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Non-Exchangeable Error Compensation for Strapdown Inertial Navigation System in High Dynamic Environment

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Abstract: Strapdown non-exchangeable error compensation technology in high dynamic environment is one of the key technologies of strapdown inertial navigation system. Mathematical platform is used in strapdown inertial navigation system instead of physical platform in traditional platform inertial navigation system, which improves reliability and reduces cost and volume of system. The maximum error source of attitude matrix solution is the non-exchangeable error of rotation due to the non-exchangeable of finite rotation of rigid bodies. The rotation non-exchangeable error reaches the maximum in coning motion, although it can be reduced by shortening the correction period and increasing the real-time calculation. The equivalent rotation vector method is used to modify the attitude to reduce the coning error in this paper. Simulation experiments show that the equivalent rotation vector method can effectively suppress the non-exchangeable error and improve the accuracy of attitude calculation.

Keywords: Strapdown, error compensation, coning motion, equivalent rotation vector, simulation experiments.

1 Introduction

Strapdown inertial navigation system uses mathematical platform-attitude matrix to replace the physical platform of traditional platform inertial navigation system, which brings many advantages to the system, such as greatly reducing the volume and cost of the whole system, facilitating installation, maintenance and replacement of inertial instruments, facilitating the use of redundancy configuration of inertial instruments to improve the system and reducing energy cost and improving reliability. Therefore, the elimination of non-exchangeable error in attitude matrix calculation and paddling effect error in velocity calculation is one of the core technologies of SINS. Because of the non-exchangeable of

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finite rotation of rigid body, the biggest error source of attitude matrix solution is the non-exchangeable error of rotation. The error of rotation non exchangeability reaches the maximum and is called coning error under coning motion. The coning error can be reduced by shortening the correction period [Hong, Lee, Chun et al. (2005)]. The correction period is too small, which will increase the real time computation of the navigation computer. Since the direction cosine method, Euler method and quaternion method all have principle errors for coning motion, people are devoted to the research of algorithms which can effectively suppress the non-exchangeable errors caused by coning motion. Using equivalent rotation vector to correct attitude is an effective way to reduce coning error [Jiang and Yu (2016); Jia, Li, Qin et al. (2017)].

Tang's orientation vector differential equation in Tang et al. [Tang and Chen (2014)] established a comprehensive theoretical basis for the calculation of attitude matrix of strapdown inertial navigation system. The rate of change of rotation vector is the sum of angular velocity vector of inertial measurement (output of gyroscope) and non-interchangeable velocity vector calculated. The latter is an important factor affecting the accuracy of strapdown inertial navigation system. Therefore, in high dynamic angular velocity environment, in order to prevent the accumulation of attitude error, the non-interchangeable velocity must be considered. The rate vector is compensated. Coning compensation algorithm is usually used to compensate non-exchangeable vectors. In reference Liu et al. [Liu and Yu (2011)] another form of rotating vector differential equation is proposed, and the inherent error of simplified form of equation commonly used in strapdown inertial navigation system is estimated. In order to improve the system accuracy, there are two choices: one is to collect the output signal of gyro at high speed, using simple coning algorithm; the other is to reduce the sampling rate of the output signal of gyro appropriately, using complex but accurate coning algorithm. Goshen et al. [Goshen and Bar (1992a, 1992b)] put forward the theory of fast loop with higher frequency for iterative calculation of equivalent vector and slow loop with lower iteration frequency for calculation of direction cosine matrix or quaternion after using equivalent rotation vector. The above theoretical research lays the classical theoretical foundation for attitude updating algorithm.

Coning compensation algorithm with high precision and less computation is always the goal pursued by people [Zhang, Chen, Fu et al. (2014); Wang, Wu, Wang et al. (2015)]. Rhee proposed a new strapdown attitude algorithm, which considers the equivalent rotation vector updating and quaternion attitude updating separately. Firstly, an optimized three-sample algorithm [Rhee, Abdel and Speyer (2004)] under coning motion was proposed. On the basis of the three subsample algorithm, Li et al. proposed the four sub algorithm [Li, Li, Zhang et al. (2018)]. These two algorithms do not utilize the gyro output signal of the previous coning compensation period. Liu et al. [Liu, Wang, Li et al. (2017); Ning, Zhang, Gui et al. (2018)] proposed a kind of algorithm which utilizes the gyro output signal of the previous compensation period. This kind of algorithm includes all possible cross-multipliers of the gyro output signal in the coning compensation calculation period, and utilizes the gyroscope in the previous period. Wang et al. [Wang, Qin, Yang et al. (2017)] proposed an idea of optimizing accuracy and minimizing computational complexity in pure coning motion environment. The cross-multiplication of multi-sensor data in the proposed algorithm has the simplest form and the optimum coefficients, and minimizes the coning compensation error [Zhang, Li, Huang et al. (2019)]. Hong et al. [Hong, Chun, Kwon et al. (2008)] proposed the optimization index of coning algorithm, and analyzed the relationship between the algorithm error after coning compensation and the power R of the compensation period [Yu, Lee and Park (1999)]. Han et al. [Han, Dou, Tong et al. (2017)] proposed the design method of attitude, velocity and position integration algorithm used in strapdown inertial navigation system, and gave the overall design requirements.

Many domestic scholars have also studied coning compensation. Reference Ham et al. [Ham and Brown (1983)] have designed a strapdown inertial navigation attitude algorithm based on the fact that the cross-multiplication of two angular increments with the same time interval contributes equally to the coning compensation under the condition of coning motion. With the same accuracy obtained by the coning compensation, the calculation amount is less; at the same time, using this characteristic, the former is deduced. An algorithm for compensating the corner increment of the compensation cycle improves the accuracy of compensation. Literature [Yang, Li, Luo et al. (2016); Yang, Liu, Meng et al. (2018)] enriches and improves the design method of new coning compensation algorithm, gives a new method for evaluating the accuracy of coning compensation algorithm, studies the residual error characteristics of paddle compensation and gives the evaluation index of the accuracy of the algorithm, and studies the similarity between coning compensation and paddle compensation, which is more comprehensive for the first time. The similarity between the two is described. References Yan et al. [Yan, Yang, Weng et al. (2017); Gao, Li, Song et al. (2018)] apply the angular incremental output of gyroscope in the current iteration cycle and the first two iteration cycles. According to the number of gyroscope samples in the current iteration cycle, three new attitude algorithms are proposed, which provides a new idea for improving the rotation vector algorithm. An improved MDS-MAP localization algorithm based on weighted clustering and heuristic merging for anisotropic wireless networks was proposed in Wang et al. [Wang, Qiu and Tu (2019)]. A new general coning compensation algorithm is deduced in Deng et al. [Deng, Sun, Ding et al. (2019)]. The performances of several general coning compensation algorithms are analyzed and compared, and some meaningful conclusions are obtained.

This paper gives the programming formula of SINS, which includes the calculation of attitude matrix, velocity, gravity, longitude and latitude. In this chapter, the coning compensation and paddle effect compensation in attitude calculation are studied in depth. A new general form of coning compensation and paddle effect compensation is proposed. The errors of coning compensation and paddle effect compensation are analyzed and compared.

2 Coning compensation method

2.1 SINS principles

Strapdown Inertial Navigation System (SINS) is an autonomous dead reckoning system which uses inertial sensors, reference direction and initial speed, position, attitude matrix and quaternion to determine the attitude, position and speed of carrier. It consists of gyroscope, accelerometer, computer and display device. Its principle block diagram is shown in Fig. 1. Because SINS does not have a physical platform, it is necessary to simulate the platform coordinate system function of SINS. This can be achieved by calculating the attitude of three orthogonally mounted gyroscopes' angular rate information through the programmed navigation program in the computer. With the mathematical platform, the specific force information of accelerometer in three directions can be transformed into the specific force information of navigation coordinate system, and then the velocity and position of the carrier can be calculated in navigation coordinate system by using integral technology. According to the different methods of selecting the mathematical platform of SINS, it can be divided into: SINS in inertial coordinate system; SINS in swimming azimuth coordinate system; SINS in free azimuth coordinate system; SINS in geographical coordinate system. In this thesis, the strapdown inertial navigation system based on geographic coordinate system is mainly studied.

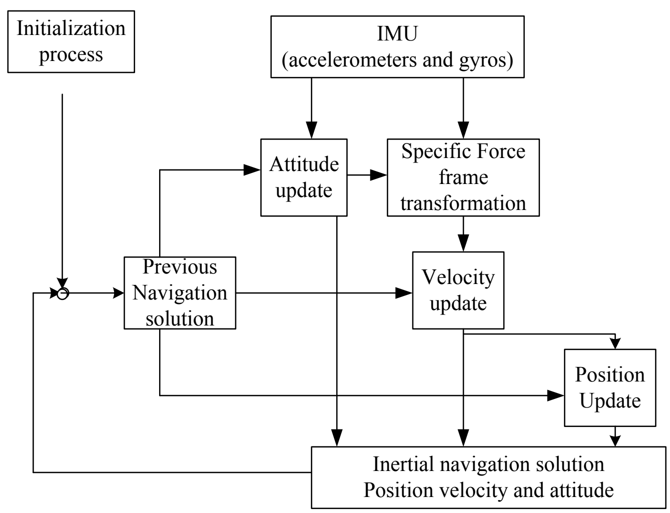


Figure 1: Schematic of inertial navigation processor

2.2 Attitude calculation with quaternion

Quaternions are hyper complex numbers consisting of one real unit and three imaginary units, which contain four real elements in the following form

$$Q = q_0 + q_1 i + q_2 j + q_3 k \tag{1}$$

where q_0, q_1, q_2, q_3 is four real numbers.

According to the rigid body rotation theory, in order to solve the attitude matrix from the carrier coordinate system to the navigation coordinate system, the following Quaternion Differential Equations need to be solved:

$$\dot{Q} = \frac{1}{2} M^*(\omega_b) Q \tag{2}$$

where

$$M^*(\omega_b) = \begin{bmatrix} 0 & -\omega_{nb}^{bx} & -\omega_{nb}^{by} & -\omega_{nb}^{bz} \\ \omega_{nb}^{bx} & 0 & \omega_{nb}^{bz} & -\omega_{nb}^{by} \\ \omega_{nb}^{by} & -\omega_{n}^{bz} & 0 & \omega_{nb}^{bx} \\ \omega_{nb}^{bz} & \omega_{nb}^{by} & -\omega_{nb}^{bx} & 0 \end{bmatrix} \quad Q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{3}$$

where ω_{nb}^{b} is the projection of the rotation angular rate of the carrier coordinate system relative to the navigation coordinate system on the carrier coordinate system can be obtained by the angular rate transformation of the gyroscope output. The transformation relations are as follows:

$$\omega_{nb}^{b} = \omega_{ib}^{b} - \omega_{ie}^{b} - \omega_{en}^{b} = \omega_{ib}^{b} - C_n^b(\omega_{ie}^{n} + \omega_{en}^{n}) \tag{4}$$

where ω_{ib}^{b} is the gyro output angle rate; C_n^b is the attitude matrix from navigation frame to body frame; ω_{ie}^{n} is the projection of the angular velocity of the earth coordinate system relative to the inertial coordinate system in the navigation coordinate system; ω_{en}^{n} is the projection of the angular rate of the navigation coordinate system relative to the earth coordinate system in the navigation coordinate system; ω_{ie}^{n}、ω_{en}^{n} can be expressed as follows:

$$\omega_{ie}^{n} = [0 \quad \omega_{ie}\cos L \quad \omega_{ie}\sin L]^T \tag{5}$$

$$\omega_{en}^{n} = [-V_n/R_n \quad V_e/R_e \quad V_e \tan L/R_e]^T \tag{6}$$

where L is the latitude; V_e, V_n is the eastern velocity and north velocity; R_n is the meridional curvature radius, R_e is Curvature radius of normal plane perpendicular to meridian.

Formula (2) can be solved by using the approach of the Bi Ka approximation.

$$Q(t) = e^{\frac{1}{2}[\Delta\theta]} Q(0) \tag{7}$$

where

$$[\Delta\theta] = \int M^*(\omega_b)dt = \begin{bmatrix} 0 & -\Delta\theta_x & -\Delta\theta_y & -\Delta\theta_z \\ \Delta\theta_x & 0 & \Delta\theta_z & -\Delta\theta_y \\ \Delta\theta_y & -\Delta\theta_z & 0 & \Delta\theta_x \\ \Delta\theta_z & \Delta\theta_y & -\Delta\theta_x & 0 \end{bmatrix} \tag{8}$$

The form of trigonometric functions is

$$Q(t) = \left\{ \cos\frac{\Delta\theta_0}{2} I + \frac{\sin\frac{\Delta\theta_0}{2}}{\Delta\theta_0}[\Delta\theta] \right\} Q(0) \tag{9}$$

where $\Delta\theta_0 = \sqrt{(\Delta\theta_x)^2 + (\Delta\theta_y)^2 + (\Delta\theta_z)^2}$, $Q(0)$ is the initial quaternion.

Due to the influence of calculation error and non-exchangeable error, the quaternion calculated by formula (9) will lose its normality, that is, its norm is no longer equal to 1, and the quaternion calculated needs to be normalized periodically. The processing formulas are as follows:

$$q_i = \frac{\hat{q}_i}{\sqrt{\hat{q}_0^2 + \hat{q}_1^2 + \hat{q}_2^2 + \hat{q}_3^2}},\ i = 0,1,2,3 \tag{10}$$

where, q_i represents the normalized quaternion, $\hat{q}_i$ represents the calculated quaternion.

The attitude matrix C_n^b can be obtained from normalized q_i substituted in Eq. (11)

$$C_n^b = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_0q_3) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\ 2(q_1q_3 + q_0q_2) & 2(q_2q_3 - q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \tag{11}$$

Attitude angle extraction for upper form can be expressed as follows:

$$\begin{bmatrix} P \\ R \\ H \end{bmatrix} = \begin{bmatrix} \arctan(\frac{C_{23}}{\sqrt{C_{21}^2 + C_{22}^2}}) \\ \arctan(-\frac{C_{13}}{C_{33}}) \\ \arctan(\frac{C_{21}}{C_{22}}) \end{bmatrix} \tag{12}$$

where P is the pitching angle, R is the rolling angle, H is the heading angle.

The initial attitude matrix can be obtained from substituting P_0, R_0, H_0 in the following formula:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} \cos R\cos H + \sin R\sin P\sin H & -\cos R\sin H + \sin P\sin R\cos H & -\cos P\sin R \\ \cos P\sin H & \cos P\cos H & \sin P \\ \sin R\cos H - \sin P\cos R\sin H & -\sin R\sin H - \sin P\cos R\cos H & \cos P\cos R \end{bmatrix} \tag{13}$$

The initial quaternion can be obtained from substituting P_0, R_0, H_0 in the following formula $Q(0)$:

$$Q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \cos\frac{P}{2}\cos\frac{R}{2}\cos\frac{H}{2} + \sin\frac{P}{2}\sin\frac{R}{2}\sin\frac{H}{2} \\ \sin\frac{P}{2}\cos\frac{R}{2}\cos\frac{H}{2} + \cos\frac{P}{2}\sin\frac{R}{2}\sin\frac{H}{2} \\ \cos\frac{P}{2}\sin\frac{R}{2}\cos\frac{H}{2} - \sin\frac{P}{2}\cos\frac{R}{2}\sin\frac{H}{2} \\ \cos\frac{P}{2}\cos\frac{R}{2}\sin\frac{H}{2} - \sin\frac{P}{2}\sin\frac{R}{2}\cos\frac{H}{2} \end{bmatrix} \tag{14}$$

Equivalent rotation vector method

In mechanics, the finite rotation of a rigid body is not exchangeable. In order to eliminate the non-exchangeable error, the equivalent rotation vector algorithm can be used. The equivalent rotation vector method is divided into two steps: (1) calculation of rotation vector. The rotation vector describes the change of the attitude of the carrier; (2) the updating of the four-element number. The four-element number describes the current position of the carrier relative to the reference coordinate system.

The general method of updating four variables is:

$$Q(t+h) = Q(t) * \Delta q \tag{15}$$

where $Q(t+h)$ and $Q(t)$ represent the body attitude quaternion at $t+h$ and t, Δq is the updating quaternion, the relationship with equivalent rotation vector Φ is as follows:

$$q(h) = \begin{bmatrix} C \\ \Phi_x S \\ \Phi_y S \\ \Phi_z S \end{bmatrix} \tag{16}$$

where $C = \cos(\frac{1}{2}\Phi_0)$, $S = \frac{1}{\Phi_0}\sin(\frac{1}{2}\Phi_0)$, $\Phi_0 = |\Phi|$ is the modulus of the rotation vector, Φ_x、 Φ_y、 Φ_z represents the project of rotation vector on axis of carrier coordinate system.

The calculation of rotation vector is based on rotating vector differential equation, which can be expressed as

$$\dot{\Phi} = \omega + \frac{1}{2}\Phi \times \omega + \frac{1}{\Phi_0^2}\left[1 - \frac{\Phi_0 \sin \Phi_0}{2(1-\cos\Phi_0)}\right]\Phi \times (\Phi \times \omega) \tag{17}$$

where, $\Phi_0 = |\Phi|$; $[\Phi \times]$ is Φ skew symmetric matrix.

The above formula can be expressed as follows when Φ is small enough

$$\dot{\Phi} = \omega + \frac{1}{2}\Phi \times \omega + \frac{1}{12}\Phi \times (\Phi \times \omega) \tag{18}$$

Neglecting the three order multiplication, only the first two items can be used in practical applications.

$$\dot{\Phi} = \omega + \frac{1}{2}\Phi \times \omega \tag{19}$$

Integrating the upper formula

$$\Phi = \int \omega dt + \frac{1}{2}\int \alpha \times \omega dt = \alpha + \delta\Phi \tag{20}$$

where

$$\alpha = \int \omega dt \tag{21}$$

$$\delta\Phi = \frac{1}{2}\int \alpha \times \omega dt \tag{22}$$

where $\delta\Phi$ is the non-reciprocal vector that needs compensation, which is also known as the conic effect. Equivalent rotation vector Φ substituting $\Delta\theta$ of quaternion differential equation, the non-exchangeable error can be eliminated.

3 Design of coning error compensation algorithm

Coning effect is a geometric phenomenon of rigid body motion. When a rigid body is subjected to ambient vibration or its angular motion, the third orthogonal axis will move around its average position in space at the same angular vibration rate in the direction of two orthogonal axes, which is called the coning motion or coning effect of the rigid body.

The coning effect of rigid body motion has important influence on strapdown inertial navigation system. The gyro used in strapdown inertial navigation system will feel the coning motion and produce corresponding measurement error. The error of strapdown inertial navigation system caused by coning effect is called coning error of the system. Therefore, in the design of strapdown inertial navigation system, how to prevent coning error and limit its propagation has become the focus of attention. In order to solve this problem, Bortz's differential equation of rotation vector establishes a new theoretical basis for calculating attitude matrix of strapdown inertial navigation system. On this basis, scholars at home and abroad have done a lot of research on coning error compensation algorithm with high efficiency and accuracy. Miller [Miller (1983)] has proposed three-sample algorithm, Lee [Lee, Mark, Tazares et al. (1990)] has proposed four-sample algorithm. Jiang et al. [Jiang and Yu (2016)] proposed using the previous cycle gyroscope output algorithm, Mussoff proposed the optimization index of coning algorithm, and analyzed the relationship between the compensated algorithm error and the power of the compensated period. In this section, based on the analysis of the above algorithms, a new general formula will be derived, and the performance of each algorithm will be analyzed and compared.

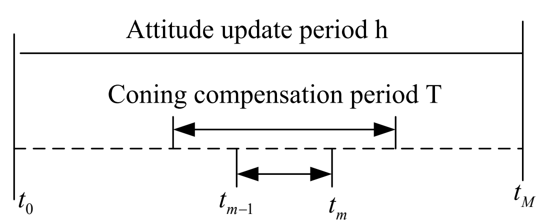


Figure 2: Time period relationship in the coning compensation

From Fig. 2, $\delta\Phi = \frac{1}{2}\int \alpha \times \omega dt$, In the case of typical coning motion, it will reach the maximum, that is to say, the typical coning motion will have the worst impact on strapdown attitude algorithm, so it needs to be compensated. In common coning compensation algorithms, attitude updating algorithm and coning compensation algorithm are generally divided into different updating cycles to calculate, in order to reduce the computational burden. The attitude updating period h is divided into smaller time interval T, which is coning compensation period (i.e., rotation vector updating period), and each coning compensation period contains several sampling data. ΔT is the data acquisition period.

Angular velocity vectors in typical coning motion is

$$\omega = \begin{bmatrix} a\Omega\cos\Omega t \\ b\Omega\sin\Omega t \\ 0 \end{bmatrix} \tag{23}$$

where ω is the projection angle velocity vector on the body frame, a , b is the angle vibration amplitude along the body frame(x axis and y axis), Ω is the angle vibration frequency. Hence

$$\alpha(t, t_{m-1}) = \int_{t_{m-1}}^{t} \omega dt = \begin{bmatrix} a(\sin\Omega t - \sin\Omega t_{m-1}) \\ b(\cos\Omega t_{m-1} - \cos\Omega t) \\ 0 \end{bmatrix} \tag{24}$$

Substituting the above formula into $\delta\Phi = \frac{1}{2}\int \alpha \times \omega dt$, the coning effect from t_{m-1} to t_m is

$$\delta\Phi_m = \frac{1}{2}\int_{t_{m-1}}^{t_m} \alpha(t, t_{m-1}) \times \omega dt = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2}ab\Omega(T - \frac{1}{\Omega}\sin\Omega T) \end{bmatrix} \tag{25}$$

The formula above represents the coning effect existing on the z axis. $\delta\Phi_m$ is the coning effect along the z direction.

4 Simulation experiments

4.1 Simulation on stationary base

Suppose the ship three-axis moving model is $\varphi = \theta = \gamma = 0^\circ$, the linear velocity is $V = 0\, m/s$, the initial longitude is 118° and the initial latitude is 32°, the constant bias of the accelerator is 50μg and the random bias is 50μg, the constant drift of the three gyroscope is $0.05^\circ/\mathrm{h}$ and the random drift $0.05^\circ/\mathrm{h}$. The initial error of angle is $\Delta\varphi_0 = \Delta\theta_0 = \Delta\gamma_0 = 0.1^\circ$, simulation time is 24 hours.

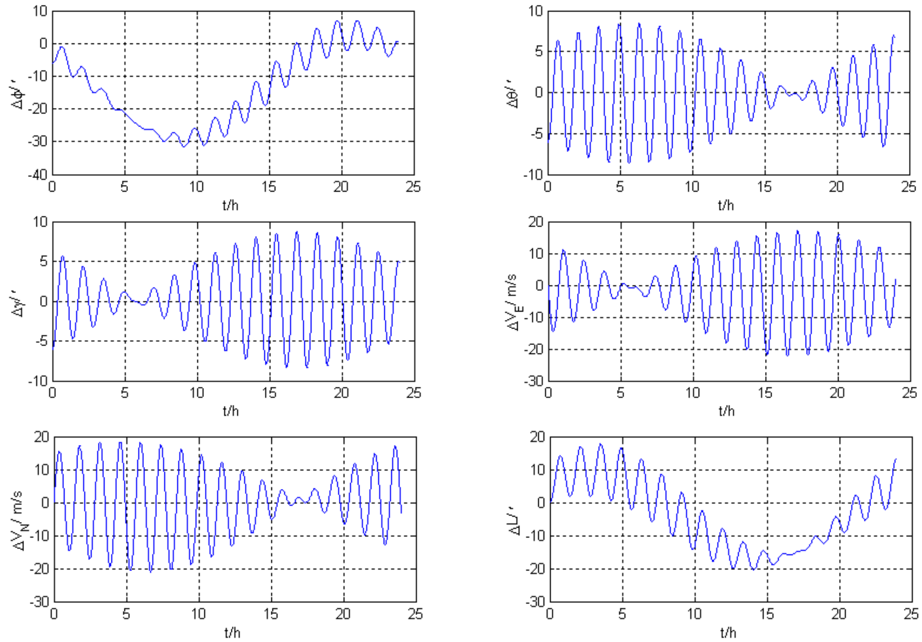


Figure 3: System error under stationery base

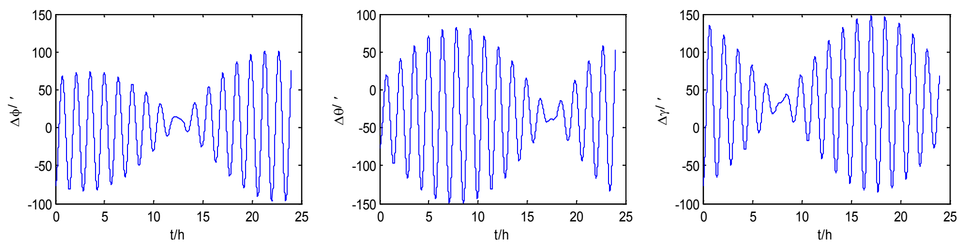
Analysis: From the simulation error curve of the static base system, it can be seen that the strapdown system is an undammed free oscillation system like the stable platform system, and there are three oscillation periods for the system error. Schuler period $T_s = 84.4\,\mathrm{min}$; earth period $T_e = 24h$; Faucault period $T_F = 2\pi / \omega_{ie} \sin L$, Foucault cycle is related to latitude. Latitude is 45° Foucault cycle is 34 hours. Foucault cycle modulates the amplitude of the Pula periodic oscillation. Foucault periodic oscillation is a kind of velocity error $2\omega_{ie} \sin L\delta v_y$ and $2\omega_{ie} \sin L\delta v_x$ aroused, If these two terms are not taken into account, the Foucault periodic oscillation is no longer included in the error characteristics, that is, the error is only Schuler periodic oscillation and Earth periodic oscillation.

If these two terms are not taken into account, the Foucault periodic oscillation is no longer included in the error characteristics, that is, the error is only Schuler periodic oscillation and Earth periodic oscillation.

4.2 Simulation on moving base

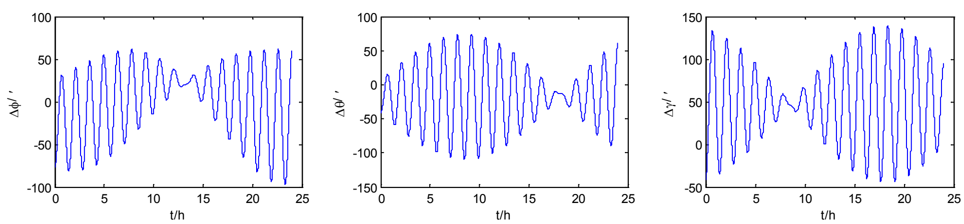
For SINS attitude updating, coning motion is the worst working environment, which will induce serious drift of mathematical platform. Therefore, coning motion is often used as the environment condition for optimization of rotation vector algorithm. That is to say, if we can ensure the minimum algorithm drift in coning motion environment, we can ensure the minimum algorithm drift in other environment. Simulation experiments were conducted under quaternion, coning compensation method 1 $(G_k = 0, k = 2,3\cdots, p = 1, N = 2)$, coning compensation method 2 $(G_k = 0, k = 2, 3\cdots, p = 2, N = 3)$ and coning compensation method 3 $(n = 2, p = 3, N = 3)$.

Coning movement simulation parameters are as follows: $\omega = 1Hz$, $\alpha = 1^\circ$, attitude update period is $T = 10ms$. The constant bias of the accelerator is $50\mu g$, random error is $50\mu g$; The constant drift of three gyro is $0.05^\circ / h$, random error is $0.05^\circ / h$, simulation time is $24h$.



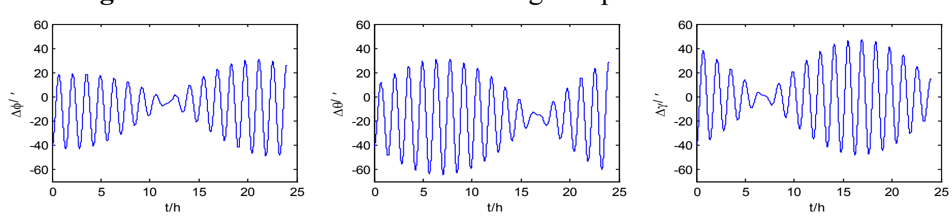
(a) heading angle error (b) pitching angle error (c) rolling angle error

Figure 4: Simulation curve with quaternion



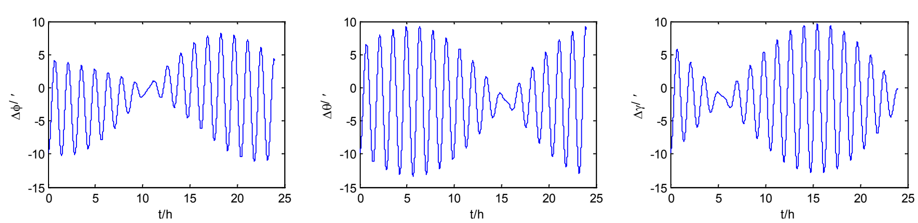
(a) heading angle error (b) pitching angle error (c) rolling angle error

Figure 5: Simulation curve with coning compensation method 1



(a) heading angle error (b) pitching angle error (c) rolling angle error

Figure 6: Simulation curve with coning compensation method 2 $(G_k = 0, k = 2, 3\cdots, p = 2, N = 3)$



(a) heading angle error (b) pitching angle error (c) rolling angle error

Figure 7: Simulation curve with coning compensation method 3 $(n = 2, p = 3, N = 3)$

5 Conclusions

The following conclusions can be drawn from the previous algorithm deduction and simulation analysis.

(1) When the vehicle is coning, the accuracy of coning error compensation algorithm is obviously higher than that of quaternion method and the heading and attitude angle errors are reduced from about 100' to about 60'.

(2) The accuracy of coning compensation algorithm 2 is higher than that of coning compensation algorithm 1. The greater the value, the higher the accuracy of the algorithm.

(3) The accuracy of coning compensation algorithm 3 is higher than that of coning compensation algorithm 2 and coning compensation algorithm 1. It shows that the coning error compensation form is more effective by considering the angular incremental output of the first two cycles and utilizing the angular variation of several sampling points in the previous cycle.

From the above analysis, it can be seen that the coning error compensation algorithm proposed in this paper has the highest accuracy among all the compensation algorithms. However, in order to improve the accuracy, considering that the amount of calculation cannot be increased too much, the strapdown inertial navigation system introduced later in this paper adopts the general algorithm proposed in this paper.

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