Resource Allocation in Edge-Computing Based Wireless Networks Based on Differential Game and Feedback Control

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Abstract: In this paper, we have proposed a differential game model to optimally solve the resource allocation problems in the edge-computing based wireless networks. In the proposed model, a wireless network with one cloud-computing center (CC) and lots of edge services providers (ESPs) is investigated. In order to provide users with higher services quality, the ESPs in the proposed wireless network should lease the computing resources from the CC and the CC can allocate its idle cloud computing resource to the ESPs. We will try to optimally allocate the edge computing resources between the ESPs and CC using the differential game and feedback control. Based on the proposed model, the ESPs can choose the amount of computing resources from the CC using feedback control, which is affected by the unit price of computing resources controlled by the CC. In the simulation part, the optimal allocated resources for users' services are obtained based on the Nash equilibrium of the proposed differential game. The effectiveness and correctness of the proposed scheme is also verified through the numerical simulations and results.

Keywords: Resource allocation, edge computing, differential game, feedback control, Nash equilibrium.

1 Introduction

With the rapid growth of big data, there are varying and unpredictable patterns of network traffics in the next generation wireless networks [Huang, Fu, Yu et al. (2018)], and more and more users services cannot being satisfied under the traditional network architecture [Imran, Zoha and Abu-Dayya (2014); Hui, Zhou, Xu et al. (2020); Su, Lin, Zhou et al. (2015); Ramadan, Tawfik and Riad (2018)]. In order to effectively solve the high traffic and increasing data problem, the next generation wireless network should use the mobile computing technologies to provide users with better and satisfied network

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services [Walkowiak, Woźniak, Klinkowski et al. (2015)]. The mobile computing technologies include the cloud computing, fog computing and edge computing [Zhang, Chen and Li (2019); Huang, Bai, Liu et al. (2018)]. As an extension of cloud computing, edge computing can provide users with faster and more convenient services at the edge of the network, which has been considered as the chief component of the next generation wireless network. The next generation wireless network can be considered as an edge-computing based wireless network [Abawajy and Hassan (2017); Jiang, Xu, Cai et al. (2014); Liu, Yang, Yan et al. (2019)], which includes the traditional cloud computing and the edge-computing. The edge-computing based wireless networks can provide computing resources, store resources, and services supports to satisfy the increasing services requirements of users [Yu, Langar, Fu et al. (2018)], and to overcome the big data challenges [Ahmed and Ahmed (2016)].

Although in the edge-computing based wireless networks, system performance can be improved because of the higher resource usage efficiency of edge computing technique [Amjad, Rabby, Sadia et al. (2017)], it still needs to optimally allocate the network resources according to the available resource level to enhance the network throughput. Meanwhile, for the edge computing services providers, sometimes there are no enough computing resources available, because the capacities of the edge computing services providers are limited [Wei, Guo, Yu et al. (2019)]. The edge computing services providers should borrow the computing resources from the cloud computing center to satisfy the increasing services demands of users. In this paper, we are interested in finding a practical mechanism to achieve optimal computing resource allocation between the cloud computing center and edge computing services providers, in order to achieve the optimal solution of computing resources in the edge-computing based networks.

The resource allocation problem has been researched by lots of researchers [Yue, Sun and Liu (2018); Wang, Jiao, Li et al. (2017); Ren, Gui, Dai et al. (2017); Song, Yau, Yu et al. (2017), Okhovvat and Kangavari (2019)]. In Yue et al. [Yue, Sun and Liu (2018)], resource allocation in mobile edge computing is utilized based on the network economics to maximize the system efficiency. The edge computing server can serve users based a single-round double auction scheme, to satisfy the rationality and truthful of the system. In Wang et al. [Wang, Jiao, Li et al. (2017)], an online algorithm based on regularization technique is given in the edge computing to produce feasible solutions for edge computing resource allocation problem. The resource allocation problem is divided into a series of subproblems that can be solved in a time slot. The series of solutions generated in each time slot thus constitute a feasible solution to the original problem. In Ren et al. [Ren, Gui, Dai et al. (2017)], a distributed resource distribution network is constructed to provide the distribution services, then the total energy consumption and every service delay is analyzed for the proposed resource distribution networks. A hierarchical caching scheme is designed based on the fuzzy C-means clustering. In Song et al. [Song, Yau, Yu et al. (2017)], an QoS-based task distributing approach is presented to achieve efficiently task management.

Based on the above literature review, we can find that only a few researchers have studied the dynamic resource allocation problem in the edge-computing based wireless networks, falling short in addressing the challenges when the network is combined by the cloud computing and edge computing. In this paper, we try to use the differential game to model the dynamic resource allocation problem in the edge-computing based wireless networks. The main contributions of our paper are as follows:

(1) In this paper, an edge-computing based wireless networks with one cloud-computing center (CC) and a set of edge-computing services providers (ESPs) is researched. The ESPs can share the computing resource from the CC, and provide the computing services to the corresponding end users. During the resource leasing process, the CC controls the unit price for loaning out the computing resource. The ESPs control their amount of the borrowed computing resource from the CC, to minimize the cost and to grantee the performance of the end users.

(2) In order to describe the dynamic characteristics of network resources, the available computing resource in the proposed wireless networks that can be allocated are formulated as a differential equation, which is the dynamic state of the whole network.

(3) The Nash equilibriums to the proposed game model can be solved based on dynamic programming, and are considered as the optimal strategies of ESPs.

The whole paper is organized as follows. Section 2 describes the system model and formulates the differential game-based resource allocation problem. Section 3 gives out the Nash equilibrium solutions to the game and analyze the proposed game and solutions. Numerical simulations are given in Section 4 and it is concluded in Section 5.

2 System model

Considering an edge-computing based wireless networks with one cloud-computing center (CC) and a set $M = \{1, 2, \ldots, M\}$ of edge-computing services providers (ESPs). The ESPs can share the computing resource from the CC, and provide the computing services to the corresponding end users $N = \{1, 2, \ldots, N\}$. Let L denote the services links set between ESPs and the corresponding users. The services links set L can be expressed as follows:

$$L = \{l_{ij}\} = \begin{bmatrix} l_{11} & l_{11} & \cdots & l_{1N} \\ l_{21} & l_{22} & \cdots & l_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ l_{M1} & l_{M2} & \cdots & l_{MN} \end{bmatrix}. \tag{1}$$

In the above equation, if $l_{ij} = 0$, there is no link between the ESP i and the user j, and if $l_{ij} = 1$, the user j is served by the ESP i. Let R_l denote the computing resource of the link l, which is the computing resource of the user j obtained from the ESP i. If the user j has a computing resource $R_l \geq 0$, then the instantaneous payment that a user pays for having a specific amount of computing resource can be defined as follows:

$$P_l(t) = \alpha_l \, ln\left(\frac{R_l(t)}{\overline{R}}\right), \tag{2}$$

where α_l for $l \in L$ is a positive parameter. The computing resource $R_l(t)$ is nonnegative and bounded by a minimum computing resource constraint $\overline{R}$. The instantaneous price becomes negative when the computing resource $R_l(t)$ is lower than the constraint $\overline{R}$. From a rational player's perspective, users always want to acquire as much computing resource as possible with a lower cost. The instantaneous payments given by the users can be considered as the profit of the ESPs.

For the ESPs, the cost during services providing can be divided into two parts. One is the computing resource cost. In order to have a specific amount of computing resource, the ESPs should buy from the CC. Then the resource cost can be considered as the resource payments from the ESPs to CC. Here, we use $c_m\ (t)$ to denote the unit price of computing resources controlled by the CC. Then the resource cost can be expressed as follows:

$$U_l\ (t) = c_m\ (t) R_l\ (t). \tag{3}$$

As the ESPs should store the computing resource locally, which will cause additional cost to the ESPs, the ESPs will have their strategies to reduce the store cost during the resource allocation process. Each ESP should have its strategies $u_l\ (t)$ to solve the computing resource store problem, and the cost for reducing the store influence can be expressed as follows:

$$I_l\ (t) = e_l u_l\ (t)^2. \tag{4}$$

Let $x\ (t) \in R^+$ denote the level of avail computing resource of the whole network at time t, and the dynamics of $x\ (t)$ is governed by the following stochastic differential equation:

$$dx\ (t) = -\left[\sum_{l\in L} \beta_l R_l\ (t) - \sum_{l\in L} \pi_l u_l\ (t)[x\ (t)]^{\frac{1}{2}} - \delta x\ (t)\right] dt - \sigma x\ (t) dz\ (t), \tag{5}$$

where σ is a noise parameter and $z(t)$ is a Wiener process. $\beta_l R_l\ (t)$ is the computing resource influences generated by link $l \in L$. $\pi_l u_l\ (t)[x\ (t)]^{\frac{1}{2}}$ is the amount of store influence reduced. δ is a parameter to denote the background noise of the system.

In the edge-computing based wireless network, each EPS controls the amount of computing resource buying from the CC to minimize the cost and to grantee the performance of the end users. The instantaneous objective of link l at time t can be expressed as:

$$\alpha_l\ ln\ \left(\frac{R_l(t)}{\bar{R}}\right) - c_m\ (t) R_l\ (t) - e_l u_l\ (t)^2 - h_l x\ (t), \tag{6}$$

where h_l is a constant parameter.

Assuming the planning horizon is $[0, T]$, each ESP seeks to minimize the integral of its instantaneous objective Eq. (6) over the planning horizon subject to the computing resource given in Eq. (5) as follows:

$$\max_{R_l(t), u_l(t)} E_0 \left\{ \int_0^T \begin{bmatrix} \alpha_l\ ln\left(\frac{R_l(t)}{\bar{R}}\right) - c_m(t) R_l(t) \\ -e_l u_l(t)^2 - h_l x(t) \end{bmatrix} \times e^{-rt} dt - g_l[\bar{x} - x(T)] e^{-rT} \right\}. \tag{7}$$

subject to

$$dx\ (t) = -\left[\sum_{l\in L} \beta_l R_l\ (t) - \sum_{l\in L} \pi_l u_l\ (t)[x\ (t)]^{\frac{1}{2}} - \delta x\ (t)\right] dt - \sigma x\ (t) dz\ (t), \tag{8}$$

where r is the discount factor. It is possible that T may be very large. At time T, the terminal value associated with the state is denoted by $g_l[\bar{x} - x\ (T)]$, where $g_l \geq 0$ and $\bar{x} \geq 0$.

3 Feedback equilibrium solutions

In this section, we discuss the solutions to the proposed game in Eqs. (7) and (8). Under a non-cooperative framework, feedback Nash equilibrium solutions to the proposed game problem in Eqs. (7) and (8) can be characterized by the following theorem [Xu, Cao, Yang et al. (2017)].

Definition 1 A set of feedback strategies $\{R_l^*(t) = \phi_l(t,x), u_l^*(t) = \mu_l(t,x)\}$ provides a Nash equilibrium to the game in Eqs. (7) and (8), if there exists suitably smooth functions $V_l(t,x)$ satisfying the following partial differential equation:

$$-V_t^{\,l}(t,x) - \frac{\sigma^2 x^2}{2} V_{xx}^l(t,x) = \left[\alpha_l \ln\left(\frac{\phi_l(t,x)}{\overline{R}}\right) - c_m(t)\phi_l(t,x) - e_l u_l^2 - h_l x\right] \times e^{-rt} + V_x^{\,l}(t,x)\left[-\beta_l \phi_l(t,x) - \sum_{k\in L,k\neq l} \beta_k R_k(t) + \sum_{k\in L,k\neq l} \pi_k u_k(t) x^{\frac{1}{2}} + \pi_l u_l x^{\frac{1}{2}} + \delta x\right] \quad (9)$$

$$V_l(T,x) = -g_l[x(T) - \overline{x}]e^{-rT}. \quad (10)$$

Performing the indicated maximization in Eq. (9) yields,

$$\mu_l(t,x) = \frac{\pi_l}{2e_l} V_x^{\,l}(t,x) e^{rt} x^{\frac{1}{2}}, \quad (11)$$

$$\phi_l(t,x) = \frac{\alpha_l}{c_m(t)}\overline{R} - \frac{\beta_l}{c_m(t)} V_x^{\,l}(t,x) e^{rt}. \quad (12)$$

Based on definition 1, we can find that there exist the optimal solutions for the resource allocation problems in Eq. (7), and the optimal solutions can be obtained through the feedback control and Nash equilibrium, using the Bellman dynamic programming theory. Meanwhile, through definition 1, we can find the optimal strategy of each ESP.

Lemma 1 The computing resource allocation problem realizes a Nash equilibrium, if the computing resource of the services link l can be expressed by,

$$\phi_l(t,x) = \frac{\alpha_l}{c_m(t)}\overline{R} - \frac{\beta_l}{c_m(t)} V_x^{\,l}(t,x) e^{rt}. \quad (13)$$

Simplifying Eq. (13), we get the optimal allocated resource can be expressed by,

$$\phi_l(t,x) = f_l + q_l V_x^{\,l}(t,x) e^{rt}, \quad (14)$$

where $f_l = \frac{\alpha_l \overline{R}}{c_m(t)}$ and $q_l = -\frac{\beta_l}{c_m(t)}$. It can be seen from Eq. (14) that the optimal allocated resource for each ESP is affected by the resource sharing price $c_m(t)$, which is controlled by the CC. Once the CC makes a decision on the price $c_m(t)$, the ESPs can control their resource requirements to minimize the cost.

Lemma 2 The systems Eqs. (9) and (10) admit a solution,

$$V_l(t,x) = [A_l(t)x + C_l(t)]e^{-rt}, \quad (15)$$

where $\{A_1(t), A_2(t), \ldots, A_L(t)\}$ satisfies the following set of constant coefficient quadratic ordinary differential equations,

$$A_l'(t) = (r-\delta)A_l(t) - A_l(t)\sum_{k\in L,k\neq l} \frac{\pi_k^2}{2e_k} A_k(t) - \frac{\pi_l^2}{4e_l} A_l(t)^2 + h_l, \quad (16)$$

$$A_l(T) = -g_l. \quad (17)$$

And $\{C_1(t), C_2(t), \ldots, C_L(t)\}$ is given by,

$$C_l'(t) = rC_l(t) - \alpha_l \ln\left(f_l + q_l A_l(t)\right) + \left[c_m(t)q_l + \beta_l f_l + \sum_{k\in L,k\neq l}\left(\beta_k f_k + \beta_k q_k A_k(t)\right)\right] A_l(t) + \beta_l q_l A_l(t)^2 + \alpha_l \ln \overline{R} + c_m(t) f_l, \quad (18)$$

$$C_l(T) = g_l \overline{x}. \quad (19)$$

Proof See “Appendix 1”.

The corresponding feedback Nash equilibrium of the game in Eqs. (7) and (8) can be obtained as follows:

$$\mu_l(t,x) = \frac{\pi_l}{2e_l} A_l(t) x^{\frac{1}{2}}, \tag{20}$$

$$\phi_l(t,x) = f_l + q_l A_l(t), \tag{21}$$

where $A_l(t)$ is given by Eqs. (16) and (17).

Lemma 3 The optimal strategies for computing resource store influence is given by,

$$\mu_l(t,x) = \frac{\pi_l}{2e_l} V_x{}^l(t,x) e^{rt} x^{\frac{1}{2}}, \tag{22}$$

where $V_x{}^l(t,x)$ can be obtained from Eq. (15).

4 Numerical simulations

In this section, we will analyze the proposed model. We consider a wireless network consisting of 1 CC, 3 ESPs and 5 end users. During the time horizon, we assume that user 1 and user 3 are simultaneously serviced by ESP 1 and ESP 3; user 2 and user 5 are serviced by ESP 2; user 4 is serviced by ESP 3. The valid link set can be expressed as $L = \{11,13,22,25,31,33,34\}$. The number results for the optimal resource allocation of each link will be given under the Matlab simulation environment. The parameters are given in Tab. 1.

Table 1: Table caption

r				0.25			
δ				0.1			
$\bar{R}$				1000			
π_l	0.15	0.20	0.25	0.30	0.35	0.40	0.45
e_l	0.10	0.12	0.14	0.16	0.18	0.20	0.22
α_l	0.5	0.55	0.6	0.65	0.7	0.75	0.8
β_l	1	1.5	2	2.5	3	3.5	4

The variations of $A_l(t)$ over the time interval $[0,10]$ are presented in Fig. 1. The different lines in Fig. 1 stand for the different links in the wireless network, which means the computing services links between the users and ESPs. As can be seen from Eq. (14), the Nash equilibrium $\phi_l(t,x)$ is a function of $A_l(t)$, variation of $A_l(t)$ will significantly reflect on $\phi_l(t,x)$, which means the optimal computing resource allocated to each link will be modified by$A_l(t)$. On the other hand, the system solution $V_l(t,x)$ will also be influenced by $A_l(t)$.

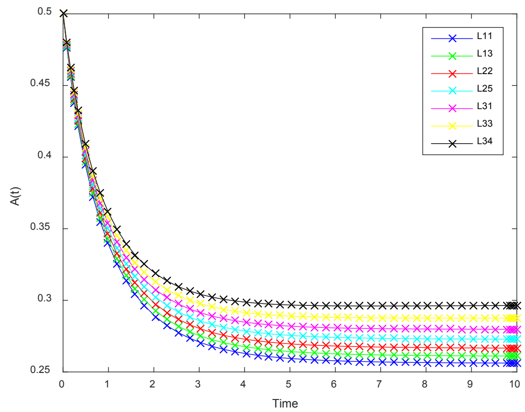

Figure 1: Values of $\boldsymbol{A_l}\ (\boldsymbol{t})$ over the time interval $[\mathbf{0},\ \mathbf{10}]$

The optimal computing resource $\phi_l(t,x)$ is shown in Fig. 2. The different lines in Fig. 2 stand for the optimal allocated computing resource for each computing services link in the wireless network. In order to formulate the relationship between the CC and ESPs, the optimal computing resource for each link between the users and ESPs is affect by the resource sharing price, which is controlled by the CC. In Fig. 2, in order to simplify the simulations process, the price for using the resource of the CC is set to be 10 unit. The optimal computing resource for each link can be obtained through the proposed scheme. Generally, the obtained Nash Equilibrium $\phi_l(t,x)$ for each services link is always converge with the time increasing. Each link has its own computing resource to minimize the services cost. Because the initial parameters setup are different among ESPs, they will have different cost based on their own requirements, and the computing resource will be different. Meanwhile, users will have different requirements on the computing services, which also effect the parameters setup and optimal computing resource allocation. We also simulate the total computing resource of each ESP, which is shown in Fig. 3. The different lines in Fig. 3 stand for the optimal total computing resource for each ESP.

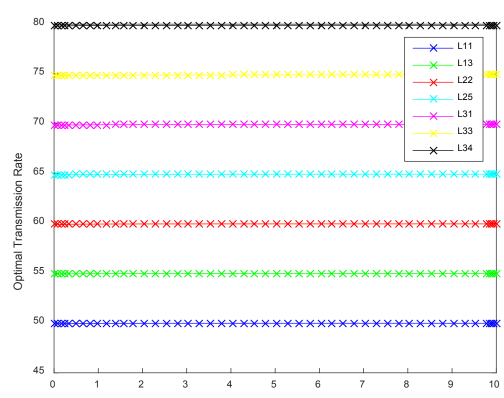

Figure 2: Values of $\boldsymbol{\phi_l(t,x)}$ over the time interval $[\mathbf{0},\mathbf{10}]$

The influence of the resource sharing price is shown in Figs. 4 and 5. In Fig. 4, the different lines stand for the different links in the wireless network, under different resource price. It can be seen from Fig. 4 that the computing resource of each link decreases when the resource sharing price increases. The Fig. 5 gives the optimal computing resource achieved by each ESP, which is defined in different drawing in the figure. Seen from the Fig. 5, the total computing resource of each ESP decreases with the increase of the resource sharing price, because the ESPs need to pay for the CC for resource sharing. When the price is increased by the CC, the ESPs would decrease the computing resource to reduce the services providing cost.

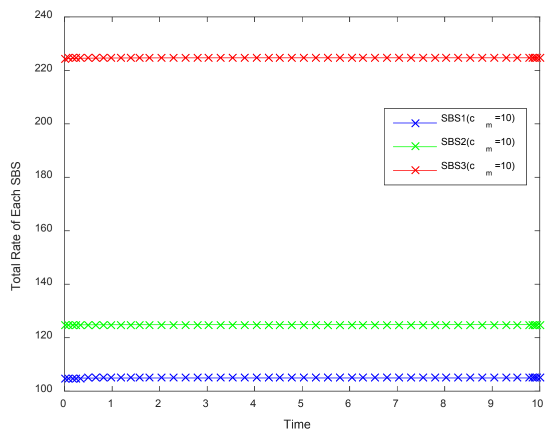

Figure 3: Total transmission rate of each SBS over the time interval $[\mathbf{0}, \mathbf{10}]$

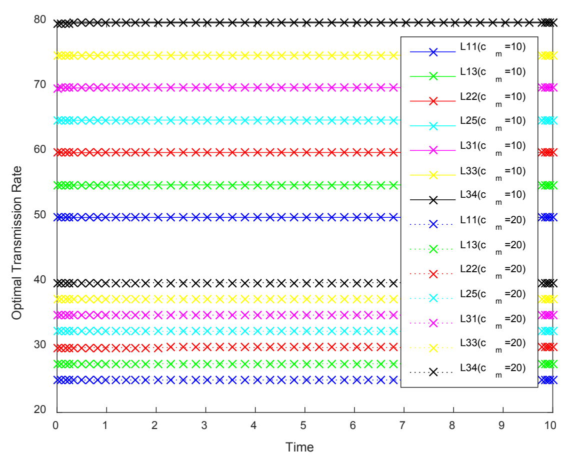

Figure 4: Variations of $\boldsymbol{\phi_l(t, x)}$ under different resource price

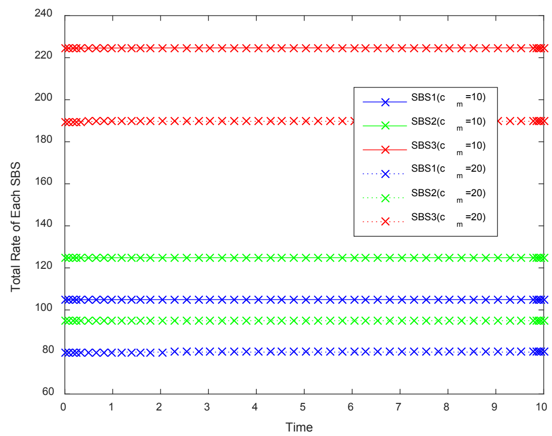

Figure 5: Total transmission rate of each SBS under different resource price

5 Conclusion

In this paper, we research the computing resource allocation problem in edge-computing based wireless network using the differential game theory, where the ESP can control their requirements of computing resource from the CC, control by the resource sharing price of the CC. The objective functions are formulated for the ESPs to achieve optimal computing resource control and the available computing resources of CC that can be leased to the ESPs are the constraints. Nash equilibriums are obtained for proposed objective functions of the ESPs. Based on the Nash equilibrium, we find that the CC's decision on the resource sharing price can affect the decisions of required computing resource. Numerical simulations are presented to illustrate the proposed mechanisms. In this paper, although we have formulated the relationship between the CC and ESPs, the resource sharing price of the CC is not controlled by a maximization/minimization process. The price is set to be some constant value for the computing resource allocation problem. In the future works, more research on the optimal control of price may introduce, to dynamically control the resource price and to dynamically affect the allocation of computing resources. Meanwhile, some more solutions, such as the cooperative solutions of the ESPs are expected.

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Appendix proof of lemma 2

Based on Eq. (15), we can get the following equations,

$$V_t{}^l(t,x) = [A_l{}'(t)x + C_l{}'(t)]e^{-rt} - r[A_l(t)x + C_l(t)]e^{-rt}, \tag{23}$$

$$V_x{}^l(t,x) = A_l(t)e^{-rt}. \tag{24}$$

Using Eqs. (11), (14), (23) and (24), system Eqs. (9) and (10) can be expressed as follows:

$$\begin{aligned} r[A_l(t)x + C_l(t)] - [A_l{}'(t)x + C_l{}'(t)] &= \alpha_l \ln\left(\frac{\left(f_l + q_l A_l(t)\right)}{\overline{R}}\right) - c_l\left(f_l + q_l A_l(t)\right) \\ -e_l\left(\frac{\pi_l}{2e_l}A_l(t)x^{\frac{1}{2}}\right)^2 - h_i x + A_l(t)&\Bigg[-\beta_l\left(f_l + q_l A_l(t)\right) - \sum_{k\in L,k\neq l}\beta_k\left(f_k + q_k A_k(t)\right) \\ &+ \textstyle\sum_{k\in L,k\neq l}\pi_k\left(\frac{\pi_k}{2e_k}A_k(t)x^{\frac{1}{2}}\right)[x]^{\frac{1}{2}} + \pi_l\left(\frac{\pi_l}{2e_l}A_l(t)x^{\frac{1}{2}}\right)x^{\frac{1}{2}} + \delta x\Bigg], \end{aligned} \tag{25}$$

$$A_l(T)x + C_l(T) = -g_l[x(T) - \overline{x}]\,. \tag{26}$$

For Eqs. (25) and (26) to hold, it is required that,

$$A_l{}'(t) = (r-\delta)A_l(t) - A_l(t)\textstyle\sum_{k\in L,k\neq l}\frac{\pi_k{}^2}{2e_k}A_k(t) - \frac{\pi_l{}^2}{4e_l}A_l(t)^2 + h_i, \tag{27}$$

$$A_l(T) = -g_l, \tag{28}$$

$$\begin{aligned} C_l{}'(t) &= rC_l(t) - \alpha_l \ln\left(f_l + q_l A_l(t)\right) + \beta_l q_l A_l(t)^2 \\ &+ \left[c_l q_l + \beta_l f_l + \textstyle\sum_{k\in L,k\neq l}\left(\beta_k f_k + \beta_k q_k A_k(t)\right)\right]A_l(t) + \alpha_l \ln\overline{R} + c_l f_l, \end{aligned} \tag{29}$$

$$C_l(T) = g_l\overline{x}. \tag{30}$$

Solving $\{A_1(t), A_2(t), \dots, A_L(t)\}$ in (27)-(28) and upon substituting them into Eqs. (29) and (30) yields a system of linear first order differential equations,

$$C_l'(t) = rC_l(t) + B_l(t), \tag{31}$$

where,

$$B_l(t) = -\alpha_l \ln\big(f_l + q_l A_l(t)\big) + \beta_l q_l A_l(t)^2$$
$$+[c_l q_l + \beta_l f_l + \textstyle\sum_{k\in L, k\neq l}\big(\beta_k f_k + \beta_k q_k A_k(t)\big)]A_l(t) + \alpha_l \ln \overline{R} + c_l f_l, \tag{32}$$

Since $C_l(t)$ is independent of $C_k(t)$ for $l \neq k$, $C_l(t)$ can be solved as:

$$C_l(t) = e^{rt}\left[\int_0^T B_l(s)e^{-rs}ds + C_l{}^0\right], \tag{33}$$

where $C_l{}^0 = g_l\overline{x}e^{-rT} - \int_0^T B_l(s)e^{-rs}ds$.

Hence Lemma 2 follows.