Computing Topological Invariants of Triangular Chandelier Lattice

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Abstract: A numerical parameter mathematically derived from the graph structure is a topological index. The topological index is the first actual choice in QSAR research and these indices are used to build the correlation model between the chemical structures of various chemicals compounds. Here, we investigate some old degree-based topological indices like Randic index, sum connectivity index, *ABC* index, *GA* index, 1st and 2nd Zagreb indices, modified second Zagreb index, redefined version of 1st, 2nd and 3rd Zagreb indices, hyper and augmented Zagreb indices, forgotten index and symmetric division degree index, and some new degree-based indices like *SK* index, SK_1 index, SK_2 index, and AG_1 index of triangular chandelier-lattice (TCL). The results are generalized by using edge partition and closed formulas for topological indices of triangular chandelier-lattice are analysed.

Keywords: Topological index, Zagreb index, graph, triangular chandelier-lattice, network.

1 Introduction

In the previous two decades topological indices of graphs are being widely applied in analyzing the topology of theoretical and computer-aided models of different physical and chemical phenomena and have found extensive usage in such different areas of scientific research as theoretical physics, chemistry, pharmacology and pharmaceutical chemistry, toxicology, engineering, computer science, sociology, geography, architecture and linguistics. A topological index is a unique mathematical quantity associated with a graph or network which is associated with different properties of the network like connectivity, stability, stress and many others. In the case of chemical graphs, topological indices predict different physiochemical properties like boiling point, biological reactivity, stress, kovat's constant. The study of topological indices is now one of the most vital research field in the chemical graph theory. Topological indices define the structure of molecules mathematically and are

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Received: 02 August 2019; Accepted: 31 August 2019.

used in the growth of qualitative structure-activity relationships (QSARs). Most frequently known invariants of such kinds are degree-based topological indices.

In this work, all molecular graphs are considered to be connected, finite and deprived of parallel edges. Let ς be a graph with n vertices and m edges. The degree of a vertex is the number of vertices adjacent to r and is signified as Δ_r. By these expressions, certain topological indices are well-defined as follows.

A much-investigated degree-based topological index is Randic index and is indicated as $\chi(\varsigma)$ and proposed by Randic [Randic (1975)] while analyzing the boiling point of paraffin. This index is certainly the most extensively applied in chemistry and pharmacology.

Definition 1.1 Consider a molecular graph ς, then the Randic index is defined as

$$\chi(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r\Delta_s}} \tag{1}$$

The sum connectivity index [Zhou and Trinajstic (2009)] is a variation of the Randic connectivity index.

Definition 1.2 Consider a molecular graph ς, then the sum connectivity index is defined as

$$\mathrm{S}(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r+\Delta_s}} \tag{2}$$

Estrada et al. [Estrada, Torres and Rodriguez (1998)] offered a degree based topological invariant of graph, called the atom-bond connectivity index. After that, he investigated the ABC index of branched alkanes [Estrada (2008)].

Definition 1.3 Consider a molecular graph ς, then the ABC index is defined as

$$\mathrm{ABC}(\varsigma)=\prod_{rs\in E(\varsigma)}\sqrt{\frac{\Delta_r+\Delta_s-2}{\Delta_r\Delta_s}} \tag{3}$$

Vukicevic et al. [Vukicevic and Furtula (2009)] presented the geometric-arithmetic index.

Definition 1.4 Consider a molecular graph ς, then the geometric-arithmetic index is defined as

$$\mathrm{GA}(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{2\sqrt{\Delta_r\Delta_s}}{\Delta_r+\Delta_s} \tag{4}$$

Definition 1.5 Fajtlowicz [Fajtlowicz (1987)] proposed harmonic index defined as

$$\mathrm{H}(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{2}{\Delta_r+\Delta_s} \tag{5}$$

For simple connected graphs and trees Zhong [Zhong (2012)] computed the minimum and maximum values of harmonic index.

Definition 1.6 The first and second Zagreb invariants [Gutman and Trinajstic (1972)] are the oldest vertex degree-based topological indices. These indices are defined as

$$\mathrm{M}_1(\varsigma)=\prod_{rs\in E(\varsigma)}[\Delta_r+\Delta_s] \tag{6}$$

$$\mathrm{M}_2(\varsigma)=\prod_{rs\in E(\varsigma)}[\Delta_r\Delta_s] \tag{7}$$

Definition 1.7 Nikolic et al. [Nikolic, Kovacevic, Milcevic et al. (2003)] introduced modified Zagreb indices. The second modified Zagreb index is defined as

$$M_2^*(\varsigma)=\prod_{rs\in E(\varsigma)}\left[\frac{1}{\Delta_r\Delta_s}\right] \tag{8}$$

After this, Hao [Hao (2011)] proposed the connection between the Zagreb and modified Zagreb indices and Hao [Hao (2012)] proposed the modified Zagreb indices of Nanotubes and Dendrimer Nanostars.

Definition 1.8 Ranjini et al. [Ranjini, Lokesha and Usha (2013)] introduced a Re-defined version of Zagreb indices $R_eM_1(\varsigma)$ $R_eM_2(\varsigma)$ and $R_eM_3(\varsigma)$ defined as

$$R_eZM_1(\varsigma)=\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r+\Delta_s}{\Delta_r\Delta_s}\right] \tag{9}$$

$$R_eZM_2(\varsigma)=\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r\Delta_s}{\Delta_r+\Delta_s}\right] \tag{10}$$

$$R_eZM_3(\varsigma)=\prod_{rs\in E(\varsigma)}\Delta_r\Delta_s[\Delta_r+\Delta_s] \tag{11}$$

Definition 1.9 Shirdel et al. [Shirdel, Rezapour and Sayadi (2013)] proposed a new vertex degree-based index named as a hyper-Zagreb index and defined as

$$HM(\varsigma)=\prod_{rs\in E(\varsigma)}[\Delta_r+\Delta_s]^2 \tag{12}$$

Gao et al. [Gao, Siddiqui, Naeem et al. (2017)] computed the closed formula of hyper Zagreb index, first and second multiple Zagreb indices and first and second Zagreb polynomials of carbon graphite and crystal structure of cubic carbon.

Definition 2.0 Furtula et al. [Furtula, Graovac, and Vukicevic (2010)] proposed a topological index named as augmented Zagreb index and defined as

$$\mathrm{AZI}(\varsigma)=\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r\Delta_s}{\Delta_r+\Delta_s-2}\right]^3 \tag{13}$$

Ali et al. [Ali, Bhatti and Raza (2017)] established inequalities between augmented Zagreb index and various other degree-based topological indices.

Definition 2.1 Inspired by the first topological index Furtula et al. [Furtula and Gutman (2015)] established a new vertex degree-based topological index named as "Forgotten index" and defined as

$$\mathrm{F}(\varsigma)=\prod_{rs\in E(\varsigma)}[\Delta_r{}^2+\Delta_s{}^2] \tag{14}$$

Ajmal et al. [Ajmal, Nazeer, Khalid et al. (2017)] computed forgotten index and forgotten polynomial of line graphs of Firecracker graph, Banana tree graph, and subdivision graph.

Definition 2.2 Vukicevic [Vukicevic (2010)] presented the symmetric division degree index and defined as

$$\mathrm{SDD}(\varsigma)=\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r}{\Delta_s}+\frac{\Delta_s}{\Delta_r}\right] \tag{15}$$

Mohanappriya et al. [Mohanappriya and Vijayalakshmi (2018)] obtained the general expression for SDD index and inverse sum index of the transformation networks.

Definition 2.3 Shigehalli et al. [Shigehalli and Kanabur (2016)] proposed new degree-based topological indices like AG_1 index, SK index, SK_1 index, and SK_2 index for Graphene.

$$AG_1(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2\sqrt{\Delta_r\Delta_s}} \tag{16}$$

$$SK(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2} \tag{17}$$

$$SK_1(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{\Delta_r\Delta_s}{2} \tag{18}$$

$$SK_2(\varsigma)=\prod_{rs\in E(\varsigma)}\left(\frac{\Delta_r+\Delta_s}{2}\right)^2 \tag{19}$$

The objective of this article is to analyze the degree-based topological indices of triangular Chandelier lattice. Computing topological indices are one of the central problems in chemical graph theory and are being extensively used quantitative structure-activity relations and network topologies. For further interesting results see: Khalid et al. [Khalid and Idrees (2018); Idrees, Said, Rauf et al. (2017); Idrees, Hussain and Sadiq (2018); Qiong and Li (2019)].

2 Main results for triangular chandelier-lattice (TCL)

Lattices are important mathematical objects which have been successfully employed to study different models in biology, chemistry, and physics. For instance, many researchers have employed the Ising and Potts models [Akin (2018); Uguz and Akin (2010)] in conjunction with the Triangular chandelier lattice (Cayley tree-like lattice).

Chandelier lattices can be realized as simple connected undirected graphs $\varsigma := (V, E)$, where V denotes the set of nodes and E denotes the relation between the nodes. Let $T^k = (V, E)$ be order k chandelier lattice with a root vertex $x(0)\in V$, where each vertex has $(k+3)$ nearest neighbors with V as the set of vertices and the set of edges . It is clear that the root vertex $x(0)$ has k the nearest neighbors (see Fig. 1). For k =3, chandelier lattice is termed as triangular chandelier lattice, similarly for k =4,5,…, we have a terminology of rectangular lattice, pentagonal lattice and so forth. Our object of interest in the present study is triangular chandelier lattice (TCL). We compute several well-established degree-based topological indices of triangular chandelier-lattice (TCL).

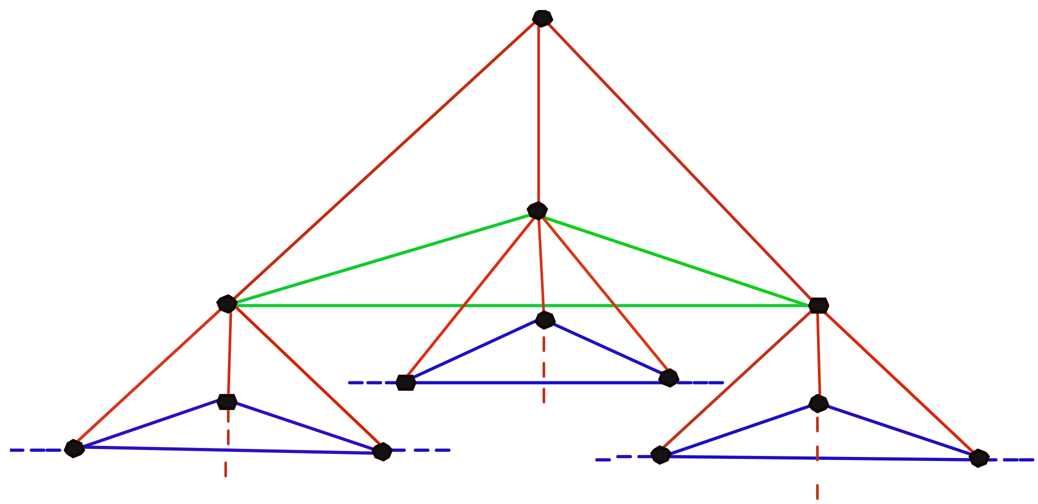

Figure 1: Triangular chandelier-lattice T_4

We use edge partition method to compute the topological indices of TCL. A TCL is said to be 1-chandelier if it comprises of a root vertex along with three neighboring vertices. A TCL is obtained from 1-chandelier by successively attaching 1-chandeliers with all vertices except the root vertex. Let c denotes the number of 1-chandeliers in a TCL. The graph of TCL is denoted by T_c has 4c+1 number of total vertices or nodes and 6c edges. Degree of root vertex is three and all the vertices in middle layers of TCL have degree 6 except the boundary layer vertices which again have degree three. So all edges of TCL has degrees of end vertices as (3,3), (3,6) and (6,6).We denote each type of edges as $E_{3,3}$, $E_{3,6}$ and $E_{6,6}$, respectively. The number of these types of edges is analyzed by considering different values of c. These results aresummarised in Tab. 1 below.

Table 1: Edge partition of TCL with respect to degrees of end vertices

Edge of the form E_{Δ_r,Δ_s}	Sum of edges
$E_{3,3}$	$2c+1$
$E_{3,6}$	$2c-4$
$E_{6,6}$	$2c-5$

Theorem 2.1

Let T_c be triangular chandelier-lattice. Then

i. The Randic index of TCL is $\chi(T_c)=\frac{2c-1}{2}+\frac{2c-4}{3\sqrt{2}}$.

ii. The sum connectivity index of TCL is $\mathrm{S}(T_c)=\frac{2c+1}{\sqrt{6}}+\frac{2c-4}{3}+\frac{2c-5}{2\sqrt{3}}$.

iii. The atom bond connectivity index of TCL is
$\mathrm{ABC}(T_c)=\frac{4c+2}{3}+\frac{(2c-4)\sqrt{7}}{3\sqrt{2}}+\frac{(2c-5)\sqrt{10}}{6}$.

iv. The geometric-arithmetic index of TCL is $\mathrm{GA}(T_c)=4(c-1)+4(c-2)\sqrt{2}$.

v. The harmonic index of TCL is $\mathrm{H}(T_c)=\frac{13c}{9}-\frac{25}{18}$.

Proof: For the triangular chandelier-lattice (TCL) graph T_c having c 1-chandeliers, we divider the edges of T_c into edges of the form E_{Δ_r,Δ_s}, where rs is an edge. We develop the edges of the form $E_{3,3}$, $E_{3,6}$ and $E_{6,6}$. In Fig. 1, $E_{3,3}$, $E_{3,6}$ and $E_{6,6}$ are colored in blue, red and green respectively. The number of edges of these forms is given in Tab. 1.

i. From Eq. (1), we have $\chi(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r\Delta_s}}$

$$\chi(T_c)=\left|E_{(3,3)}\right|\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r\Delta_s}}+\left|E_{(3,6)}\right|\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r\Delta_s}}+\left|E_{(6,6)}\right|\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r\Delta_s}}$$

$$=(2c+1)\frac{1}{\sqrt{9}}+(2c-4)\frac{1}{\sqrt{18}}+(2c-5)\frac{1}{\sqrt{36}}$$

$$=\frac{(2c+1)}{3}+\frac{(2c-4)}{3\sqrt{2}}+\frac{(2c-5)}{6}$$

$$\chi(T_c)=\frac{2c-1}{2}+\frac{2c-4}{3\sqrt{2}}.$$

ii. From Eq. (2), we have $\mathrm{S}(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r+\Delta_s}}$

$$\mathrm{S}(T_c)=\left|E_{(3,3)}\right|\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r+\Delta_s}}+\left|E_{(3,6)}\right|\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r+\Delta_s}}+\left|E_{(6,6)}\right|\prod_{rs\in E(\varsigma)}\frac{1}{\sqrt{\Delta_r+\Delta_s}}$$

$$=(2c+1)\frac{1}{\sqrt{6}}+(2c-4)\frac{1}{\sqrt{9}}+(2c-5)\frac{1}{\sqrt{12}}$$

$$\mathrm{S}(T_c)=\frac{2c+1}{\sqrt{6}}+\frac{2c-4}{3}+\frac{2c-5}{2\sqrt{3}}.$$

iii. From Eq. (3), we have $ABC(\varsigma)=\prod_{rs\in E(\varsigma)}\sqrt{\frac{\Delta_r+\Delta_s-2}{\Delta_r\Delta_s}}$

$$ABC(T_c) = \left|E_{(3,3)}\right| \prod_{rs \in E(\varsigma)} \sqrt{\frac{\Delta_r + \Delta_s - 2}{\Delta_r \Delta_s}} + \left|E_{(3,6)}\right| \prod_{rs \in E(\varsigma)} \sqrt{\frac{\Delta_r + \Delta_s - 2}{\Delta_r \Delta_s}}$$

$$+ \left|E_{(6,6)}\right| \prod_{rs \in E(\varsigma)} \sqrt{\frac{\Delta_r + \Delta_s - 2}{\Delta_r \Delta_s}}$$

$$= (2c+1)\sqrt{\frac{3+3-2}{9}} + (2c-4)\sqrt{\frac{3+6-2}{18}} + (2c-5)\sqrt{\frac{6+6-2}{36}}$$

$$= (2c+1)\frac{2}{3} + (2c-4)\frac{\sqrt{7}}{3\sqrt{2}} + (2c-5)\frac{\sqrt{10}}{6}$$

$$\mathrm{ABC}(T_c) = \frac{4c+2}{3} + \frac{(2c-4)\sqrt{7}}{3\sqrt{2}} + \frac{(2c-5)\sqrt{10}}{6}.$$

iv. From Eq. (4), we have $GA(\varsigma) = \prod_{rs \in E(\varsigma)} \frac{2\sqrt{\Delta_r \Delta_s}}{\Delta_r \Delta_s}$

$$GA(T_c) = \left|E_{(3,3)}\right| \prod_{rs \in E(\varsigma)} \frac{2\sqrt{\Delta_r \Delta_s}}{\Delta_r \Delta_s} + \left|E_{(3,6)}\right| \prod_{rs \in E(\varsigma)} \frac{2\sqrt{\Delta_r \Delta_s}}{\Delta_r \Delta_s} + \left|E_{(6,6)}\right| \prod_{rs \in E(\varsigma)} \frac{2\sqrt{\Delta_r \Delta_s}}{\Delta_r \Delta_s}$$

$$= (2c+1)\frac{2\sqrt{9}}{6} + (2c-4)\frac{2\sqrt{18}}{9} + (2c-5)\frac{2\sqrt{36}}{12}$$

$$= (2c+1) + (2c-4)2\sqrt{2} + (2c-5)$$

$$= (4c-4) + (2c-4)2\sqrt{2}$$

$$\mathrm{GA}(T_c) = 4(c-1) + 4(c-2)\sqrt{2}.$$

v. From Eq. (5), we have $\mathrm{H}(\varsigma) = \prod_{rs \in E(\varsigma)} \frac{2}{\Delta_r + \Delta_s}$.

$$\mathrm{H}(T_c) = \left|E_{(3,3)}\right| \prod_{rs \in E(\varsigma)} \frac{2}{\Delta_r + \Delta_s} + \left|E_{(3,6)}\right| \prod_{rs \in E(\varsigma)} \frac{2}{\Delta_r + \Delta_s} + \left|E_{(6,6)}\right| \prod_{rs \in E(\varsigma)} \frac{2}{\Delta_r + \Delta_s}$$

$$= (2c+1)\frac{2}{3+3} + (2c-4)\frac{2}{3+6} + (2c-5)\frac{2}{6+6} = \frac{(2c+1)}{3} + \frac{2(2c-4)}{9} + \frac{(2c-5)}{6}$$

$$\mathrm{H}(T_c) = \frac{13c}{9} - \frac{25}{18}.$$

Theorem 2.2

Let T_c be triangular chandelier-lattice. Then

i. The first Zagreb index of TCL is $M_1(T_c) = 54c - 90$.

ii. The second Zagreb index of TCL is $M_2(T_c) = 126c - 243$.

iii. The modified second Zagreb index of TCL is $M_2^*(T_c) = \frac{1}{36}(15c - 9)$.

iv. The redefined version of first Zagreb index of TCL is $R_eZM_1(T_c) = 3(c-1)$.

v. The redefined version of second Zagreb index of TCL is $R_eZM_2(T_c) = 13c - \frac{19}{2}$.

vi. The redefined version of third Zagreb index of TCL is

$$R_eZM_3(T_c) = 1296c - 2754.$$

vii. The hyper Zagreb index of TCL is $HM(T_c) = 522c - 1008$.

viii. The augmented Zagreb index of TCL is

$$\mathrm{AZI}(T_c) = 729\left(\frac{c}{32} + \frac{1}{64}\right) + \frac{11664}{343}(c-2) + 5832\left(\frac{2c}{125} - \frac{1}{25}\right).$$

ix. The forgotten index of TCL is $\mathrm{F}(T_c) = 270c - 522$.

x. The symmetric division degree index of TCL is $\mathrm{SDD}(T_c) = 13c - 18$.

Proof: Using edge partition given in Tab. 1, we deduce the following results

i. From Eq. (6), we know that $M_1(\varsigma) = \prod_{rs \in E(\varsigma)} [\Delta_r + \Delta_s]$.

$$M_1(T_c) = |E_{(3,3)}| \prod_{rs \in E(\varsigma)} [\Delta_r + \Delta_s] + |E_{(3,6)}| \prod_{rs \in E(\varsigma)} [\Delta_r + \Delta_s] + |E_{(6,6)}| \prod_{rs \in E(\varsigma)} [\Delta_r + \Delta_s]$$

$$= (2c+1)(3+3) + (2c-4)(3+6) + (2c-5)(6+6)$$

$$= (2c+1)(6) + (2c-4)(9) + (2c-5)(12)$$

$$M_1(T_c) = 54c - 90.$$

ii. From Eq. (7), we know that $M_2(\varsigma) = \prod_{rs \in E(\varsigma)} [\Delta_r \Delta_s]$

$$M_2(T_c) = |E_{(3,3)}| \prod_{rs \in E(\varsigma)} [\Delta_r \Delta_s] + |E_{(3,6)}| \prod_{rs \in E(\varsigma)} [\Delta_r \Delta_s] + |E_{(6,6)}| \prod_{rs \in E(\varsigma)} [\Delta_r \Delta_s]$$

$$= (2c+1)(3 \times 3) + (2c-4)(3 \times 6) + (2c-5)(6 \times 6)$$

$$= (2c+1)(9) + (2c-4)(18) + (2c-5)(36)$$

$$M_2(T_c) = 126c - 243.$$

iii. From Eq. (8), we know that $M_2^*(\varsigma) = \prod_{rs \in E(\varsigma)} \left[\frac{1}{\Delta_r \Delta_s}\right]$.

$$M_2^*(T_c) = |E_{(3,3)}| \prod_{rs \in E(\varsigma)} \left[\frac{1}{\Delta_r \Delta_s}\right] + |E_{(3,6)}| \prod_{rs \in E(\varsigma)} \left[\frac{1}{\Delta_r \Delta_s}\right] + |E_{(6,6)}| \prod_{rs \in E(\varsigma)} \left[\frac{1}{\Delta_r \Delta_s}\right]$$

$$= (2c+1)\left[\frac{1}{3 \times 3}\right] + (2c-4)\left[\frac{1}{3 \times 6}\right] + (2c-5)\left[\frac{1}{6 \times 6}\right]$$

$$= (2c+1)\left[\frac{1}{9}\right] + (2c-4)\left[\frac{1}{18}\right] + (2c-5)\left[\frac{1}{36}\right]$$

$$= \frac{8c+4c+3c}{36} + \frac{4-8-5}{36}$$

$M_2^*(T_c) = \frac{1}{36}(15c-9)$ ·

iv. From Eq. (9), we know that $R_eZM_1(\varsigma) = \prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r+\Delta_s}{\Delta_r\Delta_s}\right]$.

$$R_eZM_1(T_c) = \left|E_{(3,3)}\right|\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r+\Delta_s}{\Delta_r\Delta_s}\right] + \left|E_{(3,6)}\right|\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r+\Delta_s}{\Delta_r\Delta_s}\right]$$

$$+\left|E_{(6,6)}\right|\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r+\Delta_s}{\Delta_r\Delta_s}\right]$$

$$=(2c+1)\left(\frac{3+3}{3\times 3}\right)+(2c-4)\left(\frac{3+6}{3\times 6}\right)+(2c-5)\left(\frac{6+6}{6\times 6}\right).$$

$$=(2c+1)\left(\frac{6}{9}\right)+(2c-4)\left(\frac{9}{18}\right)+(2c-5)\left(\frac{12}{36}\right)$$

$$=\frac{4c}{3}+\frac{2}{3}+c-2+\frac{2c}{3}-\frac{5}{3}$$

$R_eZM_1(T_c) = 3(c-1)$.

v. From Eq. (10), we know that $R_eZM_2(\varsigma) = \prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r\Delta_s}{\Delta_r+\Delta_s}\right]$.

$$R_eZM_2(T_c) = \left|E_{(3,3)}\right|\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r\Delta_s}{\Delta_r+\Delta_s}\right] + \left|E_{(3,6)}\right|\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r\Delta_s}{\Delta_r+\Delta_s}\right]$$

$$+\left|E_{(6,6)}\right|\prod_{rs\in E(\varsigma)}\left[\frac{\Delta_r\Delta_s}{\Delta_r+\Delta_s}\right]$$

$$=(2c+1)\left(\frac{9}{6}\right)+(2c-4)\left(\frac{18}{9}\right)+(2c-5)\left(\frac{36}{12}\right).$$

$R_eZM_2(T_c) = 13c - \frac{19}{2}$ ·

vi. From Eq. (11), we know that $R_eZM_3(\varsigma) = \prod_{rs\in E(\varsigma)}\Delta_r\Delta_s[\Delta_r+\Delta_s]$ ·

$$R_eZM_3(T_c) = \left|E_{(3,3)}\right|\prod_{rs\in E(\varsigma)}\Delta_r\Delta_s[\Delta_r+\Delta_s] + \left|E_{(3,6)}\right|\prod_{rs\in E(\varsigma)}\Delta_r\Delta_s[\Delta_r+\Delta_s]$$

$$+\left|E_{(6,6)}\right|\prod_{rs\in E(\varsigma)}\Delta_r\Delta_s[\Delta_r+\Delta_s]$$

$$=(2c+1)(3\times 3)(3+3)+(2c-4)(3\times 6)(3+6)+(2\text{c}-5)(6\times 6)(6+6)$$

$$=(2c+1)(54)+(2c-4)(162)+(2\text{c}-5)(432)$$

$R_eZM_3(T_c) = 1296c - 2754$.

vii. From Eq. (12), we know that $HM(\varsigma) = \prod_{rs\in E(\varsigma)}[\Delta_r+\Delta_s]^2$.

$$HM(T_c) = |E_{(3,3)}| \prod_{rs \in E(\varsigma)} [\Delta_r + \Delta_s]^2 + |E_{(3,6)}| \prod_{rs \in E(\varsigma)} [\Delta_r + \Delta_s]^2 + |E_{(6,6)}| \prod_{rs \in E(\varsigma)} [\Delta_r + \Delta_s]^2$$

$$= (2c+1)(3+3)^2 + (2c-4)(3+6)^2 + (2c-5)(6+6)^2$$

$$= (2c+1)(6)^2 + (2c-4)(9)^2 + (2c-5)(12)^2$$

$$HM(T_c) = 522c - 1008.$$

viii. From Eq. (13), we have $\mathrm{AZI}(\varsigma) = \prod_{rs \in E(\varsigma)} \left[\frac{\Delta_r \Delta_s}{\Delta_r + \Delta_s - 2} \right]^3$.

$$\mathrm{AZI}(T_c) = |E_{(3,3)}| \prod_{rs \in E(\varsigma)} \left[\frac{\Delta_r \Delta_s}{\Delta_r + \Delta_s - 2} \right]^3 + |E_{(3,6)}| \prod_{rs \in E(\varsigma)} \left[\frac{\Delta_r \Delta_s}{\Delta_r + \Delta_s - 2} \right]^3 + |E_{(6,6)}| \prod_{rs \in E(\varsigma)} \left[\frac{\Delta_r \Delta_s}{\Delta_r + \Delta_s - 2} \right]^3$$

$$= (2c+1)\left(\frac{3\times 3}{3+3-2} \right)^3 + (2c-4)\left(\frac{3\times 6}{3+6-2} \right)^3 + (2c-5)\left(\frac{6\times 6}{6+6-2} \right)^3$$

$$= (2c+1)\left(\frac{9}{4} \right)^3 + (2c-4)\left(\frac{18}{7} \right)^3 + (2c-5)\left(\frac{36}{10} \right)^3$$

$$= (2c+1)\left(\frac{729}{64} \right) + (2c-4)\left(\frac{5832}{343} \right) + (2c-5)\left(\frac{46656}{1000} \right)$$

$$\mathrm{AZI}(T_c) = 729\left(\frac{c}{32} + \frac{1}{64} \right) + \frac{11664}{343}(c-2) + 5832\left(\frac{2c}{125} - \frac{1}{25} \right).$$

ix. From Eq. (14), we have $\mathrm{F}(\varsigma) = \prod_{rs \in E(\varsigma)} [\Delta_r{}^2 + \Delta_s{}^2]$.

$$\mathrm{F}(T_c) = |E_{(3,3)}| \prod_{rs \in E(\varsigma)} [\Delta_r{}^2 + \Delta_s{}^2] + |E_{(3,6)}| \prod_{rs \in E(\varsigma)} [\Delta_r{}^2 + \Delta_s{}^2] + |E_{(6,6)}| \prod_{rs \in E(\varsigma)} [\Delta_r{}^2 + \Delta_s{}^2]$$

$$= (2c+1)(3^2 + 3^2) + (2c-4)(3^2 + 6^2) + (2c-5)(6^2 + 6^2)$$

$$= (2c+1)(18) + (2c-4)(45) + (2c-5)(72)$$

$$\mathrm{F}(T_c) = 270c - 522.$$

x. From Eq. (15), we have $\mathrm{SDD}(\varsigma) = \prod_{rs \in E(\varsigma)} \left[\frac{\Delta_r}{\Delta_s} + \frac{\Delta_s}{\Delta_r} \right]$.

$$\mathrm{SDD}(T_c) = |E_{(3,3)}| \prod_{rs \in E(\varsigma)} \left[\frac{\Delta_r}{\Delta_s} + \frac{\Delta_s}{\Delta_r} \right] + |E_{(3,6)}| \prod_{rs \in E(\varsigma)} \left[\frac{\Delta_r}{\Delta_s} + \frac{\Delta_s}{\Delta_r} \right] + |E_{(6,6)}| \prod_{rs \in E(\varsigma)} \left[\frac{\Delta_r}{\Delta_s} + \frac{\Delta_s}{\Delta_r} \right]$$

$$= (2c+1)\left[\frac{3}{3} + \frac{3}{3} \right] + (2c-4)\left[\frac{3}{6} + \frac{6}{3} \right] + (2c-5)\left[\frac{6}{6} + \frac{6}{6} \right]$$

$$= (2c+1)[2] + (2c-4)\left[\frac{1}{2} + \frac{2}{1} \right] + (2c-5)[2]$$

$$\mathrm{SDD}(T_c) = 13c - 18.$$

Theorem 2.3

Consider a triangular chandelier-lattice T_c. Then

i. The arithmetic-geometric index AG_1 of TCL is $\mathrm{AG}_1(T_c)=(4c-4)+\frac{3c}{\sqrt{2}}-\frac{6}{\sqrt{2}}$.

ii. The SK index of TCL is $SK(T_c)=27c-45$.

iii. The SK_1 index of TCL is $\mathrm{SK}_1(T_c)=63c-126+\frac{9}{2}$.

iv. The SK_2 index of TCL is $\mathrm{SK}_2(T_c)=90c+\frac{81c}{2}-252$.

Proof: Partitioning edges according to Tab. 1, we have following results

i. From Eq. (16), we have $AG_1(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2\sqrt{\Delta_r\Delta_s}}$.

$$AG_1(T_c)=\left|E_{(3,3)}\right|\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2\sqrt{\Delta_r\Delta_s}}+\left|E_{(3,6)}\right|\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2\sqrt{\Delta_r\Delta_s}}+\left|E_{(6,6)}\right|\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2\sqrt{\Delta_r\Delta_s}}$$

$$=(2c+1)\frac{6}{2\sqrt{9}}+(2c-4)\frac{9}{2\sqrt{18}}+(2c-5)\frac{12}{2\sqrt{36}}$$

$$=(4\mathrm{c}-4)+\frac{6c-12}{2\sqrt{2}}$$

$$\mathrm{AG}_1(T_c)=(4c-4)+\frac{3c}{\sqrt{2}}-\frac{6}{\sqrt{2}}.$$

ii. From Eq. (17), we have $SK(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2}$.

$$SK(T_c)=\left|E_{(3,3)}\right|\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2}+\left|E_{(3,6)}\right|\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2}+\left|E_{(6,6)}\right|\prod_{rs\in E(\varsigma)}\frac{\Delta_r+\Delta_s}{2}.$$

$$=(2c+1)\frac{6}{2}+(2c-4)\frac{9}{2}+(2c-5)\frac{12}{2}.$$

$$=6c+3+9c-18+12c-30$$

$$SK(T_c)=27c-45.$$

iii. From Eq. (18), we have $SK_1(\varsigma)=\prod_{rs\in E(\varsigma)}\frac{\Delta_r\Delta_s}{2}$.

$$SK_1(T_c)=\left|E_{(3,3)}\right|\prod_{rs\in E(\varsigma)}\frac{\Delta_r\Delta_s}{2}+\left|E_{(3,6)}\right|\prod_{rs\in E(\varsigma)}\frac{\Delta_r\Delta_s}{2}+\left|E_{(6,6)}\right|\prod_{rs\in E(\varsigma)}\frac{\Delta_r\Delta_s}{2}$$

$$=(2c+1)\frac{9}{2}+(2c-4)\frac{18}{2}+(2c-5)\frac{36}{2}$$

$$=\frac{18c+9}{2}+18c-36+36c-90$$

$$\mathrm{SK}_1(T_c)=63c-126+\frac{9}{2}.$$

iv. From Eq. (19), we have $SK_2(\varsigma)=\prod_{rs\in E(\varsigma)}\left(\frac{\Delta_r+\Delta_s}{2}\right)^2$.

$$SK_2(T_c)=\left|E_{(3,3)}\right|\prod_{rs\in E(\varsigma)}\left(\frac{\Delta_r+\Delta_s}{2}\right)^2+\left|E_{(3,6)}\right|\prod_{rs\in E(\varsigma)}\left(\frac{\Delta_r+\Delta_s}{2}\right)^2+\left|E_{(6,6)}\right|\prod_{rs\in E(\varsigma)}\left(\frac{\Delta_r+\Delta_s}{2}\right)^2$$

$$=(2c+1)\left(\frac{6}{2}\right)^2+(2c-4)\left(\frac{9}{2}\right)^2+(2c-5)\left(\frac{12}{2}\right)^2$$

$$=18c+9+\frac{162c-324}{4}+72c-180$$

$$\mathrm{SK}_2(T_c)=90c+\frac{81c}{2}-252\cdot$$

3 Conclusion

In this paper, we investigate some well-known degree-based topological indices like Randic index, sum connectivity index, ABC index, GA index, first and second Zagreb indices, modified second Zagreb index, redefined version of first, second and third Zagreb indices, hyper Zagreb index, augmented Zagreb indices, forgotten index and symmetric division degree index are computed for triangular chandelier lattice. Moreover, some recently introduced degree-based indices like SK index, SK_1 index, SK_2 index, and AG_1 index of TCL are computed. These results, in turn, can be of great use in QSAR/QSPR studies and analyzing the network topologies and scientific models.

Funding Statement: The author(s) received no specific funding for this study.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

Akin, H. (2018): Gibbs measures of an Ising model with competing interactions on the triangular chandelier-lattice. *Condensed Matter Physics*, vol. 22, no. 2, pp. 1-14.

Ali, A.; Bhatti, A. A.; Raza, Z. (2017): Further inequalities between vertex-degree-based topological indices. *International Journal of Applied and Computational Mathematics*, vol. 3, no. 3, pp. 1921-1930.

Estrada, E.; Torres, L.; Rodriguez, L.; Gutman, I. (1998): An atom-bond connectivity index: modeling the enthalpy of formation of alkanes. *Indian Journal of Chemistry*, vol. 37A, pp. 849-855.

Estrada, E. (2008): Atom-bond connectivity and the energetic of branched alkanes. *Chemical Physics Letters*, vol. 463, no. 4-6, pp. 422-425.

Furtula, B.; Graovac, A.; Vukičević, D. (2010): Augmented Zagreb index. *Journal of Mathematical Chemistry*, vol. 48, no. 2, pp. 370-380.

Furtula, B.; Gutman, I. (2015): A forgotten topological index. *Journal of Mathematical Chemistry*, vol. 53, no. 4, pp. 1184-1190.

Gutman, I.; Trinajstic, N. (1972): Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, *Chemical Physics Letters*, vol. 17, pp. 535-538.

Gao, W.; Siddiqui, M.; Naeem, M.; Rehman, N. (2017): Topological characterization of carbon graphite and crystal cubic carbon structures. *Molecules*, vol. 22, no. 9, pp. 1496.

Hao, J. (2011): Theorems about Zagreb indices and modified Zagreb indices. *MATCH Communications in Mathematical and Computational Chemistry*, vol. 65, pp. 659-670.

Hao, J. (2012): The modified Zagreb indices of nanotubes and dendrimer nanostars, *Journal of Computational and Theoretical Nanoscience*, vol. 9, pp. 727-730.

Idrees, N.; Saif, M.; Rauf, A.; Mustafa, S. (2017): First and second Zagreb eccentricity indices of thorny graphs. *Symmetry*, vol. 9, no. 1, pp. 1-7.

Idrees, N.; Hussain, F.; Sadiq, A. (2018): Topological properties of benzenoid graphs. *University Politehnica of Bucharest Scientific Bulletin Series B-Chemistry and Materials Science*, vol. 80, no. 1, pp. 145-156.

Khalid, R.; Idrees, N. (2018): Wiener and hyper-wiener index of dutch windmill graphs. *University Politehnica of Bucharest Scientific Bulletin Series B-Chemistry and Materials Science*, vol. 80, no. 4, pp. 31-38.

Mohanappriya, G.; Vijayalakshmi, D. (2018): Symmetric division degree index and inverse sum index of transformation graph. *Journal of Physics: Conference Series*, vol. 1139, no. 1, pp. 12-48.

Nikolic, S.; Kovacevic, G.; Milicevic, A.; Trinajstic, N. (2003): The Zagreb indices 30 years after. *Croatica Chemica Acta*, vol. 76, pp. 113-124.

Qiong, K.; Li, X. (2019): Some topological indices computing results if Archimedean lattices L (4, 6, 12). *Computers*, *Materials & Continua*, vol. 58, no. 1, pp. 121-133.

Randic, M. (1975): Characterization of molecular branching. *Journal of the American Chemical Society*, vol. 97, no. 23, pp. 6609-6615.

Ranjini, P. S.; Lokesha, V.; Usha, A. (2013): Relation between phenylene and hexagonal squeeze using harmonic index. *International Journal of Graph Theory*, vol. 1, no. 4, pp. 116-121.

Shirdel, G. H.; Rezapour, H.; Sayadi, A. M. (2013): The hyper-Zagreb index of graph operations. *Iranian Journal of Mathematical Chemistry*, vol. 4, no. 2, pp. 213-220.

Shigehalli, V. S.; Kanabur, R. (2016): Computation of new degree-based topological indices of graphene. *Journal of Mathematics*, vol. 2016, pp. 1-6.

Uğuz, S; Akin, H. (2010): Phase diagrams of competing quadruple and binary interactions on Cayley tree-like lattice: triangular chandelier. *Physica A: Statistical Mechanics and its Applications*, vol. 389, no. 9, pp. 1839-1848.

Vukičević, D.; Furtula, B. (2009): Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges. *Journal of Mathematical Chemistry*, vol. 46, no. 4, pp. 1369-1376.

Vukičević, D. (2010): Bond additive modeling 2, mathematical properties of max-min rodeg index. *Croatica Chemica Acta*, vol. 83, no. 3, pp. 261-273.

Zhou, B.; Trinajstić, N. (2009): On a novel connectivity index. *Journal of Mathematical Chemistry*, vol. 46, no. 4, pp. 1252-1270.

Zhong, L. (2012): The harmonic index for graphs. *Applied Mathematics Letters*, vol. 25, no.3, pp. 561-566.