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Propagation of a Thermoelastic Wave in a Half-Space of a Homogeneous Isotropic Material Subjected to the Effect of Rotation and Initial Stress

Fatima Bayones[1], Abdelmooty Abd-Alla[2,*], Raghad Alfatta[3] and Hoda Al-Nefaie[3]

Abstract: The propagation of thermoelastic waves in a homogeneous, isotropic elastic semi-infinite space is subjected to rotation and initial stress, which is at temperature T_0 - initially, and whose boundary surface is subjected to heat source and load moving with finite velocity. Temperature and stress distribution occurring due to heating or cooling and have been determined using certain boundary conditions. Numerical results have been given and illustrated graphically in each case considered. Comparison is made with the results predicted by the theory of thermoelasticity in the absence of rotation and initial stress. The results indicate that the effect of the rotation and initial stress is very pronounced.

Keywords: Thermal stresses, thermoelasticity, wave propagation, rotation, initial stress.

1 Introduction

The subject of generalized thermoelasticity has drawn the attention of researchers due to its relevance in many practical applications. The generalized thermoelasticity theories involve hyperbolic-type governing equations and admit the finite speed of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation, these theories are referred to as generalized theories. Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermoelasticity theories are more realistic than conventional thermoelasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes such as those occurring in laser units, energy channels, nuclear reactors, etc. The phenomenon of coupling between the thermomechanical behavior of materials and magnetic behavior of materials have been studied since the19th. On generalized magneto-thermoelastic Rayleigh waves in a granular medium under influence of the gravity field and initial stress have been studied by Abd-Alla et al. [Abd-Alla, Abo-Dahab, Mahmoud et al. (2011); Abd-Alla and Mahmoud (2010)] investigated the magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylinder under the hyperbolic

[1] Department of Mathematics, Faculty of Science, Taif University, Taif, 888, Saudi Arabia.
[2] Department of Mathematics, Faculty of Science, Sohag University, Sohag, 243, Egypt.
[3] General Department of Education in Taif, Taif University, Taif, 888, Saudi Arabia.
[*] Corresponding Author: Abdelmooty Abd-Alla. Email: mohmrr@yahoo.com.

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heat conduction model. Propagation of Rayleigh waves in magneto-thermo-elastic half-space of a homogeneous orthotropic material under the effect of the rotation, initial stress and gravity field was studied by Abd-Alla et al. [Abd-Alla, Abo-Dahab, Bayones (2013); Ailawalia and Narah (2009)] discussed the effect of rotation in a generalized thermoelastic medium with hydrostatic initial stress subjected to ramp-type heating and loading.

Abouelregal [Abouelregal (2011)] has presented Rayleigh waves in a thermoelastic solid half space using dual-phase-lag model. Stoneley waves in a non-homogeneous orthotropic granular medium under the influence of gravity studied by Ahmed et al. [Ahmed (2005); Ailawalia and Narah (2009)] investigated the effect of rotation in generalized thermoelastic solid under the influence of gravity with an overlying infinite thermoelastic fluid. Amin et al. [Amin, El-Bary and Youssef (2017)] investigated the two dimensional problem of generalized thermoelastic half space subjected to moving heat source. Choudhury et al. [Choudhury, Basu and Bhattacharyya (2015)] investigated wave propagation in a rotating randomly varying granular generalized thermoelastic medium. Deswal et al. [Deswal and Choudhary (2008)] studied the two-dimensional interactions due to moving loads in generalized thermoelastic solid with diffusion. Deswal et al. [Deswal, Punia and Kalkal (2019)] investigated the propagation of waves at an interface between a transversely isotropic rotating thermoelastic solid half space and a fiber-reinforced magneto-thermoelastic rotating solid half space. Ezzat et al. [Ezzat and Youssef (2010)] proposed the three-dimensional thermal shock problem of generalized thermoelastic half-space. Kakar [Kakar (2012)] studied the effect of initial stress and gravity on Rayleigh wave propagation in non-homogeneous isotropic elastic media. Kumar et al. [Kumar and Deswal (2000)] investigated the Steady-state response of a micropolar generalized thermoelastic half-space to the moving mechanical/thermal loads. Propagation of waves in transversely isotropic micropolar generalized thermoelastic half space was studied by Kumar et al. [Kumar and Gupta (2010)]. Plane waves at an imperfectly bonded interface of two orthotropic generalized thermoelastic rotating half-spaces with two relaxation times was studied by Kumar et al. [Kumar and Singh (2009); Kumar and Chawla (2011)] proposed the wave propagation at the imperfect boundary between transversely isotropic thermodiffusive elastic layer and half-space. Kumar et al. [Kumar and Singh (2007)] studied the propagation of plane waves in thermoelastic cubic crystal material with two relaxation times. Othman et al. [Othman and Atwa (2012)] investigated the Thermoelastic plane waves for an elastic solid half-space under the hydrostatic initial stress of type III. Rossikhin [Rossikhin (1976)] investigated the propagation of plane waves in an anisotropic thermoelastic half-space. Said [Said (2016)] investigated the influence of gravity on generalized magneto-thermoelastic medium for three-phase-lag model. A reflection of a plane magneto-thermoelastic wave at the boundary of a solid half-space in presence of initial stress was studied by Sarkar et al. [Sarkar and Lahiri (2012); Sharma and Walia (2007)] investigated the effect of rotation on Rayleigh waves in piezothermoelastic half space. Sherief et al. [Sherief and Saleh (2005)] investigated the half-space problem in the theory of generalized thermoelastic diffusion. Singh et al. [Singh, Yadav and Gupta (2019)] investigated the reflection of plane waves from a micropolar thermoelastic solid half-space with impedance boundary conditions. Singh [Singh (2010)] investigated the wave propagation in an initially stressed transversely isotropic thermoelastic solid half-space. Singh et al. [Singh and

Chakraborty (2015)] discussed the three-dimensional thermoelastic problem for a half-space without energy dissipation. Vishwakarma et al. [Vishwakarma and Gupta (2014)] studied Rayleigh wave propagation: A case wise study in a layer over a half space under the effect of rigid boundary. Wang et al. [Wang, Yu and Wang (2016)] proposed the analytical solutions for elastic fields caused by eigenstrains in two joined and perfectly bounded half-spaces and related problems. Xia et al. [Xia, Tian and Shen (2014)] studied the dynamic response of two-dimensional generalized thermoelastic coupling problem subjected to a moving heat source.

In spite of all these investigations, no attempt has been made yet to study the propagation of a thermoelasticwave in a half-space of a homogeneous isotropic material under the effect of the gravity field and in contact with change coordinate system moving with input by shifting origin to the position of the input. The components of displacement, normal stress, tangential stress and temperature subjected to heat source and load moving with finite velocity are obtained by Lame's potential method. Numerical computation is performed by using a numerical technique and the resulting quantities are shown graphically. The current manuscript is devoted to investigate the propagation of wave in a homogeneous isotropic, thermoelastic medium under the effect of the rotation and initial stress. The temperature, displacement components, stresses components is obtained in the physical domain using Lame's potential method. The results obtained in this investigation are more general in the sense that some earlier published results are obtained from our result as special cases. Numerical results for temperature, displacement and stress distributions have been obtained for a stainless steel like material and presented graphically.

2 Formulation of the problem

Consider a half space $y \geq 0$, initially at the temperature T_0 and in the stress free state. A variation in temperature, displacement and stress fields will occur due to actions of external loading. Assuming that the displacement will be along x -axis, the y -axis function of space coordinates x , y and time t . The initial stress τ_{ij} are given as:

$$\tau_{xx} = \tau_{yy} = \tau_{xy} = 0 \tag{1}$$

whereτ is a function of depth. The equilibrium equations of the initial stress is in the form:

$$\frac{\partial \tau}{\partial x} = 0, \ \frac{\partial \tau}{\partial y} = 0. \tag{2}$$

The generalized equation of heat conduction is given by:

$$K\nabla^2 = \rho c_e \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2}\right) + (3\lambda + 2\mu)\alpha_t T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) \underline{\nabla}.\vec{u}. \tag{3}$$

The dynamic equation of motion under the effect of initial stress and rotation is given by:

$$(\lambda + \mu)\frac{\partial \theta}{\partial x} + \mu \nabla^2 u - \frac{p}{2}\frac{\partial^2 u}{\partial y^2} + \frac{p}{2}\frac{\partial^2 v}{\partial x \partial y} - \gamma \frac{\partial T}{\partial x} = \rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u\right], \tag{4}$$

$$(\lambda + \mu)\frac{\partial \theta}{\partial y} + \mu \nabla^2 v + \frac{p}{2}\frac{\partial^2 u}{\partial x \partial y} - \frac{p}{2}\frac{\partial^2 v}{\partial x^2} + \gamma \frac{\partial T}{\partial y} = \rho\left[\frac{\partial^2 v}{\partial t^2} - \Omega^2 v\right]. \tag{5}$$

The stress, displacement relations isgiven by:

$$\tau_{xx} = (\lambda + 2\mu + P)\frac{\partial u}{\partial x} + (\lambda + P)\frac{\partial v}{\partial y} - \gamma(T - T_0),$$

$$\tau_{yy} = (\lambda + 2\mu + P)\frac{\partial v}{\partial y} + (\lambda + P)\frac{\partial u}{\partial x} - \gamma(T - T_0), \tag{6}$$

$$\tau_{xy} = \left(\mu - \frac{p}{2}\right)\frac{\partial v}{\partial x} + \left(\mu + \frac{p}{2}\right)\frac{\partial u}{\partial y}.$$

where

$\gamma = (3\lambda + 2\mu)\alpha_t$, $\theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$, $\underline{\nabla} = \frac{\partial}{\partial x}\underline{i} + \frac{\partial}{\partial y}\underline{j}$.

Also, T is the temperature, c_e is the specific heat, ρ is the density, α_t is the coefficient of thermal expansion, τ_0 is the relaxation time, K is thermal conductivity and $\vec{u}$ = (u, v, 0) is the displacement vector, T_0 is the initial temperature, $\overline{\Omega}$ =(0, 0, Ω) is rotation vector.

Introducing following non-dimensional variables as:

$$t' = \frac{c_1^2}{k_1}t,\ \ x_i' = \frac{c_1}{k_1}x_i,\ \ \overrightarrow{u'} = \frac{c_1^3}{k_1}\frac{\rho}{(3\lambda+2\mu)\alpha_t T_0}\vec{u},\ \ \Omega' = \frac{k_1}{c_1^2}\Omega,\ \ \grave{\tau}_i = \frac{c_1^2}{k_1}\tau_i, \tag{7}$$

$$\tau'_{ij} = \frac{1}{(3\lambda+2\mu)\alpha_t T_0}\tau_{ij},\ \ T' = \frac{T}{T_0}.$$

where

$$c_1^2 = (\lambda + 2\mu)/\rho,\ k_1 = K/\rho c_e,\ \ \rho = \frac{\rho'}{\gamma T_0}.$$

Introducing the non-dimensional variables (7) into (3)-(6), we get:

$$\nabla'^2 T' = \rho c_e\left(\frac{\partial T'}{\partial t'} + \tau\frac{\partial^2 T'}{\partial t'^2}\right) + \varepsilon\left(\frac{\partial}{\partial t'} + \tau\frac{\partial^2}{\partial t'^2}\right)\underline{\nabla}.\vec{u}, \tag{8}$$

$$(\lambda + \mu)\frac{\partial \theta'}{\partial x'} + \mu\left(\frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - (\lambda + 2\mu)\frac{\partial T'}{\partial x'} - \frac{p}{2}\frac{\partial^2 \grave{u}}{\partial \grave{y}^2} + \frac{p}{2}\frac{\partial^2 \grave{v}}{\partial \grave{x}\partial \grave{y}},$$

$$= (\lambda + 2\mu)\left[\frac{\partial^2 u'}{\partial t'^2} - \Omega^2 \grave{u}\right] \tag{9}$$

$$(\lambda + \mu)\frac{\partial \theta'}{\partial y'} + \mu\left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v}{\partial y^2}\right) - (\lambda + 2\mu)\frac{\partial T'}{\partial y'} - \frac{p}{2}\frac{\partial^2 \grave{v}}{\partial \grave{x}^2} + \frac{p}{2}\frac{\partial^2 \grave{u}}{\partial \grave{x}\partial \grave{y}}$$

$$= (\lambda + 2\mu)\left[\frac{\partial^2 v'}{\partial t'^2} - \Omega^2 \grave{v}\right]. \tag{10}$$

The stress, displacement relations isgiven by:

$$\grave{\tau}_{x'x'} = \frac{\partial \grave{u}}{\partial x'} + (1 - 2c^2)\frac{\partial \grave{v}}{\partial y'} - T',$$

$$\tau'_{y'y'} = \frac{\partial v'}{\partial y'} + (1 - 2c^2)\frac{\partial u'}{\partial x'} - T',$$

$$\tau'_{x'y'} = \left(c^2 - \frac{p^*}{2}\right)\frac{\partial \grave{u}}{\partial \grave{x}} + \left(c^2 + \frac{p^*}{2}\right)\frac{\partial \grave{u}}{\partial \grave{y}} \tag{11}$$

where

$\varepsilon = \frac{(3\lambda + 2\mu)\alpha_t{}^2 T_0}{\rho c_e(\lambda+2\mu)}, \tau = \tau_0 \frac{c_1{}^2}{k_1},\ p^* = \frac{p}{\lambda+2\mu+P},\ c^2 = \frac{\mu}{\lambda+2\mu+P}.$

3 Solution of the problem

From Helmholt's theorem [Morse and Feshbach (1953)], we have

$$\vec{u} = grad\ \phi + curl\ \vec{\psi}, \vec{\psi} = (0, 0, \psi). \quad (12)$$

We may write the displacement u and v in terms of potentials as follows

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \quad (13)$$

By substituting from (13) into (8)-(10), we get

$$\left(\nabla^2 - \frac{\partial}{\partial t} - \tau \frac{\partial^2}{\partial t^2}\right) T = \varepsilon \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}\right) \nabla^2 \phi, \quad (14a)$$

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - \Omega^2\right) \phi = T, \quad (14b)$$

$$\left(\nabla^2 - c^2 \frac{\partial^2}{\partial t^2} + c^2\Omega^2\right) \psi = 0. \quad (14c)$$

We change to coordinate system moving with input by shifting origin to the position of the input

$$X'' = -mt',\ y'' = y',\ \nabla_1^2 = \frac{\partial^2}{\partial x''^2} + \frac{\partial^2}{\partial y''^2}. \quad (15)$$

where $m = v/c_1$ dimensionless loading speed and coordinates x'' and y'' move in the positive direction of x -axis with speed m'.

By substituting from (15) into (14a)-(14c), we get

$$\left(\nabla_1^2 + m \frac{\partial}{\partial x''} - \tau m^2 \frac{\partial^2}{\partial x''^2}\right) T = \varepsilon \left(-m \frac{\partial}{\partial x''} + \tau m^2 \frac{\partial^2}{\partial x''^2}\right) \nabla_1^2 \phi, \quad (16a)$$

$$\left(\nabla_1^2 - m^2 \frac{\partial^2}{\partial x''^2} - \Omega^2\right) \phi = T, \quad (16b)$$

$$\left(\nabla_1^2 - c^2 m^2 \frac{\partial^2}{\partial x''^2} + c^2\Omega^2\right) \psi = 0. \quad (16c)$$

For easy calculations, we will omit the double primes, also writing ∇ to ∇_1 , we get

$$\left(\nabla^2 + m \frac{\partial}{\partial x} - \tau m^2 \frac{\partial^2}{\partial x^2}\right) T = \varepsilon \left(-m \frac{\partial}{\partial x} + \tau m^2 \frac{\partial^2}{\partial x^2}\right) \nabla^2 \phi, \quad (17a)$$

$$\left(\nabla^2 - m^2 \frac{\partial^2}{\partial x^2} - \Omega^2\right) \phi = T, \quad (17b)$$

$$\left(\nabla^2 - c^2 m^2 \frac{\partial^2}{\partial x^2} + c^2\Omega^2\right) \psi = 0. \quad (17c)$$

Substituting from (17b) into (17a), we have

$$[\left(\nabla^2 + m \frac{\partial}{\partial x} - \tau m^2 \frac{\partial^2}{\partial x^2}\right) (\nabla^2 - m^2 \frac{\partial^2}{\partial x^2} -$$

$$+\,\varepsilon\left(m\frac{\partial}{\partial x}-\tau m^2\frac{\partial^2}{\partial x^2}\right)\nabla^2]\phi\ =0. \tag{18}$$

Now we assume the solutions of (17c) and (18) in the form

$$\psi=Be^{ikx-\sigma' y}\ ,\ \phi=Ae^{ikx-\sigma y} \tag{19}$$

where, A and B are constants , k is the wave number and σ and σ' is unknown quantities to be determined.

Using (19) in (17c) and (18), we have

$$\sigma'^2+k^2(c^2m^2-1)+c^2\Omega^2=0 \tag{20}$$

and

$$(\sigma^2+k^2(\tau m^2-1)+imk)(\sigma^2+k^2(m^2-1)-\Omega^2)$$
$$+\varepsilon(imk+\tau k^2m^2)(\sigma^2-k^2)=0. \tag{21}$$

From (19), we have

$$\sigma' = \pm\sqrt{k^2(1-c^2m^2)-c^2\Omega^2}. \tag{22}$$

For σ' to be real ,$k^2(1-c^2m^2)>c^2(\Omega^2+2\Omega imk)$,$\psi$ to be bound σ' >0

Also from (21), we have

$$\sigma_{1,2}=\pm\frac{1}{2}\sqrt{-2\alpha\pm2\sqrt{\alpha^2-4\beta}} \tag{23}$$

Also, ϕ to be bound σ >0 .

$$\alpha=(k^2(m^2-1)-\Omega^2+k^2(\tau m^2-1)+imk(1+\varepsilon)+\tau k^2m^2\varepsilon)$$
$$\beta=k^4(\tau m^2-1)(m^2-1)-\Omega^2k^2(\tau m^2-1)+imk^3(m^2-1)-imk(\Omega^2+k^2\varepsilon)-\tau k^4m^2\varepsilon$$

We can write the solution (19) in the form

$$\psi=Be^{ikx-\sigma' y},\ \ \phi=A_1e^{ikx-\sigma_1 y}+A_2e^{ikx-\sigma_2 y}. \tag{24}$$

The stress components in terms of ϕ and ψ are

$$\tau_{xx}=\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\psi}{\partial x\partial y}+(1-2c^2)\left(\frac{\partial^2\phi}{\partial y^2}-\frac{\partial^2\psi}{\partial x\partial y}\right)-T, \tag{25}$$

$$\tau_{yy}=\frac{\partial^2\phi}{\partial y^2}-\frac{\partial^2\psi}{\partial x\partial y}+(1-2c^2)\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\psi}{\partial x\partial y}\right)-T, \tag{26}$$

$$\tau_{xy}=\left(c^2-\frac{P^*}{2}\right)\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\psi}{\partial x\partial y}\right)+\left(c^2+\frac{P^*}{2}\right)\left(\frac{\partial^2\phi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial y^2}\right). \tag{27}$$

Substituting from (24) into (13), (17b) and (25)-(27), we have

$$T=\ell_1A_1e^{ikx-\sigma_{1y}}+\ell_2A_2e^{ikx-\sigma_2 y}, \tag{28}$$

$$u=\{ik[A_1e^{-\sigma_{1y}}+A_2e^{-\sigma_2 y}]-\sigma' Be^{-\sigma' y}\}e^{ikx}, \tag{29}$$

$$v=-\{\sigma_1A_1e^{-\sigma_{1y}}+\sigma_2A_2e^{-\sigma_2 y}-ikBe^{-\sigma' y}\}e^{ikx}, \tag{30}$$

$$\tau_{xx}=F_1A_1e^{ikx-\sigma_1 y}+F_2A_2e^{ikx-\sigma_2 y}+F_3Be^{ikx-\sigma' y}, \tag{31}$$

$$\tau_{yy}=V_1A_1e^{ikx-\sigma_1 y}+V_2A_2e^{ikx-\sigma_2 y}+V_3Be^{ikx-\sigma' y}, \tag{32}$$

$$\tau_{xy}=M_1A_1e^{ikx-\sigma_1 y}+M_2A_2e^{ikx-\sigma_2 y}+M_3Be^{ikx-\sigma' y}. \tag{33}$$

where

$\ell_1 = \sigma_1^2 + k^2(m^2 - 1) - \Omega^2,$

$\ell_2 = \sigma_2^2 + k^2(m^2 - 1) - \Omega^2,$

$F_1 = -k^2 + (1 - 2c^2)\sigma_1^2 - \ell_1$, $F_2 = -k^2 + (1 - 2c^2)\sigma_2^2 - \ell_2$,

$F_3 = -ik\sigma' + ik\sigma'(1 - 2c^2)$, $V_1 = \sigma_1^2 - k^2(1 - 2c^2) - \ell_1$, $V_2 = \sigma_2^2 - k^2(1 - 2c^2) - \ell_2,$

$V_3 = ik\sigma' - ik\sigma'(1 - 2c^2)$, $M_1 = -\left(k^2\left(c^2 - \frac{P^*}{2}\right) + ik\sigma_1\left(c^2 + \frac{P^*}{2}\right)\right)$,

$M_2 = -\left(k^2\left(c^2 - \frac{P^*}{2}\right) + ik\sigma_2\left(c^2 + \frac{P^*}{2}\right)\right), M_3 = -\left(ik\sigma'\left(c^2 - \frac{P^*}{2}\right) - \sigma'^2\left(c^2 + \frac{P^*}{2}\right)\right).$

4 Boundary conditions

The boundary conditions at the interface $y = 0$ subjected to an arbitrary normal force P_1 are:

$$\tau_{xx}(x,0) = P_1 e^{ikx},$$
$$\tau_{xy}(x,0) = 0, \tag{34}$$
$$\frac{\partial T(x,o)}{\partial x} + hT(x,0) = 0.$$

where, P_1 is the magnitude of mechanical force, k is the wave number.

Now, using the boundary conditions to determine the constants A_1 ,A_2 and B, we have

$A_1 = \frac{-P_1 M_3 \ell_2 (h+ik)}{G}, A_2 = \frac{P_1 M_3 \ell_1 (h+ik)}{G}, \quad B = \frac{P_1 (h+ik)(M_1\ell_2 - M_2\ell_1)}{G}$,

$G = F_1 M_3 \ell_2 (h + ik) + F_2 M_3 \ell_1 (h + ik) + F_3 (h + ik)(M_1 \ell_2 - M_2 \ell_1).$

5 Numerical results and discussion

The material chosen in this work of stainless steel, the physical data are given [Datta (1986)]

$\rho = 7.97\ 10^3.,$

$\alpha_t = 13.2x10^{-6} \deg^{-1}, c_e = 0.560x10^3.,\ T_0 = 293.1K,$

$\lambda = 9.3x10^{10}\,Nm^{-1}, \mu = 8.4\ 10^{10}\,Nm^{-1}, P = 2., P_1 = 1., m = 4.25,$

$k_1 = 50., c_1 = 0.2.,\ h = 0.002.$

Considering the above physical data, non-dimensional field variables have been evaluated and results are presented in the form of graphs at different positions of y at x =1.0. the graphical results for the displacement components , the stress components, and temperature are shown in Figs. 1 to 7, respectively:

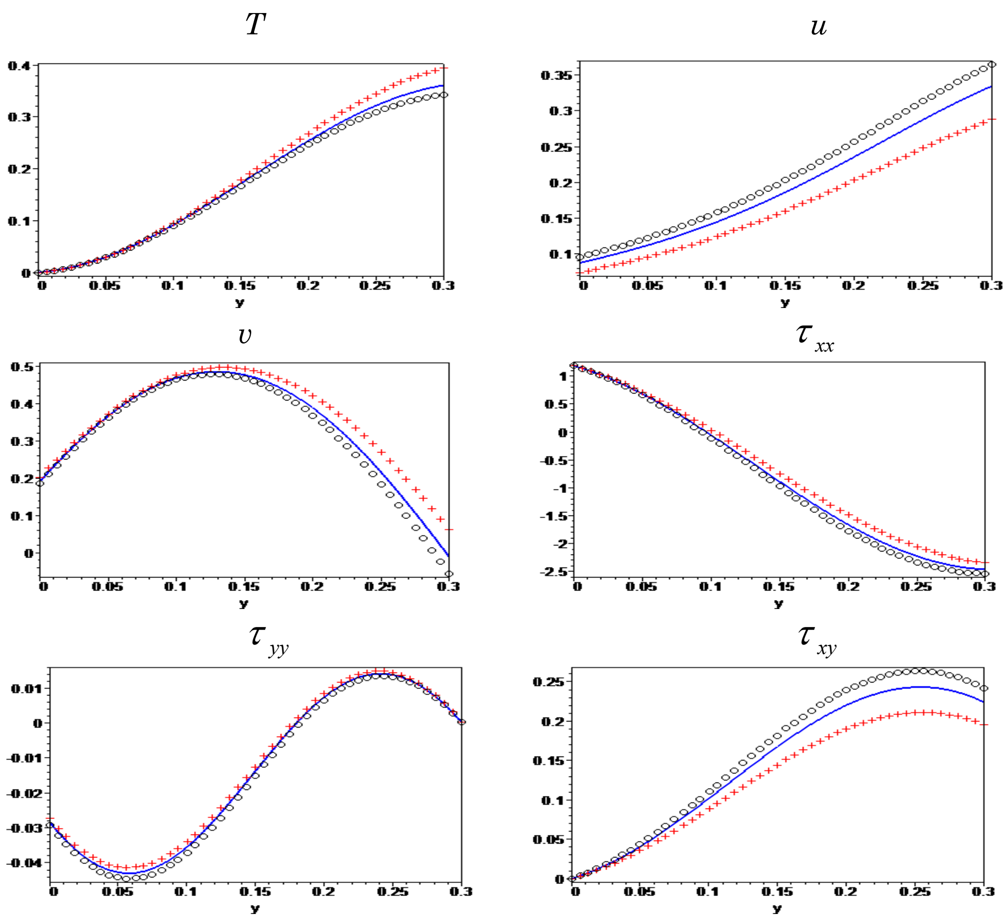

Figure 1: Variation of rotation Ω on *T, u, v,* τ_{xx}, τ_{yy} and τ_{xy} with respect to y

Ω=1 oooooooo, *Ω=2* _______ , *Ω=3* +++++++

Fig. 1 shows the variations of the value of temperature T, the value of displacement components u, v, value of normal stresses τ_{xx}, τ_{yy} and value of shear stress τ_{xy} with respect to axial y for different values of rotation Ω and axial y for the moving load source. The values of temperature, the value of displacement components u, v and value of normal stresses τ_{xx}, τ_{yy} increase with increasing of rotation, while value of shear stress and axial displacement v decrease with increasing of rotation, as well it decreases with increasing of axial y in the whole range of the axial y, except the value of normal stress τ_{yy} has oscillatory behavior due to moving load source concerned we see that oscillate is more in left side and less in right side., it is noticed that the temperature, shear stress and normal stress satisfied the boundary conditions.

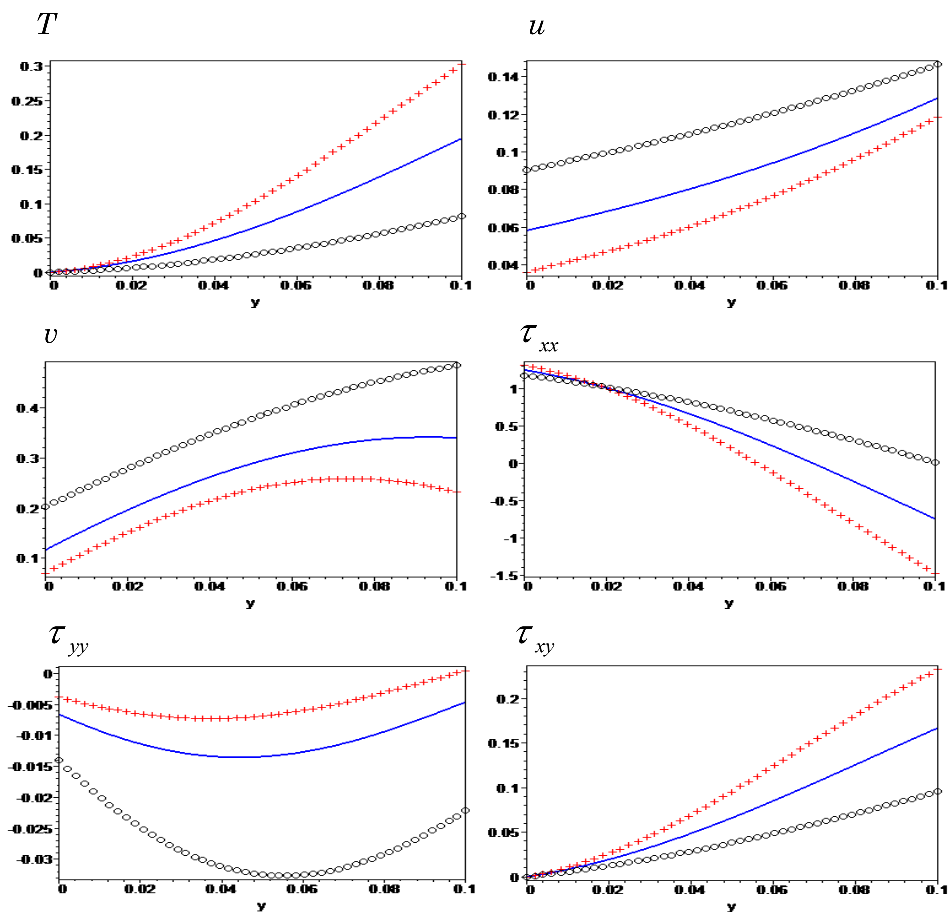

Figure 2: Variation of wave number k on *T, u, v,* τ_{xx} , τ_{yy} and τ_{xy} with respect to y

k=*2 oooooooo, k=3* ________ *, k=4* ++++++

Fig. 2 shows the variations of the value of temperature T, the value of displacement components u, v, value of normal stresses τ_{xx}, τ_{yy} and value of shear stress τ_{xy} with respect to axial y for different values of wave number k and axial y for the moving load source. The values of temperature, the value of normal stress τ_{yy} and value of shear stress τ_{xy} increase with increasing of wave number, while value of normal stress τ_{xx} and displacement components u, v decrease with increasing of wave number, as well it decreases with increasing of axial y in the whole range of the axial y, except the value of normal stress τ_{yy} has oscillatory behavior due to moving load source concerned we see that oscillate is more in left side and less in right side., it is noticed that the temperature, shear stress and normal stress satisfied the boundary conditions.

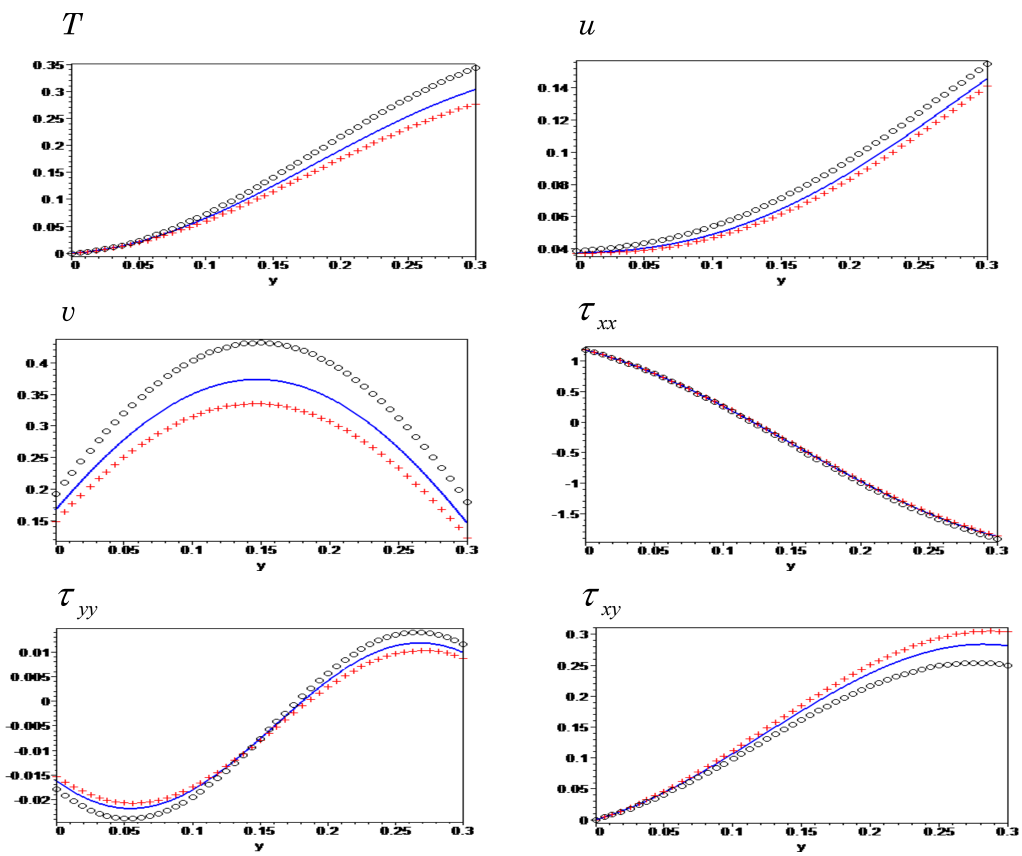

Figure 3: Variationof initial stress P on T, u, v, τ_{xx}, τ_{yy} and τ_{xy} with respect to y

$P = 3 * 10^{10} oooooooo,\ P = 6 * 10^{10}$________ , $P = 9 * 10^{10}$+++++++

Fig. 3 shows the variations of the value of temperature T, the value of displacement components u, v, value of normal stresses τ_{xx}, τ_{yy} and value of shear stress τ_{xy} with respect to axial y for different values of initial stress P and axial y for the moving load source. The values of temperature and value of displacement components decrease with increasing of initial stress, while value of shear stress τ_{xy} increases with increasing of initial stress, as well as, the normal stress τ_{yy} ncreases and decreases with increasing of initial stress, while the initial stress there is no effect on the normal stresss τ_{xx}, as well it decreases with increasing of axial y in the whole range of the axial y, except the value of normal stress τ_{yy} has oscillatory behavior due to moving load source concerned we see that oscillate is more in left side and less in right side., it is noticed that the temperature, shear stress and normal stress satisfied the boundary conditions.

6 Special cases

6.1 If the rotation is neglected

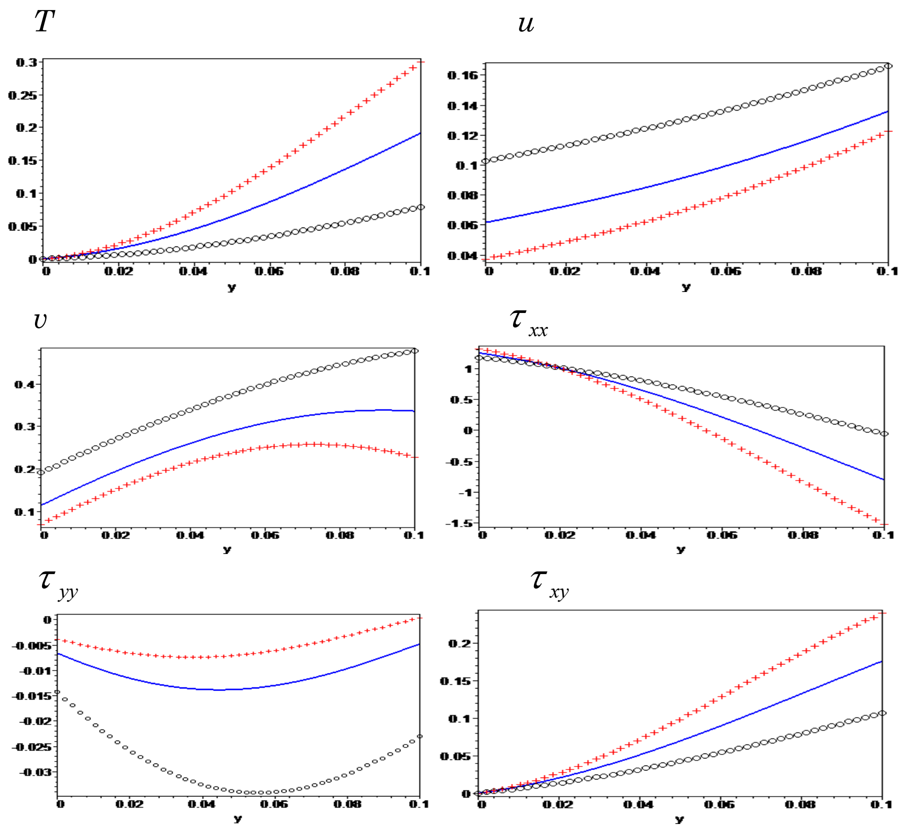

Figure 4: Variationof wave number k on *T, u, v,* τ_{xx} , τ_{yy} and τ_{xy} with respect to y
k=*2 oooooooo, k=3* ________ *, k=4* +++++++

Fig. 4 shows the variations of the value of temperature T, the value of displacement components u, v, value of normal stresses τ_{xx}, τ_{yy} and value of shear stress τ_{xy} with respect to axial y for different values of wave number k and axial y for the moving load source in the neglecting of the rotation. The values of temperature and value of stresses components τ_{yy}, τ_{xy} increase with increasing of wave number, while the value of displacement components u, v and normal stress τ_{xx} decrease with increasing of wave number, as well it increases and decreases with increasing of axial y in the whole range of the axial y . It is noticed that the temperature, shear stress and normal stress satisfied the boundary conditions.

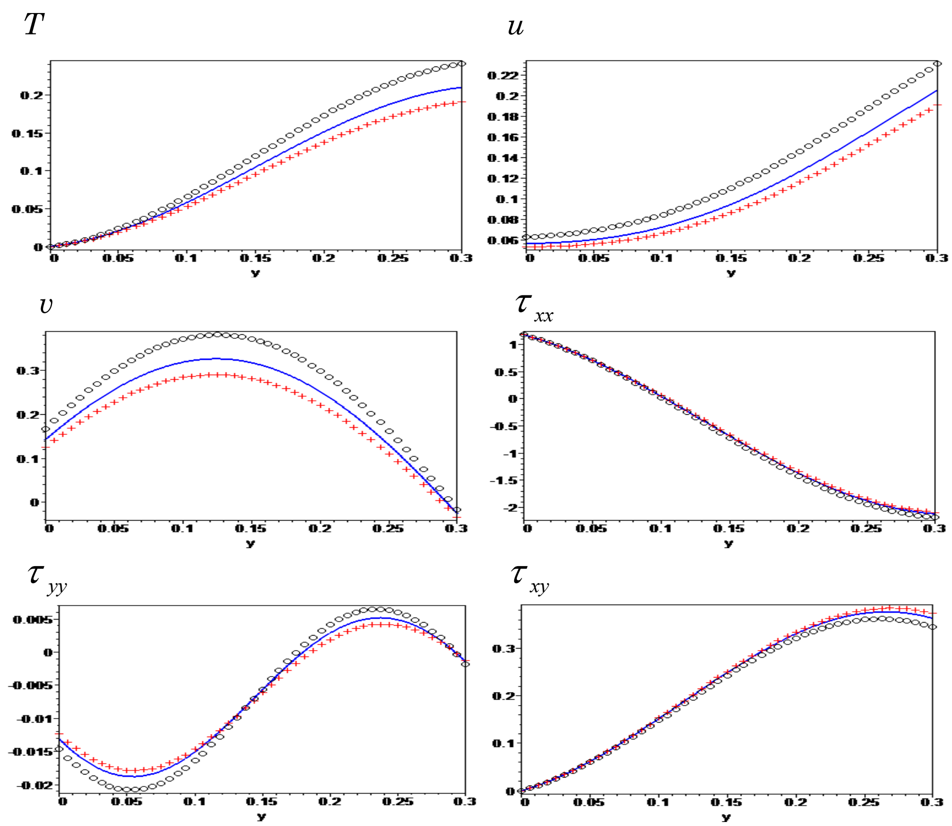

Figure 5: Variationof initial stress P on T, u, v, τ_{xx}, τ_{yy} and τ_{xy} with respect to y
$P = 3 * 10^{10}$ *oooooooo*, $P = 6 * 10^{10}$_______ , $P = 9 * 10^{10}$+++++++

Fig. 5 shows the variations of the value of temperature T, the value of displacement components u, v, value of normal stresses τ_{xx}, τ_{yy} and value of shear stress τ_{xy} with respect to axial y for different values of initial stress P and axial y for the moving load source in the neglecting of the rotation. The values of temperature and value of displacement components u, v decrease with increasing of initial stress, while value of shear stress τ_{xy} increases with increasing of initial stress, as well as, the normal stress τ_{yy} ncreases and decreases with increasing of initial stress, while the initial stress there is no effect on the normal stresss τ_{xx}, as well it decreases with increasing of axial y in the whole range of the axial y, except the value of normal stress τ_{yy} has oscillatory behavior due to moving load source concerned we see that oscillate is more in left side and less in right side., it is noticed that the temperature, shear stress and normal stress satisfied the boundary conditions.

6.2 If the initial stress is neglected

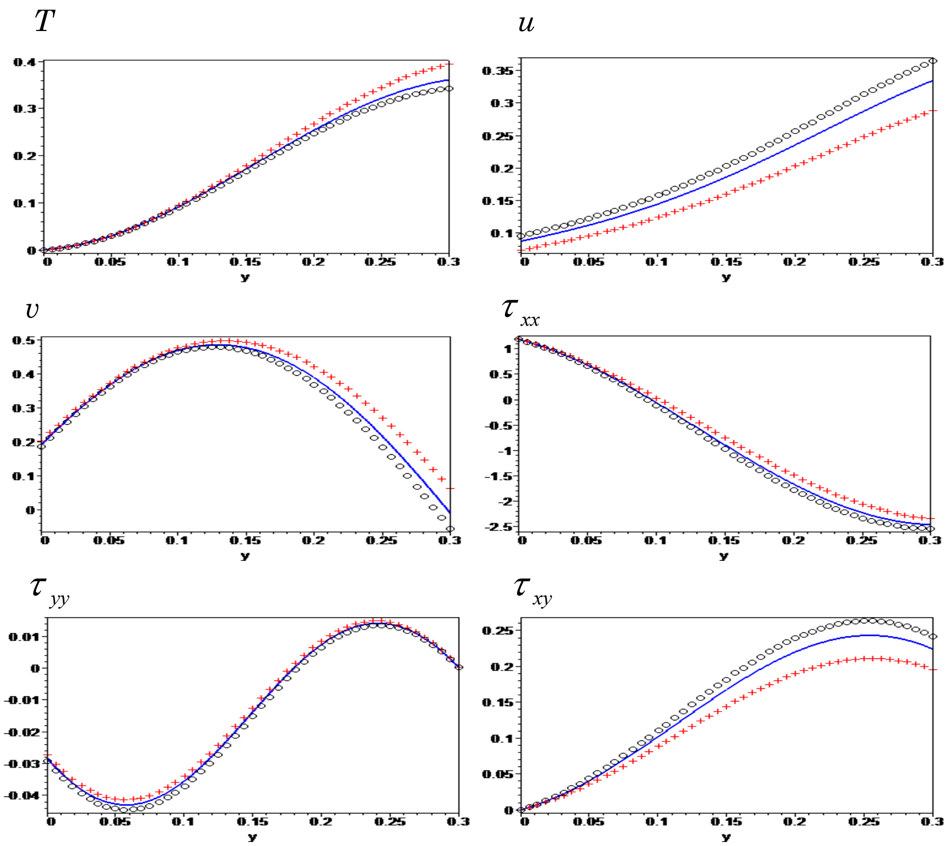

Figure 6: Variationof rotation Ω on *T, u, v,* τ_{xx}, τ_{yy} and τ_{xy} with respect to y
Ω=*1 oooooooo,* Ω=*2* ________ *,* Ω=*3* +++++++

Fig. 6 shows the variations of the value of temperature T, the value of displacement components u, v, value of normal stresses τ_{xx}, τ_{yy} and value of shear stress τ_{xy} with respect to axial y for different values of rotation Ω and axial y for the moving load source in the neglecting of the initial stress. The values of temperature,axial displacement component v and normal stresses components τ_{xx}, τ_{yy} increase with increasing of rotation, while values of radial displacement u and shear stress τ_{xy} decrease with increasing of rotation, as well as, the normal stress τ_{yy} increases and decreases with increasing of rotation, while it increases and decreases with increasing of axial y in the whole range of the axial y, except the value of normal stress τ_{yy} has oscillatory behavior due to moving load source concerned we see that oscillate is more in left side and less in right side.,it is noticed that the temperature, shear stress and normal stress satisfied the boundary conditions.

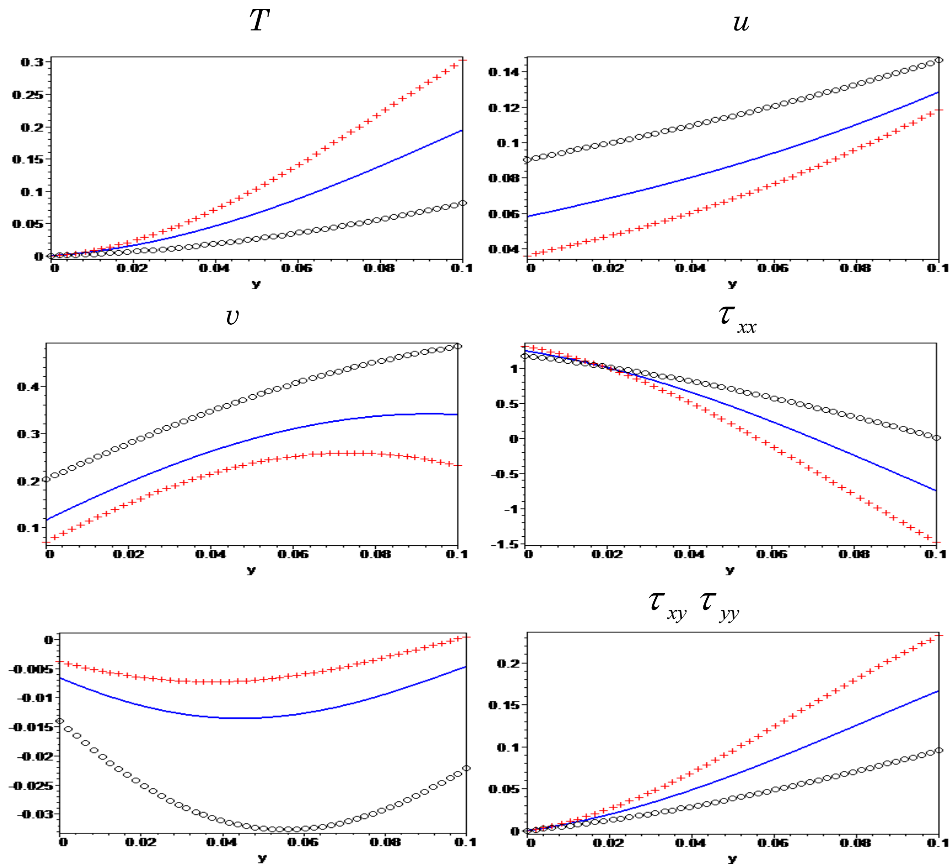

Figure 7: Variation of wave number k on *T, u, v,* τ_{xx} , τ_{yy} and τ_{xy} with respect to y k=*2 oooooooo, k=3* ________ *, k=4* +++++++

Fig. 7 shows the variations of the value of temperature T, the value of displacement components u, v, value of normal stresses τ_{xx}, τ_{yy} and value of shear stress τ_{xy} with respect to axial y for different values of wave number k and axial y for the moving load source in the neglecting of the initial stress. The values of temperature, normal stress τ_{yy} and shearstresse components τ_{xy} increase with increasing of wave number, while values of radial displacement u and shear stress τ_{xy} decrease with increasing of wave number, while it increases and decreases with increasing of axial y in the whole range of the axial y. It is noticed that the temperature, shear stress and normal stress satisfied the boundary conditions.

7 Conclusion

Behavior of displacement, stresses and temperature in a homogeneous, isotropic, generalized thermoelastic medium with initial stress and rotation. The theoretical and numerical results reveal that all the considered parameters, namely, rotation, initial stress and wave number have significant effects on the field variables. According to the above analysis, we can conclude the following points:

1. The method which used in the present article is applicable to a wide range of problems in thermodynamics and thermoelasticity.
2. The presence of rotation, initial stress and wave number play a significant role in all the physical quantities. The temperature, displacement components and stress components of all the physical quantities decrease or increase while the rotation, initial stress and wave number increases.
3. Deformation of a body depends on the nature of the moving load source applied as well as the type of boundary conditions.
4. The results presented in this paper will be very helpful for researchers concerning with material science, designers of new materials, low-temperature physicists, as well as for those working on the development thermoelasticity and in practical situations as in geophysics. The methods used in the present article are applicable to a wide range of problems in thermodynamics and thermoelasticity.
5. Study of the phenomenon of rotation is also used to improve the conditions of oil extractions.

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