Sound & Vibration

Tech Science Press

Synthesized AI LMI-based Criterion for Mechanical Systems

Jcy Chen[1,*], Wc Chen[1], Tim Chen[1], Alex Wilson[2], N. Fadilah Jamaludin[3], Nertrand Kapron[1], Tim Chen[4,5] and John Burno[5]

[1]Faculty of Information Technology, Ho Chi Minh City University, Ho Chi Minh City, Vietnam
[2]Department of Data Science, RMIT University, Melbourne, VIC, Australia
[3]School of Physical and Mathematical Technology, Nanyang Technological University, 21 Nanyang Link, Singapore, Singapore
[4]Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[5]Heidelberg Academy of Sciences, Heidelberg, Baden-Württemberg, Germany
*Corresponding Author: Jcy Chen. Email: jcychen@hcmu.edu.vn, jc343965@gmail.com.

Abstract: This paper proposes a novel artificial intelligence sythethized controller in the mechanical system which has high speed computation because of the LMI type criterion. The proposed membership functions are adopted and stabilization criterion of the closed-loop T-S fuzzy systems are obtained through a new parametrized LMI (linear matrix) inequality which is rearranged by machine learning membership functions.

Keywords: Fuzzy energy equation; artificial intelligence; large-scale systems; fuzzy model

1 Introduction

For an ever-changing society and scientific technology, there are many models for large-scale systems with complex structures that have been proposed during the past 30 years. However, the differential equations used to describe the large-scale systems generally have very high dimensions, making it seemingly more and more difficult to solve practical stability problems. The general guiding principle for dealing with the stability problems that rise from large-scale systems is to first appropriately decompose the overall composite large-scale system into several isolated subsystems and coupling systems (or so-called interconnected systems), and then use some algebraic relations, the integral relations between the suitable Lyapunov functions or Cauchy matrices and coupling items of the isolated low-dimensional subsystems to determine the stability conditions for the overall composite large-scale system. The mathematical models of many physical and engineering systems are frequently of high dimension, or possessing interacting dynamic phenomena. The information processing and requirements to experiment with these models for control purposes are usually excessive. It is therefore natural to seek techniques that can reduce the computational effort. The methodologies of large-scale systems provide such techniques through the manipulation of system structure in some way. Thus, there has been considerable interest in the research area of modeling, analysis, optimization and control of large-scale systems [1]. Recently, many approaches have been used to investigate the stability and stabilization of large-scale systems, as proposed in the literature [2, 3].

During the past several years, fuzzy-rule-based modeling has become an active research field because of its unique merits in solving complex nonlinear system identification and control problems. This approach can obtain more flexibility and more effective capability of handling and processing uncertainties in complicated and ill-defined systems. Unlike conventional modeling, fuzzy rule-based modeling is essentially a multimodel approach in which individual rules are combined to describe the global behavior of the system [4].

2 System Description and Stability Analysis

Consider a fuzzy large-scale system F which consists of J interconnected fuzzy subsystems. The jth fuzzy subsystem is of the following form:

$$x_j(k+1) = \sum_{i=1}^{r_j} h_{ij}(k)A_{ij}x_j(k) + \sum_{\substack{n=1\\n\neq j}}^{J} C_{nj}x_n(k) \tag{1}$$

where A_{ij} is a constant matrix with appropriate dimensions, $x_j(k)$ is the state vector, C_{nj} is the interconnection between the nth and jth subsystems, r_j is the number of fuzzy implications and $h_{ij}(k)$ is the normalized weight. Each isolated subsystem (i.e., $C_{nj} = 0$) of F is represented by a Takagi-Sugeno fuzzy model composed of a set of fuzzy implications and the final output of this fuzzy model is described as [5]

$$x_j(k+1) = \sum_{i=1}^{r_j} h_{ij}(k)A_{ij}x_j(k). \tag{2}$$

Lemma 1 [5]: The jth isolated subsystem (2) is asymptotically stable if there exists a common positive definite matrix such that

$$Q_{ij} \equiv {A_{ij}}^T P_j A_{ij} - P_j < 0 \tag{3}$$

Based on Lemma 1, a stability criterion is derived below to guarantee the asymptotic stability of the fuzzy large-scale system F.

Theorem 1: The fuzzy large-scale system F is asymptotically stable, if each isolated subsystem of F is asymptotically stable and the following inequality is fulfilled [6-10]:

$$\begin{aligned}\sum_{i=1}^{r_j} h_{ij}^2(k)\lambda_M(Q_{ij}) + \sum_{i<f}^{r_j} h_{ij}(k)h_{fj}(k)[\lambda_M(Q_{ij}) + \lambda_M(Q_{fj})]\\ + \sum_{\substack{n=1\\n\neq j}}^{J}\left[\sum_{i=1}^{r_j} h_{ij}(k)m_{ijn} + \sum_{i=1}^{r_j} h_{in}(k)m_{inj} + (J-1)\lambda_M(P_n)\left\|C_{jn}\right\|^2\right] < 0\end{aligned} \tag{4}$$

Proof: Let the Lyapunov function for the fuzzy large-scale system F be defined as

$$V(k) = \sum_{j=1}^{J} V_j(k) = \sum_{j=1}^{J} x_j(k)P_jx_j(k) \tag{5}$$

Taking the backward difference of V(k), we have

$$\begin{aligned}\Delta V(k) &= V(k+1) - V(k)\\ &= \sum_{j=1}^{J}\left\{\left[\sum_{i=1}^{r_j} h_{ij}(k)A_{ij}x_j(k) + \phi_j(k)\right]^T P_j\left[\sum_{f=1}^{r_j} h_{fj}(k)A_{fj}x_j(k) + \phi_j(k)\right] - x_j^T(k)P_jx_j(k)\right\}\\ &= m_1 + m_2 + m_3 + m_4 + m_5,\end{aligned} \tag{6}$$

where

$$
\begin{aligned}
m_1 + m_2 + m_3 \equiv & \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)x_j^T(k)A_{ij}^T P_j C_{nj} x_n(k) + \sum_{j=1}^{J}\sum_{i\neq f}^{r_j} h_{ij}(k)h_{fj}(k)x_j^T(k)(A_{ij}^T P_j A_{fj} - P_j)x_j(k) \\
& + \sum_{j=1}^{J}\sum_{i=f=1}^{r_j} h_{ij}^2(k)x_j^T(k)(A_{ij}^T P_j A_{ij} - P_j)x_j(k) \le \sum_{j=1}^{J}\sum_{i=1}^{r_j} h_{ij}^2(k)\lambda_M(Q_{ij})x_j^T(k)x_j(k) \\
& + \sum_{j=1}^{J}\sum_{i<f}^{r_j} h_{ij}(k)h_{fj}(k)[\lambda_M(Q_{ij}) + \lambda_M(Q_{fj})]x_j^T(k)x_j(k) \\
& + \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)\|x_j(k)\|\|A_{ij}^T P_j C_{nj}\|\|x_n(k)\|
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
m_4 + m_5 \equiv & \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)x_n^T(k)C_{nj}^T P_j A_{ij} x_j(k) + \sum_{j=1}^{J}\phi_j^T(k)P_j\phi_j(k) \\
& \le \sum_{j=1}^{J}\sum_{n\neq j}^{J}(J-1)\lambda_M(P_n)\|C_{jn}\|^2\|x_j(k)\|^2 + \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)\|x_n(k)\|\left\|C_{nj}^T P_j A_{ij}\right\|\|x_j(k)\|,
\end{aligned} \tag{8}
$$

Then we have

$$
\begin{aligned}
m_3 + m_4 \le & \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)\|x_j(k)\|\left[\left\|A_{ij}^T P_j C_{nj}\right\| + \left\|C_{nj}^T P_j A_{ij}\right\|\right]\|x_n(k)\| \\
& \le \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)m_{ijn}\left[\|x_j(k)\|^2 + \|x_n(k)\|^2\right] \\
& = \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{ij}(k)m_{ijn}\|x_j(k)\|^2 + \sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{n\neq j}^{J} h_{in}(k)m_{inj}\|x_j(k)\|^2.
\end{aligned} \tag{9}
$$

That means

$$
\begin{aligned}
\Delta V(k) \le & \sum_{j=1}^{J} x_j^T(k)\Bigg\{\sum_{i=1}^{r_j} h_{ij}^2(k)\lambda_M(Q_{ij}) + \sum_{i<f}^{r_j} h_{ij}(k)h_{fj}(k)[\lambda_M(Q_{ij}) + \lambda_M(Q_{fj})] \\
& + \sum_{n\neq j}^{J}\left[\sum_{i=1}^{r_j} h_{ij}(k)m_{ijn} + \sum_{i=1}^{r_j} h_{in}(k)m_{inj} + (J-1)\lambda_M(P_n)\|C_{jn}\|^2\right]\Bigg\}x_j(k).
\end{aligned}
$$

Evolved Bat Algorithm (EBA) is proposed based on the bat echolocation fuzzy complex system in the natural world. Unlike other swarm intelligence algorithms, the strong point of EBA is that it only has one parameter, which is called the medium, needs to be determined before employing the algorithms to solve problems. Choosing different medium determines different searching step size in the evolutionary process. In this study, we choose the air to be the medium because it is the original existence medium in the natural environment where bats live. The operation of EBA can be summarized as follows:

The artificial agents are spread into the solution space by randomly assigning coordinates to them. The artificial agents are moved. A random number is generated and then it is checked whether it is larger than the fixed pulse emission rate. If the result is positive, the artificial agent is moved using the random walk process. $x_j^T = x_j^{T-1}$+D and $x_j^T x_j$ indicates the coordinate of the i-th artificial agent at the t-th iteration, x_j^{T-1} represents the coordinate of the i-th artificial agent at the last iteration, and D is the moving distance that the artificial agent goes in this iteration. D $=\gamma\cdot\Delta$T and γ is a constant corresponding to the medium chosen in the experiment, and $\Delta \mathrm{T} \in (-1, 1)$ is a random number. $\gamma = 0.17$ is used in our experiment because the chosen medium is air. β is a random number; x_{best} indicates the coordinate of the near best solution found so far throughout all artificial agents; and x_j^{T-1} represents the new coordinates of the artificial agent after the operation of the random walk process.

3 Mechanical System as an Example

First, let us look at the case where a four-dimensional nonlinear system is decomposed into an isolated system consisting of two two-dimensional subsystems that are coupled to each other:

$$\frac{d}{dt}\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{42} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} + \begin{pmatrix} f1 \\ f2 \\ f3 \\ f4 \end{pmatrix}, \tag{10}$$

wherein, $[t_k,\ t_{k+1}]$ for

$$k = 0, 1,\ 2,\ \ldots,\ N-1. \tag{11}$$

Therefore, we have

$$\mathbf{A}_{15} = \begin{bmatrix} 0.1414 & -0.0252 \\ 1 & 0 \end{bmatrix},\ \mathbf{A}_{16} = \begin{bmatrix} 0.0816 & -1.2695 \\ 1 & 0 \end{bmatrix},$$

$$\mathbf{B}_1 = \mathbf{B}_2 = \cdots = \mathbf{B}_9 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \mathbf{B}_{10} = \begin{bmatrix} -0.1172 \\ 0 \end{bmatrix},\ \mathbf{B}_{11} = \begin{bmatrix} 0.0266 \\ 0 \end{bmatrix},\ \mathbf{B}_{12} = \begin{bmatrix} -0.0120 \\ 0 \end{bmatrix},$$

$$\mathbf{B}_{13} = \begin{bmatrix} -0.0906 \\ 0 \end{bmatrix},\ \mathbf{B}_{14} = \begin{bmatrix} -0.1292 \\ 0 \end{bmatrix},\ \mathbf{B}_{15} = \begin{bmatrix} 0.0147 \\ 0 \end{bmatrix},\ \mathbf{B}_{16} = \begin{bmatrix} -0.1025 \\ 0 \end{bmatrix}. \tag{12}$$

We say that Eq. (12) is an isolated subsystem belonging to Eq. (10). Rewrite Eq. (10) as

$$u(k) = \frac{\sum_{i=1}^{2} M_i(k)\mathbf{K}_i\mathbf{x}(k)}{\sum_{i=1}^{2} M_i(k)} = M_1(k)\boldsymbol{K}_1\boldsymbol{x}(k) + M_2(k)\boldsymbol{K}_2\boldsymbol{x}(k) \tag{13}$$

Therefore, Eq. (13) can be regarded as an interconnected system including the highly coupled Eq. (12).

Generally, it is considered proper to use ordinary differential equations to describe the large-scale systems

$$\dot{x}_r(t) = \sum_{j=1}^{n} \alpha_j(t) F(x_r,\ \ t,\ \ \beta_j), \tag{14}$$

Suppose that Eq. (14) can be decomposed into an interconnected system consisting r subsystems

$$0 \le \alpha_j(t) \le 1, \sum_{j=1}^{n} \alpha_j(t) = 1. \tag{15}$$

Then, an isolated subsystem corresponding to Eq. (15) is

$$\begin{array}{ll} \alpha_1(t_0) = \alpha_1(t_1) = \cdots = \alpha_1(t_{N-1}) = 0.3 & \beta_1(t_0) = \beta_1(t_1) = \cdots = \beta_1(t_{N-1}) = 2a \\ \alpha_2(t_0) = \alpha_2(t_1) = \cdots = \alpha_2(t_{N-1}) = 0.5 & \beta_2(t_0) = \beta_2(t_1) = \cdots = \beta_2(t_{N-1}) = -a \\ \alpha_3(t_0) = \alpha_3(t_1) = \cdots = \alpha_3(t_{N-1}) = 0.2 & \beta_3(t_0) = \beta_3(t_1) = \cdots = \beta_3(t_{N-1}) = a. \end{array} \tag{16}$$

We constantly assume that Eqs. (14) and (16) are the uniqueness conditions for satisfying the relevant solutions.

If Eq. (15) is regarded as a system under structural perturbation, then, a specific concept that utilizes the stability of Eq. (16) to survey the stability of Eq. (15) has already appeared in the general perturbation theory; however, when it is applied to a large-scale system, it seems to become more successful.

The reason for this is that Eq. (16) is a mechanical combination of some isolated low-dimensional subsystems, wherein the subsystems are decoupled from one another. In addition, it is much easier to study the stability of Eq. (16) than that of Eq. (15). This is why; the relevant studies on the stability of large-scale system proposed in prior literature most commonly used isolated subsystems.

The stability criterion (4) for Eq. (16) can appropriately perform; therefore, we can predict the stability of the large-scale system by means of the stability of isolated subsystems. However, the stability of both systems is not equivalent. The fact that the former is unstable, but the latter can become stable has made people realize that using the stability of isolated subsystems to predict the stability of the large-scale system is only a sufficient but not a necessary condition. The study of the stability of isolated subsystems is indeed not the primary objective, it is nothing but a means to an end and thus we propose a series of improvements. Under these circumstances, the correct formulation is to utilize certain information from isolated subsystems as well as the relationship between relational structures in order to achieve the purpose of determining the stability of large-scale systems.

Choosing the most suitable decomposition method for contributing to the problem-solving efforts is proposed as follows. A practical mechanical system composed of three interconnected subsystems which are described as follows.

We can solve this problem by the proposed theorem for the following 3 subsystems.

$$\begin{array}{l} \begin{cases} \dot{x}_{11}(t) = x_{21}(t) \\ \dot{x}_{21}(t) = \dfrac{5.1\sin(x_{11}(t)) - 0.028x_{21}^2(t)\sin(2x_{11}(t)) - 0.2\cos(x_{11}(t))u_1(t)}{0.37 - 0.056\cos(x_{11}(t))} \end{cases} \\ \begin{cases} \dot{x}_{12}(t) = x_{22}(t) \\ \dot{x}_{22}(t) = -6.5\sin(x_{22}(t)) + u_2(t) \end{cases} \\ \begin{cases} \dot{x}_{13}(t) = x_{23}(t) \\ \dot{x}_{23}(t) = \dfrac{5.6\sin(x_{13}(t)) - 0.03x_{23}^2\sin(2x_{13}(t)) - \cos(x_{13}(t))u_3(t)}{0.4 - 0.06\cos(x_{13}(t))} \end{cases} \end{array} \tag{17}$$

In order to stabilize the nonlinear large-scale system N, three model-based fuzzy controllers which are designed via the concept of PDC scheme.

In accordance with Remark 1, appropriate bounding matrices are chosen as

$$H_{q1} = \begin{bmatrix} 0.4 & 0.33 \\ 0.012 & 0.414 \end{bmatrix},\ H_{q2} = \begin{bmatrix} 0.94 & 0.53 \\ 0.3 & 0.6 \end{bmatrix},\ H_{q3} = \begin{bmatrix} 0.7 & 0.33 \\ 0.162 & 0.384 \end{bmatrix},\ \delta_{ijfj} = I. \tag{18}$$

4 Conclusions

For the purpose of fulfilling the stability conditions of Theorem 1, selecting the proper common positive definite matrix and the control force becomes the key problem to be dealt with. In this paper, we use EBA to discover the proper solutions. In this case, the obtained solutions can be classified into two categories: feasible and infeasible. It means that designing the fitness function in a binary operation form is a simpler way to answer to the need of this application. In this paper, the fitness function is designed based on the stability criterion derived from the LMI conditions via the Lyapunov function approach. The AND logical operation is employed in the fitness function for examining the solutions to produce the binary classification results on the discovered solutions.

References

1. Mahmoud, M. S., Hassan, M. F., Darwish, M. G. (1985). *Large-Scale Control Systems*. New York: Marcel Dekker.
2. Wang, W. J., Mao, L. G. (1997). Stabilization and estimation for perturbed discrete time-delay large-scale systems. *IEEE Transactions, 42,* 1277–1282.
3. Yan, X. G., Dai, G. Z. (1998). Decentralized output feedback robust control for nonlinear large-scale systems. *Automatica, 34(11),* 1469–1472. DOI 10.1016/S0005-1098(98)00090-9.
4. Yen, J., Wang, L. (1999). Simplifying fuzzy rule-based models using orthogonal transformation methods. *IEEE Transactions, Part B, 29,* 13–24.
5. Tanaka, K., Ikeda, T., Wang, H. O. (1998). A unified approach to controlling chaos via an LMI-based fuzzy control systems design. *IEEE Transactions, Part I, 45,* 1021–1040.
6. McDaid, A. J., Mace, B. R. (2016). A robust adaptive tuned vibration absorber using semi-passive shunt electronics. *IEEE Transactions and Electron, 63,* 5069–5077.
7. Nečásek, J., Václavík, J., Marton, P. (2016). Digital synthetic impedance for application in vibration damping. *Review of Scientific Instruments, 87(2),* 024704. DOI 10.1063/1.4942085.
8. Pérez-Díaz, J., Valiente-Blanco, I., Cristache, C. (2016). Z-Damper: A new paradigm for attenuation of vibrations. *Machines, 4(2),* 12. DOI 10.3390/machines4020012.
9. Shen, W., Zhu, S., Zhu, H., Xu, Y. L. (2016). Electromagnetic energy harvesting from structural vibrations during earthquakes. *Smart Structures and Systems, 18(3),* 449–470. DOI 10.12989/sss.2016.18.3.449.
10. Yan, B., Zhang, S., Zhang, X., Wang, C., Wu, C. (2017). Self-powered electromagnetic energy harvesting for the low power consumption electronics: Design and experiment. *International Journal of Applied Electromagnet Mechanics, 54(1),* 1–11. DOI 10.3233/JAE-160043.