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Robust EM Algorithm for Iris Segmentation Based on Mixture of Gaussian Distribution

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ABSTRACT
Density estimation via Gaussian mixture modelling has been successfully applied to image segmentation. In this paper, we have learned distributions mixture model to the pixel of an iris image as training data. We introduce the proposed algorithm by adapting the Expectation-Maximization (EM) algorithm. To further improve the accuracy for iris segmentation, we consider the EM algorithm in Markovian and non Markovian cases. Simulated data proves the accuracy of our algorithm. The proposed method is tested on a subset of the CASIA database by Chinese Academy of Sciences Institute of Automation-Iris-Twins. The obtained results have shown a significant improvement of our approach compared to the standard version of EM algorithm and the classical segmentation method.

KEYWORDS: CASIA Iris images database, EM algorithm Mixture of Distributions, Segmentation.

1 INTRODUCTION

IRIS recognition has several advantages over other biometric technologies [1]. Compared with others, such as face, speech and finger recognition, iris recognition is considered the most reliable because of its uniqueness and stability during the life of the individual. Iris segmentation plays an important role in biometric technology research. A major obstacle for iris image segmentation is the noise produced while taking the picture. the noises such as occlusion, reflections, etc… can affect the performance of the segmentation process. To improve the segmentation results, we need to maximize the iris data within the picture. And to do so, we present in this paper a novel statistical technique to segment the iris. Our models are based on EM algorithm that utilizes the mixture of Gaussian distributions family. Mixtures of Gaussian distributions are well recognized in image segmentation. Mixtures of Gaussian are flexible to accommodate various shapes of continuous distributions and able to capture leptokurtic, skewed and multimodal characteristics. In this paper, we modeled the eye image by a finite mixture of Gaussian distributions. In order to estimate the mixture parameters, we propose an extension of the EM algorithm [2] method. The EM algorithm iterates between two steps: an expectation (E) where a created function for log-likelihood is evaluated using the current estimate for the parameter and a maximization (M) step in which the estimates are maximized. In the following sections, we introduce the proposed EM algorithm called EMG algorithm. Then, we compare the behavior of the mixture-estimated model. A novel segmentation process is proposed by using the finite mixture of Gaussian distributions. Finally, we discuss results and conclude.

2 ESTIMATION OF MIXTURE MODELS

MIXTURE models play an important role in many statistical problems such as for example in biostatistics, image segmentation. Many works have been devoted to estimation of the density: finite mixture, nonparametric maximum likelihood estimation [3] give estimates for both mixing distribution and number of components. In parametric models, when number of components is fixed, the EM algorithm introduced by Dempter et al [3] has been widely used and extended in the literature.

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We consider the following model: let $X_1,........,X_N$ be an independent identically distributed random sample from a mixture with K components with density f defined as:

$$f(x)=\sum_{k=1}^{K}\pi_k f(x/\theta_k) \tag{1}$$

where $\pi_1,...,\pi_K$ are the mixing proportions (prior classification probabilities for each class), probability density $f(x/\theta_k)$ is the conditional function for data from class k, each θ_k is the set of parameters defining the kh component, and $\Theta=\{\theta_1,......,\theta_K,\pi_1,.......,\pi_K\}$ is the complete set of parameters needed to specify the mixture. Being probabilities, the π_k must satisfy:

$$0\le\pi_k\le 1 \text{and} \sum_{k=1}^{K}\pi_k=1 \tag{2}$$

In many works, the conditional probability density function is assumed as Gaussian distribution [4, 5]. Here, we are interesting in finding estimation by maximum likelihood of *f*. Before presenting the EM algorithm for mixture density estimation, we intend to define the EM algorithm.

3 EM ALGORITHM FOR MIXTURE DENSITY ESTIMATION

WE will briefly explain the steps of the EM algorithm for the mixture density estimation [6, 7]. The EM algorithm is a general iterative technique for computing maximum-likelihood (ML) when the observed data can be regarded as incomplete. In maximum-likelihood estimation [7], the unknown parameter Θ is estimated so that the log-likelihood functions:

$$L(\theta)=\sum_{i=1}^{N}\log\left[f(x_i)\right]=\sum_{i=1}^{N}\log\left[\sum_{k=1}^{K}\pi_k f(x_i/\theta_k)\right] \tag{3}$$

is maximized by using a set X= $\{x_1..............x_N\}$ of observable samples drawn independently according to the density *f(x)*. Accordingly, an estimate of Θ can be obtained as a solution of the likelihood equation given by:

$$\partial L(\Theta)/\partial\Theta=0 \tag{4}$$

Unfortunately in mixture density models, likelihood equations are usually nonlinear, which means that the general analytical solution of the log-likelihood equation may not exist. The usual EM algorithm consists of an E-step and a M-step is proposed to resolve this problem. Suppose that Θ^n denotes the estimation of Θ obtained after the nth iteration of the algorithm.

Then at the (n+1) th iteration, the E-step computers the expected complete data log-likelihood function:

$$Q(\Theta,\Theta^n)=\sum_{i=1}^{N}\sum_{k=1}^{K}\tau_k^n(x_i)\log\left[\pi_k f(x_i/\theta_k)\right] \tag{5}$$

where $\tau_k^n(x_i)$ is a posterior probability and is computed as:

$$\tau_k^n(x_i)=\frac{\pi_k^n f(x_i/\theta_k^n)}{\sum_{k=1}^{K}\pi_k^n f(x_i/\theta_k^n)} \tag{6}$$

The M-step maximizes $Q(\Theta,\Theta^n)$ function with respect to Θ to obtain the new parameter value Θ^{n+1}.

From the maximization in the M-step, the following constraints are derived:

$$\pi_k^{(n+1)}=\frac{1}{N}\sum_{i=1}^{N}\tau_k^n(x_i) \tag{7}$$

In the case of Gaussian mixture model Table 1, the estimator of $\theta_k=(\mu_k, \sigma_k^2)$ where μ_k represents the mean and σ_k^2 is the variance of the class at the (n+1) th iteration is given by:

$$\hat{\mu}_j^{n+1}=\frac{\sum_{i=1}^{N}x_i\hat{\pi}_j^n(x_i)}{\sum_{i=1}^{N}\hat{\pi}_j^n(x_i)} \tag{8}$$

$$\left[\hat{\sigma}_k^2\right]^{n+1}=\frac{\sum_{i=1}^{N}\hat{\pi}_k^n(x_i)(x_i-\hat{\mu}_k^{n+1})(x_i-\hat{\mu}_k^{n+1})^T}{\sum_{i=1}^{N}\hat{\pi}_k^n(x_i)} \tag{9}$$

EM algorithm is highly dependent on initialization. It is, in general, extremely difficult to set a good initial parameter value. In our method, we initialize the mixture parameters by K-means algorithm [8]. We extend the EM algorithm by using Gaussian distributions to introduce our new algorithm called EMG.

4 GAUSSIAN DISTRIBUTION

4.1 Gaussian Density

HERE, we are interesting to apply the EM. After presenting the EM algorithm for mixture density estimation, we intend to present. Our mixture model is based on Gaussian density function given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \quad (10)$$

4.2 Gaussian mixture curve

Since the general form of probability, function can be expressed in terms of the standard distribution, all subsequent formulas in this section are given for the standard form of the function. The following is the three Gaussian function plots *f1, f2, f3* (Figure 1).

$$f(x) = \pi 1 f1 + \pi 2 f2(x)\, \pi 3 f3 \quad (11)$$

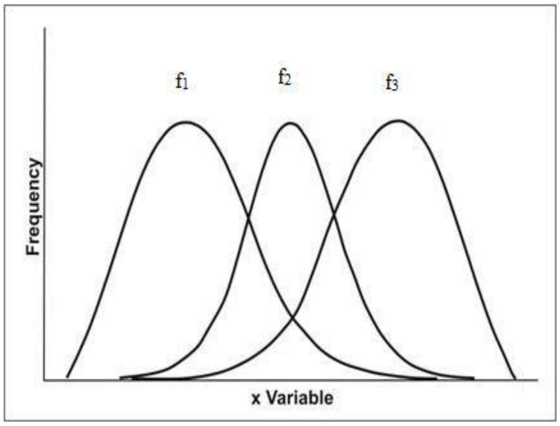

Figure 1. Mixture with three Gaussian distributions curve.

We use the finite mixture of Gaussian distributions. It is the classical model and it is used as a reference of comparison knowing that we are treating pixels iris images that belong to [0, 255] . It has a smooth shape. Therefore, we introduce the finite of mixture model composed by three Gaussian component distributions.

5 PROPOSED METHOD

FIRSTLY, mixture Gaussian distribution family is proposed to overcome the impact of noise in the images. By incorporating the EM algorithm amongst neighborhood pixels and non neighborhood pixels, the proposed algorithm is constructed based on a comparison of probabilities in order to segment the iris in both proposed environment of pixels.

The proposed iris segmentation algorithm using EMG divides the task of finding the accurate eye image into three regions. The approximate papillary circular denoted by R1, also detecting the iris surrounded between the inner and outer limbic boundary denoted by R2. The rest of the eye image such us the sclera, the eyelid, is presented by R3 as shown in Figure 2.

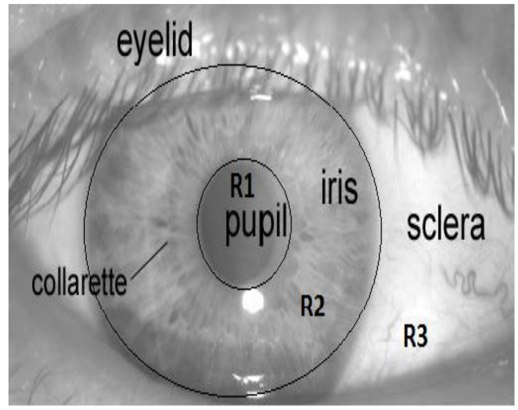

Figure 2. Modelling the eye image.

In order to segment an iris image using the mixture of Gaussian distributions model, we propose to divide the iris image into three parts. This section illustrates the estimation procedure using the simulated casia iris data. First, we generate 100 observations iris images for testing. We, then, apply the Chi-Squared test to choose one of the two algorithms in order to give the most accurate segmentation results for each different eye image. The two models for each image are the Mixture Markovian Gaussian algorithm and the Mixture independent Gaussian algorithm. Knowing that we divided the image into three parts, we estimate, for each model, three sets of parameters. Each image is then segmented using the two models in both the Markovian case and the independent case. The best model will be choosing due to our learning system. The Chi-square test is used to obtain a learning system that facilitates the proposed statistical model of segmentation process. The following are the major steps that constitute our segmentation algorithm.

Step 1 Initialization:

We apply K-Means method in order to obtain an initial segmentation of the eye in three regions R_1, R_2, R_3.

Therefore the number of segmentation regions is fixed to K=3. As shown in Figure 2, the desired three regions R_1, R_2, R_3, represent respectively as follow: the papillary, the iris, and the surrounding area of the iris.

Step 2 Chi-squared Test:

We apply Chi-squared test to choose the best density distribution *fi* of each region Ri.

The three regions R1, R2, R3 are treated during the segmentation process by using respectively a mixture of *f1, f2, f3*. Each region is affected by the adequate distribution in order to have the most reliable segmentation.

Consequently, each pixel of the eye image follows our mixture model given by the equation (1), with K=3.

Step 3 Parameters Estimation:

We estimate the mixture parameters by applying the EMG algorithm.

We try to create a learning system that makes the best distribution choice for each region.

Step 4 Estimation:

We apply the Bays formula, in order to segment the eye image in to 3 regions. Finally repeat this step until convergence. Figure 3 represents a segmentation framework to cover up all the mentioned hybrid segmentation steps.

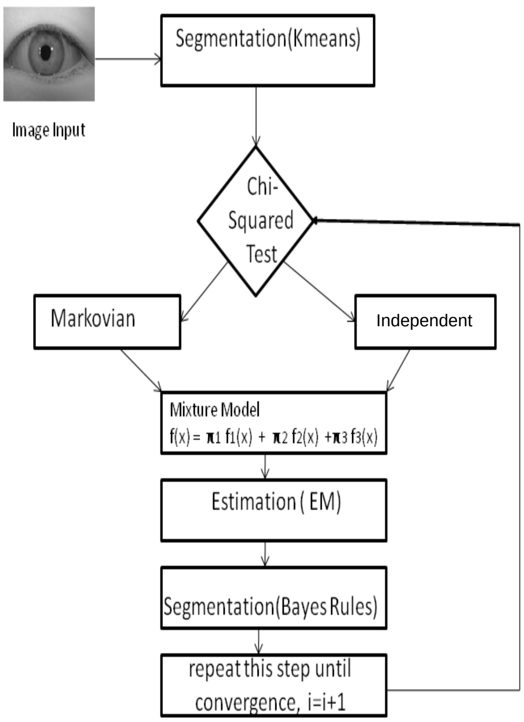

Figure 3. **Proposed segmentation framework.**

6 EXPERIMENTAL RESULTS AND EVALUATION

The comparison results demonstrate that the proposed algorithm can produce higher accuracy segmentation and has stronger ability of noising, especially in the area within the Markovian environment.

Graphical results show that the EMG algorithm performs good segmentation result. Our experimental results show that while segmenting the eye images by the EMG for Mixture Markovian Gaussian algorithm, segmented iris images are obviously close to the original ones. Some exceptions exist to prove the accuracy of the EMG segmentation using the independent algorithm (Figure 2). The Comparison tools used in this experimentation for each algorithm are the Kullback-Leibler Divergence (KL), the MSE (Mean Squared Errors) followed by the mean and the standard deviation (Std). The performance of the proposed EMG algorithm within the Markovian [9,10,11] environment is confirming by the computation of the Kullback-Leibler divergence, MSE test. The number of classes in our mixture is chosen to be equal to 3 by default because we consider the iris image is a supervised area. That assumption can fit nicely with some iris input and prevent the image from losing much of its sharpness and details. Meanwhile, the value of K-means can be tuned to be selected a little smaller or larger to tolerate the noise of some particular cases. The K-means selection is set up in this considered application of our model to fixed value because it can overcome most of iris data and it can affect the level of complexity in the initial segmentation step. However the infixed K-means value can lead to higher complexity in the proposed model, by considering a Chi-squared test to separate the different region for each iris image from the data set which, would add more complexity and lead the estimations results more accurate for the particular cases. The following is a sample of some segmented data (Figure 4).

Original Image	Markovian Segmentation	Independent Segmentation

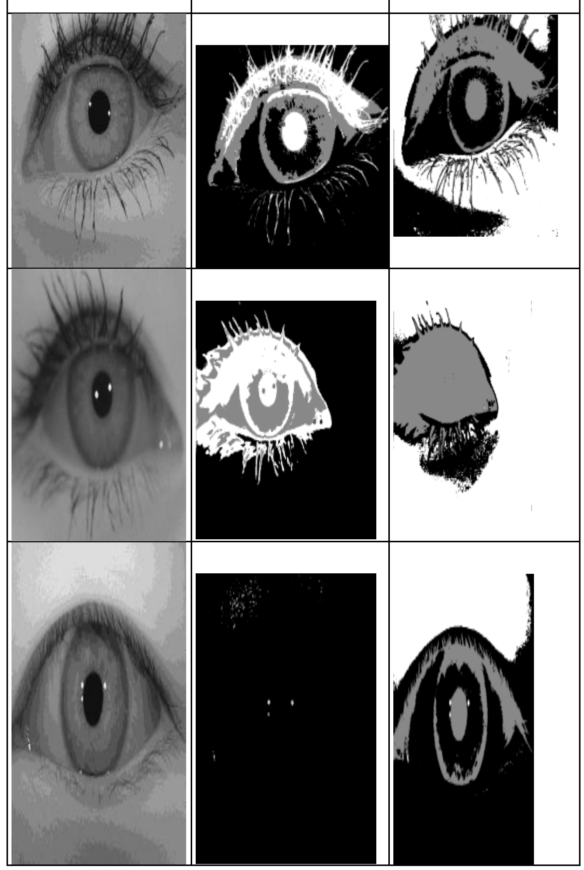

Figure 4. **Illustration of Iris Cassia Segmentation Result.**

To confirm the accuracy of EMG, our statistical results are based on minimal Kullback-Leibler distances (KL) (mean and Kullback-Leibler distance for each class), mean (MSE) and Std (MSE).

We calculate KL between the empirical density and the estimated density of the image in the Markovian and independent environments. By observing the founding in (Fig 4), we can mention that in both environments, results are comparable.

It should be noted that all these two comparison algorithms can only overcome the impact of noise in the image without considering intensity in homogeneity estimation. Therefore, in this experiment, we apply both algorithms on supervised images selected from cassia iris database containing different levels of noise, but no intensity in homogeneity iris image provides full three regions data volumes which have been simulated using three sequences (K=3) and a variety of slice thicknesses, noise levels, and levels of intensity non-uniformity. In our experiments, we use the Kullback-Leibler divergence images with different levels of noise. To make comparison, all algorithms are initialized by using 100 existing selected Casia iris data for segmentation approaches, including three EMG-based algorithms Markovian and EMG based algorithms independent. Summary of parameter setting and procedures for each comparison algorithm are listed in (Figure 4 and Figure 5). Within the Markovian environment, the KL distance calculated by mixture of Gaussian model distributions is less than the KL distance calculated by the mixture of Gaussian model distributions in the independent environment. This can lead us to mention that the segmentation process using mixture of Gaussian distributions within the Markovian gives better results than the independent environment due to the effects of the weighted neighboring pixels.

We calculate the KL between the distributions of the original image and the segmented image for each model in each case. The results are divided into two sets of KL for each iteration.

These algorithms are also applied to the segmentation of the same 100 iris images, in which the segmentation accuracy is measured in terms of the average MSE and graphical. To prove the performance of our EMG model verses the classical model, we compute the mean squared errors MSE graphical representation (Figure 6) followed by the numeric results of the men (MSE) and standard deviation as shown in (Table 2).

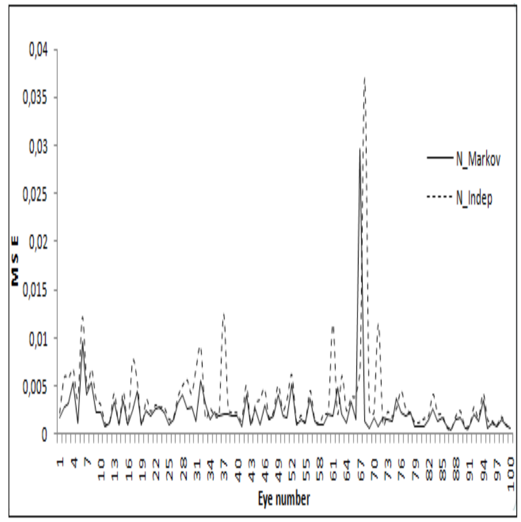

Figure 5. **MSE between the Markovian and the independent environment.**

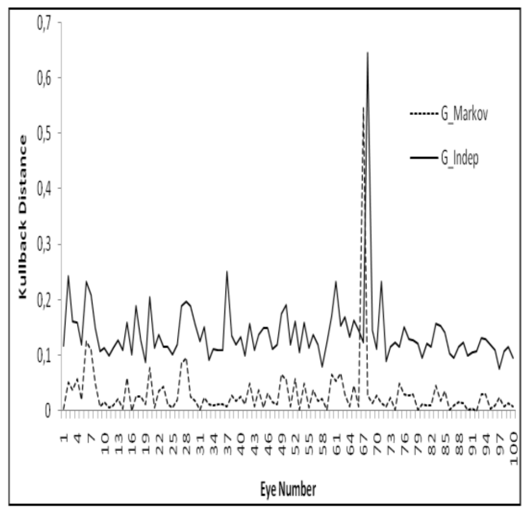

Figure 6. **Class distances between the Markovian and the independent environment.**

As shown in (Figure 7) the Markovian curve is obviously higher than the independent curve. Theses Graphical variations results proof the performance of the EMG in the Markovian case.

In the Markovian case, according to (Table 2), the MSE calculated for EMG in the independent environment is slightly lower than the MSE calculated for EMG in the Markovian environment (0, 0023 < 0, 0037), which means that MEG model is slightly better within the Markovian environment. The standard deviation mean Std (MSE) results (Table 2) confirm the performance of the proposed algorithm.

Table 1. Mean (MSE) and Std (MSE)

Eye Num	G - Markov	G - Indep
Mean (MSE)	0.0023606375	0.0037261213
Std(MSE)	0.003099956	0.004189272

Segmenting the iris using EMG algorithm tuned CASIA image type may fail partially in the Markovian cases. This is explains when the independent algorithm behaves better than the Markovian algorithm in few particular cases. Both visual and quantitative comparisons show that the proposed EMG algorithm in the Markovian environment is so robust for the selected cassia iris data and produce more accurate segmentation results except for few exceptions.

The novelties of our algorithm are the idea of segmenting the iris by using probabilistic techniques. Our algorithm is characterized by the creation of Gaussian mixture distributions in both environments the Markovian and the independent one. Due to the proposed learning system, a mixture of Gauss algorithm is used to segment the iris for each eye image. For each iris segmentation process the most adequate environment is chosen where the EMG can fit better with each eye image and give better segmentation results (Figure 5, Figure 6). Probabilistic adequate segmentation provides results by our new approach that is proved by EMG. The probabilistic segmentation method using the EMG algorithm is enhanced in the Markovian case. But, we cannot deny that the independent environment can deliver better segmentation results in few cases.

7 CONCLUSION AND PERSPECTIVE

In this paper, the mixture of Gaussian distribution model is used in order to segment the iris image. Our objective is to introduce a novel probabilistic algorithm EMG to segment the iris based on EM algorithm. It is an extension of the classical Expectation-Maximization (EM) algorithm. EMG is developed for iris images segmentation within two different environments: The Markovian case and the independent case of pixels.

The proposed approach uses different steps. First, K-Means method is set up to initialize the parameters of each mixture model. Second, choosing the optimal environment for each iris input. and creating the most fitted Gaussian mixture model for the desired observation. Then, estimating the parameters by EMG, and finally starting the segmenting process until convergence.

Therefore, the proposed EMG algorithm is flexible and easy to implement. To segment each iris image, a specific EMG mixture is created. This robustness made from EMG a novelty in the iris segmentation field in general.

A possible way to extend this work is to benefit from other distributions mixture models. We can extend the EM algorithm by: inverse-gamma, Gamma, Laplace… and analyze the segmentation results in the independent case and the Markovian case.

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