

## On the Modeling of Non-Classical Problems Involving Liquid Jets and Films and Related Heat Transfer Processes

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**Abstract:** Non-classical subjects relating to the dynamics of jet and film flow and related heat transfer processes are considered. These problems, which are relevant to several technological applications, cannot completely be addressed in the frame of the canonical Navier-Stokes equations. The first example deals with the formation of a film flow as a result of the hydraulic shock of a vertical jet impinging on a horizontal plane. The effective thickness of the film resulting from the hydraulic shock is much less than the value obtained using the conventional approach (relying on the assumption of smooth flow), while the corresponding speed is much higher. The next case is a jet penetrating into a pool of another liquid, which under certain conditions can form a jet head of a fungoid shape. When the jet penetrates the pool, it expands sharply, and this situation is another example of circumstances where classical models are flawed. Moreover, by virtue of the intense radiating heat transfer taking place at the front of the jet, a high-temperature jet can penetrate a pool of evaporating coolant, like a “jet in a bag of steam”. These exotic problems are modeled and explained in the present article along with experimental data used for model validation. Several new hydrodynamic phenomena discovered over recent years are presented together with their practical applications, in particular, the modes of soliton-like and shock-wave decay of jets during vibration and electromagnetic resonant decomposition of the jet into droplets of a given size. These phenomena are relevant to the areas of granulation of metal melts, cooling of molten corium during postulated severe accidents in nuclear power plants and in other industrial and technological devices and processes.

**Keywords:** Jet, film thickness, flow, hydraulic shock, penetration pool, vapor, mushroom.

### 1 Introduction to the problem and goal of the paper

Non-classical problems of jet and film flow and their parametrically controlled decay into droplets of a given size have been collected for decades in the development of various industrial and technological applications. The first area of research was related to metallurgy, tokamak and new materials science, producing unique materials from amorphous or close to amorphous granules [Binetskiy, Vodyanuk, Kazachkov et al.

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(1985); Bundy, Gorislavets, Kazachkov et al. (1986); Demyanenko, Gorislavets, Kazachkov et al. (1989)]; Glukhenkyi, Kazachkov, Kolesnichenko et al. (1988); Kazachkov (2015, 2016); Kolesnichenko, Kazachkov, Vodyanuk et al. (1988); Kolesnichenko, Kuchinskyi, Kazachkov et al. (1992); Kuchinskyi, Kazachkov, Kolesnichenko et al. (1989); Malakhov, Kazachkov, Kolesnichenko et al. (1989); Vodyanuk, Ivanochok, Kazachkov et al. (1990)]. For this, a theory of parametrically controlled decay of a jet of liquid metal and film flow was developed [Kazachkov (1985, 1986, 1989, 1991, 1996); Kolesnichenko, Kazachkov, Vodyanuk et al. (1988); Kazachkov and Kolesnichenko (1986, 1991)].

Another area of research was related to the simulation of thermal-hydraulic processes in the passive protection system at nuclear power plants during postulated severe accidents. One of these problems of multiphase systems was associated with the problem of the penetration of liquid metal jets into the pool of volatile coolant: the maximum length of penetration, the instability of jets and their decay into droplets and fragments, rapid solidification and re-melting due to internal heat release, etc. [Park, Kazachkov, Sehgal et al (2000); Kazachkov and Moghaddam (2008); Kazachkov (2011)].

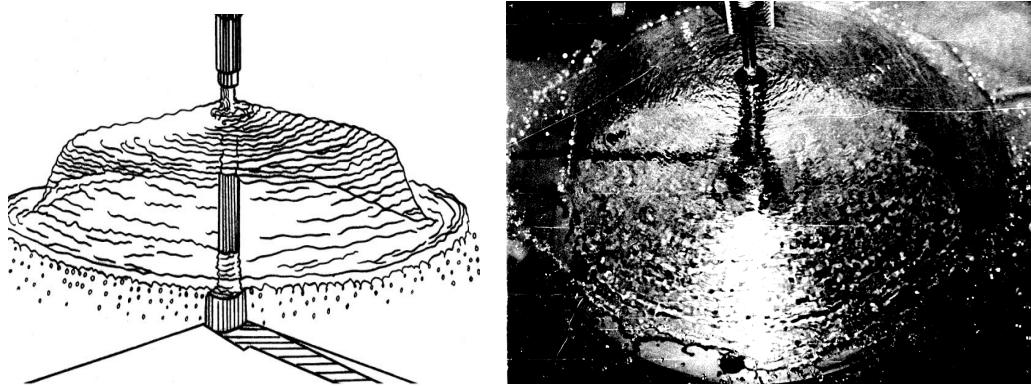
The paper is organized as follows. First the problem about the film flow created by the hydraulic shock of the vertical jet with horizontal plate is considered in detail and the basic parameters of the film flow are determined. Then a few new phenomena of the parametrically controlled film flow disintegration into the drops of given size are presented in short. Afterwards a few interesting phenomena occurring in the process of the jet penetration into the pool of other liquid are investigated and explained. The accomplishments in the considered problems, which are impossible for complete description in the frame of the Navier-Stokes equations revealed some important challenges for further study.

## **2 Creation of the film flow and parametric control**

### ***2.1 Creation of the film flow due to hydraulic shock of the jet with plate***

Film flows of liquid have long been known for various applications and physical processes, including natural and even in everyday life (film formed in the kitchen when a jet of water gets from a tap on a spoon). Some examples of film flow are shown in Fig. 1 (scheme and some photos):

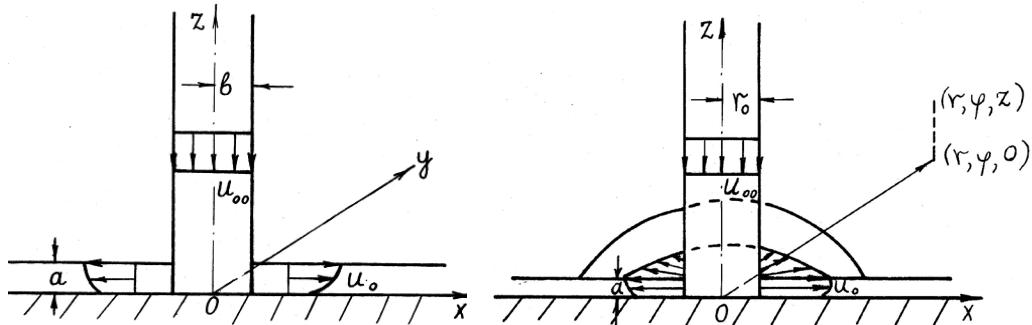
If we consider the creation of a film flow within the framework of the continuity hypothesis and the Navier-Stokes equations, then, based on the mass conservation equation, we can obtain a film that propagates in the radial direction on a plate that divides the vertical jet flow into a film flow that is not thin, contrary to what is seen from the experiments, starting with the aforementioned experience in the kitchen. This is because hydraulic shock is not described in the framework of the Navier-Stokes equations. So first, let us start with the phenomenon of the formation of a thin film flow.



**Figure 1:** Drawing of film flow (left) and photo of water film flow (right)

### 2.2 Parameters of the film flow created by hydraulic shock of a jet with plate

The formation of the flow of thin film, of course, is a very exciting phenomenon. The vertical jet turns into a horizontal flow of thin film when the jet velocity is significant, but the horizontal plate (disk) is fixed. First, simplify the problem by considering a flat jet (Fig. 2 to the left), which actually gives a result that is close to round jet (Fig. 2 to the right). If we consider the transformation of a jet into a film flow without analyzing the hydraulic shock, we will get a big contradiction with observations both in speed and in film thickness, with the exception of some smooth, slow jets. For hydraulic impact of a vertical jet on a horizontal disk, it is necessary to consider the laws of conservation of mass and momentum.



**Figure 2:** Structural scheme of plane film (left) and radially spreading film (right)

The mass of a part of the jet fluid, which decelerates sharply from a high jet velocity to a zero disc velocity, during a short impact time  $\Delta t$  is  $\rho c \Delta t S$ , where  $\rho$  is the density of the liquid,  $c$  is the sound velocity in this fluid (e.g., for water it is about 1500 m/s),  $S$ -the cross-sectional area of the jet [Pokrovskii (1972)]. Because of the symmetry of the flow, only half of the jet stream and film are considered. This mass, multiplied by the speed of the jet, gives a momentum of the decelerated part of the jet caused by the impact of the jet on the disk. Thus, the first major difference from conventional hydrodynamics is the slowing down of a part of a jet's fluid at the speed of sound when struck on a fixed plate

from its velocity to zero in a short time  $\Delta t$ . Sharply slowed mass of the jet  $\rho c \Delta t S$  determines the action of the jet on the plate during hydraulic shock. Despite the small  $\Delta t$ , it is much more than with a smooth flow, because the speed of sound in most cases is much higher than the speed of the jet flow.

The momentum of the decelerated part of the jet is equal to the impulse of the force created by the jet on the disk due to the impact:  $PS\Delta t$ . Then  $P$  is calculated as pressure due to the collision of the disk and the jet. The equation of conservation of momentum for the impact of a moving jet with a fixed disk is equal to

$$P \cdot b \cdot \Delta t = \rho \cdot c \cdot \Delta t \cdot b \cdot u_{00}, \quad (1)$$

where  $S = b \cdot 1$  (jet width multiplied per unit distance in the direction perpendicular to the plane of the figure,  $y$ ). Then it follows from (1) that it is much higher than the normal action of the jet on the plate by a smooth flow, when it is equal to the kinetic energy of the jet  $P = 0.5 \rho \cdot u_{00}^2$ . For example, for a jet of water with a speed of 1 m/s, the pressure in the impact is about  $1.5 \cdot 10^6 \text{ N/m}^2$ , which is 3000 times higher than the pressure of the same jet flow created on the plate ( $500 \text{ N/m}^2$ ) without impact (smooth flow). Similar phenomena occur in other situations, for example, waves in a storm, beating against a barrage on the seashore (they jump much higher than waves in the ocean).

Using the above, we can calculate the velocity in the film flow created by the jet after its impact on the disk. The momentum conservation for the film flow is:

$$\rho \cdot c \cdot u_{00} = 0.5 \rho \cdot u_0^2. \quad (2)$$

From Eq. (2) follows

$$u_0 = \sqrt{2c \cdot u_{00}}, \quad a = b \cdot u_{00} / u_0, \quad \text{or} \quad a = b \sqrt{0.5 u_{00} / c}. \quad (3)$$

It was computed in the above conditions as follows:  $u_0 \approx 55 \text{ m/s}$ ,  $a \approx 0.018 \cdot b$ , where from yields  $a \approx 0.09 \text{ mm}$  for  $b=5 \text{ mm}$  (jet of 1 sm width, 1 m/s velocity creates film flow less than 0.1 mm thickness and a speed 55 m/s!). The viscous dissipation of flow energy was neglected in calculations; therefore these values are a little overestimated but still exciting. The ratio of various forces significantly depends on the film thickness, physical properties of liquid or metal melt, type and intensity of the external influences that define a variety of the flow modes. An important role is played by the capillary, viscous, electromagnetic forces, etc. [Kazachkov (1989, 2015)].

For the radially spreading film flow (the round jet creates the one), the mass conservation of a film is  $\rho 2\pi b a u_0 = \rho \pi b^2 u_{00}$  ( $a$ -film thickness,  $b$ -jet radius). Then from (3) yields:

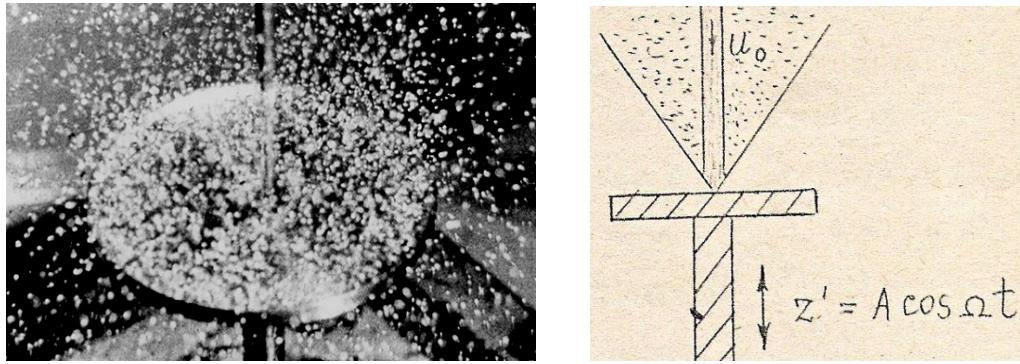
$$a = 0.5 b u_{00} / u_0, \quad (4)$$

where it is seen that the radial film has the same velocity as the flat one (created by flat jet), but its thickness in twice less. Moreover, a film flow is spreading radially, therefore its thickness  $\varepsilon$  is decreasing with the radial coordinate growing according to the mass conservation equation:  $\rho 2\pi b a u_0 = \rho 2\pi r \varepsilon u$ , where  $r$ -radial coordinate (distance from the film flow origin, circle),  $\varepsilon$ -film thickness, and  $u$ -its velocity on a distance  $r$  from origin. Then  $\varepsilon u = b a u_0 / r$ . The momentum equation yields

$$\rho \pi b a u_0^2 = \rho \pi r \varepsilon u^2, \quad \varepsilon = ab/r, \quad (5)$$

where from follows that a film velocity is the same as previously got (3) for the flat film flow ( $u=u_0$ ), while a thickness is varying starting from the origin  $r=b$  following the correlation  $\varepsilon=ab/r$ , thus, substantially, in a contrast to the flat film flows.

Analysis of the mathematical model (3)-(5) thus obtained shows that the limit value for the velocity of a jet is  $u_{00}=c$ , when the model keeps valid, the Eq. (3) gives  $a=b\sqrt{0.5} \approx 0.7b$ , and (5)- $a=0.35b$ . Afterwards, for supersonic jet, the flow regime is absolutely different (shock-wave jet flow), and a film flow is not created this way. For supersonic jet flow there are conical shock waves on the head of a jet similar to the ones observed by us in the experimental study by high vibration acceleration (vibration Euler number  $Eu_g = g_v r_0 / u_0^2$  was about 100) [Kazachkov (1989, 2015); Glukhenkyi, Kazachkov, Kolesnichenko et al. (1988)], shown in Fig. 3 to the right.



**Figure 3:** The soliton-like (left) and the shock-wave (right) regimes of film flows

Here  $g_v = A\Omega^2$  - the acceleration due to vibration developed by the vibrator, where  $A, \Omega$  - the amplitude and frequency of vibrations.

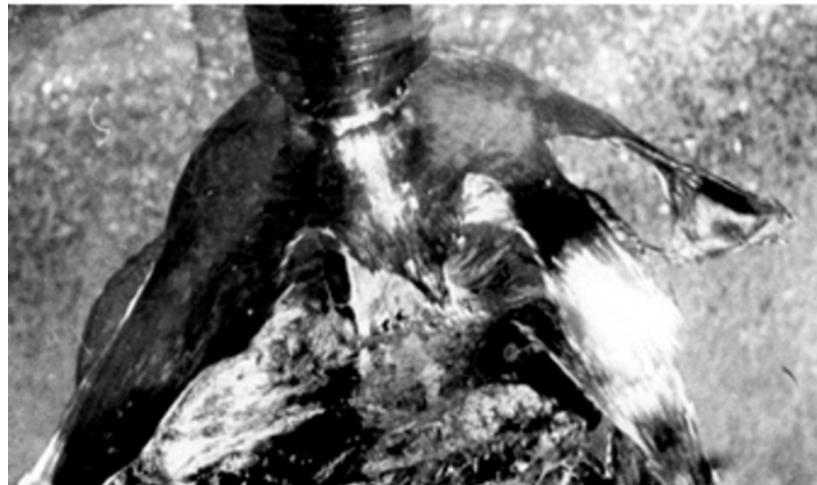
### **2.3 Three new phenomena of controlled film disintegration into drops of given size**

The film flow can be disintegrated into the drops of given size depending on a frequency of the parametric excitation (e.g., vibration of the disk or alternating electromagnetic field for the metal melts), with the comparably low energy consumption. The first new phenomenon discovered and tested by us is the vibrational soliton-like disintegration of film flow as in Fig. 3 (left). The solitons on the disk throw off the drops like a series of the unit jets do in a phase of vibration modulation. When the vibration acceleration grows, the film flow is shortened moving to the origin of the film flow-vertical jet. The second discovered new phenomenon is the shock wave on a jet, at the high vibration Euler numbers (10-100 depending on the parameters) [Kazachkov and Kolesnichenko (1986, 1991)] shown in Fig. 3 (right).

The correlation (3) reveals the supersonic limit for velocity of film flow  $u_0 = c\sqrt{2} \approx 1.42c$ , with a transition of a film flow through the sound barrier ( $u_0 = c$ ) under subsonic velocity of a jet:  $u_{00} = 0.5c$ , and with account of a friction on a disk-by a little prevailing of this

velocity. The lower limit of the jet's velocity for creation of a film flow corresponds to the viscous dissipation of a film on a disk. This is a hard question because we do not know the minimal velocity of a jet capable to create a shock on a disk (problem!).

The theory of film flows, developed and experimentally studied in Kazachkov et al. [Kazachkov (1985, 1986, 1989, 1991, 1996); Kazachkov and Kolesnichenko (1986, 1991); [Kolesnichenko, Kazachkov, Vodyanuk et al. (1988)]], revealed the features of film flows controlled by electromagnetic fields and vibrations of the disk, on which the film is formed. The mechanism of the film flow formation was not considered. It is explored in this paper. Fig. 4 shows a film flow of gallium in a nitrogen atmosphere. Vacuuming of the working chamber was insignificant (up to 0.1 mbar) and the chamber was filled with a nitrogen. As can be seen from Fig. 4, under such conditions, a thin film of strong oxides is formed on a surface of film flow, which destroys the film flow due to the Marangoni instability. The nozzle was 2.5 mm in diameter; velocity of a jet was 2 m/s. According to (3) the velocity and thickness of the film flow was  $u_0=106$  m/s,  $a/b=0.0045$ ,  $b=1.25$  mm,  $a=5.5$   $\mu\text{m}$ .



**Figure 4:** Gallium film flow in a weakly controlled nitric atmosphere (up to 0.1 mbar)

The density of gallium is  $\rho=6095$  kg/m<sup>3</sup>, therefore, the pressure of a jet as a result of the hydraulic shock with the disk is approximately  $P_{hs}=34242$  kN/m<sup>2</sup> (nearly 342 bar), whereas for a regime of smooth flow, such velocity forms a pressure  $P=12.19$  kN/m<sup>2</sup>. This explains our surprising of the fact that the experiment with pulsed electromagnetic inductor did show no effect on the film flow.

The computed force of a jet on the disk because of hydraulic shock is  $F=P_{hs}S=53.5$  N, that is about 5.4 kilogram force. Hence, for the pulse inductor of such force, respectively, in 1 ms and 1  $\mu\text{s}$ , the energy is 53.5 kJ and 53.5 MJ. During 1 ms and 1  $\mu\text{s}$ , the film, respectively, spans a distance of 10.6 cm and 1.06 mm. Therefore, it is obvious that one could notice the effect of the pulse inductor on the film flow only in the second case, because the film during this time falls apart at this distance, in the first case. However, in the second case, the inductor of too high power is required compared to the one used by us.

Only an application of the electromagnetic inductor creating the running wave is inducing a modulated electromagnetic force in a film flow, which is capable to control the film flow, especially-in a resonance regime. The most important is not amplitude of the force but a frequency of an external action coinciding with the frequency of Eigen oscillations.

The film flow is mechanically divided on the jets on a disk with the radial channels on its surface. These channels on the surface of disk are seen under inductor (the big one in Fig. 5 located over disk). Fig. 5 presents resonant regime of electromagnetically controlled disintegration of the film flow into the drops of nearly constant size computed by the first harmonics of the Eigen oscillations of the small jets on the disk surface. This is the third new phenomenon discovered by us. Electromagnetic inductor on the nozzle creates the running wave

$$h=h_m(z,r)\exp(i(kr+m\varphi-\omega t)), \quad (6)$$

where  $k, m$  are the wave numbers by coordinates  $r, \varphi$  of the cylindrical coordinate system,  $\omega$  is the frequency of electromagnetic field, which can be controlled by the special poly-harmonic power supply,  $h$ - vertical component of the strength of magnetic field,  $h_m$ - its amplitude. Such electromagnetic wave induces the circle currents in a film flow, which interaction with the field results in the radial ponderomotive forces controlling the film decay. The controlling electromagnetic force in the film is modulated in the direction of the film flow. The drops are first ejected slightly up, and then fall down by the ballistic trajectories.



**Figure 5:** Electromagnetically controlled gallium film decay in the resonant regime

#### 2.4 Calculation of the parameters of film flows

According to (3), (4), the speed of sound determines the thickness and velocity of the film flow formed because of the hydraulic shock of the vertical jet with a surface of the disk. Therefore, now we make estimates for the different liquids and melts. The speed of sound is the constant of a given substance in gases, liquids and isotropic solids. It is determined by elasticity and density by the formula:

$$c = \sqrt{E / \rho}, \quad (7)$$

being minimal in gases (under normal conditions in the air  $c=330-340$  m/s, depending on temperature  $T$ : the higher  $T$ , the higher the speed). Here  $E$ -the Young's modulus of elasticity. Value  $c$  is maximal in solids ( $c=6000$  m/s in steels), intermediate-in liquids

( $c=1500$  m/s in water).  $M=u/c$  is the Mach number. Flow is supersonic by  $M>1$ . The speed of sound is independent of frequency in an ideal gas but weakly depends on the frequency in all real physical processes. This is a function of the square root of temperature, while does almost not depend on pressure and density in given gas.

Let us take the following average values according to Babichev et al. [Babichev, Babushkina, Bratkovskii et al. (1991)] for the speed of sound in aluminum (Al), gallium (Ga), copper (Cu), tin (Sn), lead (Pb), zinc (Zn), which were used in our experimental studies and in the production of granules: 4700, 2800, 3460, 2400, 1800, 2800 (m/s). Then from the formulas (3)-(5), the following approximate expressions for calculating the film velocity and its thickness are written:

$$u_0 = \alpha \sqrt{u_{00}}, \quad a/b = \gamma \sqrt{u_{00}}, \quad (8)$$

where  $\alpha, \gamma$  ( $\gamma=1/\alpha$ ) are the constants of the material presented in the Tab. 1, where it is evident that water and mercury are close by these indicators, while the other metal melts are very different, especially aluminum, which has a speed of sound almost twice as high as water: with the same parameters of the jet, the aluminum film will be twice as fast and thinner than the corresponding film of water. For a radial film, according to (5), the constant  $\gamma$  is twice less than that presented in Tab. 1. Velocity and thickness of the film flow depending on the velocity of jet flow are given in Tab. 2.

**Table 1:** The values of speed of sound and parameters  $\alpha, \gamma$  for melts and water

Liquid	Al	Cu	Ga, Zn	Sn	Pb	Water	Hg
$c, \text{m/s}$	4700	3460	2800	2400	1800	1500	1450
$\alpha, \sqrt{\text{m/s}}$	97.0	83.2	74.8	69.3	60	54.8	53.9
$\gamma, \sqrt{\text{s/m}}$	0.01	0.012	0.013	0.014	0.017	0.018	0.019

**Table 2:** Velocity  $u_0$  (m/s) and  $(a/b) \cdot 10^3$  for film flows depending on jet's velocity

$u_{00}, \text{m/s}$	Al	Cu	Ga, Zn	Sn	Pb	Water	Hg
0.001	3.07/0.32	2.63/0.38	2.37/0.41	2.19/0.44	1.90/0.54	1.73/0.57	1.70/0.60
0.01	9.7/1.0	8.32/1.2	7.48/1.3	6.93/1.4	6/1.7	5.48/1.8	5.39/1.9
0.1	30.7/3.2	26.3/3.8	23.7/4.1	21.9/4.4	19.0/5.4	17.3/5.7	17.0/6.0
1.0	97/10	83.2/12	74.8/13	69.3/14	60/17	54.8/18	53.9/19
10.0	307/32	263/38	237/41	219/44	190/54	173/57	170/60
100	970/100	832/120	748/130	693/140	600/170	548/180	539/190
1000	3070/320	2630/380	2370/410	2190/440	<b>1900/540</b>	<b>1730/570</b>	<b>1700/600</b>

Velocity of the film flow, depending on the velocity of jet forming it, can be seen directly from here. Bold underlined the exceeding the speed of sound for a given medium. Thus, for the melt of aluminum at  $u_{00}=0.001$  m/s, the film velocity is  $u_0=3.07$  m/s, then at  $u_{00}=1000$  m/s, respectively,  $u_0=3070$  m/s, the impressive film velocity, and it is subsonic

for the aluminum. In this case,  $a/b=0.32$  and 320 respectively, which means that a film thickness of 1.6 microns ( $a = 1.6 \cdot 10^{-6}$  m) for a jet of 10 mm thick ( $b = 5$  mm) at  $u_{00}=0.001$  m/s, and  $a=1.6$  mm-at  $u_{00}=1000$  m/s. In the latter case, this does not seem like a film, but a simple split of the flow according to the equations of smooth fluid flow, although a small compression of the jet on the disk is still observed.

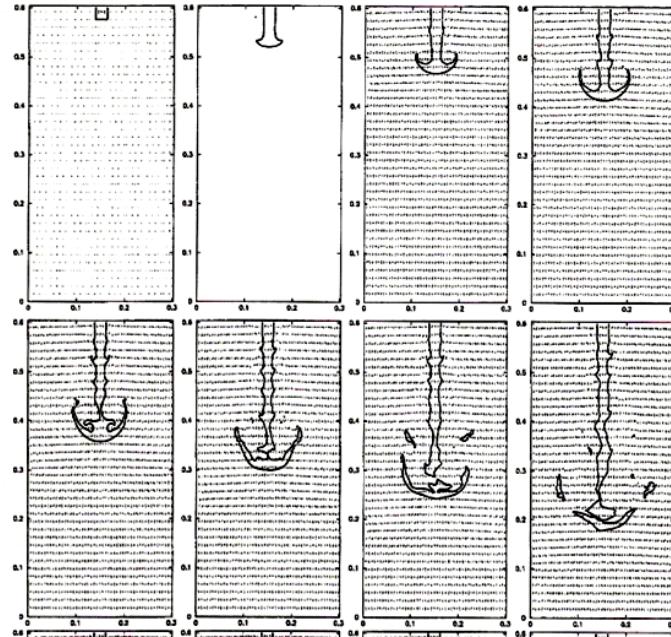
Obviously, the micron thickness of the film is only possible with the application of jets with a radius by order around 1 mm, when the corresponding thickness of the film will be  $a = 0.08$   $\mu\text{m}$  at  $u_{00}=0.001$  m/s,  $a = 0.25$   $\mu\text{m}$  at  $u_{00}=0.01$  m/s,  $a = 0.8$   $\mu\text{m}$  at  $u_{00}=0.1$  m/s and  $a = 2.5$   $\mu\text{m}$  at  $u_{00}=1$  m/s. In the latter case, according to (5), at a distance of 2.5 radii of the jet will be  $\varepsilon=1$   $\mu\text{m}$ , and at a distance of 25 radii of the jet, the thickness of the film  $\varepsilon=0.1$   $\mu\text{m}$ . Thin high-pressure jets are used, for example, in surgery [Early clinical experience (2002)] as so-called unique water scalpels with the thin water jets instead of metal scalpels.

Wang et al. [Wang, Dandekar, Bustos et al. (2018)] wrote that they developed a model for the decomposition of a thin film of fluid into individual droplets, which takes into account the dependence of its acceleration on time, for example, by collisions of liquid droplets with a solid surface. They experimentally investigated the phenomenon of spattering droplets of various liquids by means of high-speed shooting, which at the present time became possible and allows observing high-speed processes. The authors aim to simulate in detail the behavior of various chemical fluids, such as aerosols, and to “establish epidemiology and health care in physics and mathematics.” So, to prevent the disease, they decided: “We want to give recommendations based on the science that was tested in the laboratory.” From a practical point of view, they even consider it possible to show the risks of contamination in the vicinity of infected, protected equipment optimized to protect hospital workers from specific types of microorganisms, and better predictions of how diseases are moving through the population. Very simplified look at complex physical phenomena that have been exploring for more than a century and still are not fully understood.

### **3 Penetration of a jet into the pool of other liquid**

#### ***3.1 Fungoid shape head of the jet penetrating pool of other liquid***

The problems of jet penetration into the pool of another fluid were considered by many scientists for different applications: [Eichelberger (1956); Carleone, Jameson and Chou (1977); Hirsch (1979); Saito (1988); Bui, Dinh and Sehgal (1997); Park, Kazachkov and Sehgal (2000); Kazachkov and Moghaddam (2008)], etc. Illustration a jet of model melt penetrating the pool calculated at the Royal Institute of Technology using the SIPHRA code [Bui, Dinh and Sehgal (1997)] presented in Fig. 6. The diameter of the jet was 25 mm, the initial speed 3 m/s.



**Figure 6:** Penetration of jet into pool of other liquid with forming the fungoid shape head

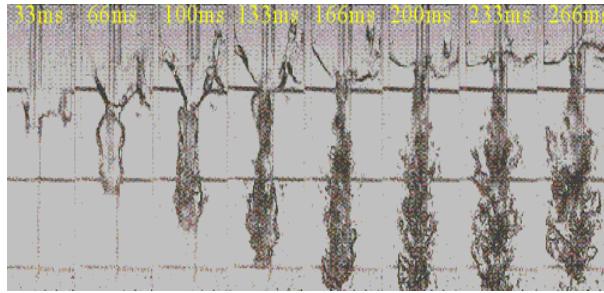
The model melt had a kinematic viscosity coefficient and a surface tension coefficient about 3 times less than a melt of nuclear fuel. Disintegration of the jet was approximately at a distance of 0.3-0.4 m. As seen from Fig. 6, the jet is getting some kind of fungoid shape head due to hydraulic shock of a jet with a free surface of the pool. Here the hydraulic shock is not as strong as for the above-considered shock of the jet with plate but still under some conditions it is remarkable and must be accounted. Further penetration of the jet with the fungoid shape head creates an additional resistance.

In this case, the features of a jet are clearly visible, associated with appearance of the fungoid shape head of a jet, which is formed at the initial moment as a result of the impact of jet on a fixed surface of liquid, by shock and capillary forces. The initial propagation of the jet is pronouncedly symmetric and subject to the axially symmetric perturbations of the jet's thickness. The thickened head creates significant resistance to the jet's penetration, as a result of which its deceleration increases. After separation of the fungoid shape head, it is obvious that the jet experiences, mainly, bending perturbations of the axis, leading to its disintegration into fragments. The phenomenon requires more detail study. Batchelor [Batchelor (1967)] has given the equation to compute the momentum losses by a shock of the jet on a liquid pool surface at the initial moment of a jet penetration when it touches a liquid pool. Using that equation one can compute the abrupt change of a jet velocity at the entrance to a pool.

### **3.2 Bifurcation of the jet penetrating the pool**

The dynamics of penetration phenomenon for the jet of corium melt into the pool of water or another coolant has been investigated by a number of scientists and engineers

but the problem still remains. It requires more detail study, especially for the thick jets, which penetrate a pool of other liquid under the dominated inertia, drag and buoyancy forces. For the thin jets, it was shown [Park, Kazachkov and Sehgal (2000); Kazachkov and Moghaddam (2008)], etc. that their instability depends on the bending perturbations of the axis, while the penetration behaviours for the thick jet revealed that except density ratio and Froude number there is one important parameter concerning the point of bifurcation where the jet abruptly changes its radius nearly three times as shown in Fig. 7 by experimental study of Park et al. [Park, Kazachkov and Sehgal (2000)]:



**Figure 7:** Experimental results by thick jet penetration into the pool

Let us consider the Bernoulli equation and the mass conservation equation for the jet:

$$S_1 \left[ (\rho_1 - \rho_2)hg + 0.5\rho_1 v_1^2 \right] = 0.5\rho_1 u_0^2 S_0, \quad \rho_1 v_1 S_1 = \rho_1 u_0 S_0, \quad (9)$$

where  $S$  is the area of the jet's cross section. Index 0 denotes the initial state while the index 1 denotes some current state afterwards. Then in a dimensionless form, retaining the same symbols:

$$S_1 \left[ 2h(1 - \rho_{2/1}) / Fr + v_1^2 \right] = 1, \quad S_1 v_1 = 1. \quad (10)$$

The equation array (10) has the following solution:

$$S_1 = \frac{Fr}{4h(1 - \rho_{2/1})} \left[ 1 \pm \sqrt{1 - 8h(1 - \rho_{2/1}) / Fr} \right], \quad v_1 = 1 / S_1. \quad (11)$$

There are two possible values for the jet's radius with the point of bifurcation:

$$h = \frac{Fr}{8(1 - \rho_{2/1})}.$$

After this point the solution (11) does not exist in real numbers, therefore the jet can change its radius abruptly between these two available values. The jet starts penetration into the pool with initial cross-section  $S_1 = 1$ . Analysis of (11) shows that for a small depth of penetration or, more common,  $8h(1 - \rho_{2/1}) \ll Fr$ , it goes to:

$$S_1 \approx 1 \text{ or } S_1 \approx \frac{Fr}{2h(1 - \rho_{2/1})} \gg 1.$$

There is no reason for a jet to change its cross section area 1 to the bigger one because the jet momentum directs mainly along its axis. But then, due to instability causing by the

free surface perturbations and by a loss of momentum, the jet's area may change at any moment. It requires complete instability and bifurcation analysis being a subject of separate study. The jet should change from  $S_1=1$  to  $S_2=2$  at the point

$$h_1 = \frac{Fr}{8(1-\rho_{2/1})} = \frac{1}{8Ri},$$

when further existence of the two possible jet's radii is impossible. Here  $Ri$  is the Richardson number (the ratio of the momentum and buoyancy forces of a jet).

Substituting  $S_1=2$  into (10) gives  $v_1=0.5$ . The jet is going from  $h=h_0$  to  $h_1=1/(8Ri)$  and during this time its radius is growing from 1 to  $r_1=\sqrt{2}$ , when the jet velocity becomes  $v_1=0.5$ , e.g., for the density ratio 0.1 the total depth of a jet penetration into a pool up to this point is computed as  $h_0 + h_1 \approx 5.5 + 13.9 \approx 19.4$  [Kazachkov and Moghaddam (2008)].

From (11) a jet's cross-section at the depth of penetration of  $h = h_0$  is as follows:

$$S_1 = 0.5\rho_{1/2}(1 \pm \sqrt{1 - 4\rho_{2/1}}), \quad (12)$$

where from the following two available solutions follow for the density ratio 0.1:

$$S_1 \approx 1.15, \quad r_1 \approx 1.07, \quad v_1 \approx 0.87, \quad \text{or} \quad S_1 \approx 8.87, \quad r_1 \approx 2.98, \quad v_1 \approx 0.11,$$

so that the first set of parameters is close to the assumptions above, while the other set of parameters is a possible solution, which may occur abruptly at the point of bifurcation  $h=h_1$  due to instability of the jet when any regular solution, as shown by (11), (12) does not exist. Experimental data in Fig. 7 support this analysis.

As concern to mechanism of such bifurcation with abrupt changes of the radius of a jet, we assume that changes of the momentum due to loss of energy have some delay in the process of penetration. Therefore, the jet switches abruptly from one to another radius, as described, to fit the new momentum of the jet.

### **3.3 High-temperature jet penetrating the pool of volatile coolant**

The model for the high-temperature jet penetrating the pool of volatile coolant based on the momentum equation is written in the form [Moghaddam and Kazachkov (2010)]:

$$\rho_1 h \frac{dV_1}{dt} = g(\rho_1 h - \rho_2 x) - \alpha \rho_2 V_1^2 - \beta \rho R T_1, \quad (13)$$

where the cross-sectional area  $S_1=\pi a^2$  is omitted. Here  $g$  is acceleration due to gravity,  $h$  is the length of jet,  $\rho$ -density of vapor,  $T_1$ -temperature of vapor,  $R$ -the universal gas constant,  $\alpha$ - coefficient of the drag force (depends on the jet form and flow regime). The non-linear differential Eq. (13) with the initial conditions:

$$t=0, \quad x=0, \quad \frac{dx}{dt} = V_n, \quad (14)$$

where  $V_n$  is the initial velocity of a penetration for the jet with account of shock on the surface of pool at entrance. The Cauchy problem (13), (14) results in the jet's phase trajectories in the following dimensionless form:

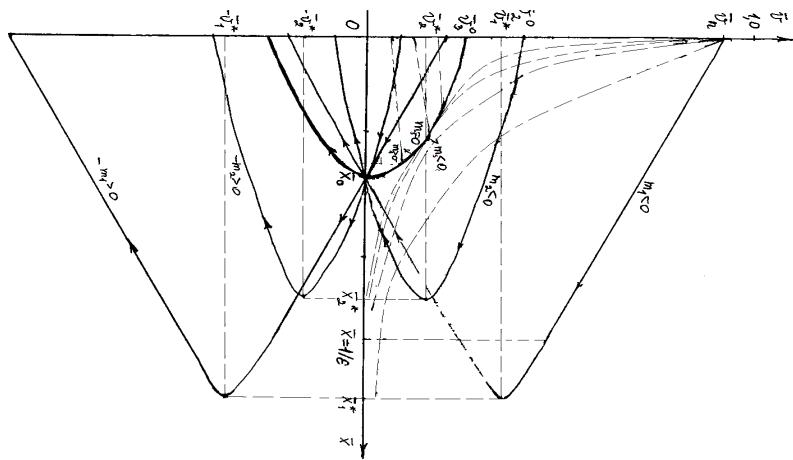
$$\frac{d\bar{v}}{d\bar{x}} = -\frac{\varepsilon \alpha \rho_{21} Fr^2 \bar{v}^2 + \varepsilon \rho_{21} \bar{x} + \bar{b} - 1}{Fr^2 \bar{v}}, \quad \bar{x} = 0, \quad \bar{v} = \bar{v}_n. \quad (15)$$

The phase portrait of a jet penetrating pool of volatile coolant is given in Fig. 8, which contains all available regimes of the jet penetration into the pool in a concentrated form including the case of high-temperature jet, which vaporizes the coolant due to radiative high heat flux. In this case the jet is going in the “vapor sack”, so that study of the penetration regime is complex due to many interacting heat and mass transfer dynamic processes with varying boundary conditions.

## 4 Discussions

Despite the abundance of work on the flow and heat transfer of thin films over the past decades and earlier, e.g., [Akamine, Okamoto, Gee et al. (2018); Teamah and Khairat (2015); Guo and Green (2015); Molana and Banooni (2013); Johnson and Gray (2011); Fujimoto, Suzuki, Hama et al. (2011); Eggers and Villermaux (2008); De Paz and Jubran (2008); Shu and Wilks (1972)], etc. the phenomenon of film formation during the hydraulic shock of the vertical jet with a fixed horizontal plane has not been previously studied. Only in the brochure [Pokrovskii (1972)], it was analyzed together with a number of other critical processes like an explosion under water and so on. This problem was considered in detail in this article together with a few other non-classical problems of the jet and film flows and heat transfer.

Usually, all theoretical and experimental work on films spreading at rather high velocities over the surface or in a free space begins with the presence of such a film, without analyzing its formation as a result of the collision of a vertical jet with a horizontal or inclined surface. The stability of the films, the processes of heat transfer and hydraulic jump, when the film abruptly goes into the normal mode of flow of the liquid layer have been studied. The aforementioned works of ours since 1985 were also carried out in a similar way, however, it was soon discovered that the stage of film formation was very important and should be studied separately.



**Figure 8:** Phase portrait of a jet penetrating pool of volatile coolant

## 5 Conclusions

A few problems of the heat and mass transfer in the jet/film flows were considered, which was not studied in the frame of conventional Navier-Stokes equations. The thin film flow is created due to hydraulic shock of the vertical jet with horizontal plate. The obtained parameters differ a lot from the ones got in classic theory. The three new phenomena discovered and tested were described: soliton-like and shock-wave regimes of the film flow disintegration into the drops of given size; and the electromagnetic resonant disintegration of the liquid metal film.

Also a few new phenomena were considered for the jet penetrating pool of other liquid. There are two features connected to the initial step of the jet penetration. Due to the shock of the jet with surface of the pool, the head of fungoid shape is created under close densities of the jet and pool. And as shown by experimental study supported by theoretical analysis, the jet is not smoothly changing its radius by penetration as by classic theory forecasted but abruptly, which may be explained by delay in reaction of the jet on the momentum decrease.

And the last case of the jet penetrating the pool of volatile coolant in the “vapor sack” is complex due to interaction of a few different factors and permanently changing boundary conditions. Some simple phase portrait of the jet was proposed to analyze the general parameters of a jet in different cases, e.g., depth of penetration depending on initial conditions and physical situations.

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