

A Multi-Level Threshold Method for Edge Detection and Segmentation Based on Entropy

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Abstract: The essential tool in image processing, computer vision and machine vision is edge detection, especially in the fields of feature extraction and feature detection. Entropy is a basic area in information theory. The entropy, in image processing field has a role associated with image settings. As an initial step in image processing, the entropy is always used the image's segmentation to determine the regions of image which is used to separate the background and objects in image. Image segmentation known as the process which divides the image into multiple regions or sets of pixels. Many applications have been development to enhance the image processing. This paper utilizes the Shannon entropy to achieve edge detection process and segmentation of the image. It introduces a new method of edge detection for 2-D histogram and Shannon entropy based on multilevel threshold. The method utilizes the gray value and the average gray value of the pixels to achieve the two dimensional histogram. The current method has apriority in comparison to some upper classical methods. The experimental results exhibited that the proposed method could capture a moderate quality and execution time better than other comparative methods, particularly in the largest images size. The proposed method offers good results when applied with images of different sizes from the civilization of ancient Egyptians.

Keywords: Multi-level threshold, edge detection, 2D histogram, entropy.

1 Introduction

Edge detection is one of the most important areas in image processing, image pattern recognition, computer vision, image analysis, and in human vision. There are many applications use the edge detection such as the feature extraction, registration, segmentation and identification of objects in a scene [Kim, Kim, Lee et al. (2009)]. Edge detection refers to the process which define the changes that occur in pixels gray value for an image. These changes produced from the characteristics of the different

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scene such as changes in material properties, abruptness in depth, variations in scene illumination and discontinuities in surface orientation.

The shapes of the objects expressed by determining the edges of the image. So edges are very important feature of image segmentation. Segmentation defined as the process that divides the image into multiple regions [Vaithyanathan, Karthikeyan and Venkatraman (2014)]. Thresholding is an important technique for image segmentation that it extracts the target from the background [Muthukrishnan and Radha (2011); Ramadevi, sridevi, poornima et al. (2010)]. Usually the image segmented as bi level or multilevel thresholding. Bi level separates the image into two classes but the multi-level separates the image into several classes [Djerou, Khelil, Dehimi et al. (2012)]. Some examples of thresholding involve cell images [Kumar, Kumar and Sharma (2014); Naya, Sahu and Mohammed (2013)], and knowledge representation [Kuznetsov and Poelmans (2013)]. Other thresholding applications for document image analysis that extracts printed characters [Kasturi, O'grman and Govindaraju (2002); Dixit and Shirdhonkar (2015)], musical scores, graphical content, logos and map processing that is used to find lines, legends, characters [Chiang, Leyk and Knoblock (2014)]. Additionally thresholding can be applied in quality inspection of materials [Vacho, Balazi, Paulovic et al. (2018); Fridousi and Parveen (2014)]. In Lu et al. [Lu and Wong (2017)], explained non-destructive testing applications such as eddy current images, laser scanning confocal microscopy [Sophian, Tian, Taylor et al. (2001)], extraction of edge fields [Venkatesh and Rosin (1995)], spatio-temporal segmentation of video images [Jiang, Zhang, Wang et al. (2015)]

Thresholding methods is classified into two methods parametric and non-parametric. The first type applies Gaussian distribution on grey levels of each group therefore an estimation of the Gaussian distribution parameters can be found as shown in Kikawa et al. [Kikawa, Shatalov, Kloppers et al. (2016)]. Nonparametric methods find the thresholds that divide the gray-levels in an optimal style upon some discriminating criterion such as the entropy measures [Suresh and Anitha (2017)]. In El-Sayed [El-Sayed (2011)] proposed a new technique for edge detection in images depend on entropic threshold. In Rajini [Rajini (2019)] provided an efficient process depend on improvement image thresholding rely on two dimension Tsallis entropy.

This paper is organized as follows: Section 2 introduce the idea for detecting of the edges by using filter "3×3" and apply the image to this filter. Section 3 describe the main concept of the entropy for images. Section 4 presents the fundamental definition of the threshold value with different levels. Section 5 offer the effectiveness of the methods when it applied on many of images that have different sizes. Finally the conclusion of this paper is introduced in Section 6.

2 Detecting of the edges

In Fig. 1, The spatial filter mask 3×3 specific as a matrix "R" of size "m×n". where "m=2α+1" and "n=2β+1", (α, β) are nonzero positive integers. Fig. 2 illustrates the image under the above mask how it looks.

The first step of the edge detection is the classification of all pixels which are suitable for the criterion of homogeneousness. Second step is detection all pixels on the borders between the areas that different in the homogeneous.

$R(-1, -1)$	$R(-1, 0)$	$R(-1, 1)$
$R(0, -1)$	$R(0, 0)$	$R(0, 1)$
$R(1, -1)$	$R(1, 0)$	$R(1, 1)$

Figure 1: Filter mask 3×3

$f(x - 1, y - 1)$	$f(x - 1, y)$	$f(x - 1, y + 1)$
$f(x, y - 1)$	$f(x, y)$	$f(x, y + 1)$
$f(x + 1, y - 1)$	$f(x + 1, y)$	$f(x + 1, y + 1)$

Figure 2: Image under the mask 3×3

In the proposed planner, first step creates a binary image by using suitable threshold value using Shannon entropy. In Fig. 3 the method applied on this the binary image window. Set all window coefficients equal to one except the center equal to x .

1	1	1
1	x	1
1	1	1

Figure 3: Binary image window

To find all the probabilities of each central pixel in the image under the window, move the top window on the whole image. Then the entropy is estimated of each central pixel under the window from this relationship $E(cpix) = -p_c \ln(p_c)$. Where $cpix$ is the central pixel of binary image under the window and p_c is the probability of this pixel. The entropy of the central pixel equal to zero when the probability equal to one. If the gray level of all pixels under the window are homogeneous then $p_c = 1$, $E = 1$. Therefore, the central pixel is not an edge. Tab. 1 show the entropy for each other possibilities of central pixel under the window.

As shown in Tab. 1, in case $p_c \leq \frac{6}{9}$, the diversity for gray level of pixels under the window is high, but in cases $p_c = \frac{7}{9}$ and $8/9$, the diversity for gray level of pixels under the window is low. So, in these cases the central pixel is not an edge pixel.

Table 1: p and E of central pixel under window

p	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9}$
E	0.2441	0.3342	0.3662	0.3604	0.3265	0.2703	0.1955	0.1047	0.0

3 Concept of the amount of information for image

The fundamental idea of the amount of information for image in information theory is the randomness in a signal or in a random event [Al-Halabi (2012)]. One of the simplest examples in the definition of entropy is flipping a coin and communicating the result. According to this definition a random event v that occurs with the probability $p(v)$ can be expressed as follows:

$$I(v) = \log\left[\frac{1}{p(v)}\right] = -\log[p(v)] \quad (1)$$

where $I(v)$ is the self-information of event v , $p(v)$ is the probability of this event. The relationship between the event and the probability is inverse relationship, if $p(v) = 1$ then $I(v) = 0$. This show we cannot gain any information. In addition, if the event frequently occurs, then no information resulting from this event. The base of the logarithm determines the unit that measured the information. The unit of information is bit if the base of the logarithm is two.

Commonly the entropy can be obtained from the probability distribution. Consequently the gray value of the pixel is located at the point (x, y) in a digital image $d(x, y)$, where $x = 1, 2, \dots, X$, $y = 1, 2, \dots, Y$ and $(X \times Y)$ is the image size. Suppose that the probability distribution for an image with w gray levels are $p_i = p_1, p_2, \dots, p_w$. This distribution is classified in two probabilities, one for the object (class o) and the other for the background (class B) are given by:

$$p_o : \frac{p_1}{p_o}, \frac{p_2}{p_o}, \dots, \frac{p_t}{p_o} \quad (2)$$

$$p_B : \frac{p_{t+1}}{p_B}, \frac{p_{t+2}}{p_B}, \dots, \frac{p_w}{p_B}$$

where

$$p_o = \sum_{i=1}^t p_i, \quad p_B = \sum_{i=t+1}^w p_i \quad (3)$$

All the set of probabilities must satisfy the requirement $\sum_{i=1}^w p_i = 1, 0 \leq p_i \leq 1$

The average resulting information is called Shannon entropy $s(z)$ that can be described as follows:

$$s(z) = -\sum_{i=1}^w p_i \ln(p_i) \quad (4)$$

When the system divided into two subsystems o and B , the Shannon entropy has the property additivity, in this case, the equation can be used:

$$s(o + B) = s(o) + s(B) \quad (5)$$

4 Threshold value and two dimensional histogram

4.1 Definition of threshold value

Threshold value is used to obtain new dataset contains only two values. The data values can be replaced with one in the output when data values are less than the threshold value. Otherwise the data values can be replaced with other output values [Kato and Ahern (2011)]. In gray level image, incoming data ranges from [0, 255]. After applying the threshold the resulting values are only 0 and 255, thus the resulting image is binary image. Suppose that T is the threshold. Any Pixel in $f(x, y) < T$ is called background pixel, otherwise called object pixel. From the previous, the relationship between the threshold value, the gray levels, and the local property of these pixels can be provided as follows: $T = T[x, y, q(x, y), g(x, y)]$, where $q(x, y)$ refers to some of local property of the pixels and $g(x, y)$ refers to the gray level for the pixel (x, y) . There are several kinds of threshold, one kind is called local which depends on $q(x, y)$, $g(x, y)$. Another kind is called global when depends on $g(x, y)$. There are another kind is called adaptive or dynamic threshold which depends on the position of the pixel (x, y) and $g(x, y)$ at the pixel position.

4.2 Bi- level and multi-level threshold

Usually, the threshold is used to segment an image into Bi-level or multi-level thresholding. In the Bi level thresholding the pixels of an image divided into two classes. In the multi-level threshold divided the image into several classes [El-Sayed (2013); Sparavigna (2015)], let us assume that the threshold in Bi level is t for the gray levels and there are two classes o and B and their probability distribution are pO , pB illustrated in Eq. (3). Then the Shannon entropies for each distribution are given by:

$$s^o(t) = -\sum_{i=1}^t \frac{p_i}{p_o} \ln \frac{p_i}{p_o}, s^B(t) = -\sum_{i=t+1}^w \frac{p_i}{p_B} \ln \frac{p_i}{p_B} \quad (6)$$

The total entropy is:

$$s(t) = s^o(t) + s^B(t) \quad (7)$$

In the multi-level thresholding, let us assume an image having w of gray levels and several thresholding applied on this image. The Shannon entropies for each distribution and the total entropy can be written as follows:

$$\begin{aligned} s^1(t) &= -\sum_{i=1}^{t_1} \frac{p_i}{p_1} \ln \frac{p_i}{p_1} \\ s^2(t) &= -\sum_{i=t_1+1}^{t_2} \frac{p_i}{p_2} \ln \frac{p_i}{p_2}, \dots, \\ s^n(t) &= -\sum_{i=t_{n-1}}^w \frac{p_i}{p_n} \ln \frac{p_i}{p_n} \end{aligned} \quad (8)$$

$$s(t) = s^1(t) + s^2(t) + s^3(t) + \dots + s^n(t) \quad (9)$$

4.3 2D histogram based shannon entropy

Let digital image with size $M \times N$. $f(x, y)$ denote to the Gray value pixel at point (x, y) . $g(x, y)$ is the average gray value at point (x, y) . To calculate the two dimensions histogram firstly compute the average gray value $g(x, y)$. To do so the average gray value of 3×3 neighborhood of each pixel is computed as follows:

$$g(x, y) = \frac{1}{9} \sum_{m=-1}^1 \sum_{n=-1}^1 f(x+m, y+n) \quad (10)$$

$f(x, y)$ and $g(x, y)$ are used to establish 2-D histogram as illustrate in Eq. (11):

$$p(i, j) = \frac{nij}{ntot} \{f(x, y) = i, g(x, y) = j\}, i, j = 0, 1, \dots, 255 \quad (11)$$

where $p(i, j)$ indicate the 2-D histogram function, $ntot$ is the size of images, nij indicates the pixel number of which gray value is i and which average Gray value in the neighborhood is j . Threshold value can be get from a vector (c, d) where $c \in f(x, y)$ which refers to the threshold of the gray Level of the pixel. While $d \in g(x, y)$ which refers to the threshold of the average gray level for the neighboring pixels. By Using $p(i, j)$ that achieved from Eq. (11), the surface consists of one valley and two peaks. The foreground and background compatible with the peaks that maybe isolated by determination the vector (c, d) . Where the total of two classes entropies can be maximized by this vector. The domain of the histogram is divided into 4 quadrants, see Fig. 4.

(0,0)	(255,0)
$p2(c, d)$	$p1(c, d)$
$p3(c, d)$	$p4(c, d)$
(0,255)	(255,255)

Figure 4: Binary image window

The first area specified by $[c+1, 255] \times [0, d]$, the second area specified by $[0, c] \times [0, d]$, the third area denoted by $[0, c] \times [d+1, 255]$ and the fourth space specified by $[c+1, 255] \times [d+1, 255]$. $p1(c, d)$ and $p3(c, d)$ areas are ignored in the calculation because it contains noise and doesn't include important information. The second and fourth areas are containing the object and the background. They are independent distributions, in

each case the probability value must be normalized so that the total probability in each area equal to one. the normalization completed by using the next Eq. (12).

$$p_o(c, d) = \sum_{i=0}^c \sum_{j=0}^d p(i, j), \quad pB(c, d) = \sum_{i=c+1}^{255} \sum_{j=d+1}^{255} p(i, j) \quad (12)$$

The values $p2(c, d)$ and $p4(c, d)$ can be approximated by ignoring the first and third areas in calculations as follows:

$p4(c, d) \approx 1 - p2(c, d)$ and 2-D Shannon entropy s of an image is obtained from this equation:

$$s = - \sum_{i=0}^{255} \sum_{j=0}^{255} p(i, j) \ln p(i, j) \quad (13)$$

The function that compute the optimum threshold (c_{opt}, d_{opt}) is:

$$c_{opt}(q), d_{opt}(q) = Arg \max s_q^{o+B}(c, d) = Arg \max [s_q^o(c, d) + s_q^B(c, d)] \quad (14)$$

where q is the real number that indicate to the entropic index, in this case it equal to one. In the following steps we prove the value that found by Shannon's method when $q \rightarrow 1$ equal to the same threshold value.

$$\begin{aligned} \lim_{q \rightarrow 1} s_q^{o+B}(c, d) &= \lim_{q \rightarrow 1} [s_q^o(c, d) + s_q^B(c, d) + (1 - q) \cdot s_q^o(c, d) \cdot s_q^B(c, d)] \\ &= \lim_{q \rightarrow 1} [s_q^o(c, d) + s_q^B(c, d)] \\ &= \lim_{q \rightarrow 1} \left[\frac{1}{q-1} \left(1 - \sum_{i=0}^c \sum_{j=0}^d \left(\frac{p(i, j)}{p2(c, d)} \right)^q \right) \right] \\ &+ \lim_{q \rightarrow 1} \left[\frac{1}{q-1} \left(1 - \sum_{c+1}^{255} \sum_{d+1}^{255} \left(\frac{p(i, j)}{p2(c, d)} \right)^q \right) \right] \\ &= - \lim_{q \rightarrow 1} \frac{d}{dq} \left(\sum_{i=0}^c \sum_{j=0}^d \left(\frac{p(i, j)}{p2(c, d)} \right)^q \right) \\ &- \lim_{q \rightarrow 1} \frac{d}{dq} \left(\sum_{i=c+1}^{255} \sum_{j=d+1}^{255} \left(\frac{p(i, j)}{1 - p2(c, d)} \right)^q \right) \\ &= - \lim_{q \rightarrow 1} \sum_{i=0}^c \sum_{j=0}^d \frac{d}{dq} \left(\frac{p(i, j)}{p2(c, d)} \right)^q \\ &- \lim_{q \rightarrow 1} \sum_{i=c+1}^{255} \sum_{j=d+1}^{255} \frac{d}{dq} \left(\frac{p(i, j)}{1 - p2(c, d)} \right)^q \end{aligned}$$

But, $\frac{d}{dq}(a^q) = e^{q \ln a} \cdot \ln a$, therefore $\lim_{q \rightarrow 1} \frac{d}{dq}(a^q) = e^{\ln a} \cdot \ln a = a \cdot \ln a$, hence,

$$\begin{aligned} \lim_{q \rightarrow 1} s_q^{o+B}(c, d) &= - \sum_{i=0}^c \sum_{j=0}^d \left(\frac{p(i, j)}{p2(c, d)} \right) \ln \left(\frac{p(i, j)}{p2(c, d)} \right) \\ &\quad - \sum_{i=c+1}^{255} \sum_{j=d+1}^{255} \left(\frac{p(i, j)}{p2(c, d)} \right) \ln \left(\frac{p(i, j)}{p2(c, d)} \right) \\ &= s^o(c, d) + s^B(c, d) \end{aligned}$$

El-Sayed et al. [El-Sayed and Abd-El Hafeez (2011)] proposed new edge detection based on the 1-D Shannon entropy in gray level images. We generalized this work, so we proposed method based on 2-D Shannon entropy with Bi-level and multi-level thresholding according to 4.2, 4.3 and the Eqs. (8), (9), (13). Now Shannon 2-D threshold algorithm can be described to determine the optimal threshold t_{opt} as follows:

Algorithm 1: threshold value selection (2-D Shannon threshold)

1-Input the digital gray image I of size $M \times N$.

2-Assume that $f(x, y)$ be the original value of the pixel at the point (x, y) ,

where $x = 1 \dots M, y = 1 \dots N$

3-Calculate $g(x, y)$ which define the average gray value in 3×3 neighborhood on the pixel (x, y) from this equation:

$$g(x, y) = \frac{1}{9} \sum_{m=-1}^1 \sum_{n=-1}^1 f(x+m, y+n)$$

4-Compute $z(i, j)$ that define the common probability function

$$z(i, j) = \frac{nij}{ntot}, i, j = 0, 1, \dots, 255$$

5-When $q = 1$, the optimum threshold Calculated as

$$(t_{opt}(1), t_{opt}(1)) = Arg \max [s_q^o(t, t) + s_q^B(t, t)]$$

6-output: the optimal threshold value t_{opt} of image I when $q \rightarrow 1$

End algorithm 1

Algorithm 2: edge detection of 2-D Shannon.

1-input the gray image I of size $(M \times N)$, which calculated from Algorithm 1.

2-make a binary image, for x, y , if $I(x, y) \leq t_{opt}$, then $f(x, y) = 0$, else

800×713	0.04126	0.03130	0.03059	0.01280	0.01280	0.01494	0.01707	0.02276
1017×564	0.04410	0.03486	0.03201	0.01280	0.01280	0.01992	0.01636	0.02774
1026×564	0.03984	0.04126	0.03628	0.01423	0.01494	0.01992	0.01921	0.02632
1037×564	0.04126	0.03343	0.03486	0.01352	0.01280	0.01850	0.01707	0.02205
1078×564	0.04197	0.03628	0.03841	0.01423	0.01352	0.01707	0.01992	0.02703
1090×564	0.04410	0.03913	0.03913	0.01707	0.01850	0.01636	0.01778	0.02917
720×966	0.04339	0.03770	0.03984	0.01707	0.01636	0.02063	0.01992	0.02774
1619×564	0.05691	0.05193	0.05406	0.02419	0.02134	0.02632	0.03272	0.03272
784×1174	0.06545	0.05549	0.05406	0.02276	0.02276	0.02561	0.02988	0.03272
900×1200	0.07469	0.06402	0.06616	0.02561	0.02419	0.02703	0.03201	0.04482
1110×1772	0.13302	0.11880	0.11880	0.04410	0.03984	0.05193	0.06260	0.06758
1494×1755	0.19136	0.14369	0.14654	0.05975	0.05833	0.06118	0.09532	0.08323
2359×1632	0.26605	0.21768	0.21910	0.10315	0.08110	0.10742	0.12804	0.12307

The proposed scheme used the best characters of Shannon entropy in order to compute local and global threshold. Tab. 2 presents a comparison between the proposed method and the other seven classical edge detection methods based on the CPU time in seconds. It is Clear that the execution of the proposed method takes large time than other methods with small images sizes. But the execution time of proposed method is less than Canny, LoG and Zerocross methods with large images sizes, see Fig. 5. In general, the proposed method works well done for different gray scale digital images with comparison of time execution by seconds of other edge detection approaches. Fig. 6 show the average time of the proposed algorithm and Canny, LoG, Zerocross, approxCanny, Prewitt, Roberts and Sobel algorithms.

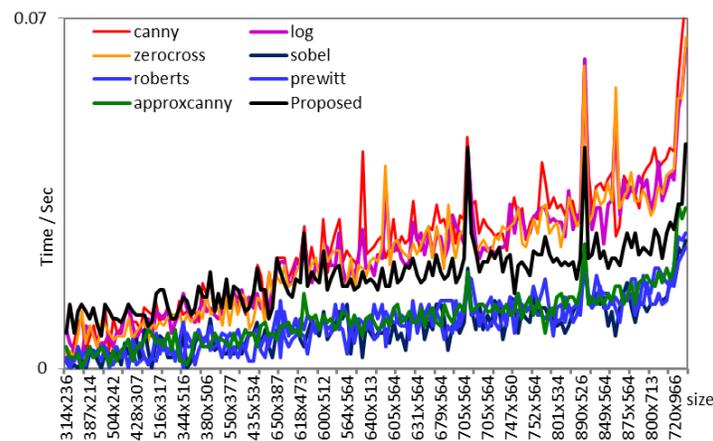


Figure 5: CPU time of classical edge detection methods and proposed method

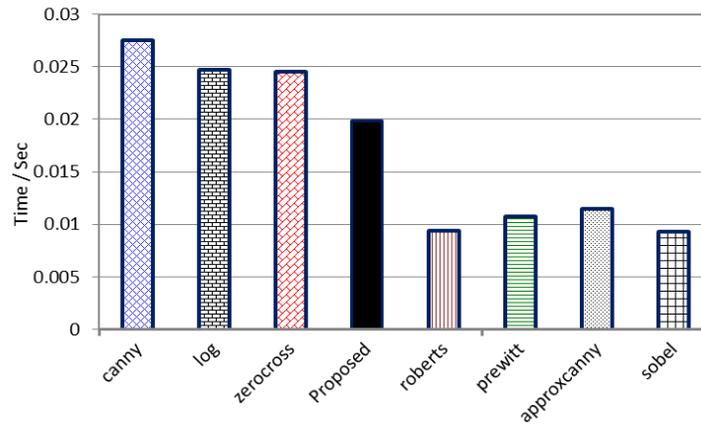
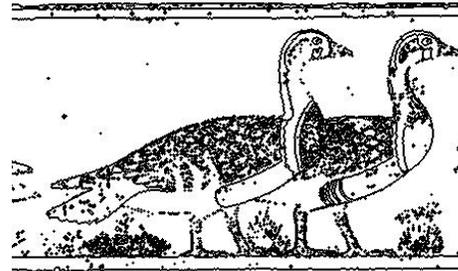


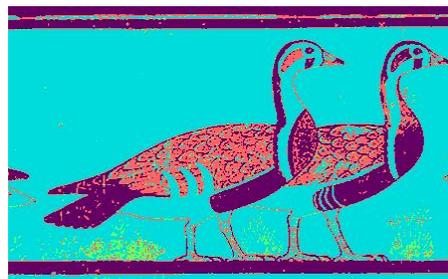
Figure 6: Average time of the proposed algorithm and common algorithms



(a) Original image



(b) T1=158, T2=82, T3=171



(c) segmented with T1, T2, T3

Figure 7: AnEg01 (279×450)

Figs. 7-13 show some experimental results of the proposed method for edge detection, the quality of edge detector with threshold values and the segmentation results of different sizes of images. The algorithm works effectively for different gray level images detector for output images.



(a) Original image



(b) T1=132, T2=83, T3=160



(c) segmented with T1, T2, T3

Figure 8: AnEg02 (813×564)

(a) Original image



(b) T1=126, T2=72, T3=161



(c) segmented with T1, T2, T3

Figure 9: AnEg03 (800×494)

(a) Original image



(b) T1=129, T2=67, T3=156



(c) segmented with T1, T2, T3

Figure 10: AnEg04 (640×457)



(a) Original image



(b) $T_1=112, T_2=61, T_3=141$



(c) segmented with T_1, T_2, T_3

Figure 11: AnEg05 (800×494)



(a) Original image



(b) $T_1=119, T_2=70, T_3=158$



(c) segmented with T_1, T_2, T_3

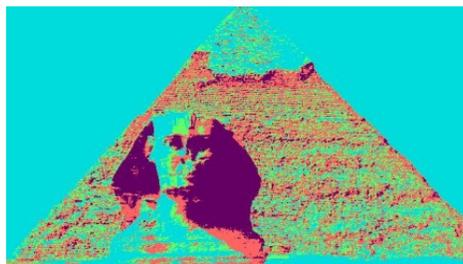
Figure 12: AnEg06 (588×564)



(a) Original image



(b) $T_1=137, T_2=89, T_3=172$



(c) segmented with T1, T2, T3

Figure 13: AnEg07 (279×450)

6 Conclusion

In this research the one dimension Shannon entropy technique is generalized to two dimension Shannon entropy by using bi-level and multi-level threshold. This paper utilizes the Shannon entropy to achieve edge detection process and segmentation of the image. Tested database of images is used to compare the proposed method with most of known edge detection methods such as Canny, LoG, Sobel, Prewitt, Zerocross, Roberts and approxCanny. In order to achieve the results, the execution times in seconds are presented for a proposed method and the other of edge detection methods. It has been clear that the proposed method works in a good way for different gray scale images and different sizes with moderate time.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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