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Numerical Analysis of Stochastic Vector Borne Plant Disease Model

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Abstract: We are associating the solutions of stochastic and deterministic vector borne plant disease model in this manuscript. The dynamics of plant model depends upon threshold number P^*. If $P^* < 1$ then condition helpful to eradicate the disease in plants while $P^* > 1$ explains the persistence of disease. Inappropriately, standard numerical systems do not behave well in certain scenarios. We have been proposed a structure preserving stochastic non-standard finite difference system to analyze the behavior of model. This system is dynamical consistent, positive and bounded as defined by Mickens.

Keywords: Vector borne plant model, stochastic numerical systems, stability.

1 Literature survey

Diseases spread by vectors are expanding due to environmental factors. Malaria, dengue and other large-scale diseases which are spread by vector are increasing in the present industrial world. Many new diseases such as west Nile and Lyme have appeared in different parts of United States [Jeger, Madden and Bosch (2009)]. Those who have contributed in preparing this chapter have scrutinized different factors, responsible for the emergence and resurgence of these diseases, which are spread by vectors. So that such knowledge could be used to predict outbreaks of such diseases in the future and the spread of vectors could also be anticipated in different geographical regions. The chapter begins with summary of the keynote address delivered in a workshop in the University of Hawaii. Different social and economic effects influenced the sudden appearance of vector borne diseases in the past thirty years. The speaker pointed to the changing factors of the spread of different diseases such as malaria, dengue and yellow fever. He also highlighted different factors responsible for the emergence of these vector-borne diseases and analyzed the ramifications of these elements on human health. He pointed out that modern means of transportation had caused easy movement of vectors from one part of

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the world to another which resulted in dispersion of diseases, while it was not easy in the past. Gabler is of the opinion that it is necessary to check the free transportation of vectors and other pathogens through modern means of transportation in order to prevent the emergence of these diseases. Fish of Yale university emphasized on the fact that constant growth of tick-borne diseases in north eastern parts of the United States are linked with the eradication of forests in that region; he described this in his presentation in Yale university [Grilli and Holt (2000); Madden, Jeger and Bosch (2000)]. Lynne disease was not known previously but it became the most ubiquitous disease within twenty years. Moreover, black legged deer ticks served as a vector for Ana plasma phago-cytophilic, a bacterium responsible for causing illness resembling flue which is called human granulocytic anaplasmosis. The developed insects of these species feed only on a deer while the immature insects feed and transmit diseases causing bacteria to the human. The reduction in agriculture and forest area in the north eastern part of the United States in the past few decades have provided conducive environment for the growth and development of white-tailed deer, other ticks they carry and also bacteria's they carry. Fish observed that many foreign tick-borne arboviruses could infect numerous human feeding kinds of ticks present in United States.

Plant diseases caused by vectors have many environmental and epidemiological characteristics with human beings and animals. But they are examined and studied separately. Rodrigo Almeida from the university of California, Berkeley is of the view that new approach on the nature of diseases caused by vectors could be acquired by sharing of instruments and knowledge among different research communities through plant system [Blum, Bresson, Zahid et al. (2018); Severns, Sackett, Farberet et al. (2019)]. Large number of experiments can be executed with variety of hosts, vectors and pathogens, which could be helpful in addressing ecological and evolutionary theories on pathogen range and efficiency of transmission connected with many kinds of diseases caused by arthropod. In Africa, the early warning system checked a major outbreak of rift valley fever, because the association was strong enough to develop risk maps that forecast the outbreak of disease in 2007 [Keeling and Rohani (2008); Shoji and Ozaki (1997)]. These predictions may enable to anticipate the changes of dispersion of diseases on world level. This helped the health and the agricultural authorities to monitor and control the diseases also reducing the cost.

Jonathan Patz from the university of Wisconsin, Madison has made two contributions and also co-authored discussed the probable consequences of global climate change on the appearance of diseases caused by vectors. Patz and Olson, who wrote the first paper, gave an outline of the consequences of climate change on the danger of emergence of disease at both global and indigenous level. Subsequently Patz and Uejio, added the detailed proofs of the effects of climate change on Lyme disease and WNV, the most dominant diseases caused by vectors in North America. Pathogens born in vectors are prone to changes in climate because they affect the survival of vector, its reproduction, feeding and biting, reproduction by replication and incubation and the effectiveness of pathogen transmission in various hosts. The writers' analyses prove that average rise in global temperature would widen the territorial range of malaria in Africa and would increase the probability of emergence of dengue worldwide. However, they concentrated more on higher chances of the emergence of disease in local environments caused by practices of land use. For example, deforestation, cultivation and the building of dams. The writers

have endorsed an opinion that assessment of risk for diseases caused by vectors should include meticulous analysis of the repercussions of land use on climate and weather, ecosystem, diversity of living beings. Vector-borne diseases were the major health challenges in the beginning of twentieth century [Cunniffe and Gilligan (2011); Jeger, Holt, Bosch et al. (2004)]. Later other diseases caused by vectors were highlighted as major causes of disease in humans and domestic animals. With the better understanding of the history and causes of disease, it became possible to prevent and check these diseases with high success. After the second world war, prevention and control of vectors increased because of the invention of new chemicals such as insecticides, drugs and vaccines. Most of the major vector borne diseases controlled by 1960 while more serious efforts were directed to control those diseases which were not controlled at that time by new drugs and vaccines [Cai and Li (2010); Meng and Li (2010)].

A lot of papers have presented on vector borne plant disease dynamics in Madden et al. [Madden, Jeger and Bosch (2000)]. Stochastic differential equation models play an essential role in many branches of applied sciences such as industries, including population dynamics, finance, mechanics, medicine and biology as they provide an extra degree of realism compared to their deterministic counterpart [Bayram, Partal and Buyukoz (2018)]. The main question of this article is to restore the dynamical properties of model by developed stochastic systems [Mickens (1994); Mickens (2005); Mickens (2005); Cresson and Pierret (2014)]? So, we shall construct the implicitly driven explicit system for the given model under the rules presented by Mickens.

This article based on the following sections:

In section two, we have explained the deterministic plant disease model and its states. In section three, we have presented the stochastic numerical systems for plant disease model and its stability analysis. In fifth section, we have presented the results discussion and coming directions.

2 Deterministic plant model

In this way, we consider the deterministic plant model [Shi, Zhao and Tang (2014)]. Let, an arbitrary time 't', the whole plants population is divided to various classes like $S(t)$ represents susceptible plant hosts, $I(t)$ represents infected plant hosts, $R(t)$ represents recovered plant hosts. Similarly, the vector (insects) population is subdivided into two compartments namely $X(t)$ which represents density of the susceptible vectors and $Y(t)$ represents density of the infected. The communication dynamics of plant model as shown in Fig. 1.

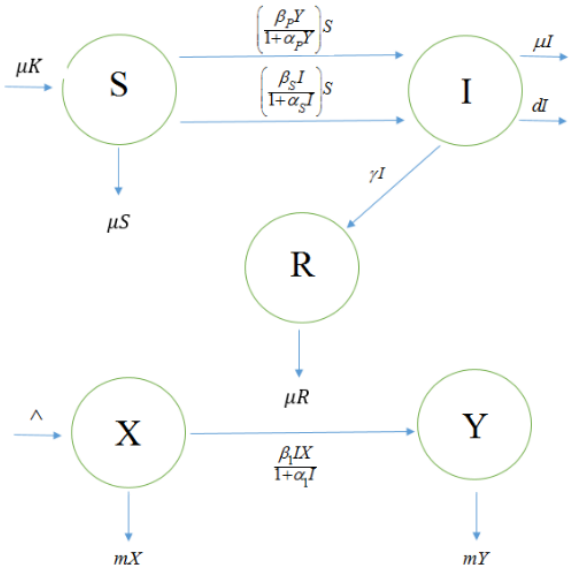


Figure 1: Flow map of plant disease model

The model parameters are described as K (denote the sum of total plant hosts), N (denotes the sum of total insect vectors density), β_1 (denotes the infection ratio), β_p (denotes the biting rate of an infected vector), β_s (denotes the infection incidence of plant model), α_1 (denotes the force of infection host), α_p (denotes the force of infection vector), α_s (denotes the ratio of infection of plants), γ (denotes the force of recovered hosts), μ (denotes the natural death rate of plant hosts), Λ (denotes the birth or immigration of insect vectors), m (denotes the natural death rate of insect vectors) and d (denotes the disease-induced mortality of infected hosts).

The governing equations of plant model are given below as

$$\left.\begin{aligned}
\frac{dS(t)}{dt} &= \mu K - \mu S(t) - \left(\frac{\beta_P Y(t)}{1+\alpha_p Y(t)} + \frac{\beta_S I(t)}{1+\alpha_S I(t)}\right) S(t)\\
\frac{dI(t)}{dt} &= \left(\frac{\beta_p Y(t)}{1+\alpha_P Y(t)} + \frac{\beta_S I(t)}{1+\alpha_s I(t)}\right) S(t) - \omega I(t).\\
\frac{dR(t)}{dt} &= \gamma I(t) - \mu R(t).\\
\frac{dX(t)}{dt} &= \Lambda - \frac{\beta_1 I(t) X(t)}{1+\alpha_1 I(t)} - mX(t).\\
\frac{dY(t)}{dt} &= \frac{\beta_1 I(t)}{1+\alpha_1 I(t)}\left(\frac{\Lambda}{m} - Y(t)\right) - mY(t)
\end{aligned}\right\} \quad (1)$$

where $N = X(t) + Y(t)$ and $S(t) + I(t) + R(t) = K$.

The reduced form of plant model (1) is

$$\left.\begin{aligned}\frac{dS(t)}{dt} &= \mu(K - S(t)) - \left(\frac{\beta_P Y(t)}{1+\alpha_P Y(t)} + \frac{\beta_S I(t)}{1+\alpha_S I(t)}\right) S(t) + dI(t),\\ \frac{dI(t)}{dt} &= \left(\frac{\beta_P Y(t)}{1+\alpha_P Y(t)} + \frac{\beta_S I(t)}{1+\alpha_S I(t)}\right) S(t) - \omega I(t),\\ \frac{dY(t)}{dt} &= \frac{\beta_1 I(t)}{1+\alpha_1 I(t)}\left(\frac{\Lambda}{m} - Y(t)\right) - mY(t),\end{aligned}\right\} \quad (2)$$

where $\omega = d + \mu + \gamma$.

2.1 Equilibria of plant model

The equilibria of plant model (2) can be categorized as:

Disease-free equilibrium, $D_1 = (K, 0, 0)$.

Endemic equilibrium, $E_1 = (S^o, I^o, Y^o)$.

$S^o = K - \left(1 + \frac{\gamma}{\mu}\right) I^o$, $I^o = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ and $Y^o = \frac{\beta_1 \Lambda I^0}{m\beta_1 I^0 + m^2(I + \alpha_1 I^0)}$.

where,

$P^* = \frac{\beta_s K}{\omega} + \frac{\beta_1 \beta_p \Lambda K}{m^2 \omega}$,

$A = (\mu + \gamma)\left(\beta_p \beta_1 \alpha_s \Lambda + m\beta_1 \beta_s + m^2 \beta_s \alpha_1 + \beta_s \beta_p \beta_1 \Lambda\right)$,

$B = (\mu + \gamma)(\beta_P \beta_1 \Lambda + \beta_s m^2) - \mu K\left(m\beta_1 \beta_s \alpha_1 \beta_s m^2 \beta_p \beta_s \beta_1 \Lambda + \beta_p \beta_1 \alpha_s \Lambda\right) + \mu\omega\left(m\beta_1 + \alpha_1 m^2 + \alpha_p \beta_1 \Lambda + m^2 \alpha_s\right)$

and $C = \mu m^2 \omega (1 - P^*)$.

Note that P^* is the reproductive number of the plant model (2) which will decide that weather disease will persists or it will die out. The reproductive number has important role in the disease dynamics of plants.

3 Stochastic plant model

The deterministic plant model (2) can be extended to stochastic differential equations (SDEs) as follows.

Let $V = [S(t), I(t), Y(t)]^T$ We are supposed to calculate the expectations $E^*[\Delta V]$ and $E^*[\Delta V \Delta V^T]$. The transition probabilities have presented in Tab. 1.

Table 1: Possibilities in the process of plant model

Transition	Probabilities
$(\Delta V)_1 = [1 \quad 0 \quad 0]^T$	$P_1 = \mu K \Delta t$
$(\Delta V)_2 = [-1 \quad 0 \quad 0]^T$	$P_2 = \mu S(t) \Delta t$
$(\Delta V)_3 = [-1 \quad 1 \quad 0]^T$	$P_3 = (\frac{\beta_P Y(t)}{1+\alpha_P Y(t)} + \frac{\beta_S I(t)}{1+\alpha_s I(t)}) S(t) \Delta t$
$(\Delta V)_4 = [0 \quad -1 \quad 0]^T$	$P_4 = \omega I(t) \Delta t$
$(\Delta V)_5 = [0 \quad 0 \quad 1]^T$	$P_5 = \frac{\beta_1 I(t)}{1+\alpha_1 I(t)} (\frac{\Lambda}{m}) \Delta t$
$(\Delta V)_6 = [0 \quad 0 \quad -1]^T$	$P_6 = \left(\frac{\beta_1 I(t)}{1+\alpha_1 I(t)} + m\right) Y(t) \Delta t$

$E^*[\Delta V] = \sum_{i=1}^{6} P_i\,(\Delta V)_i$.

$$\text{Expectation} = E^*[\Delta V] = \begin{bmatrix} \mu K - \mu S - (\frac{\beta_P Y(t)}{1+\alpha_P Y(t)} + \frac{\beta_S I(t)}{1+\alpha_S I(t)})S(t) \\ \left(\frac{\beta_P Y(t)}{1+\alpha_p Y(t)} + \frac{\beta_s I(t)}{1+\alpha_s I(t)}\right) S(t) - \omega I(t) \\ \frac{\beta_1 I(t)}{1+\alpha_1 I(t)}\left(\frac{\Lambda}{m}\right) - \left(\frac{\beta_1 I(t)}{1+\alpha_1 I(t)} + m\right) Y(t) \end{bmatrix} \Delta t.$$

$\text{Var} = E^*[\Delta V \Delta V^T] = \sum_{i=1}^{n} P_i\,[(\Delta V)_i][(\Delta V)_i]^T$.

$$E^*[\Delta V \Delta V^T] = \begin{bmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{bmatrix} \Delta t.$$

where, $V_{11} = \mu K \Delta t + \mu S \Delta t + \left(\frac{\beta_P Y(t)}{1+\alpha_P Y(t)} + \frac{\beta_S I(t)}{1+\alpha_s I(t)}\right) S(t)$, $V_{12} = -\left(\frac{\beta_P Y(t)}{1+\alpha_P Y(t)} + \frac{\beta_S I(t)}{1+\alpha_s I(t)}\right) S(t)$, $V_{13} = 0$, $V_{21} = -\left(\frac{\beta_P Y(t)}{1+\alpha_P Y(t)} + \frac{\beta_S I(t)}{1+\alpha_s I(t)}\right) S(t)$, $V_{22} = \left(\frac{\beta_P Y(t)}{1+\alpha_P Y(t)} + \frac{\beta_S I(t)}{1+\alpha_s I(t)}\right) S(t) + \omega I(t)$, $V_{23} = 0$

$V_{31} = 0$, $V_{32} = 0$, $V_{33} = \frac{\beta_1 I(t)}{1+\alpha_1 I(t)}\left(\frac{\Lambda}{m}\right) + \left(\frac{\beta_1 I(t)}{1+\alpha_1 I(t)} + m\right) Y(t)$.

So, we can write,

$\frac{dY(t)}{dt} = V(Y(t), t) + L(Y(t), t)\frac{dB(t)}{dt}$.

where, $f(V(t), t) = \frac{E^*[\Delta V]}{\Delta t}$ and $L(V(t), t) = \sqrt{\frac{E^*[\Delta V \Delta V^T]}{\Delta t}}$ So,

$$dV(t) = f(V(t), t)dt + L(V(t), t)dB(t). \quad (3)$$

The Eq. (3) is called stochastic differential equation of plant model (2) with initial conditions $V(0) = V_o = [700, 200, 10]^T$, $0 \le t \le T$.

3.1 Euler maruyama system

The given system could be presented in [Maruyama (1955)] and biological data has been presented [Shi, Zhao and Tang (2014)]. (see Tab. 2).

Table 2: Values of Parameters

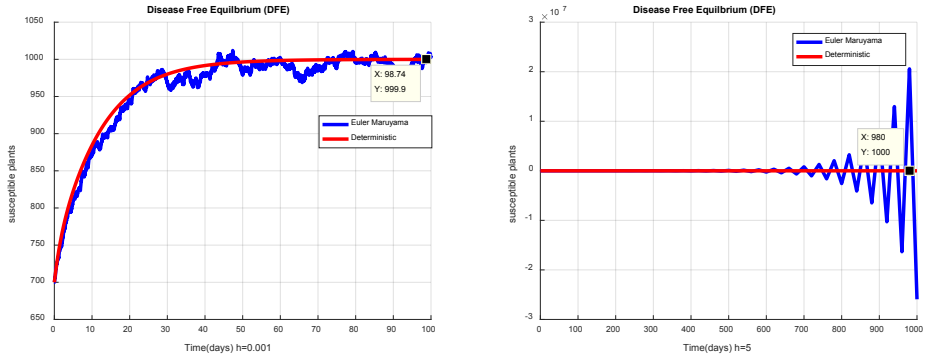
	Values (Days)	
Parameters	**VFE**	**EE**
K	1000	1000
β_1	0.0025	0.01
β_P	0.0025	0.02
β_S	0.0001	0.01
α_1	0.1	0.1
α_P	0.2	0.2

α_S	0.2	0.2
γ	0.4	0.4
μ	0.1	0.1
Λ	5	5
m	0.3	0.3
d	0.1	0.1
σ_1	0.004	0.004
σ_2	0.003	0.003
σ_3	0.02	0.02

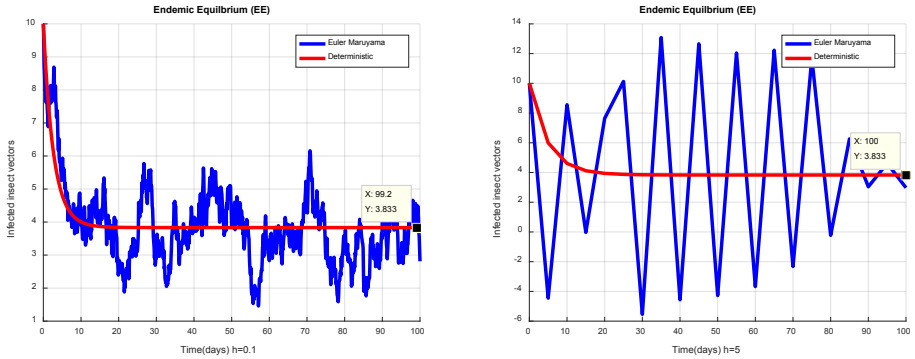
System (3) could be written as follows:

$V_{n+1} = V_n + f(V_n, t)\Delta t + L(V_n, t)\Delta B(t).$

where 'Δt' denotes the time step size. The disease-free equilibrium is $D_1 = (1000,0,0)$ and the reproductive number $P^* = 0.7454 < 1$ means disease is controlled in the population of plants. The endemic equilibrium is $E_1 = (569.7,86.07,3.833)$ and the reproductive number $P^* = 35.1852 > 1$ means disease is endemic in the population of plants [Bayram, Partal and Buyukoz (2018)].



(a) **(b)**



(c) **(d)**

Figure 2: (a) Susceptible plants at $h = 0.001$ for DFE **(b)** Susceptible plants at $h = 5$ for DFE **(c)** Infected insect vectors at $h = 0.1$ for EE **(d)** Infected insect vectors at $h = 5$ for EE

3.2 Non-parametric perturbation of stochastic plant model

One more method to constitute SDEs from the deterministic ODEs that we shall insert the stochasticity in each equation of system (2) as follows [Allen and Burgin (2000); Allen (2007); Allen, Allen, Arciniega et al. (2008); Raza, Arif and Rafiq (2019)]:

$$\left.\begin{aligned} dS(t) &= \left[\mu\big(K - S(t)\big) - \left(\frac{\beta_P Y(t)}{1+\alpha_p Y(t)} + \frac{\beta_S I(t)}{1+\alpha_s I(t)}\right) S(t)\right] dt + \sigma_1 S(t)\Delta B_1(t) \\ dI(t) &= \left[\left(\frac{\beta_P Y(t)}{1+\alpha_p Y(t)} + \frac{\beta_S I(t)}{1+\alpha_s I(t)}\right) S(t) - \omega I(t)\right] dt + \sigma_2 I(t)\Delta B_2(t) \\ dY(t) &= \left[\frac{\beta_1 I(t)}{1+\alpha_1 I(t)}\left(\frac{\Lambda}{m} - Y(t)\right) - mY(t)\right] dt + \sigma_3 Y(t)\Delta B_3(t) \end{aligned}\right\} \quad (4)$$

where σ_1, σ_2 and σ_3 are stochasticity of each compartment of plant model and $B_j(t), (j = 1,2,3)$ are the independent Brownian motions [Raza, Arif and Rafiq (2019); Raza, Arif, Rafiq et al. (2019); Arif, Raza, Rafiq et al. (2019); Arif, Raza, Rafiq et al. (2019)]. Hence, we introduced a more advanced stochastic numerical system to discover the solution of model.

3.2.1 Stochastic euler system

The given model (4) could be written as follows [Raza, Arif and Rafiq (2019); Raza, Arif, Rafiq et al. (2019); Arif, Raza, Rafiq et al. (2019); Arif, Raza, Rafiq et al. (2019)]:

$$\left.\begin{aligned} S^{n+1}(t) &= S^n(t) + h\left[\mu\big(K - S^n(t)\big) - \left(\frac{\beta_p Y^n(t)}{1+\alpha_p Y^n(t)} + \frac{\beta_s I^n(t)}{1+\alpha_s I^n(t)}\right) S^n(t) + \sigma_1 S^n(t)\Delta B_1(t)\right] \\ I^{n+1}(t) &= I^n(t) + h\left[\left(\frac{\beta_p Y^n(t)}{1+\alpha_p Y^n(t)} + \frac{\beta_s I^n(t)}{1+\alpha_s I^n(t)}\right) S^n(t) - \omega I^n(t) + \sigma_2 I^n(t)\Delta B_2(t)\right] \\ Y^{n+1}(t) &= Y^n(t) + h\left[\frac{\beta_1 I^n(t)}{1+\alpha_1 I^n(t)}\left(\frac{\Lambda}{m} - Y^n(t)\right) - mY^n(t) + \sigma_3 Y^n(t)\Delta B_3(t)\right] \end{aligned}\right\} \quad (5)$$

where h is step length of time. We use MATLAB program for numerical experiments taking biological data given in Shi et al. [Shi, Zhao and Tang (2014)]. (see Tab. 2).

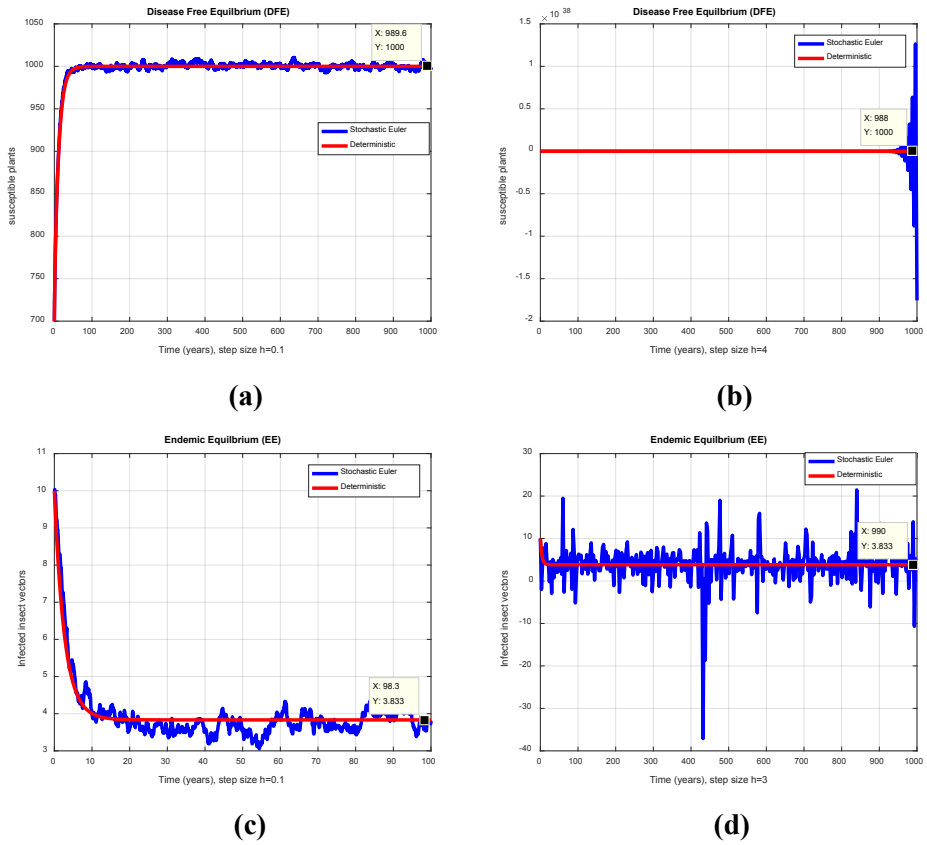


Figure 3: (a) Susceptible plants at $h = 0.1$ for DFE **(b)** Susceptible plants at $h = 4$ for DFE **(c)** Infected insect vectors at $h = 0.1$ for EE **(d)** Infected insect vectors at $h = 3$ for EE

3.2.2 Stochastic runge kutta system

The given model (4) could be written as follows [Raza, Arif and Rafiq (2019); Raza, Arif, Rafiq et al. (2019); Arif, Raza, Rafiq et al. (2019); Arif, Raza, Rafiq et al. (2019)]:

First stage

$$A_1 = h\left[\mu\left(K - S^n(t)\right) - \left(\frac{\beta_P Y^n(t)}{1+\alpha_p Y^n(t)} + \frac{\beta_S I^n(t)}{1+\alpha_s I^n(t)}\right) S^n(t) + \sigma_1 S^n(t)\Delta B_1(t)\right]$$

$$B_1 = h\left[\left(\frac{\beta_P Y^n(t)}{1+\alpha_p Y^n(t)} + \frac{\beta_S I^n(t)}{1+\alpha_s I^n(t)}\right) S^n(t) - \omega I^n(t) + \sigma_2 I^n(t)\Delta B_2(t)\right]$$

$$C_1 = h\left[\frac{\beta_1 I^n(t)}{1+\alpha_1 I^n(t)}\left(\frac{\Lambda}{m} - Y^n(t)\right) - mY^n(t) + \sigma_3 Y^n(t)\Delta B_3(t)\right]$$

Second stage

$$A_2 = h\left[\mu\left(K - \left(S^n(t) + \frac{A_1}{2}\right)\right) - \left(\frac{\beta_P\left(Y^n(t)+\frac{C_1}{2}\right)}{1+\alpha_P\left(Y^n(t)+\frac{C_1}{2}\right)} + \frac{\beta_S\left(I^n(t)+\frac{B_1}{2}\right)}{1+\alpha_S\left(I^n(t)+\frac{B_1}{2}\right)}\right)\left(S^n(t) + \frac{A_1}{2}\right) + \sigma_1\left(S^n(t) + \frac{A_1}{2}\right)\Delta B_1(t)\right]$$

$$B_2 = h\left[\left(\frac{B_P\left(Y^n(t)+\frac{C_1}{2}\right)}{1+\alpha_p\left(Y^n(t)+\frac{C_1}{2}\right)} + \frac{\beta_s\left(I^n(t)+\frac{B_1}{2}\right)}{1+\alpha_s\left(I^n(t)+\frac{B_1}{2}\right)}\right)\left(S^n(t) + \frac{A_1}{2}\right) - \omega\left(I^n(t) + \frac{B_1}{2}\right) + \sigma_2\left(I^n(t) + \frac{B_1}{2}\right)\Delta B_2(t)\right]$$

$$C_2 = h\left[\frac{\beta_1\left(I^n(t)+\frac{B_1}{2}\right)}{1+\alpha_1\left(I^n(t)+\frac{B_1}{2}\right)}\left(\frac{\Lambda}{m} - \left(Y^n(t) + \frac{C_1}{2}\right)\right) - m\left(Y^n(t) + \frac{C_1}{2}\right) + \sigma_3\left(Y^n(t) + \frac{C_1}{2}\right)\Delta B_3(t)\right]$$

Third stage

$$A_3 = h\left[\mu\left(K - \left(S^n(t) + \frac{A_2}{2}\right)\right) - \left(\frac{\beta_P\left(Y^n(t)+\frac{C_2}{2}\right)}{1+\alpha_P\left(Y^n(t)+\frac{C_2}{2}\right)} + \frac{\beta_S\left(I^n(t)+\frac{B_2}{2}\right)}{1+\alpha_S\left(I^n(t)+\frac{B_2}{2}\right)}\right)\left(S^n(t) + \frac{A_2}{2}\right) + \sigma_1\left(S^n(t) + \frac{A_2}{2}\right)\Delta B_1(t)\right]$$

$$B_3 = h\left[\left(\frac{B_P\left(Y^n(t)+\frac{C_2}{2}\right)}{1+\alpha_p\left(Y^n(t)+\frac{C_2}{2}\right)} + \frac{\beta_s\left(I^n(t)+\frac{B_2}{2}\right)}{1+\alpha_s\left(I^n(t)+\frac{B_2}{2}\right)}\right)\left(S^n(t) + \frac{A_2}{2}\right) - \omega\left(I^n(t) + \frac{B_2}{2}\right) + \sigma_2\left(I^n(t) + \frac{B_2}{2}\right)\Delta B_2(t)\right]$$

$$C_3 = h\left[\frac{\beta_1\left(I^n(t)+\frac{B_2}{2}\right)}{1+\alpha_1\left(I^n(t)+\frac{B_2}{2}\right)}\left(\frac{\Lambda}{m} - \left(Y^n(t) + \frac{C_2}{2}\right)\right) - m\left(Y^n(t) + \frac{C_2}{2}\right) + \sigma_3\left(Y^n(t) + \frac{C_2}{2}\right)\Delta B_3(t)\right]$$

Fourth stage

$$A_4 = h\left[\mu\left(K - \left(S^n(t) + \frac{A_3}{2}\right)\right) - \left(\frac{\beta_P\left(Y^n(t)+\frac{C_3}{2}\right)}{1+\alpha_P\left(Y^n(t)+\frac{C_3}{2}\right)} + \frac{\beta_S\left(I^n(t)+\frac{B_3}{2}\right)}{1+\alpha_S\left(I^n(t)+\frac{B_3}{2}\right)}\right)\left(S^n(t) + \frac{A_3}{2}\right) + \sigma_1\left(S^n(t) + \frac{A_3}{2}\right)\Delta B_1(t)\right]$$

$$B_4 = h\left[\left(\frac{B_P\left(Y^n(t)+\frac{C_3}{2}\right)}{1+\alpha_p\left(Y^n(t)+\frac{C_3}{2}\right)} + \frac{\beta_s\left(I^n(t)+\frac{B_3}{2}\right)}{1+\alpha_s\left(I^n(t)+\frac{B_3}{2}\right)}\right)\left(S^n(t) + \frac{A_3}{2}\right) - \omega\left(I^n(t) + \frac{B_3}{2}\right) + \sigma_2\left(I^n(t) + \frac{B_3}{2}\right)\Delta B_2(t)\right]$$

$$C_4 = h\left[\frac{\beta_1\left(I^n(t)+\frac{B_3}{2}\right)}{1+\alpha_1\left(I^n(t)+\frac{B_3}{2}\right)}\left(\frac{\Lambda}{m} - \left(Y^n(t) + \frac{C_3}{2}\right)\right) - m\left(Y^n(t) + \frac{C_3}{2}\right) + \sigma_3\left(Y^n(t) + \frac{C_3}{2}\right)\Delta B_3(t)\right]$$

Final stage

$$\left.\begin{aligned} S^{n+1}(t) &= S^{n}(t) + \frac{1}{6}[A_1 + 2A_2 + 2A_3 + A_4] \\ I^{n+1}(t) &= I^{n}(t) + \frac{1}{6}[B_1 + 2B_2 + 2B_3 + B_4] \\ Y^{n+1}(t) &= Y^{n}(t) + \frac{1}{6}[C_1 + 2C_2 + 2C_3 + C_4] \end{aligned}\right\} \quad (6)$$

where h is time step size.

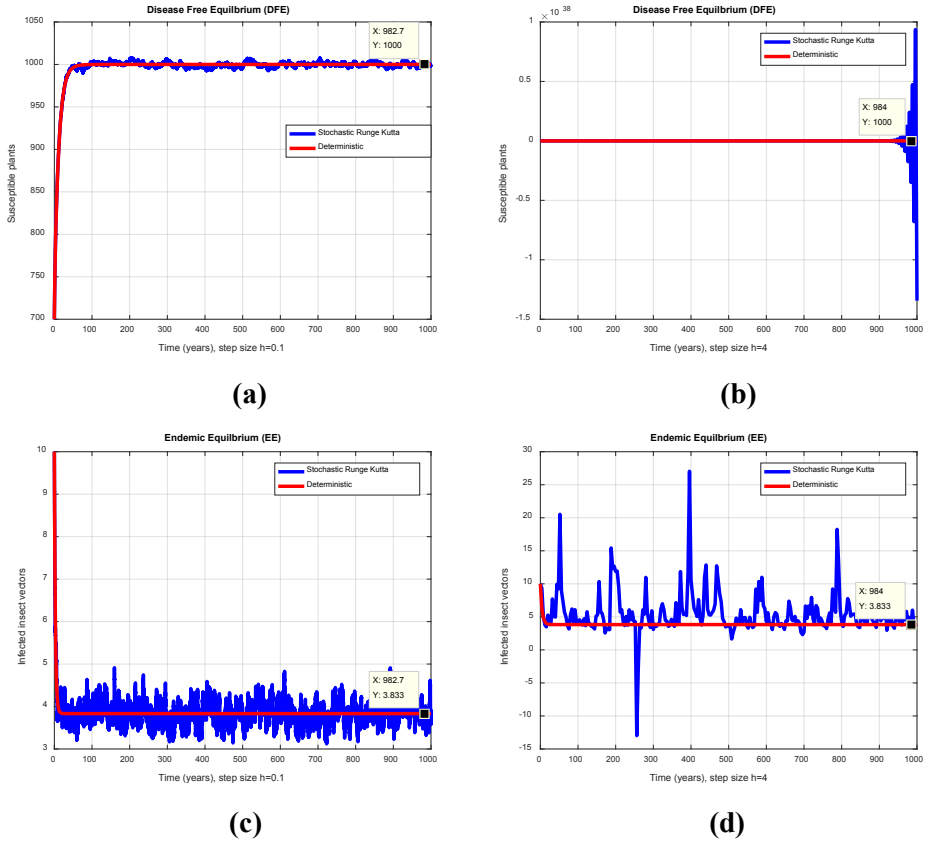


Figure 4: (a) Susceptible plants at $h = 0.1$ for DFE **(b)** Susceptible plants at $h = 4$ for DFE **(c)** Infected insect vectors at $h = 0.1$ for EE **(d)** Infected insect vectors at $h = 4$ for EE

3.2.3 Stochastic NSFD system

The given model (4) could be written as follows [Raza, Arif and Rafiq (2019); Raza, Arif, Rafiq et al. (2019); Arif, Raza, Rafiq et al. (2019, 2019)]:

$$\left.\begin{aligned} S^{n+1}(t) &= \frac{S^n(t)+h\mu K+h\sigma_1 S^n(t)\Delta B_1(t)}{1+h\mu+h\left(\frac{\beta_P Y^n(t)}{1+\alpha_P Y^n(t)}+\frac{\beta_S I^n(t)}{1+\alpha_S I^n(t)}\right)} \\ I^{n+1}(t) &= \frac{I^n(t)+h\left(\frac{\beta_P Y^n(t)}{1+\alpha_P Y^n(t)}+\frac{\beta_S I^n(t)}{1+\alpha_S I^n(t)}\right)S^n(t)+h\sigma_2 I^n(t)\Delta B_2(t)}{1+h\omega} \\ Y^{n+1}(t) &= \frac{Y^n(t)+h\frac{\beta_1 I^n(t)}{1+\alpha_1 I^n(t)}\left(\frac{\Lambda}{m}\right)+h\sigma_3 Y^n(t)\Delta B_3(t)}{1+h\left(\frac{\beta_1 I^n(t)}{1+\alpha_1 I^n(t)}+m\right)} \end{aligned}\right\} \quad (7)$$

where "h" is time step size.

3.2.4 Stability analysis

We consider the following as:

$$F = \frac{S(t)+h\mu K+h\sigma_1 S(t)\Delta B_1(t)}{1+h\mu+\frac{h\beta_P Y(t)}{1+\alpha_P Y(t)}+\frac{h\beta_S I(t)}{1+\alpha_S I(t)}}$$

$$G = \frac{I(t)+h\left(\frac{\beta_P Y(t)S(t)}{1+\alpha_P Y(t)}+\frac{\beta_S I(t)S(t)}{1+\alpha_S I(t)}\right)+h\sigma_2 I(t)\Delta B_2(t)}{1+h\omega}$$

$$H = \frac{Y(t)+\frac{h\beta_1 I(t)\Lambda}{(1+\alpha_1 I(t))m}+h\sigma_3 Y(t)\Delta B_3(t)}{1+\frac{h\beta_1 I(t)}{1+\alpha_1 I(t)}+hm}$$

We define, The Jacobian matrix J as follows:

$$J = \begin{bmatrix} \frac{\partial F}{\partial S(t)} & \frac{\partial F}{\partial I(t)} & \frac{\partial F}{\partial Y(t)} \\ \frac{\partial G}{\partial S(t)} & \frac{\partial G}{\partial I(t)} & \frac{\partial G}{\partial Y(t)} \\ \frac{\partial H}{\partial S(t)} & \frac{\partial H}{\partial I(t)} & \frac{\partial H}{\partial Y(t)} \end{bmatrix}$$

where, $\frac{\partial F}{\partial S(t)} = \frac{1+h\sigma_1\Delta B_1(t)}{1+h\mu+\frac{h\beta_P Y(t)}{1+\alpha_P Y(t)}+\frac{h\beta_S I(t)}{1+\alpha_S I(t)}}$, $\frac{\partial F}{\partial I(t)} = -\frac{\left(S(t)+h\mu K+h\sigma_1 S(t)\Delta B_1(t)\right)\frac{h\beta_S}{(1+\alpha_S I(t))^2}}{\left(1+h\mu+\frac{h\beta_P Y(t)}{1+\alpha_P Y(t)}+\frac{h\beta_S I(t)}{1+\alpha_S I(t)}\right)^2}$ and

$\frac{\partial F}{\partial Y(t)} = -\frac{\left(S(t)+h\mu K+h\sigma_1 S(t)\Delta B_1(t)\right)\frac{h\beta_P}{(1+\alpha_P S(t))^2}}{\left(1+h\mu+\frac{h\beta_P Y(t)}{1+\alpha_P Y(t)}+\frac{h\beta_S I(t)}{1+\alpha_S I(t)}\right)^2}$. $\frac{\partial G}{\partial S(t)} = \frac{h\left(\frac{\beta_P Y(t)}{1+\alpha_P Y(t)}+\frac{\beta_S I(t)}{1+\alpha_S I(t)}\right)}{1+h\omega}$,

$\frac{\partial G}{\partial I(t)} = \frac{1+hS(t)\left[\frac{\beta_S}{(1+\alpha_S I(t))^2}\right]+h\sigma_2\Delta B_2(t)}{1+h\omega}$ and $\frac{\partial G}{\partial Y(t)} = \frac{hS(t)\beta_P}{(1+\alpha_P Y(t))^2(1+h\omega)}$.

$\frac{\partial H}{\partial S(t)} = 0$, $\frac{\partial H}{\partial I(t)} = \frac{\left(1+\frac{h\beta_1 I(t)}{1+\alpha_1 I(t)}+hm\right)\frac{h\Lambda}{m}\left[\frac{\beta_1}{(1+\alpha_1 I(t))^2}\right]-\left[Y(t)+\frac{h\beta_1 I(t)\Lambda}{(1+\alpha_1 I(t)m)}+h\sigma_3 Y(t)\Delta B_3(t)\right]\left[\frac{h\beta_1}{(1+\alpha_1 I(t))^2}\right]}{\left(1+\frac{h\beta_1 I(t)}{1+\alpha_1 I(t)}+hm\right)^2}$ and

$\frac{\partial H}{\partial Y(t)} = \frac{1+\frac{h\beta_1 I(t)\Lambda}{(1+\alpha_1 I(t))m}+h\sigma_3 Y(t)\Delta B_3(t)}{1+\frac{h\beta_1 I(t)}{1+\alpha_1 I(t)}+hm}$.

By using the disease-free equilibrium $D_1 = (K, 0, 0)$ we have

$$J\,(K,0,0)=\begin{bmatrix} \frac{1+h\sigma_1\Delta B_1(t)}{1+h\mu} & -\frac{Kh\beta_s(1+h\mu+h\sigma_1\Delta B_1(t))}{(1+h\mu)^2} & -\frac{hK\beta_{p(1+h\mu+h\sigma_1\Delta B_1(t))}}{(1+h\mu)^2} \\ 0 & \frac{1+hK\beta_s+h\sigma_2\Delta B_2(t)}{1+h\omega} & \frac{hK\beta_p}{1+h\omega} \\ 0 & \frac{h\Lambda\beta_1}{m(1+hm)} & \frac{1+h\sigma_3\Delta B_3(t)}{1+hm} \end{bmatrix}$$

The eigen value of Jacobean matrix as follows:

$$\lambda_1=\frac{1+h\sigma_1\Delta B_1(t)}{1+h\mu}<1,$$

because, the perturbations as σ_1,σ_2 and σ_3 are small noise disturbances with Brownian motions $B_j(t),(j=1,2,3)$ in each compartment of plant model. So, each stochastic term $\sigma_j.(j=1,2,3)<\mu$, where the parameter μ is the natural birth rate of plants [Shi, Zhao and Tang (2014)].

$$J=\begin{bmatrix} \frac{1+hK\beta_s+h\sigma_2\Delta B_2(t)}{1+h\omega} & \frac{hK\beta_p}{1+h\omega} \\ \frac{h\Lambda\beta_1}{m(1+hm)} & \frac{1+h\sigma_3\Delta B_3(t)}{1+hm} \end{bmatrix}.$$

$P_1=$ Trace of Jacobean matrix.

$P_2=$ Determinant of Jacobean matrix.

$$P_1=\frac{1+hK\beta_s+h\sigma_2\Delta B_2(t)}{1+h\omega}+\frac{1+h\sigma_3\Delta B_3(t)}{1+hm}.$$

$$P_2=\frac{(1+hK\beta_s+h\sigma_2\Delta B_2(t))(1+h\sigma_3\Delta B_3(t))}{(1+h\omega)(1+hm)}-\frac{h^2K\Lambda\beta_1\beta_K}{m(1+h\omega)(1+hm)}.$$

3.2.5 Lemma

The given equation$\lambda^2-P_1\lambda+P_2=0$, $|\lambda_i|<1, i=1,2$; if and only if subsequent situations are satisfied [Brauer and Chavez (2001)]:

(i) $1+P_1+P_2>0$

(ii) $1-P_1+P_2>0$

(iii) $P_2<1.$

(i). $1+P_1+P_2>0$

$\because 1>0, P_1>0, To\ prove\ P_2>0.$

$$\Rightarrow \frac{(1+hK\beta_s+h\sigma_2\Delta B_2)(1+h\sigma_3\Delta B_3)}{(1+h\omega)(1+hm)}-\frac{h^2K\Lambda\beta_1\beta_K}{m(1+h\omega)(1+hm)}>0.$$

$$\Rightarrow \frac{(1+hK\beta_s+h\sigma_2\Delta B_2)(1+h\sigma_3\Delta B_3)}{(1+h\omega)(1+hm)}>\frac{h^2K\Lambda\beta_1\beta_K}{m(1+h\omega)(1+hm)}$$

$$\Rightarrow m+mh\sigma_3\Delta B_3+mhK\beta_s+mh^2K\beta_s\sigma_3\Delta B_3+mh\sigma_2\Delta B_2+mh^2\sigma_2\sigma_3\Delta B_2\Delta B_3>h^2K\Lambda\beta_1\beta_k.$$

$$\Rightarrow h^2[K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3]-h[m\sigma_2\Delta B_2+mK\beta_s+m\sigma_3\Delta B_3]<m.$$

$$\Rightarrow h^2-h\frac{[m\sigma_2\Delta B_2+mK\beta_s+m\sigma_3\Delta B_3]}{[K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3]}<\frac{m}{[K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3]}.$$

$$\Rightarrow (h)^2 - 2(h)\left(\frac{m\sigma_2\Delta B_2+mK\beta_s+m\sigma_3\Delta B_3}{K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3}\right)+$$
$$\left[\frac{m\sigma_2\Delta B_2+mK\beta_s+m\sigma_3\Delta B_3}{2(K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_3)}\right]^2 < \left[\frac{m\sigma_2\Delta B_2+mK\beta_s+m\sigma_3\Delta B_3}{2(K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3)}\right]^2 +$$
$$\frac{m}{[K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3]}.$$
$$\Rightarrow \left[\frac{m\sigma_2\Delta B_2+mK\beta_s+m\sigma_3\Delta B_3}{K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3}-h\right]^2 < \left[\frac{m\sigma_2\Delta B_2+mK\beta_s+m\sigma_3\Delta B_3}{2(K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3)}\right]^2 +$$
$$\frac{m}{[K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3]}$$

where "h" is any step size and always positive.

(ii). $1 - P_1 + P_2 > 0$

$$\Rightarrow 1 - \frac{1+hK\beta_s+h\sigma_2\Delta B_2}{1+h\omega} - \frac{1+h\sigma_3\Delta B_3}{1+hm} + \frac{(1+hK\beta_s+h\sigma_2\Delta B_2)(1+h\sigma_3\Delta B_3)}{(1+h\omega)(1+hm)} - \frac{h^2K\Lambda\beta_1\beta_K}{m(1+h\omega)(1+hm)} > 0.$$

$$\Rightarrow m(1+hm)(1+h\omega) - m(1+hm)(1+hK\beta_s+h\sigma_2\Delta B_2) - m(1+h\sigma_3\Delta B_3)(1+h\omega) + m\big(1+hK\beta_s+h\sigma_2\Delta B_2(t)\big)(1+h\sigma_3\Delta B_3) - h^2K\Lambda\beta_1\beta_K > 0.$$

$$\Rightarrow [m+hm\omega+hm^2+h^2m^2\omega] - m[1+hK\beta_s+h\sigma_2\Delta B_2(t)+hm+h^2Km\beta_s+h^2m\sigma_2\Delta B_2] - m[1+h\sigma_3\Delta B_3+h\omega+h^2\omega\sigma_3\Delta B_3] + m[1+h\sigma_3\Delta B_3+hK\beta_s+h^2K\beta_s\sigma_3\Delta B_3+h\sigma_2\Delta B_2+h^2\sigma_2\sigma_3\Delta B_2\Delta B_3] - h^2K\Lambda\beta_1\beta_k > 0.$$

$$\Rightarrow m+hm\omega+hm^2+h^2m^2\omega-m-mhK\beta_s-mh\sigma_2\Delta B_2-hm^2 - h^2Km^2\beta_s-h^2m^2\sigma_2\Delta B_2-m-mh\sigma_3\Delta B_3-mh\omega-mh^2\omega\sigma_3\Delta B_3 + m+mh\sigma_3\Delta B_3+mhK\beta_s+mh^2K\beta_s\sigma_3\Delta B_3+mh\sigma_2\Delta B_2 + mh^2\sigma_2\sigma_3\Delta B_2\Delta B_3-h^2K\Lambda\beta_1\beta_k > 0.$$

$$h^2(m^2\omega-Km^2\beta_s-m^2\sigma_2\Delta B_2-m\omega\sigma_3\Delta B_3+mK\beta_s\sigma_3\Delta B_3+m\sigma_2\sigma_3\Delta B_2\Delta B_3-K\Lambda\beta_1\beta_k) > 0.$$

$$\Rightarrow h^2 > 0.$$

$$\Rightarrow h > 0.$$

where "h" is any step size and always positive.

(iii). $P_2 < 1$

$$\Rightarrow \frac{(1+hK\beta_s+h\sigma_2\Delta B_2)(1+h\sigma_3\Delta B_3)}{(1+h\omega)(1+hm)} - \frac{h^2K\Lambda\beta_1\beta_K}{m(1+h\omega)(1+hm)} < 1.$$

$$\Rightarrow m[1+h\sigma_3\Delta B_3+hK\beta_s+h^2K\beta_s\sigma_3\Delta B_3h\sigma_2\Delta B_2+h^2\sigma_2\sigma_3\Delta B_2\Delta B_3]-h^2K\Lambda\beta_1\beta_k < m(1+hm)(1+h\omega).$$

$$\Rightarrow m+h[m\sigma_2\Delta B_2+mK\beta_s+m\sigma_3\Delta B_3]+h^2[mK\beta_s\sigma_3\Delta B_3+m\sigma_2\sigma_3\Delta B_2\Delta B_3] - h^2K\Lambda\beta_1\beta_k < +m[1+h\omega+hm+h^2m\omega].$$

$$\Rightarrow h^2[mK\beta_s\sigma_3\Delta B_3+m\sigma_2\sigma_3\Delta B_2\Delta B_3-K\Lambda\beta_1\beta_k]+h[m\sigma_2\Delta B_2+mK\beta_s+m\sigma_3\Delta B_3] < h[m\omega+m^2]+h^2m^2\omega.$$

$\Rightarrow h^2[m^2\omega + K\Lambda\beta_1\beta_k - mK\beta_s\sigma_3\Delta B_3 - m\sigma_2\sigma_3\Delta B_2\Delta B_3] + h[m^2 + m\omega - m\sigma_3\Delta B_3 - mK\beta_s - m\sigma_3\Delta B_3] > 0.$

$\Rightarrow (h)^2 + h\frac{m^2+m\omega-m\sigma_3\Delta B_3-mK\beta_s-m\sigma_3\Delta B_3}{m^2\omega+K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3} > 0.$

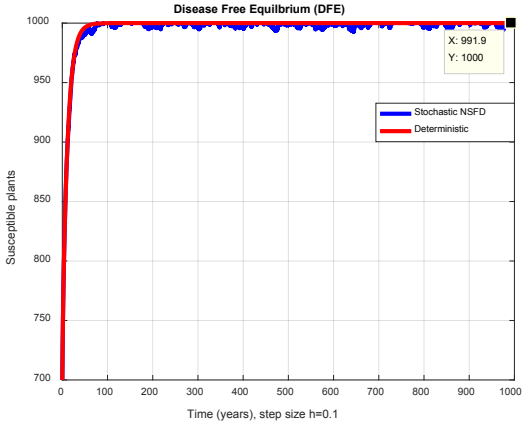
$\Rightarrow (h)^2 + 2h\left[\frac{(m^2+m\omega-m\sigma_3\Delta B_3-mK\beta_s-m\sigma_3\Delta B_3)}{2(m^2\omega+K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3)}\right] +$
$\left[\frac{(m^2+m\omega-m\sigma_3\Delta B_3-mK\beta_s-m\sigma_3\Delta B_3)}{2(m^2\omega+K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3)}\right]^2 >$
$\left[\frac{(m^2+m\omega-m\sigma_3\Delta B_3-mK\beta_s-m\sigma_3\Delta B_3)}{2(m^2\omega+K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3)}\right]^2.$

$\Rightarrow \left[\left(\frac{(m^2+m\omega-m\sigma_3\Delta B_3-mK\beta_s-m\sigma_3\Delta B_3)}{2(m^2\omega+K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3)}\right) + h\right]^2 >$
$\left[\frac{(m^2+m\omega-m\sigma_3\Delta B_3-mK\beta_s-m\sigma_3\Delta B_3)}{2(m^2\omega+K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3)}\right]^2.$

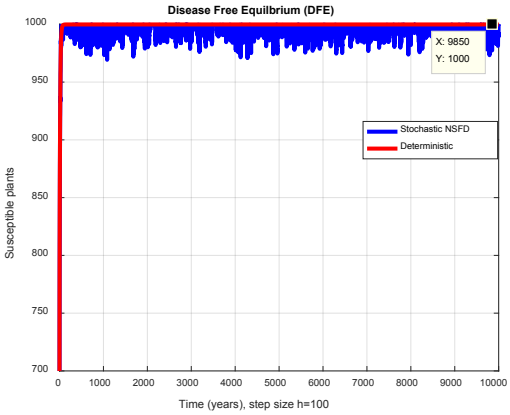
$\Rightarrow \left(\frac{(m^2+m\omega-m\sigma_3\Delta B_3-mK\beta_s-m\sigma_3\Delta B_3)}{2(m^2\omega+K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3)}\right) + h >$
$\left(\frac{(m^2+m\omega-m\sigma_3\Delta B_3-mK\beta_s-m\sigma_3\Delta B_3)}{2(m^2\omega+K\Lambda\beta_1\beta_k-mK\beta_s\sigma_3\Delta B_3-m\sigma_2\sigma_3\Delta B_2\Delta B_3)}\right).$

$\Rightarrow h > 0.$

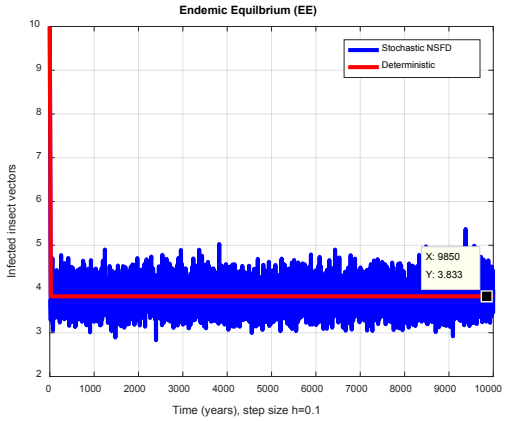
This condition is always valid [Shi, Zhao and Tang (2014)] So, the suggested frame work of stochastic nonstandard finite difference system is linearizable about the equilibria of the model.



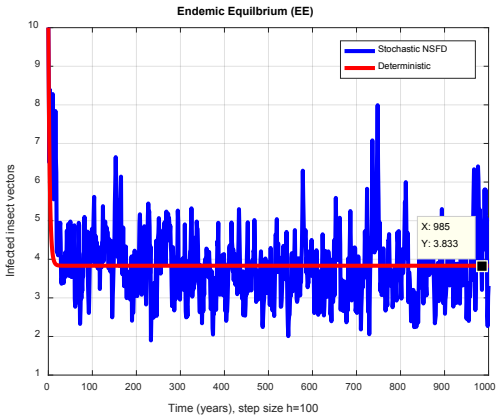
(a)



(b)



(c)



(d)

Figure 5: (a) Susceptible plants at h = 0.1 for DFE **(b)** Susceptible plants at h = 100 for DFE **(c)** Infected insect vectors at h = 0.1 for EE **(d)** Infected insect vectors at h = 100 for EE

4 Results and discussion

The euler maruyama system meets the factual steady states of the plant model as exposed in Fig. 2. Consequently for any time step size, Euler Maryuama system fails to work and shows unboundeness and negativity of solution.

In Fig. 3, the stochastic euler system meets the factual steady states of the plant model. But, in certain values of data proved to be negativity and unboundednes of model.

In Fig. 4, the stochastic runge kutta system converges the factual steady states of the plant model. But, in certain values of data proved to be negativity and unboundednes of model. Finally the above mentioned systems fails for any time step size. Henceforth, the stochastic systems do not preserving the continuous structure of model.

Our claim stochastic NSFD system works for any time step size and adpoted all the dynamical properties of model presented in Mickens et al. [Mickens (1994); Mickens (2005)] as shown in Fig. 5.

5 Conclusion and future framework

We can conclude that deterministic analysis of plant model is less reliable methodology as related to stochastic analysis of plant model. When the time step size is very small then the explicit numerical systems behave well but there is the probability that it may diverge at some particular values of time step size, also the fundamental properties of continuous dynamical system may be loosed.

In the stochastic framework, circumscribed by Mickens et al. [Mickens (1994, 2005, 2005)] the imperative properties such as dynamical consistency and positivity are conserved by stochastic NSFD.

Our keen interest in the future will be to apply the stochastic NSFD system to sophisticated stochastic diffusion and stochastic delay epidemic models. Furthermore, the recommended numerical analysis of this work may be used to enhance the fractional order dynamical system [Baleanu, Jajarmi, Bonyah et al. (2018); Jajarmi and Baleanu (2018); Singh, Kumar and Baleanu (2019)].

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