

## Introduction to the Special Issue on Numerical Methods for Differential and Integral Equations

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Differential and integral equations have a major role in many applied areas of science and engineering, For example, physics, chemistry, astronomy, biology, mechanics, electronic, economics, potential theory, electrostatics. So, the obtaining of solutions of these equations is important. Since the mentioned equations are usually difficult to solve analytically, numerical methods are required. Therefore, this Special Issue contributes to new numerical methods for solving the above-mentioned equations and thus, many problems in science and engineering can be solved using new numerical methods analyzed in this special issue.

This special issue is constructed by four guest editors who have important contributions in this field of work. Twenty-four papers were submitted to this special issue and each of them was reviewed by at least two referees. Thirteen of them were accepted to be published in line with the referees' suggestions.

These papers are interested in ordinary and partial differential equations, fractional differential equations, integral and integro-differential equations and some modelling problems. The methods used in these articles are numerical methods such as integral transform method, the quadratic finite element method, second-order time discrete scheme, the weighted and shifted Grünwald difference (WSGD) methods, the local discontinuous Galerkin (LDG) finite element method, discrete fractional method, alternating direction implicit finite volume element method, the finite difference method-like method, Galerkin approximation, matrix-collocation method and the Galerkin-like method.

We can summarize the published articles in this special issue as follows:

In the paper titled "Analytical and numerical investigation for the DMBBM equation" by Alharbi et al. [1], authors consider the nonlinear dispersive modified Benjamin-Bona-Mahony (DMBBM) equation which is a class of partial differential equations. Authors solve the DMBBM equation by means of the adaptive moving mesh PDEs (MMPDEs) method. In fact, the exact solution of the mentioned equation is computed by the aid of the extended Jacobian elliptic function expansion method.

Faraz et al. [2] present a study of "Integral transform method for a porous slider with magnetic field and velocity slip". This article is related to the injection of a viscous fluid in the existence of a transverse uniform magnetic field to decrease the sliding drag. This problem is characterized by system of Navier Stokes equations which is a system of partial differential equations. By using the integral transform method, the solutions are obtained by converting system of Navier Stokes equations into coupled equations.



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In the article “A numerical algorithm based on quadratic finite element for two-dimensional nonlinear time fractional thermal diffusion model” presented by Zhang et al. [3], Two-Dimensional Nonlinear Time Fractional Thermal Diffusion Model which corresponds to a class of partial differential equations is solved by utilizing a numerical scheme based on quadratic finite element. Indeed, the suggested method is obtained by combining the quadratic finite element method in space and a second-order time discrete scheme.

Shuaib’s et al. [4] study a modelling problem in their paper “Fractional analysis of viscous fluid flow with heat and mass transfer over a flexible rotating disk”. Authors investigate viscous fluid flow together with heat and mass transfer on a pliable rotating disk. The modelling problem corresponds to a system of ordinary differential equations. By using the similarity transformation, the problem is reduced to first order differential equation and thus the model is generalized by the help of the Caputo derivative sense.

Wang [5] presents the article “On Caputo-type cable equation: analysis and computation”. In this paper, Wang studies special case of nonlinear time fractional cable equation. Firstly, the existence, uniqueness, and regularity of the solution are investigated and then numerical solutions are obtained by utilizing the weighted and shifted Grünwald difference (WSGD) methods/the local discontinuous Galerkin (LDG) finite element methods.

In the paper entitled “Modelling of energy storage photonic medium by wave length based multivariable second-order differential equation” studied by Binesh [6], mathematical modelling regarding wave length-dependent of the differential energy change of a photon is studied. The multivariable second-order differential equation is obtained by means of transformations for energy and velocity. Making numerical analysis, specific solutions are estimated for different values of parameters in the considered model.

Khan et al. [7] investigate the tuberculosis infection cases in Pakistani province of Khyber Pakhtunkhwa from 2002 to 2017 in their article “A discrete model of TB dynamics in Khyber Pakhtunkhwa-Pakistan”. The stability analysis of the model is characterized by differential equation in the study. Also, a discrete fractional model is given by using the sense of Caputo fractional derivative. The numerical results of the model are gained for various parameters and their effect on the model is discussed.

In paper titled “An ADI finite volume element method for a viscous wave equation with variable coefficients” studied by Su et al. [8], the authors consider a viscous wave equation with variable coefficients and they solve this equation by combining the merits of finite volume element method and alternating direction implicit method. The proposed method is called as alternating-direction implicit (ADI) finite volume element method which is based on rectangular partition and bilinear interpolation.

In the next two papers, a method which is local in nature like finite difference methods is constructed by utilizing radial kernels and this method is applied to different two problems. One of them is on special partial differential equation “RBF-FD method for some dispersive wave equations and their eventual periodicity” presented by Uddin et al. [9]. Other one is on nonlinear partial integro-differential equations “RBF based localized method for solving nonlinear partial integro-differential equations” studied by Uddin et al. [10].

Yüzbaşı et al. [11] develop a Galerkin-like fractional approach to solve the Bagley-Torvik equation which is an ordinary fractional differential equation in the paper entitled “A Galerkin-type fractional approach for solutions of Bagley-Torvik equations”. By using this technique, the approximate solution is obtained in the form of a polynomial in the variable  $t = x^\alpha$ . Here,  $\alpha$  is selected to be a positive real parameter. Firstly, the solution form is written in the matrix form and it is substituted into the problem. And then the fractional problem is reduced to a system of linear algebraic equations by the use of inner product and initial conditions. Hence, the solutions of the last system give us the coefficients of the approximate solution of Bagley-Torvik equation.

In this paper “Crank-Nicolson ADI Galerkin finite element methods for two classes of Riesz space fractional partial differential equations” introduced by Chen [12], a numerical method is applied to two classes of fractional partial differential equations. The method is constructed by combining the Crank-Nicolson method in temporal direction, and the efficient alternating direction implicit Galerkin finite element methods.

Yıldız et al. [13] introduce a collocation technique based on Bell Polynomials for solving Fredholm Integro-Differential Equations in their article “Bell polynomial approach for the solutions of Fredholm integro-differential equations with variable coefficients”. The method converts the problem to a system of algebraic equations. Furthermore, error estimation technique is presented and comparisons are made with other methods.

**Conflicts of Interest:** The authors declare that they have no conflicts of interest to report regarding the present study.

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