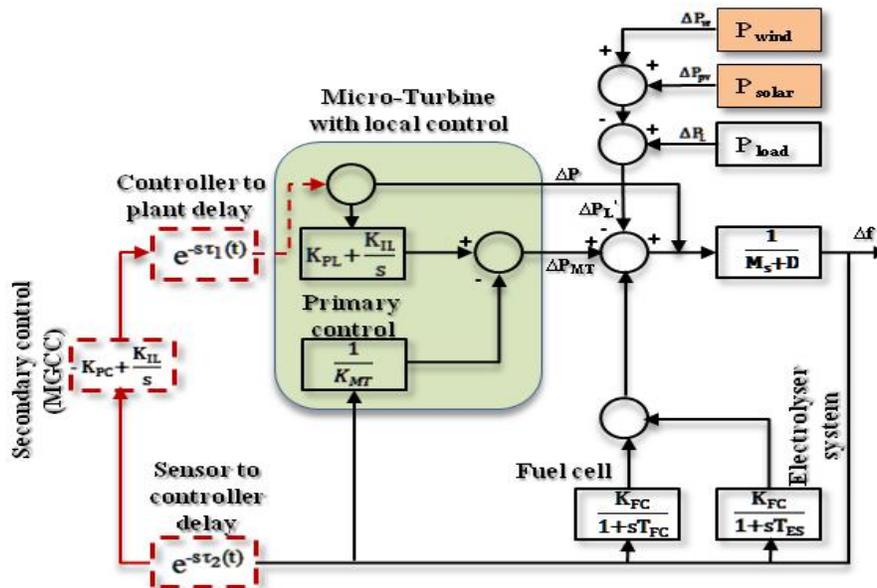


Figure 2: Block diagram of micro grid with communication delays

Delay-dependent criteria are basically sufficient conditions that would calculate the upper bound of the time-varying delays inside which the closed-loop system is asymptotically stable in the sense of Lyapunov. These stability criteria are usually derived using classical Lyapunov-Krasovskii (LK) energy functional approach wherein a positive-definite energy functional (quantifying the energy acquired by the dynamic system on a perturbation) is constructed and time derivative of the functional is bounded using appropriate bounding techniques [Zamora and Srivastava (2010); Elgerd (1971)]. The negative definiteness of the time-derivative of the LK functional guarantees the asymptotical stability of the closed loop control system [Wu, He and She (2010)]. Subsequently, the criterion evolves in the form of a convex optimization problem with Linear Matrix Inequality (LMI) constraints.

In this paper, the time-delayed micro-grid system is modelled in state space approach as a retarded delay differential equation. The feedback loop delays are assumed to have similar characteristics. Hence, they are aggregated into single time varying delay  $\tau(t)$ . The LK functional based stability analysis computes the stable margin of the time delay for various subsets of PI controller parameters [Sönmez and Ayasun (2018)]. Finally, a standard benchmark micro-grid system is considered to validate the effectiveness of presented criterion.

In this paper, an appropriate positive definite LK functional is constructed, and its time derivative is bounded using Reciprocal Complex Combination (RCC) lemma. Another novelty of the paper is that the time varying load perturbations are also considered in proposed delay dependent stability analysis. The perturbations are assumed to be norm-bounded and scaling factors are introduced to quantify the magnitude of load perturbations. To the best of author knowledge, delay dependent stability problem of network controlled (cyber-physical) micro grid system with time varying network delay and load perturbations have not been addressed so far. This has motivated us to address the problem.

### 3 Mathematical model of the proposed system

The mathematical model of various components of the micro-grid control system shown in Fig. 2 [Ramakrishnan, Vijeswaran and Manikandan (2019)] are detailed in this section.

#### 3.1 Gas-based micro turbine

The transfer function model of the micro-turbine is developed taking into account the linear speed drop characteristics between the power and frequency. This transfer function is given as follows:

$$G_{MT}(s) = \frac{\Delta P_{MT}}{\Delta f} = \frac{-1}{K_{MT}} \quad (1)$$

where  $\Delta f$ ,  $\Delta P_{MT}$  and  $\Delta K_{MT}$  represent frequency deviation, change in output power and drop characteristics of the micro-turbine, respectively.

#### 3.2 Fuel cell and electrolyzer

A fuel cell with an electrolyser system is utilized to compensate for real power imbalance when the local controller of the micro turbine becomes less effective for substantial variations in load. A part of the wind power is utilized by the aqua electrolyser to produce

hydrogen for fuel cell. The transfer function model of fuel cell and electrolyser are given as follows:

$$G_{FC}(s) = \frac{\Delta P_{FC}}{\Delta f} = \frac{K_{FC}}{1+sT_{FC}} \quad (2)$$

$$G_{ES}(s) = \frac{\Delta P_{ES}}{\Delta f} = \frac{K_{ES}}{1+sT_{ES}} \quad (3)$$

where  $\Delta P_{FC}$ ,  $K_{FC}$  and  $T_{FC}$  represent the change in output power, the gain, time constant of the fuel cell, respectively, while  $\Delta P_{ES}$ ,  $K_{ES}$  and  $T_{ES}$  denote similar variables of the electrolyser.

### **3.3 Extended load**

The extended load demand  $\Delta P'_L$  consists of housing load, wind power and PV generation; it is expressed as follows:

$$\Delta P'_L = \Delta P_L - \Delta P_{PV} - \Delta P_W \quad (4)$$

The dynamics of PV and wind power are considered as nonlinear perturbations in the test system employed for study. These uncertainties do substantially affect delay margins results.

### **3.4 Local and central controller**

The local and micro-grid central controller ( $G_{LC}(s)$  and  $G_{CC}(s)$  respectively) are configured with PI control law. The controller transfer functions are given below:

$$G_{LC}(s) = K_{PL} + \frac{K_{IL}}{s} \quad (5)$$

$$G_{CC}(s) = K_{PC} + \frac{K_{IC}}{s} \quad (6)$$

where  $K_{PX}$  and  $K_{IX}$  represent proportional and integral gains of the controllers.

### **3.5 Power system**

The power system with its connected conventional (high inertia) generators is modeled as a first order transferfunction given by

$$G_p(s) = \frac{1}{sM+D} \quad (7)$$

where M and D is moment of inertia and damping constant respectively.

### **3.6 State-space model**

The total system including central and local controller including network-induced delay is shown in Fig. 2 [Ramakrishnan, Vijeswaran and Manikandan (2019)]. The state-space model of the closed-loop system shown in Fig. 2 can be derived in the following autonomous framework:

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau(t)) + \omega(\cdot) \quad (8)$$

$$x(t) = \phi(t), t \in [-\max(\tau), 0], \quad (9)$$

where the system matrices  $A \in \mathbb{R}^{5 \times 5}$  and  $A_d \in \mathbb{R}^{5 \times 5}$  are as follows:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & K_{IC} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ 0 & 0 & -\frac{1}{T_{FC}} & 0 & \frac{K_{FC}}{T_{FC}} \\ 0 & 0 & 0 & -\frac{1}{T_{ES}} & \frac{K_{ES}}{T_{ES}} \\ 0 & \frac{1}{M} & \frac{1}{M} & -\frac{1}{M} & \frac{D}{M} \end{bmatrix} \quad (10)$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

With state vector  $x(t) \in R^{5 \times 1}$  being  $x(t) = [K_{IC} \int \Delta f(t) dt \Delta P_{MT}(t) \Delta P_{FC}(t) \Delta P_{ES}(t) \Delta f(t)]^T$ . The elements of matrices (in terms of system parameters) are given below:

$$a_{21} = 0,$$

$$a_{22} = \frac{1}{1+K_{PL}} \left[ -K_{IL} - \frac{1}{MK_{MT}} \right],$$

$$a_{23} = \frac{1}{1+K_{PL}} \left[ \frac{K_{PL}}{T_{FC}} - K_{IL} - \frac{1}{MK_{MT}} \right],$$

$$a_{24} = \frac{1}{1+K_{PL}} \left[ -\frac{K_{PL}}{T_{ES}} + K_{IL} + \frac{1}{MK_{MT}} \right],$$

$$a_{25} = \frac{1}{1+K_{PL}} \left[ -\frac{K_{PL} \cdot K_{FC}}{T_{FC}} + \frac{K_{PL} \cdot K_{ES}}{T_{ES}} + \frac{D}{MK_{MT}} \right],$$

$$d_{21} = -\frac{K_{IL}}{1+K_{PL}}$$

$$d_{22} = -\frac{K_{PL} \cdot K_{PC}}{M(1+K_{PL})}$$

$$d_{23} = -\frac{K_{PL} \cdot K_{PC}}{M(1+K_{PL})}$$

$$d_{24} = \frac{K_{PL} \cdot K_{PC}}{M(1+K_{PL})}$$

$$d_{25} = \frac{1}{(1+K_{PL})} \left[ -K_{IL} K_{PC} + \frac{K_{PL} K_{PC} D}{M} - K_{PL} K_{PC} \right]$$

The time-varying delay satisfies the following condition:

$$0 \leq \tau(t) \leq \tau; \dot{\tau}(t) \leq \mu < 1, \quad (12)$$

where  $\tau$  upper bound of the time-varying is delay and  $\mu$  is upper bound of its derivative.

#### 4 Results and discussion

The proposed result of this paper is derived by modeling unknown external power system load disturbance as a nonlinear time-varying perturbation for existing and delayed state vector. The term  $\omega(\cdot)$  represents the uncertainties caused due to the intermittent

penetration of solar and wind power to the micro-grid; it is a function of time and state-variables as described below:

$$\omega(\cdot)=\omega(t,x(t),x(t-\tau(t))) \tag{13}$$

It is assumed to satisfy the following condition:

$$\|\omega(\cdot)\| \leq \alpha \|x(t)\| + \beta \|x(t - \tau(t))\| \tag{14}$$

where  $\alpha \geq 0$  and  $\beta \geq 0$  are known scalars. A more generalized version of the condition (14), which is used in this paper, is given by

$$\omega^T(\cdot)\omega(\cdot) \leq \alpha^2 x^T(t)G^T Gx(t) + \beta^2 x^T(t-\tau(t))F^T Fx(t - \tau(t)) \tag{15}$$

where G and F are known constant matrices of appropriate dimensions. The problem addressed in this paper is stated below:

**Problem:** To develop a less conservative robust stability criterion in LMI framework to ascertain delay-dependent stability [Yang, He and Wang (2019)] of the networked micro-grid system described by the state-space model (8) with the load disturbance satisfying the condition (14), and network-induced time-delay (12) using Lyapunov-Krasovskii functional approach [Park, Ko and Jeong (2011)].

The stability criterion with delay for the cyber physical micro-grid system (8) with time-varying delay (12) and external load perturbations (14) is given in the following theorem

**Theorem 1.** The cyber physical micro-grid system (8) satisfying (12) and (14) is asymptotically stable in the view of Lyapunov, if there exist real symmetric matrices P, Q<sub>1</sub>, Q<sub>2</sub> and R; scalar  $\epsilon \geq 0$ ; free matrix S of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \geq 0; \\ P > 0; Q_i > 0, i=1,2; R > 0, \\ \Pi < 0. \\ \text{where} \\ \Pi = e_1 P e_5^T + e_5 P e_1^T + e_1(Q_1 + Q_2 + \epsilon \alpha^2 G^T G) e_1^T - e_2(- (1-\mu)Q_1 + \epsilon \beta^2 F^T F) e_2^T - e_3 Q_2 e_3^T \\ - e_4(\epsilon I) e_4^T + e_5(\tau^2 R) e_5^T - \begin{bmatrix} e_3^T & -e_2^T \\ e_1^T & -e_2^T \end{bmatrix}^T \begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \begin{bmatrix} e_3^T & -e_2^T \\ e_1^T & -e_2^T \end{bmatrix} \tag{16}$$

With

$$\begin{aligned} e_1 &= [I \ 0 \ 0 \ 0]^T, \\ e_2 &= [0 \ I \ 0 \ 0]^T, \\ e_3 &= [0 \ 0 \ I \ 0]^T, \\ e_4 &= [0 \ 0 \ 0 \ I]^T, \\ e_5 &= (A e_1^T + A_d e_2^T + e_4^T)^T. \end{aligned}$$

**Proof:** Consider the Lyapunov-Krasovskii functional  $V(t) = \sum_{i=1}^3 V_i(t)$  with:

$$V_1(t) = x^T(t) P x(t), \tag{17}$$

$$V_2(t) = \int_{t-\tau}^t x^T(s) Q_1 x(s) ds + \int_{t-\tau}^t x^T(s) Q_2 x(s) ds, \tag{18}$$

$$V_3(t) = \tau \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s) R \dot{x}(s) ds d\theta, \quad (19)$$

where  $P$ ,  $Q_i$ ,  $i=1, 2$  and  $R$  are real symmetric positive definite matrices. Define  $\delta(t) = [x^T(t) \ x^T(t-\tau(t)) \ x^T(t-\tau) f^T(\cdot)]^T$ . In LK functional time derivative  $\dot{V}_i(t)$ ,  $i=1$  to 3 through the path of (8) is as follows:

$$\dot{V}_1(t) = \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) = \delta^T(t) (e_5 P e_1^T + e_1 P e_5^T) \delta(t) \quad (20)$$

The time derivative of  $V_2(t)$  is given by

$$\dot{V}_2(t) = x^T(t) (Q_1 + Q_2) x(t) - (1 - \dot{\tau}(t)) x^T(t - \tau(t)) Q_1 x(t - \tau(t)) \quad (21)$$

Since  $\dot{\tau}(t) \leq \mu < 1$ ,  $\dot{V}_2(t)$  of (21) is expressed as an inequality as follows:

$$\begin{aligned} \dot{V}_2(t) &\leq x^T(t) (Q_1 + Q_2) x(t) \\ &\quad - (1 - \mu) x^T(t - \tau(t)) Q_1 x(t - \tau(t)) - x^T(t - \tau) Q_2 x(t - \tau) \end{aligned} \quad (22)$$

which in other words is expressed as

$$\dot{V}_2(t) \leq \delta^T(t) (e_1 (Q_1 + Q_2) e_1^T) \delta(t) - \delta^T(t) (e_2 (1 - \mu) Q_1 e_2^T + e_3 Q_2 e_3^T) \delta(t) \quad (23)$$

The time derivative of  $V_3(t)$  is given by

$$\dot{V}_3(t) = \dot{x}^T(t) (\tau^2 R) \dot{x}(t) - \tau \int_{t-\tau}^t \dot{x}^T(s) R \dot{x}(s) ds \quad (24)$$

On application of reciprocal convex combination lemma [10], (24) is presented as an inequality as below:

$$\dot{V}_3(t) \leq \delta^T(t) \left( (e_4 (\tau^2 R) e_4^T) - \begin{bmatrix} e_2^T & -e_3^T \\ e_1^T & -e_2^T \end{bmatrix}^T \begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \begin{bmatrix} e_2^T & -e_3^T \\ e_1^T & -e_2^T \end{bmatrix} \right) \delta^T \quad (25)$$

With following condition holding good

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \geq 0. \quad (26)$$

Now for any  $\epsilon \geq 0$ , from (15), following condition holds good:

$$-\epsilon f(\cdot)^T f(\cdot) + \epsilon (\alpha^2 x^T(t) G^T G x(t) + \beta^2 x^T(t - \tau(t)) F^T F x(t - \tau(t))) \geq 0,$$

Which is expressed using augmented state vector  $\delta(t)$  as follows:

$$\delta^T(t) (-e_4 (\epsilon I) e_4^T + e_1 (\alpha^2 G^T G) e_1^T + e_2 (\beta^2 F^T F) e_2^T) \delta(t) \geq 0 \quad (27)$$

On combining (20), (23) and (25) together with positive quantity (27). The following condition is obtained

$$\dot{V}(t) \leq \sum_{i=1}^3 \dot{V}_i(t) + \delta^T(t) (-e_4 (\epsilon I) e_4^T + e_1 (\alpha^2 G^T G) e_1^T + e_2 (\beta^2 F^T F) e_2^T) \delta(t) \quad (28)$$

This condition (28) is quadratically expressed as follows:

$$\dot{V}(t) \leq \delta^T(t) \Pi \delta(t) \quad (29)$$

Now, if the condition  $\Pi < 0$  and (26) held collectively, then for a small scalar  $\sigma > 0$  such that  $\dot{V}(t) \leq -\sigma \|x(t)\|^2$ , that denotes that the uncertain system in (8) satisfying (12) is asymptotically stable in the view of Lyapunov. Thus Theorem 1 is proved.

**Remark 1.** For cyber physical micro-grid LFC systems with load perturbations the delay-dependent stability criterion for determining the stability of the system can be derived

from the result by using  $\alpha = \beta = 0$  [Ramakrishnan and Ray (2015); Zamora and Srivastava (2010)]. The above condition is presented in the corollary as follows:

**Corollary.** The cyber physical micro-grid system (8) fulfilling (12) and  $f(\cdot)=0$  is asymptotically stable in the view of Lyapunov, with a condition if there exist real symmetric matrices P, Q<sub>1</sub>, Q<sub>2</sub> and R; scalar  $\epsilon \geq 0$ ; free matrix S of appropriate dimensions such that the following LMIs hold

$$\begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \geq 0.$$

$$P > 0; Q_i > 0, i = 1, 2; R > 0, \bar{\Pi} < 0$$

where

$$\begin{aligned} \bar{\Pi} = & \bar{e}_1 P \bar{e}_4^T + \bar{e}_4 P \bar{e}_1^T + \bar{e}_1 (Q_1 + Q_2) \bar{e}_1^T - \bar{e}_2 (-(1 - \mu) Q_1) \bar{e}_2^T - \bar{e}_3 Q_2 \bar{e}_3^T + \bar{e}_4 (\tau^2 R) \bar{e}_4^T \\ & - \begin{bmatrix} \bar{e}_3^T & -\bar{e}_2^T \\ \bar{e}_1^T & -\bar{e}_2^T \end{bmatrix}^T \begin{bmatrix} R & S \\ S^T & R \end{bmatrix} \begin{bmatrix} \bar{e}_3^T & -\bar{e}_2^T \\ \bar{e}_1^T & -\bar{e}_2^T \end{bmatrix}, \end{aligned} \quad (30)$$

With

$$\bar{e}_1 = [I \ 0 \ 0]^T,$$

$$\bar{e}_2 = [0 \ I \ 0]^T,$$

$$\bar{e}_3 = [0 \ 0 \ I]^T,$$

$$\bar{e}_4 = (A \bar{e}_1^T + A_d \bar{e}_2^T)^T.$$

**Remark 2.** If no restriction is applied on the upper bound of the delay-derivative, then the delay-dependent stability can be determined by setting Q<sub>1</sub>=0 in Theorem 1 and Corollary.

## 5 Simulation results

The standard benchmark system [3] is referred and parameter values are selected as, M=10, D=1, K<sub>MT</sub>=0.04, K<sub>FC</sub>=1, T<sub>FC</sub>=4, K<sub>ES</sub>=1, T<sub>ES</sub>=1, K<sub>PL</sub>=1, K<sub>IL</sub>=1. The maximum delay bound  $\bar{\tau}$  for this system for various set of centralized controller parameters (K<sub>IC</sub> and K<sub>PC</sub>) is ascertained by the criterion as stated in Theorem 1 is presented in Tab. 1. From the table, it is clear that as the magnitude of nonlinear perturbation increase (depicted by increase in  $\alpha$  and  $\beta$ ), maximum delay bound that the closed-loop system can withstand without losing stability decreases. This clearly indicates that delay-dependent stability of the system is susceptible to the magnitude of perturbations in load. The analytical results are validated using simulation based study. The intermittent power from renewable sources such as wind and solar is represented as white noise model in the simulation based study.

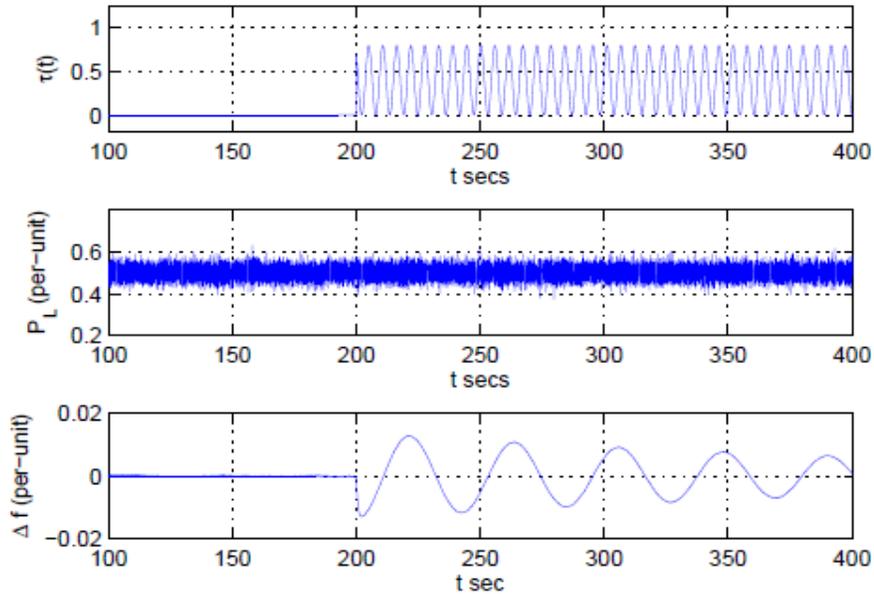
With K<sub>PC</sub>=1 and K<sub>IC</sub>=0.8, and the P<sub>L</sub>' set to mean value of 0.5 for the entire study. With zero time-delay, the system is assumed to be at the equilibrium state with  $\Delta f(t)=0$ . In the presence of time-delay, with the factors quantifying the magnitude of load perturbations viz.,  $\alpha$  and  $\beta$  are both set at 0.05, in accordance to Tab. 1, the closed-loop system is stable up to  $\tau=0.896$  secs.

The closed-loop system is introduced with a time-varying delay at t=200 secs and performance of the system (evolution of the incremental variable  $\Delta f(t)$  vs. t) is analyzed.

The evolution of the variable  $\Delta f(t)$  vs.  $t$  for  $\tau=0.8$  secs (stable operation),  $\tau=0.896$  secs (marginally stable operation) and  $\tau=1$  secs (unstable operation) presented in Fig. 3, Fig. 4 and Fig. 5 respectively validates the analytical result.

**Table 1:** Maximum upper delay bound for  $F=G=0.1$   $I_{5 \times 5}$

$K_{pc}$	$\alpha = 0 ;$ $\beta = 0$		$\alpha = 0 ;$ $\beta = 0.05$		$\alpha = 0.05 ;$ $\beta = 0.05$	
	$K_{IC}\tau^*$		$K_{IC}$	$\tau^*$	$K_{IC}$	$\tau^*$
1.0	0.2	9.718	0.2	6.398	0.2	5.196
	0.4	5.198	0.4	3.063	0.4	2.283
	0.6	3.630	0.6	1.961	0.6	1.341
	0.8	2.835	0.8	1.427	0.8	0.896
	1.0	2.355	1.0	1.117	1.0	0.646
2.0	0.2	12.105	0.2	9.329	0.2	8.372
	0.4	6.959	0.4	4.881	0.4	4.194
	0.6	4.944	0.6	3.263	0.6	2.695
	0.8	3.876	0.8	2.438	0.8	1.942
	1.0	3.215	1.0	1.942	1.0	1.496



**Figure 3:** Stable operating condition for  $\tau = 0.8$  secs

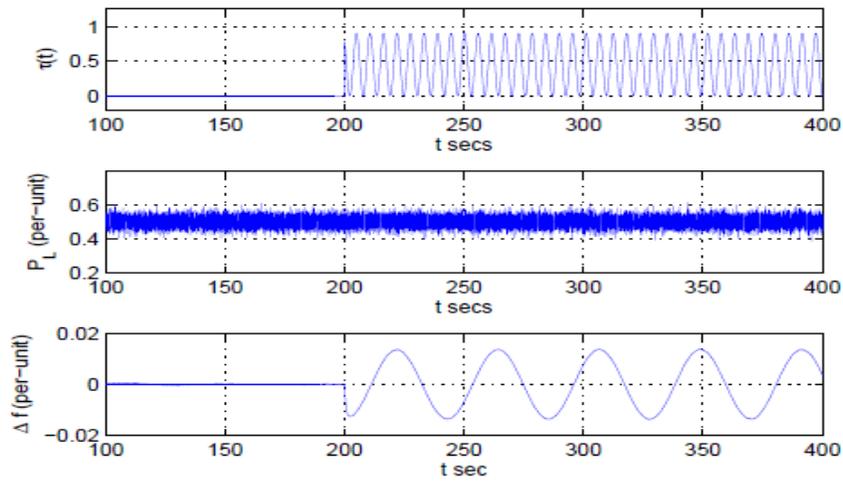


Figure 4: Marginally stable operating conditions for  $\tau = 0.896$  secs

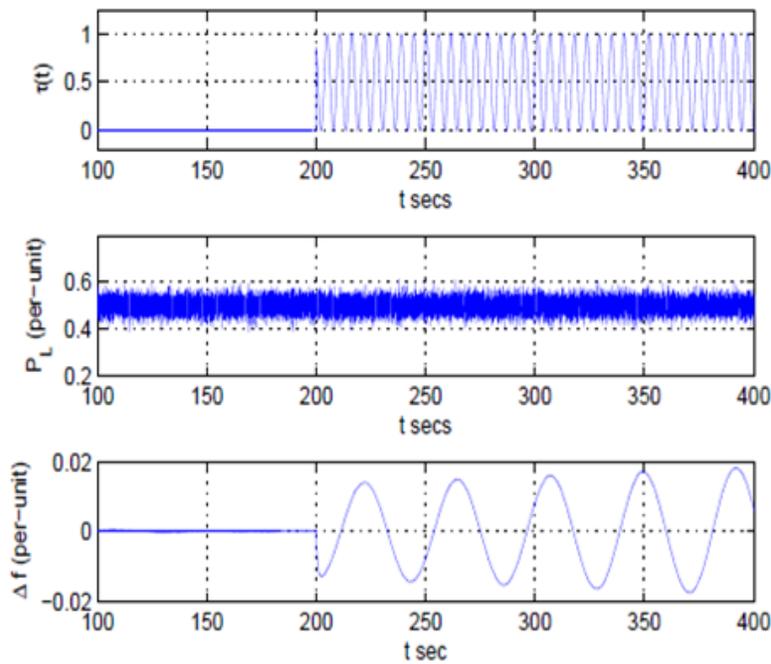


Figure 5: Unstable operating condition for  $\tau = 1.0$  secs

### 6 Conclusions

In this paper, by employing the classical Lyapunov-Krasovskii functional approach combined with reciprocal convex combination lemma, a new criterion is presented to ascertain delay-dependent stability of cyber physical micro-grid system with time-varying network delays and nonlinear load perturbation. The presented stability criterion

is expressed in LMI framework. The effectiveness of the proposed analysis is validated on a standard benchmark micro-grid system for various sub-sets of controller parameters. By varying scaling factor of perturbation term, magnitude of load perturbation is varied. Simulation results are presented to corroborate the effectiveness of the stability criterion derived in this paper. The possibility of extending proposed work for time varying system parameters will be explored as future work.

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