Mathematical Analysis of Novel Coronavirus (2019-nCov) Delay Pandemic Model

Muhammad Naveed[1], Muhammad Rafiq[2], Ali Raza[3], Nauman Ahmed[4], Ilyas Khan[5, *], Kottakkaran Sooppy Nisar[6] and Atif Hassan Soori[1]

Abstract: In this manuscript, the mathematical analysis of corona virus model with time delay effect is studied. Mathematical modelling of infectious diseases has substantial role in the different disciplines such as biological, engineering, physical, social, behavioural problems and many more. Most of infectious diseases are dreadful such as HIV/AIDS, Hepatitis and 2019-nCov. Unfortunately, due to the non-availability of vaccine for 2019-nCov around the world, the delay factors like, social distancing, quarantine, travel restrictions, holidays extension, hospitalization and isolation are used as key tools to control the pandemic of 2019-nCov. We have analysed the reproduction number $\mathbf{R_{nCov}}$ of delayed model. Two key strategies from the reproduction number of 2019-nCov model, may be followed, according to the nature of the disease as if it is diminished or present in the community. The more delaying tactics eventually, led to the control of pandemic. Local and global stability of 2019-nCov model is presented for the strategies. We have also investigated the effect of delay factor on reproduction number $\mathbf{R_{nCov}}$. Finally, some very useful numerical results are presented to support the theoretical analysis of the model.

Keywords: 2019-nCov, delay model, reproduction number, stability analysis, numerical results.

1 Introduction

Corona viruses belong to the family of corona viridae. These viruses are as small in size as 65-125 nm. They are single stranded RNA with single nucleus material, 22-26 kilo bases in size. Other branches of corona virus family are alpha (α), beta (β), gamma (γ) and delta (δ). Influenza and middle east respiratory syndrome corona virus (MERS-

[1] Department of Mathematics, Air University, Islamabad, Pakistan.

[2] Faculty of Engineering University of Central Punjab, Lahore, Pakistan.

[3] Department of Mathematics, National College of Business Administration and Economics, Lahore, Pakistan.

[4] Department of Mathematics and Statistics, The University of Lahore, Lahore, Pakistan.

[5] Faculty of Mathematics and Statistics, Ton Duc Thang University, Ho Chi Minh, 72915, Vietnam.

[6] Department of Mathematics, College of Arts and Science at Wadi Aldawaser, Prince Sattam Bin Abdulaziz University, Alkharj, Saudi Arabia.

[*] Corresponding Author: Ilyas Khan. Email: ilyaskhan@tdtu.edu.vn.

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COV) are the most common lungs infections that may cause severe damage to the respiratory system. They may cause even lungs failure. Initially, the disease was considered to be an animal disease only, because the hosts were mainly camels, bats and monkeys. Later on, it was transmitted to human beings and now it is a worldwide catastrophe. Shereen et al. [Shereen, Khan, Bashir et al. (2020)] found the characteristics, regions and transmission, of corona virus in human population. Currently, in Wuhan, in December 2019, a new Chinese business state caused to spread a deadly corona virus (2019-nCov). The death toll is continuously rise. In the first 50 days, the virus affected the 70,000 individuals, among them. The virus is reported from the beta group of corona viruses. Chinese researchers have dubbed the virus as Wuhan virus or novel Corona virus 2019 (2019-nCov). The international committee of the red cross (ICRC) has named the viruses as SARS-COV-2, COVID-19. Tahir et al. [Tahir, Shah, Zaman et al. (2019)] presented the middle east respiratory syndrome (MERS) corona virus outbreak in humans. Zhao et al. [Zhao and Chen (2020)] studied the outbreak of corona virus in China by using mathematical strategies. Africa, America, South-East Asia, Europe, the Eastern Mediterranean and the western pacific is the worst affected region due to corona virus. To date, about thirty million people worldwide infected with the virus, according to the world health organization (WHO). The reported deaths are approximately one lac and nighty four thousand and reported recovered individuals are seven lac and sixty-five thousand. In Pakistan, eleven thousand carriers of 2019-nCov and two hundred and thirty-seven deaths are reported. Unfortunately, the availability of testing kits and lab equipment is about two percent. Therefore, policy makers implemented delay tactics to control the global epidemic, like social distancing, self-quarantine and travel restriction etc. Now the pandemic of 2019-nCov is a global issue announced by World Health Organization (WHO). Shim et al. [Shim, Tariq, Choi et al. (2020)] presented the transmission potential and severity of corona virus in South Korea. Kucharski et al. [Kucharski, Russell, Diamond et al. (2020)] found some control techniques by using the mathematical model of corona virus. Jiang et al. [Jiang, Coffee, Bari et al. (2020)] presented artificial intelligence structure for data driven prediction of corona virus. Gawanmeh et al. [Gawanmeh, Pervaz and Hasan (2018)] found probabilistic analysis of electrocardiogram heart signal to devise the control strategy. Li et al. [Li, Chao and Zhang (2019)] contributed a lot, in the emotion classification based, on brain wave. Wang et al. [Wang, Li, Zou et al. (2020)] presented a novel image classification approach in internet models. Zhang et al. [Zhang, Xiong, Huang et al. (2018)] have presented the idea of delay dependent stability on neural networking models. Lin et al. [Lin, Zhao, Gao et al. (2020)] discussed the model of corona virus in individuals and also included the government reactions in this regard. Baleanu et al. [Baleanu, Raza, Rafiq et al. (2019)] made the analysis of influenza with stochastic perturbation. Raza et al. [Raza, Arif and Rafiq (2019)] analyzed the dynamics of gonorrhea disease by using stochastic inputs and treatment effect. Mathematical modelling of corona virus model by introducing the delay factors in the system of differential equations, is in good agreement to the real phenomena. In this analysis, the given reproduction number has significant role in describing the nonlinear dynamics of biological engineering and many more physical nonlinear problems. If the reproduction number is less than one, then it indicates that 2019-nCov has been controlled. On the other hand, if the reproduction number is greater than one, then it indicates that 2019-nCov has been continuously increasing. In this model, we have introduced the element of delay. The delay factors are quarantine, place of isolation or vaccination etc. In common epidemiological models, if infection rate is controlled then the disease converges towards the stable positions. In current situation of 2019-nCov, control of infection is nearly impossible, so far have to use the delaying tactics, to overcome the pandemic of 2019-nCov, like social distancing, quarantine, isolation etc. Fortunately, the delay factors or delaying tactics in the modeling are self-standing and independent of all other types of transmission rates. The strategy of our paper is as follows:

In Section 2, we discussed 2019-nCov model with time delay effect. In Section 3, we discussed equilibria of the model. In Section 4, we discussed the reproduction number and its key role describing the 2019-nCov model. In Section 5, we discussed the local and global stability of the model. In Section 6, we discussed numerical results to strengthen the theoretical analysis of the model. In last section, conclusion and future results are presented.

2 Model formulation

In this paper, we have considered the coronavirus (2019-nCov) pandemic model, proposed in human's population. The whole population is represented with $N(t)$ and is divided into the five compartments as follows: For any time t, the susceptible humans represented as $S(t)$, exposed humans represented as $E(t)$, symptomatic infected humans represented as $I(t)$, asymptomatic infected represented as $A(t)$ and recovered humans represented as $R(t)$. Simply the dynamics of the infection in the population is described through the nonlinear delay differential equations and flow chart is shown in Fig. 1.

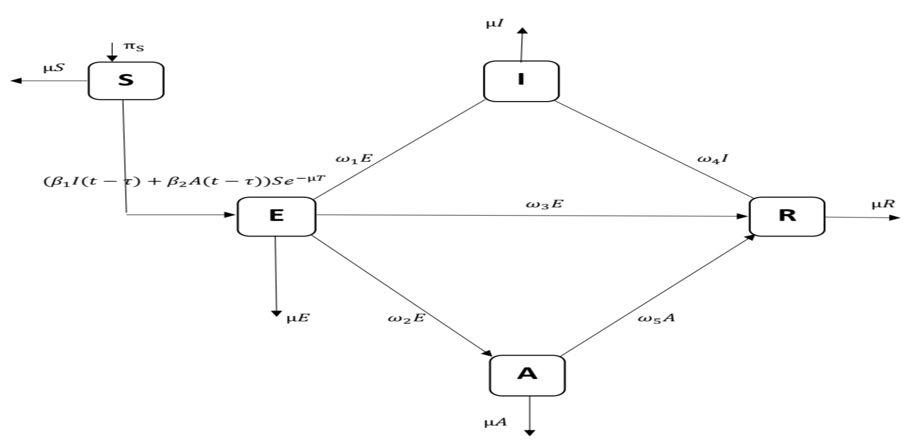


Figure 1: Flow map (2019-nCov) delay model

The parameters of the delayed model are described as follows: π_s is the recruitment rate of humans, μ is the mortality rate with natural incidences or due to virus infection, β_1 is the infection rate of symptomatic humans, β_2 is the infection rate of asymptomatic humans, ω_1 is the interaction rate of exposed humans with symptomatic infected humans, ω_2 is the interaction rate of exposed humans with asymptomatic infected humans, ω_3 is the rate of exposed humans who recovered from virus due to natural immunity, ω_4 is the rate of symptomatic carriers who recovered after quarantine, ω_5 is the rate of quarantine or isolation or vaccination of asymptomatic infected humans. The given model is based on the following assumptions: considering two ways of dispersion of virus as symptomatic and asymptomatic carriers who make bilinear incidence rate with susceptible humans. Without loss of generality, all types of other interactions have been ignored. The system of delayed differential equations of the model is represented as below:

$$\frac{dS}{dt} = \pi_s - \big(\beta_1 I(t-\tau)S(t-\tau) + \beta_2 A(t-\tau)S(t-\tau)\big)e^{-\mu\tau} - \mu S(t). \quad (1)$$

$$\frac{dE}{dt} = \big(\beta_1 I(t-\tau)S(t-\tau) + \beta_2 A(t-\tau)S(t-\tau)\big)e^{-\mu\tau} - (\omega_1 + \omega_2 + \omega_3 + \mu)E(t). \quad (2)$$

$$\frac{dI}{dt} = \omega_1 E(t) - (\omega_4 + \mu)I(t). \quad (3)$$

$$\frac{dA}{dt} = \omega_2 E(t) - (\omega_5 + \mu)A(t). \quad (4)$$

$$\frac{dR}{dt} = \omega_3 E(t) + \omega_4 I(t) + \omega_5 A(t) - \mu R(t). \quad (5)$$

The initial conditions $\psi = (\psi_1, \psi_2, \psi_3, \psi_4, \psi_5)$ for the Eq. (1) to Eq. (5) are defined in the Banach space as

$$\mathrm{C}_+ = \big\{\psi \epsilon \mathrm{C}[-\tau, 0], R_+^5 : \psi_1(0) = S(0), \psi_2(0) = E(0), \psi_3(0) = I(0), \psi_4(0) = A(0), \psi_5(0) = R(0)\big\}.$$

where, $R_+^5 = \{S, E, I, A, R \epsilon R^5 : S \geq 0, E \geq 0, I \geq 0, A \geq 0, R \geq 0\}$. We assume $\psi_i(0) > 0, (i = 1,2,3,4,5)$ due to biological background of the model. The total dynamics of Eqs. (1) to (5) is obtained by adding the five equations as follows:

$$\frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dA}{dt} + \frac{dR}{dt} \leq \pi_s - \mu N \text{ and } S + E + I + A + R = N.$$

$$\frac{dN}{dt} \leq \pi_s - \mu N.$$

The feasible region of Eqs. (1) to (5) is as follows:

$\Omega = \{S(t), E(t), I(t), A(t), R(t) \epsilon R_+^5 : N(t) \leq \frac{\pi_s}{\mu}\}$. The initial value problem, $\psi' = \pi_s - \mu\psi$, with $\psi(0) = N(0)$ has solution $\psi(t) = k_1 e^{-\mu t} + \frac{\pi_s}{\mu}$ and $\lim_{t\to\infty} \psi(t) = \frac{\pi_s}{\mu}$. Therefore, $N(t) \leq \psi(t)$ which shows that $\lim_{t\to\infty} Sup\, N(t) \leq \frac{\pi_s}{\mu}$. Thus, all solutions of Eqs. (1) to (5) lie in the feasible region Ω. The feasible region is positive and bounded for the model. Hence, the region Ω is positive invariant.

2.1 Equilibria of model

The Eqs. (1) to (5) admits two equilibria states in the feasible region Ω. A 2019-nCov free equilibrium of the Eqs. (1) to (5) as follows:

$$E_1 = (S^1, E^1, I^1, A^1, R^1) = (\frac{\pi_s}{\mu}, 0,0,0,0).$$

Also, 2019-nCov recurring equilibrium of the Eqs. (1) to (5) as follows:

$$E_2 = (S^*, E^*, I^*, A^*, R^*).$$

where, $S^* = \frac{(\omega_1+\omega_2+\omega_3+\mu)(\omega_4+\mu)(\omega_5+\mu)}{\beta_1\omega_1(\omega_5+\mu)e^{-\mu\tau}+\beta_2\omega_2(\omega_4+\mu)e^{-\mu\tau}}$,

$E^* = \frac{\pi_s - \mu S^*}{(\omega_1+\omega_2+\omega_3+\mu)}$, $I^* = \frac{\omega_1 E^*}{\omega_4+\mu}$, $A^* = \frac{\omega_2 E^*}{\omega_5+\mu}$ and $R^* = \frac{\omega_3 E^* + \omega_4 I^* + \omega_5 A^*}{\mu}$.

2.2 Basic reproduction number

Driekmann et al. [Driekmann, Heesterbeek and Roberts (2009)] presented the idea of reproduction number by using next generation matrix method. From the Eqs. (1) to (5), we apply the next generation matrix method in order to calculate the reproduction number R_{nCov}. We have taken the infectious and recovered human population from the Eqs. (1) to (5), along with the 2019-nCov free equilibrium as follows:

$$\begin{bmatrix} E' \\ I' \\ A' \\ R' \end{bmatrix} = \begin{bmatrix} 0 & \frac{\beta_1 \pi_s e^{-\mu\tau}}{\mu} & \frac{\beta_2 \pi_s e^{-\mu\tau}}{\mu} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E \\ I \\ A \\ R \end{bmatrix} -$$

$$\begin{bmatrix} \omega_1+\omega_2+\omega_3+\mu & 0 & 0 & 0 \\ -\omega_1 & \omega_4+\mu & 0 & 0 \\ -\omega_2 & 0 & \omega_5+\mu & 0 \\ -\omega_3 & -\omega_4 & -\omega_5 & \mu \end{bmatrix} \begin{bmatrix} E \\ I \\ A \\ R \end{bmatrix}.$$

where, $A = \begin{bmatrix} 0 & \frac{\beta_1 \pi_s e^{-\mu\tau}}{\mu} & \frac{\beta_2 \pi_s e^{-\mu\tau}}{\mu} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and

$$B = \begin{bmatrix} \omega_1+\omega_2+\omega_3+\mu & 0 & 0 & 0 \\ -\omega_1 & \omega_4+\mu & 0 & 0 \\ -\omega_2 & 0 & \omega_5+\mu & 0 \\ -\omega_3 & -\omega_4 & -\omega_5 & \mu \end{bmatrix}.$$

$$AB^{-1} = \begin{bmatrix} \frac{\pi_s e^{-\mu\tau}(\omega_2\beta_2(\omega_4+\mu)+\omega_1\beta_1(\omega_5+\mu))}{\mu(\omega_1+\omega_2+\omega_3+\mu)(\omega_4+\mu)(\omega_5+\mu)} & \frac{\beta_1 \pi_s e^{-\mu\tau}}{\mu(\omega_4+\mu)} & \frac{\beta_2 \pi_s e^{-\mu\tau}}{\mu(\omega_5+\mu)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The spectral radius of AB^{-1} is denoted as $R_{nCov} = \frac{\pi_s e^{-\mu\tau}(\omega_2\beta_2(\omega_4+\mu)+\omega_1\beta_1(\omega_5+\mu))}{\mu(\omega_1+\omega_2+\omega_3+\mu)(\omega_4+\mu)(\omega_5+\mu)}$.

3 Stability analysis

In this section, stability analysis of the delayed model from Eqs. (1) to (5) at both 2019-nCov free equilibrium and 2019-nCov recurring equilibrium will be discussed, to check local and global dynamical behaviour of the corona virus, as follows:

3.1 Local stability

For the local stability, at both equilibria of the delayed model, we will prove the following well known results as follows:

Theorem: For given $\tau > 0$, the Eqs. (1) to (5) is said to be locally asymptotical stable (LAS) at 2019-nCov free equilibrium $E_1 = (S^1, E^1, I^1, A^1, R^1)$, which is contained in region Ω if $R_{nCov} < 1$. Otherwise the Eqs. (1) to (5) is unstable if $R_{nCov} > 1$.

Proof: The Jacobean matrix for the Eqs. (1) to (5) at E_1 is evaluated as follows:

$$J(E_1) = \begin{bmatrix} -\mu & 0 & -\frac{\beta_1 \pi_s e^{-\mu\tau}}{\mu} & -\frac{\beta_2 \pi_s e^{-\mu\tau}}{\mu} & 0 \\ 0 & -(\omega_1+\omega_2+\omega_3+\mu) & \frac{\beta_1 \pi_s e^{-\mu\tau}}{\mu} & \frac{\beta_2 \pi_s e^{-\mu\tau}}{\mu} & 0 \\ 0 & \omega_1 & -\omega_4-\mu & 0 & 0 \\ 0 & \omega_2 & 0 & -\omega_5-\mu & 0 \\ 0 & \omega_3 & \omega_4 & \omega_5 & -\mu \end{bmatrix}$$

The following eigen values of Jacobean matrix $J(E_1)$ are obtained:

$\lambda_1 = -\mu < 0, \quad \lambda_2 = -\mu < 0$ and

$$|J(E_1) - \lambda I| = \begin{vmatrix} -(\omega_1+\omega_2+\omega_3+\mu)-\lambda & \frac{\beta_1 \pi_s e^{-\mu\tau}}{\mu} & \frac{\beta_2 \pi_s e^{-\mu\tau}}{\mu} \\ \omega_1 & -(\omega_4+\mu)-\lambda & 0 \\ \omega_2 & 0 & -(\omega_5+\mu)-\lambda \end{vmatrix} = 0.$$

Put $a_1 = (\omega_1+\omega_2+\omega_3+\mu) > 0$, $a_2 = \frac{\beta_1 \pi_s e^{-\mu\tau}}{\mu} > 0$, $a_3 = \frac{\beta_2 \pi_s e^{-\mu\tau}}{\mu} > 0$, $a_4 = \omega_4 + \mu > 0$, $a_5 = \omega_5 + \mu > 0$.

$$|J(E_1) - \lambda I| = \begin{vmatrix} -a_1-\lambda & a_2 & a_3 \\ \omega_1 & -a_4-\lambda & 0 \\ \omega_2 & 0 & -a_5-\lambda \end{vmatrix} = 0$$

$\lambda^3 + \lambda^2[a_5 + a_1 + a_4] + \lambda[a_5(a_1 + a_4) + a_1 a_4 - a_2\omega_1 - \omega_2 a_3] + (a_1 a_4 a_5 - \omega_2 a_3 a_4 - a_5 a_2 \omega_1) = 0.$

By using the Routh-Hurwitz Criterion of 3rd order polynomial as,

$[a_5 + a_1 + a_4] > 0,$

$(a_1 a_4 a_5 - \omega_2 a_3 a_4 - a_5 a_2 \omega_1) > 0$, if $\frac{\omega_2 a_3 a_4 + a_5 a_2 \omega_1}{a_1 a_4 a_5} < 1,$

by putting the values, we have $R_{nCov} = \frac{\pi_s e^{-\mu\tau}(\omega_2\beta_2(\omega_4+\mu)+\omega_1\beta_1(\omega_5+\mu))}{\mu(\omega_1+\omega_2+\omega_3+\mu)(\omega_4+\mu)(\omega_5+\mu)} < 1$

and $[a_5 + a_1 + a_4][a_5(a_1 + a_4) + a_1 a_4 - a_2\omega_1 - \omega_2 a_3] > [a_1 a_4 a_5 - \omega_2 a_3 a_4 - a_5 a_2 \omega_1]$, if $R_{nCov} < 1$.

So, all eigen values are negative. Hence, by Routh Hurwitz criteria E_1 is locally asymptotical stable (LAS).

If $R_{nCov} > 1$, that is

$\frac{\pi_s e^{-\mu\tau}(\omega_2\beta_2(\omega_4+\mu)+\omega_1\beta_1(\omega_5+\mu))}{\mu(\omega_1+\omega_2+\omega_3+\mu)(\omega_4+\mu)(\omega_5+\mu)} > 1,$

$\pi_s e^{-\mu\tau}(\omega_2\beta_2(\omega_4 + \mu) + \omega_1\beta_1(\omega_5 + \mu)) > \mu(\omega_1 + \omega_2 + \omega_3 + \mu)(\omega_4 + \mu)(\omega_5 + \mu).$

$-\mu(\omega_1 + \omega_2 + \omega_3 + \mu)(\omega_4 + \mu)(\omega_5 + \mu) + \pi_s e^{-\mu\tau}(\omega_2\beta_2(\omega_4 + \mu) + \omega_1\beta_1(\omega_5 + \mu)) > 0.$

Then $\lambda_3, \lambda_4, \lambda_5 > 0$. Hence, E_1 is unstable.

Theorem: For given $\tau > 0$, the Eqs. (1) to (5) is said to be locally asymptotical stable (LAS) at 2019-nCov present equilibrium $E_2 = (S^*, E^*, I^*, A^*, R^*)$, which is contained in region Ω if $R_{covid} > 1$. Otherwise the Eqs. (1) to (5) is unstable if $R_{covid} < 1$.

Proof: The Jacobean matrix for the Eqs. (1) to (5) at E_2 is calculated as follows:

$$J(E_2) = \begin{bmatrix} -(\beta_1 I^* + \beta_2 A^*)e^{-\mu\tau} - \mu & 0 & -\frac{\beta_1 \pi_s e^{-\mu\tau}}{\mu} & -\frac{\beta_2 \pi_s e^{-\mu\tau}}{\mu} & 0 \\ (\beta_1 I^* + \beta_2 A^*)e^{-\mu\tau} & -(\omega_1 + \omega_2 + \omega_3 + \mu) & \frac{\beta_1 \pi_s e^{-\mu\tau}}{\mu} & \frac{\beta_2 \pi_s e^{-\mu\tau}}{\mu} & 0 \\ 0 & \omega_1 & -\omega_4 - \mu & 0 & 0 \\ 0 & \omega_2 & 0 & -\omega_5 - \mu & 0 \\ 0 & \omega_3 & \omega_4 & \omega_5 & -\mu \end{bmatrix}$$

The following eigen values of Jacobean matrix $J(E_2)$ are obtained:

$\lambda_1 = -\mu < 0$ and

$$|J(E_2) - \lambda I| = \begin{vmatrix} -b_1 - \mu - \lambda & 0 & -b_2 & -b_3 \\ b_1 & -b_4 - \lambda & b_2 & b_3 \\ 0 & \omega_1 & -b_5 - \lambda & 0 \\ 0 & \omega_2 & 0 & -b_6 - \lambda \end{vmatrix} = 0$$

where,

$b_1 = (\beta_1 I^* + \beta_2 A^*)e^{-\mu\tau} > 0$, $b_2 = \beta_1 S^* e^{-\mu\tau} > 0$, $b_3 = \beta_2 S^* e^{-\mu\tau} > 0$, $b_4 = \omega_1 + \omega_2 + \omega_3 + \mu > 0$, $b_5 = \omega_4 + \mu > 0$, $b_6 = \omega_5 + \mu > 0$.

$\lambda^4 + (b_1 + b_4 + b_5 + b_6 + \mu)\lambda^3 + (b_3\omega_2 - b_1 b_6 - b_6\mu - b_5 b_6 - b_4 b_6 + b_2\omega_1 - b_1 b_5 - b_1 b_4 - b_5\mu - b_4\mu - b_4 b_5)\lambda^2 + (b_3 b_5 \omega_2 + b_3\mu\omega_2 + b_2 b_6 \omega_1 - b_1 b_5 b_6 - b_1 b_4 b_6 - b_5 b_6 \mu - b_4 b_6 \mu - b_4 b_5 b_6 + b_2\mu\omega_1 - b_1 b_4 b_5 - b_4 b_5 \mu)\lambda + (b_3 b_5 \mu\omega_2 + b_2 b_6 \mu\omega_1 + b_1 b_4 b_5 b_6 + b_4 b_5 b_6 \mu) = 0$.

So, $m_0\lambda^4 + m_1\lambda^3 + m_2\lambda^2 + m_3\lambda + m_4 = 0$.

where, $m_1 = (b_1 + b_4 + b_5 + b_6 + \mu)$,

$m_2 = (b_3\omega_2 - b_1 b_6 - b_6\mu - b_5 b_6 - b_4 b_6 + b_2\omega_1 - b_1 b_5 - b_1 b_4 - b_5\mu - b_4\mu - b_4 b_5)$,

$m_3 = (b_3 b_5 \omega_2 + b_3\mu\omega_2 + b_2 b_6 \omega_1 - b_1 b_5 b_6 - b_1 b_4 b_6 - b_5 b_6 \mu - b_4 b_6 \mu - b_4 b_5 b_6 + b_2\mu\omega_1 - b_1 b_4 b_5 - b_4 b_5 \mu)$,

$m_4 = (b_3 b_5 \mu\omega_2 + b_2 b_6 \mu\omega_1 + b_1 b_4 b_5 b_6 + b_4 b_5 b_6 \mu)$.

By using the Routh-Hurwitz Criterion of 4th order polynomial as,

$m_0 > 0, m_1 > 0, m_1 m_2 - m_0 m_3 > 0$,

$(m_1 m_2 - m_0 m_3)m_3 - {m_1}^2 m_4 > 0$ and $m_4 > 0$ only if $R_{nCov} > 1$.

So, its eigen values are negative. Hence, by Routh Hurwitz criteria E_2 is locally asymptotically stable (LAS).

3.2 Global stability

For the global stability, at both equilibria of the delayed model, we will prove the following well known results as follows:

Theorem: For given $\tau > 0$, the Eqs. (1) to (5) is said to be globally asymptotically stable (GAS) at 2019-nCov free equilibrium $E_1 = (S^1, E^1, I^1, A^1, R^1)$, which is contained in region Ω if $R_{nCov} < 1$. Otherwise unstable.

Proof: We consider the Volterra- type Lyapunov function $U: \Omega \to R$ defined as

$$U = \left(S - S^1 - S^1 log\frac{S}{S^1}\right) + E + I + A + R$$

$$\frac{dU}{dt} = (1 - \frac{S^1}{S})\frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dA}{dt} + \frac{dR}{dt}$$

$$\frac{dU}{dt} = \left(1 - \frac{S'}{S}\right)(\pi_s - \beta_1 ISe^{-\mu\tau} - \beta_2 ASe^{-\mu\tau} - \mu S) + \beta_1 ISe^{-\mu\tau} + \beta_2 ASe^{-\mu\tau} - \omega_1 E - \omega_2 E - \omega_3 E - \mu E + \omega_1 E - \omega_4 I - \mu I + \omega_2 E - \omega_5 A - \mu A + \omega_3 E + \omega_4 I + \omega_5 A - \mu R$$

$$\frac{dU}{dt} = (S' - S)\left(\frac{\pi_s}{S} - \beta_1 Ie^{-\mu\tau} - \beta_2 Ae^{-\mu\tau} - \mu\right) + \beta_1 ISe^{-\mu\tau} + \beta_2 ASe^{-\mu\tau} - \mu E - \mu I - \mu A - \mu R.$$

Since, $E_1 = (S^1, E^1, I^1, A^1, R^1)$ is a 2019-nCov free equilibrium, so for Eqs. (1) to (5), $\frac{dS^1}{dt} = \frac{dE^1}{dt} = \frac{dI^1}{dt} = \frac{dA^1}{dt} = \frac{dR^1}{dt} = 0$, gives

$$\mu = \frac{\pi_s}{S^1} - \beta_1 I^1 e^{-\mu\tau} - \beta_2 A^1 e^{-\mu\tau}.$$

$$\frac{dU}{dt} = (S - S^1)\left(\frac{\pi_s}{S} - \beta_1 Ie^{-\mu\tau} - \beta_2 Ae^{-\mu\tau} - \frac{\pi_s}{S^1} + \beta_1 I^1 e^{-\mu\tau} + \beta_2 A^1 e^{-\mu\tau}\right) + \beta_1 ISe^{-\mu\tau} + \beta_2 ASe^{-\mu\tau} - \mu E - \mu I - \mu A - \mu R.$$

$$\frac{dU}{dt} = \frac{-\pi_s (S - S^1)^2}{SS'} - \beta_1 (S - S^1)(I - I^1)e^{-\mu\tau} - \beta_2 (S - S^1)(A - A^1)e^{-\mu\tau} - \mu I(1 - \frac{\beta_1 Se^{-\mu\tau}}{\mu}) - \mu A(1 - \frac{\beta_2 Se^{-\mu\tau}}{\mu}) - \mu E - \mu R.$$

$$\frac{dU}{dt} = \frac{-\pi_s (S - S^1)^2}{SS'} - \beta_1 (S - S^1)(I - I^1)e^{-\mu\tau} - \beta_2 (S - S^1)(A - A^1)e^{-\mu\tau} - \mu I(1 - \frac{\beta_1 S^1 e^{-\mu\tau}}{\mu}) - \mu A(1 - \frac{\beta_2 S^1 e^{-\mu\tau}}{\mu}) - \mu E - \mu R.$$

$\Rightarrow \frac{dU}{dt} \le 0$ for$R_{nCov} < 1$, and $\frac{dU}{dt} = 0$ only if $S = S^1, \quad E = I = A = R = 0$. Therefore, the only trajectory of the Eqs. (1) to (5) on which $\frac{dU}{dt} = 0$ is E_1. Hence, by Lasalle invariance principle, E_1 is globally asymptotically stable (GAS) in Ω.

Theorem: For given $\tau > 0$, the Eqs. (1) to (5) is said to be globally asymptotically stable (GAS) at 2019-nCov present equilibrium $E_2 = (S^*, E^*, I^*, A^*, R^*)$, which is contained in region Ω if $R_{nCov} > 1$. Otherwise unstable.

Proof: We consider the Volterra- type Lyapunov function $V: \Omega \to R$ defined as:

$$V = K_1\left(S - S^* - S^* log\frac{S}{S^*}\right) + K_2\left(E - E^* - E^* log\frac{E}{E^*}\right) + K_3\left(I - I^* - I^* log\frac{I}{I^*}\right) + K_4\left(A - A^* - A^* log\frac{A}{A^*}\right) + K_5\left(R - R^* - R^* log\frac{R}{R^*}\right).$$

where, K_i: $(i = 1, 2, 3, 4, 5)$ are positive constants to be chosen later.

$$\frac{dV}{dt} = K_1\left(1-\frac{S^*}{S}\right)\frac{dS}{dt} + K_2\left(1-\frac{E^*}{E}\right)\frac{dE}{dt} + K_3\left(1-\frac{I^*}{I}\right)\frac{dI}{dt} + K_4\left(1-\frac{A^*}{A}\right)\frac{dA}{dt} + K_5\left(1-\frac{R^*}{R}\right)\frac{dR}{dt}$$

$$\frac{dV}{dt} = K_1\left(\frac{S-S^*}{S}\right)(\pi_s - \beta_1 ISe^{-\mu\tau} - \beta_2 ASe^{-\mu\tau} - \mu S) + K_2\left(\frac{E-E^*}{E}\right)(\beta_1 ISe^{-\mu\tau} + \beta_2 ASe^{-\mu\tau} - (\omega_1+\omega_2+\omega_3+\mu)E) + K_3\left(\frac{I-I^*}{I}\right)[\omega_1 E - (\omega_4+\mu)I] + K_4\left(\frac{A-A^*}{A}\right)[\omega_2 E - (\omega_5+\mu)A] + K_5\left(\frac{R-R^*}{R}\right)(\omega_3 E + \omega_4 I + \omega_5 A - \mu R).$$

$$\frac{dV}{dt} = K_1(S-S^*)\left(\frac{\pi_s}{S} - \beta_1 Ie^{-\mu\tau} - \beta_2 Ae^{-\mu\tau} - \mu\right) + K_2(E-E^*)\left[\frac{\beta_1 ISe^{-\mu\tau}}{E} + \frac{\beta_2 ASe^{-\mu\tau}}{E} - (\omega_1+\omega_2+\omega_3+\mu)\right] + K_3(I-I^*)\left[\omega_1\frac{E}{I} - (\omega_4+\mu)\right] + K_4(A-A^*)\left[\omega_2\frac{E}{A} - (\omega_5+\mu)\right] + K_5(R-R^*)\left(\omega_3\frac{E}{R} + \omega_4\frac{I}{R} + \omega_5\frac{A}{R} - \mu\right).$$

Since, $E_2 = (S^*, E^*, I^*, A^*, R^*)$ is a 2019-nCov present equilibrium, so for Eqs. (1) to (5), $\frac{dS^*}{dt} = \frac{dE^*}{dt} = \frac{dI^*}{dt} = \frac{dA^*}{dt} = \frac{dR^*}{dt} = 0$, gives $\mu = \frac{\pi_s}{S^*} - \beta_1 I^* e^{-\mu\tau} - \beta_2 A^* e^{-\mu\tau}$, $(\omega_1+\omega_2+\omega_3+\mu) = \frac{\beta_1 ISe^{-\mu\tau}}{E^*} + \frac{\beta_2 ASe^{-\mu\tau}}{E^*}$, $(\omega_4+\mu) = \omega_1\frac{E}{I^*}$, $(\omega_5+\mu) = \omega_2\frac{E}{A^*}$, $\mu = \omega_3\frac{E}{R^*} + \omega_4\frac{I}{R^*} + \omega_5\frac{A}{R^*}$.

$$\frac{dV}{dt} = K_1(S-S^*)\left(\frac{\pi_s}{S} - \beta_1 Ie^{-\mu\tau} - \beta_2 Ae^{-\mu\tau} - \frac{\pi_s}{S^*} + \beta_1 I^* e^{-\mu\tau} + \beta_2 A^* e^{-\mu\tau}\right) + K_2(E-E^*)\left[\frac{\beta_1 ISe^{-\mu\tau}}{E} + \frac{\beta_2 ASe^{-\mu\tau}}{E} - \frac{\beta_1 ISe^{-\mu\tau}}{E^*} - \frac{\beta_2 ASe^{-\mu\tau}}{E^*}\right] + K_3(I-I^*)\left[\omega_1\frac{E}{I} - \omega_1\frac{E}{I^*}\right] + K_4(A-A^*)\left[\omega_2\frac{E}{A} - \omega_2\frac{E}{A^*}\right] + K_5(R-R^*)(\omega_3\frac{E}{R} + \omega_4\frac{I}{R} + \omega_5\frac{A}{R} - \omega_3\frac{E}{R^*} - \omega_4\frac{I}{R^*} - \omega_5\frac{A}{R^*}).$$

$$\frac{dV}{dt} = -K_1\frac{\pi_s(S-S^*)^2}{SS^*} - K_1\beta_1 e^{-\mu\tau}(S-S^*)(I-I^*) - K_1\beta_2 e^{-\mu\tau}(S-S^*)(A-A^*) - K_2\frac{\beta_1 ISe^{-\mu\tau}(E-E^*)^2}{EE^*} - K_2\frac{\beta_2 ASe^{-\mu\tau}(E-E^*)^2}{EE^*} - K_3\frac{\omega_1 E(I-I^*)^2}{II^*} - K_4\frac{\omega_2 E(A-A^*)^2}{AA^*} - K_5\frac{\omega_3 E(R-R^*)^2}{RR^*} - K_5\frac{\omega_4 I(R-R^*)^2}{RR^*} - K_5\frac{\omega_5 A(R-R^*)^2}{RR^*}$$

$$\frac{dV}{dt} = -K_1\frac{-\pi_s(S-S^*)^2}{SS^*} - K_1(S-S^*)e^{-\mu\tau}[\beta_1(I-I^*) + \beta_2(A-A^*)] - K_2\frac{(E-E^*)^2}{EE^*}Se^{-\mu\tau}(\beta_1 I + \beta_2 A) - K_3\frac{\omega_1 E(I-I^*)^2}{II^*} - K_4\frac{\omega_2 E(A-A^*)^2}{AA^*} - K_5\frac{(R-R^*)^2}{RR^*}(\omega_3 E + \omega_4 I + \omega_5 A).$$

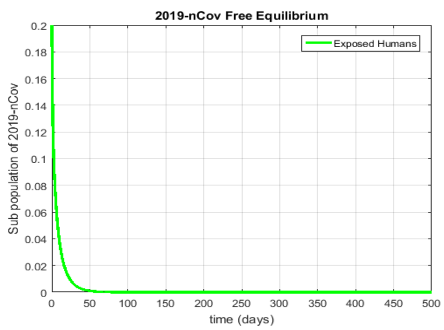
For $K_1 = K_2 = K_3 = K_4 = K_5 = 1$, we have

$$\frac{dV}{dt} = \frac{-\pi_s(S-S^*)^2}{SS^*} - (S-S^*)e^{-\mu\tau}[\beta_1(I-I^*) + \beta_2(A-A^*)] - \frac{(E-E^*)^2}{EE^*}Se^{-\mu\tau}(\beta_1 I + \beta_2 A) - \frac{\omega_1 E(I-I^*)^2}{II^*} - \frac{\omega_2 E(A-A^*)^2}{AA^*} - \frac{(R-R^*)^2}{RR^*}(\omega_3 E + \omega_4 I + \omega_5 A) \le 0.$$

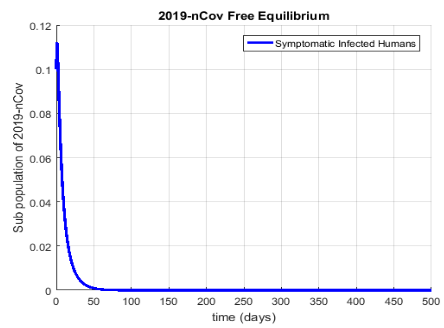
$\Rightarrow \frac{dV}{dt} \le 0$ for $R_{nCov} > 1$, and $\frac{dV}{dt} = 0$ only if $S = S^*, I = I^*, A = A^*, R = R^*$. Therefore, the only trajectory of the Eqs. (1) to (5) on which $\frac{dV}{dt} = 0$ is E_2. Hence, by Lasalle's invariance principle, E_2 is globally asymptotically stable (GAS) in Ω.

4 Numerical results

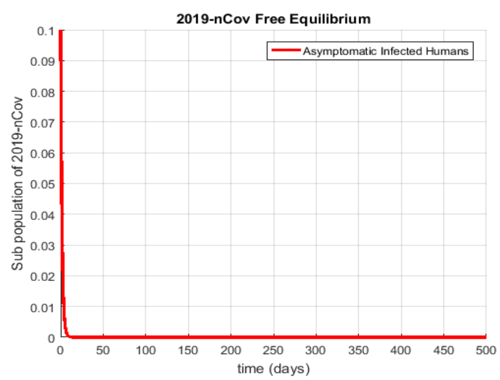
The numerical solutions of Eqs. (1) to (5) verify the dynamical behavior of the model by using different values of the parameters. Khan et al. [Khan and Atangana (2020)] selected the parametric values as follows:
$\pi_s = 0.5$, $\beta_1 = 1.05$, $\beta_2 = 1.05$, $\mu = 0.5$, $\omega_1 = 0.47876$, $\omega_2 = 0.000398$, $\omega_3 = 0.0854302$, $\omega_4 = 0.09871$, $\omega_5 = 0.1243$, for $R_{nCov} < 1$. For $R_{nCov} > 1$, $\pi_s = 0.5$, $\beta_1 = 1.05$, $\beta_2 = 1.05$, $\mu = 0.5$, $\omega_1 = 0.47876$, $\omega_2 = 1.000398$, $\omega_3 = 0.0854302$, $\omega_4 = 0.09871$, $\omega_5 = 0.1243$. by using different non-negative initial conditions $S(0) = 0.5$, $E(0) = 0.2$, $I(0) = 0.1$, $A(0) = 0.1$, $R(0) = 0.1$. In Fig. 2, we have plotted each compartment of the delayed model without time delay factor, for 2019-nCov free equilibrium.



(a)



(b)



(c)

Figure 2: Time plots of Eqs. (1) to (5) for different parameters as $\pi_s = 0.5$, $\beta_1 = 1.05$, $\beta_2 = 1.05$, $\mu = 0.5$, $\omega_1 = 0.47876$, $\omega_2 = 0.000398$, $\omega_3 = 0.0854302$, $\omega_4 = 0.09871$, $\omega_5 = 0.1243$, $\tau = 0$, by using initial conditions and $R_{nCov} = 0.7223 < 1$

In Fig. 3, we have plotted each compartment of the delayed model without time delay factor for 2019-nCov present equilibrium.

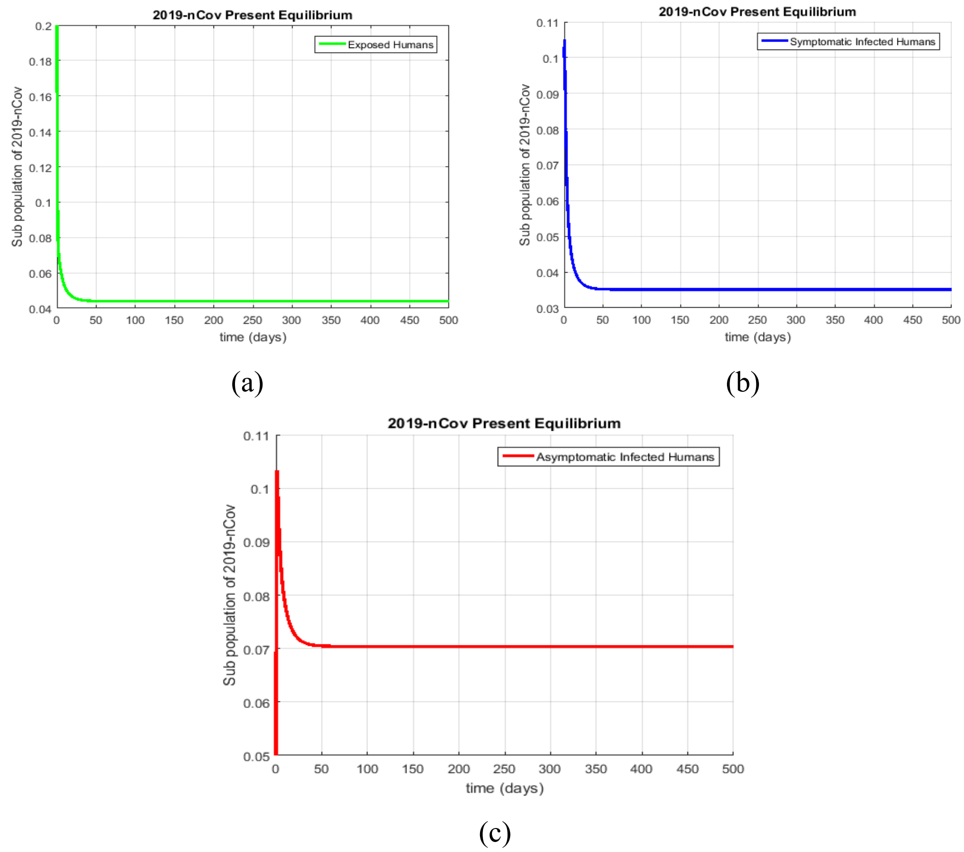


Figure 3: Time plots of Eqs. (1) to (5) for different parameters as $\pi_s = 0.5$, $\beta_1 = 1.05$, $\beta_2 = 1.05$, $\mu = 0.5$, $\omega_1 = 0.47876$, $\omega_2 = 1.000398$, $\omega_3 = 0.0854302$, $\omega_4 = 0.09871$, $\omega_5 = 0.1243$, $\tau = 0$, by using the initial conditions and $R_{nCov} = 1.1659 > 1$

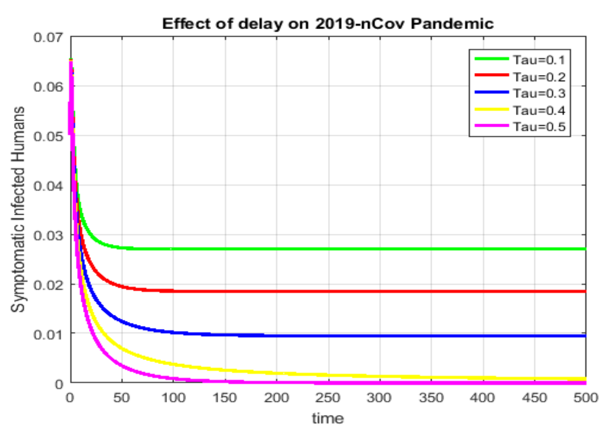


Figure 4: Time plots of Eqs. (1) to (5) for different parameters as $\pi_s = 0.5$, $\beta_1 = 1.05$, $\beta_2 = 1.05$, $\mu = 0.5$, $\omega_1 = 0.47876$, $\omega_2 = 1.000398$, $\omega_3 = 0.0854302$, $\omega_4 = 0.09871$, $\omega_5 = 0.1243$, $\tau > 0$, by using the initial conditions and $R_{nCov} = 1.1659 > 1$

In Fig. 4, we observe that, the increase in delaying tactics or delayed term, results in the reduction of infected populace without any change in the transmission rate. Even though, exponential decrease in the symptomatic infected humans may be observed by the increase in delaying tactics. Eventually, symptomatic infected humans become zero as depicted in Fig. 4, when τ=0.31. So, it means population becomes 2019-nCov free and $R_{nCov} = 0.9985 < 1$. According to given real data, if we use the delaying tactics like, social distancing, quarantine, travel restrictions, holidays extension, hospitalization and isolation for approximately one hundred and thirteen days ($\tau = 0.31\ year$) then we can overcome the pandemic of 2019-nCov.

4.1 Effect of delay factor τ on reproduction number R_{nCov}

In Fig. 5, we have presented the comparison of delay factor and reproduction number. We have concluded that an increase in delaying tactics could change 2019-nCov present equilibrium to 2019-nCov free equilibrium.

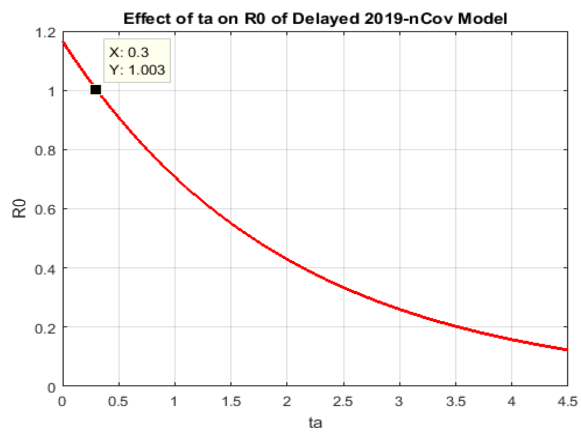


Figure 5: Comparison of delay factor and reproduction number

5 Conclusion and directions

Mathematical modelling of epidemiological diseases with the effect of time delay is an important tool to study the disease dynamics. All over the world, we have observed key strategies for overcoming the current disaster of 2019-nCov, delaying tactics or delayed factors. The best uses of the delaying tactics, reduce the 2019-nCov rapidly. The most effective tools for the delay factors are quarantine, isolation, social distancing, immigration restrictions. However, according to the given data, we can use delaying tactics for approximately one hundred and thirteen days to obtain the desired outcomes. Thus, symptomatic infected humans ultimately converge to zero as shown in Fig. 4. For the future work, we can extend this idea to many epidemic diseases and other biological problems. Also, we shall introduce more models of 2019-nCov, in which quarantine, hospitalization, restriction on immigrants' compartments for humans will be considered. This could be a more authentic way to study the 2019-nCov with delay strategy. Furthermore, direction we shall extend this idea to non-linear coupled multiplex networks with multi links and time delay effect as presented by Zhou et al. [Zhou, Tan, Yu et al.

(2019)]. Also, the delay analysis may be extended to fixed time neural dynamics as presented by Yu et al. [Yu, Liu, Xiao et al. (2019)].

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