A Genetic Algorithm to Solve Capacity Assignment Problem in a Flow Network

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Abstract: Computer networks and power transmission networks are treated as capacitated flow networks. A capacitated flow network may partially fail due to maintenance. Therefore, the capacity of each edge should be optimally assigned to face critical situations—i.e., to keep the network functioning normally in the case of failure at one or more edges. The robust design problem (RDP) in a capacitated flow network is to search for the minimum capacity assignment of each edge such that the network still survived even under the edge's failure. The RDP is known as NP-hard. Thus, capacity assignment problem subject to system reliability and total capacity constraints is studied in this paper. The problem is formulated mathematically, and a genetic algorithm is proposed to determine the optimal solution. The optimal solution found by the proposed algorithm is characterized by maximum reliability and minimum total capacity. Some numerical examples are presented to illustrate the efficiency of the proposed approach.

Keywords: Flow network, capacity assignment, network reliability, genetic algorithms.

1 Introduction

The capacity assignment problem (CAP) is generally defined as the search for the optimal capacities that minimize the delay in the network and maximize its reliability [Zantuti (2008)]. Capacity assignment plays an important role in various industries, including communication networks [Chang and Gavish (1993); Ahlert, Corsten and Gössinger (2009)] and shipping industries [Wang and Yun (2013); Hung and Chen (2013)].

The maximum capacity of each component (arc or node) is essential in evaluations of the system reliability of a stochastic-flow network (SFN). Many papers have introduced algorithms or methods based on minimal paths or minimal cuts to evaluate the reliability of SFNs [Lin, Jane and Yuan (1995); Lin (2001, 2002); Lin and Yeh (2015)]. These studies are based on knowing the capacity of each component in advance. The maximum capacity of each component has been discussed by Chen et al. [Chen and Lin (2008)].

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The analysis of a network structure has an effect on the capacity assignment strategy, as this analysis helps determine the critical nodes (the reliability value of an SFN equals zero when the critical node has zero capacity). Consequently, the best solution to the CAP must be found in order to decrease the number of critical nodes.

Chen et al. [Chen and Lin (2008)] studied the CAP for SFNs with node failure-where the node has several states or capacities and may fail. They also studied the reliability of an SFN in the case of an existing critical node. Later, Chen et al. [Chen and Lin (2010)] defined the CAP as a robust design problem for an SFN and proposed an algorithm to solve the problem. Chen [Chen (2012)] discussed the robust design problem for an SFN in the case of each edge having several capacities and the potential to fail, proposing an algorithm to determine the minimum capacity assignment of each edge so that the network can still function. Chen [Chen (2012)] also stated that the problem is an NP-hard problem and proposed an exact algorithm to solve it.

The capacity assignment in a stochastic-flow network is known to be NP-hard [Ball (1986); Chen and Lin (2010)], a relatively fast optimization algorithm would be beneficial. A genetic algorithm (GA) is a heuristic search method used in optimization problems. To solve reliability optimization problems, many based GA approaches were proposed such as [Younes and Hassan (2011)]. Also, GA is used to solve multiple objective optimization problems, [Taboada, Espiritu and Coit (2008); Coello and Christiansen (2000); Deb, Pratap, Agarwal et al. (2000); Azaron, Perkgoz, Katagiri et al. (2009); Hassan and Abdou (2018)]. Therefore, this paper presents a GA to solve the capacity assignment problem. The objective is to minimize the total capacities while meeting the network reliability requirement.

The rest of this paper is organized as follows. Section 2 sets forth the preliminaries. Section 3 presents the problem formulation. Then, Section 4 explains the proposed genetic algorithm (GA). Section 5 provides the steps of the entire algorithm. Section 6 includes several examples demonstrating the usability of the proposed approach. Finally, Section 7 draws conclusions and outlines possibilities for future work.

2 Preliminaries

2.1 Network reliability R_d

Given the demand d and the set of minimal paths (mp), the system reliability R_d is defined as Lin [Lin (2001)]:

$$R_d = Pr\{X|V(X) \geq d\} \quad (1)$$

where X is a lower boundary point for d and $V(X)$ is the maximum flow under X, $X=(x_1, x_2, \ldots, x_m)$.

X can be deduced from $F=(f_1, f_2, \ldots, f_m)$ through the following equation:

$$x_i = \sum_{j=1}^{m}\{f_j|a_i \in mp_j\} \text{ for each } i = 1, 2, \ldots, n \quad (2)$$

2.2 Critical edge

The arc a_i is said to be critical if and only if the network reliability $R_{d,i}$ is zero, where $R_{d,i}$ is the reliability of the given SFN when zero capacity is assigned to a_i (failed).

2.3 Assumptions

The following assumptions should be satisfied for the given SFN:

(i) Each node is perfect and has an infinite capacity.

(ii) The capacity of each arc has an integer-valued random variable.

(iii) The arcs are perfect and unlimited in capacity.

(iv) Flow of the given SFN satisfies the flow-conservation law [Ford and Fulkerson (1962)].

(v) The arcs are statistically independent.

3 Problem formulation

Let $M = (M^1, M^2, \dots, M^n)$ as assigned capacities to the set of edges $(a_1, a_2, \dots, a_n)$. The mathematical formulation of the problem is:

Minimize $\mathbb{C} = \sum_i^n M^i$ (3)

Subject to: $R_d \geq R_0$ (4)

where R_d is reliability corresponding to the assigned capacities under demand d and R_0 is the network reliability requirement. M^i ranges from 1 to d, except for critical edges [Lin (2008)], $M^i = d$.

4 The genetic algorithm (GA)

The following subsections describe the different components of the presented GA.

4.1 Representation, crossover and mutation

The chromosome *M* is represented by a string of length *n*, where *n* is the number of edges, as follows:

$$(M^1, M^2, \dots, M^n)$$

Figure 1: Chromosomal representation

The one-cut point crossover is used to generate two new offspring, and a simple mutation process is used to mutate the offspring. Figs. 2 and 3 show the crossover process and the mutation process, respectively.

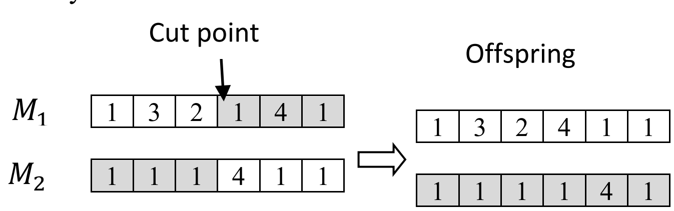

Figure 2: Crossover process

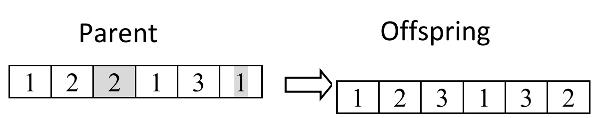

Figure 3: Mutation process

4.2 Fitness function

The following penalty function is used as a fitness function [Coit and Smith (1996); Altiparmak, Dengiz and Smith (1997); Gen and Cheng (2000)]:

$$\mathbb{C}(M)=\mathbb{C}(M)+\delta[M_{max}(R_d(M)-R_{min})]^2 \tag{3}$$

where M_{max} is the maximum capacity of M, and

$$\delta=\begin{cases}0, & if R_d(M)\geq R_{min}\\ 1, & otherwise\end{cases} \tag{4}$$

The fitness function is

$$Fit(M)=\mathbb{C}_{max}(M)-\mathbb{C}_{obj}(M) \tag{5}$$

where $\mathbb{C}_{max}(M)$ is the maximum value of Eq. (9) for the current population.

4.3 Selection process

The algorithm uses the roulette wheel selection mechanism to select new parents [Hassan (2020)], and the selection of a chromosome is based on its fitness value.

5 Entire algorithm

The following steps describe the GA used to solve the capacity assignment problem:

Start

Input the information of the network: number of edges, their reliabilities, the demand d, and paths.
Set the GA parameters.
Generate initial population.
Evaluate the initial population and calculate Fit(M) for each chromosome.
while g<= g_{max}, do
while $\wp$<= $\wp_{size}$, do
Select two parents by using roulette wheel selection.
Generate new child by applying crossover and mutation according to λ and μ, respectively.
$\wp$ = $\wp$+1;
End do.
Evaluate the current population.
g:= g+1;
End do.

End

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6 Studied cases

We used MATLAB 2013a to implement the presented algorithm. It is applied to three network examples. The GA parameters were g_{max}=100, $\wp_{size}$=20, λ=0.90, and μ=0.10.

The network in Fig. 4 includes four nodes and six edges [Chen (2012)]. The available reliabilities of the edges are 0.91, 0.93, 0.94, 0.93, 0.87, and 0.91. The best solutions found for R and C are 0.9997 and 22, respectively. Two other network examples were also studied, as shown in Figs. 5 and 6 [Chen (2012)]. The best solutions found are listed in Tab. 1.

Table 1: The best solution found for each network

Studied Network	R_{min}	M	$\mathbb{C}(M)$	$R_d(M)$
Four-node	0.9	5 5 1 1 4 5	21	0.9987
	0.99	5 3 2 1 5 5	21	0.9972
	0.999	5 5 1 1 5 5	22	0.9997
Eight-node	0.9	4 4 1 1 1 1 1 1 1 1 1 1 2 2	22	0.9591
	0.99	4 4 1 1 1 1 1 1 1 1 2 2 1 1	22.0028	0.9637
	0.999	4 4 1 1 1 1 1 1 1 1 1 1 2 2	22.0064	0.9591*
Thirteen-node	0.999	4 4 4 4 4 3 3 4 4 4 4 4 1 1 2 1 1 4 4 4 4 4 1 1 1 1 3 3 4 4 4 4 4	102.0028	0.9560*

*The R_d (M) values for eight-node and thirteen-node networks are less than those found by Chen [Chen (2012)], but the values were verified as correct for the corresponding M.

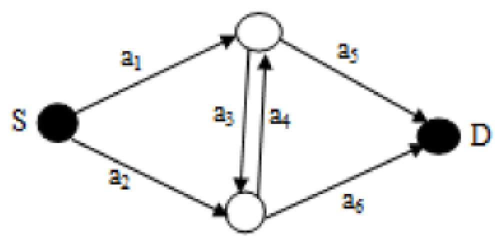

Figure 4: Four-node network with 6 edges

7 Conclusions

The capacity assignment problem was discussed in this paper and was formulated mathematically. A genetic algorithm was proposed to solve the problem. The presented algorithm successfully determined the optimal capacities with minimum total capacities and maximum reliability. The critical edges were treated in a special manner by being assigned capacity values equal to the demand. In comparison with Chen [Chen 2012], this paper finds the best capacity distribution to all studied cases. The proposed solution approach may be applied to various problem related to the capacity assignment problem.

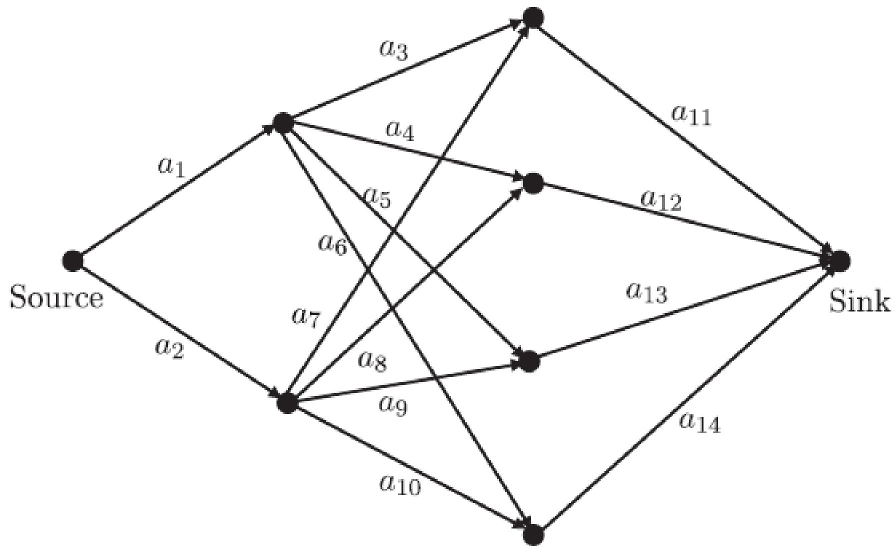

Figure 5: Eight-node network with 14 edges

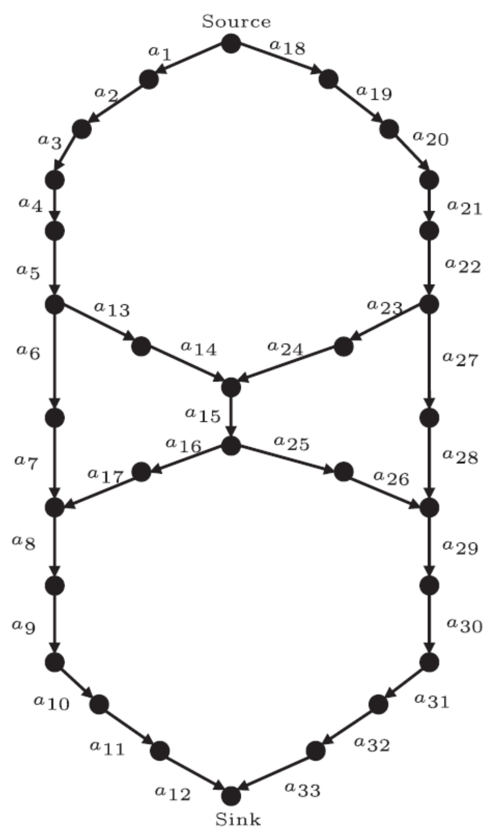

Figure 6: Thirteen-node network with 33 edges

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