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Article

Based on Compressed Sensing of Orthogonal Matching Pursuit Algorithm Image Recovery

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Abstract: Compressive sensing theory mainly includes the sparsely of signal processing, the structure of the measurement matrix and reconstruction algorithm. Reconstruction algorithm is the core content of CS theory, that is, through the low dimensional sparse signal recovers the original signal accurately. This thesis based on the theory of CS to study further on seismic data reconstruction algorithm. We select orthogonal matching pursuit algorithm as a base reconstruction algorithm. Then do the specific research for the implementation principle, the structure of the algorithm of AOMP and make the signal simulation at the same time. In view of the OMP algorithm reconstruction speed is slow and the problems need to be a given number of iterations, which developed an improved scheme. We combine the optimized OMP algorithm of constraint the optimal matching of item selection strategy, the backwards gradient projection ideas of adaptive variance step gradient projection method and the original algorithm to improve it. Simulation experiments show that improved OMP algorithm is superior to traditional OMP algorithm of improvement in the reconstruction time and effect under the same condition. This paper introduces CS and most mature compressive sensing algorithm at present orthogonal matching pursuit algorithm. Through the program design realize basic orthogonal matching pursuit algorithms, and design realize basic orthogonal matching pursuit algorithm of one-dimensional, two-dimensional signal processing simulation.

Keywords: Compressed sensing; sarse transform; orthogonal matching pursuit; image recovery

1 Introduction

It is well known that conventional signal sampling is based on the Nyquist sampling theorem. In order not to lose the information of the signal, the signal is accurately reconstructed, and when the signal is acquired, the sampling frequency is greater than twice the highest frequency in the signal. However, as the acquisition capabilities of various signal processing systems continue to increase, the amount of data that needs to be processed later increases rapidly. The limitations of Nyquist's theorem put higher demands on the processing power of the system, and also give corresponding hardware. The design of the facility poses great challenges. How to deal with this data efficiently and save the storage space and transmission cost to the greatest extent has become one of the main bottlenecks for the further development of the information field.

In fact, the Nyquist sampling theorem is a sufficient condition for the accurate reconstruction of the signal rather than a necessary condition. The Nyquist sampling theorem is not the only and optimal sampling theory. Therefore, how to break through the extraction, processing, fusion, storage, and transmission of information based on Nyquist sampling theorem is the key to the development of information field.

In 2004, Donoho, Candes, Romnerg and Tao proposed the Compressive Sensing (CS) theory for sparse signals [1]. In the following years, the theory developed rapidly and laid the foundation for solving the above problems. Different from the traditional signal processing method, the compressed sensing theory is based on spatial transformation, and the random observation matrix is used as a means to optimize the solution as a recovery signal. Compressed sensing theory properly compresses the data while acquiring the signal. The sampling frequency is lower than the Nyquist sampling frequency, which reduces the sampling data, saves storage space, and contains enough information to pass the appropriate the reconstruction algorithm accurately, reconstructs a particular image or signal. It combines traditional data acquisition and compression into one, and does not require complex data encoding algorithms, making it ideal for applications that require small devices. Signal sparse reconstruction and compressed sensing theory have great practical value and application prospects, and have become a new research direction in the signal field.

2 Calculation Problem of Compressed Sensing

The compressed sensing method solves the problem of complex image reconstruction. Compressed sensing imaging is divided into two processes: one is the compression and sampling process of signal, the other is sparse representation of the signal. This theory can realize the low-speed compression sampling of signals, and it has been widely concerned by scholars at home and abroad in recent years. Compressed sensing theory maps high-dimensional original signals to low-dimensional observation data through uncorrelated linear observations, and then accurately recovers the original signals through sparse recovery algorithms, which can avoid a series of problems caused by Nyquist high-speed sampling. X is a one-dimensional signal of length N, the spatial representation of the vector is R^N, the elements are $[n]$ ($n=$ 1, 2, … N). All space vectors of R^N space can be expressed vectors $\{\Psi_i\}_{i=1}^{N}$. In the above expression, assuming that these base vectors are orthogonal, the basic matrix is obtained by vector transformation, and any vector can be represented by Eq. (1).

$$X = \Psi\Theta \tag{1}$$

In the Eq. (1) Θ expressed the projection coefficient, and $\Theta = [\theta_i] = [\langle X, \Psi_i \rangle]$ is formula derivation of the $N \times 1$ matrix. Therefore, this is the equivalent representation of this signal X and Θ, and the signal X in the time domain, Θ is the simplified equivalent transformation representation of signals in domain Ψ. There are less nonzero variables in Θ matrix, so we call this matrix compressible. In a more general sense, the compressible signal can be represented by the coefficient matrix K [2]. The signal that can be compressed on the basis of certain orthogonally presents exponential attenuation in a certain order, while the large coefficient is relatively small, while the small coefficient is many. The process of realizing compression by matrix transformation is called transform coding. Signal sparseness is the basis of compressed sensing theory [3]. Most natural signals do not directly show this sparseness, but they will only contain a small number of non-zero components under a certain transformation basis after mathematical processing [4]. We also call this kind of signals sparseness. The purpose of sparse representation of signal is to find a way of transformation, which makes the signal sparse. The sparse representation of signals can be divided into three categories: orthogonal transform basis, multi-scale analysis and redundant dictionary. Classical orthogonal transform bases such as discrete cosine transform, discrete wavelet transform and discrete Fourier transform have been widely used in image compression, image de-noising, super-resolution imaging and other fields [5]. Because of the characteristics of simple structure, low complexity and fast algorithm calculation, orthogonal transform bases are suitable for large-scale data operation fields. In contrast, the complexity of multi-scale analysis method is relatively high. In addition, there is a redundant dictionary representation method. Through training a series of images, we can get a dictionary matrix which can better show the sparsely of images. The effect of redundant dictionary for sparse representation is better, but this method is lack of perfect theoretical support as shown in Fig. 1.

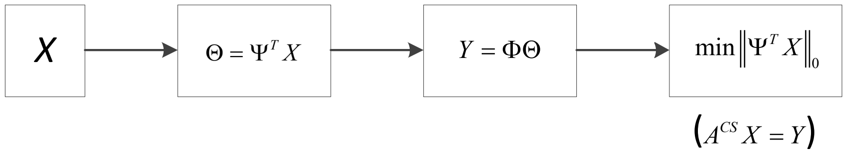

Figure 1: Signal reconstruction block diagram of compressed sensing

2.1 Sparse Representation of the Signal

The first step of compressed sensing, that is, for signal $X \in R^N$, how to find an orthogonal basis or tight frame Ψ, so that its representation on Ψ is sparse, that is, the sparse representation of the signal [6–8]. The sparse representation means that the transform coefficient vector of the signal X under the orthogonal basis is $\Theta = \Psi^T X$, and if for $0 < p < 2$ and $R > 0$, these coefficients satisfy the Eq. (2).

$$\left\| \Theta \right\|_P \equiv \left(\sum_i \left| \theta_i \right|^P \right)^{1/P} \leq R \tag{2}$$

Then the coefficient vector is sparse in some sense. How to find the optimal sparse domain Θ [9]. This is the basis and premise of the application of compressed sensing theory [10]. Only by selecting the appropriate base representation signal can the signal sparsely be ensured, thus ensuring the signal recovery accuracy. When studying the sparse representation of a signal, the sparse representation of the transform base can be measured by the transform coefficient decay rate [11–12]. Candes and Tao studies have shown that a signal with a power-velocity attenuation can be recovered using the theory of compressed sensing, and the reconstruction error satisfies Eq. (3) [13].

$$E = \left\| \hat{X} - X \right\|_2 \leq C_r \cdot (K / \log N)^{-6r} \tag{3}$$

Reference [14] pointed out that the Fourier coefficient of the smooth signal, the wavelet coefficient, the total variation norm of the bounded, the Gabor coefficient of the oscillating signal, and the Curvelet coefficient of the image signal with discontinuous edges are all sufficiently sparse. The signal can be recovered by the theory of compressed sensing [15]. How to find or construct an orthogonal basis suitable for a class of signals to obtain the most sparse representation of the signal is a problem to be further studied. Peyre extends the condition that the transform base is an orthogonal base to an orthogonal base dictionary composed of multiple orthogonal bases [16]. That is, in an orthogonal base dictionary, adaptively find the optimal orthogonal basis that can approximate a certain signal feature, find an orthogonal basis that is most suitable for the signal characteristics according to different signals, and transform the signal to obtain the most Sparse signal representation.

Another hot spot for the study of sparse representations is the sparse decomposition of signals under redundant dictionaries. This is a new signal representation. The radical function is replaced by an over complete redundant function library called a redundant dictionary [17]. The elements in the dictionary are called atoms. The choice of the dictionary should be as close as possible to the structure of the approximating signal, and its composition can be without any limitation. The K term atom with the best linear combination is found from the redundancy dictionary to represent a signal called a sparse approximation or a highly nonlinear approximation of the signal. From the perspective of nonlinear approximation, the sparse approximation of a signal consists of two levels: one is to pick a good or best base from a given base library according to the objective function; the other is to pick the best from this good base. The *K* item combination [18]. Therefore, the research on sparse representation of signals under redundant dictionary focuses on two aspects: (1) how to construct a redundant dictionary suitable for a certain type of signal; and (2) how to design a fast and efficient sparse decomposition algorithm.

In the construction of redundant dictionaries, the literature proposed the use of local Cosine basis to characterize the local frequency domain characteristics of the sound signal; the band let basis is used to characterize the geometric edges in the image; other basis functions with different shapes can also be attributed. Into the dictionary, such as Gabor base suitable for characterizing texture, Curvelet base suitable for contouring, and so on. In the design of the sparse decomposition algorithm, the MP (Matching Pursuit) algorithm based on the greedy iterative idea shows great superiority, but not the global optimal solution. Donoho later proposed a basis tracking (BP) algorithm [19]. The BP algorithm has the advantage of global optimization, but the computational complexity is extremely high. Later, a series of improved algorithms based on greedy iterative ideas, such as orthogonal matching pursuit algorithm (OMP) and segment matching (STOMP) algorithm, appeared.

2.2 Signal Observation Matrix

How to design a stable $M \times N$ dimensional observation matrix Φ which is not related to the transform base Ψ, and ensure that the important information is not destroyed when the sparse vector Θ is reduced from *N* to *M*. This is the problem to be solved in the second step, that is, the signal. Low speed sampling problem.

In the theory of compressed sensing, after transforming the sparse coefficient vector $\Theta = \Psi^T X$ of the signal, it is necessary to design the observation part of the compressed sampling system, which is developed around the observation matrix Φ. The purpose of the observer is to sample the *M* observations and ensure that the signal *X* of length *N* or the equivalent sparse coefficient vector Θ of the base Ψ can be reconstructed. Obviously, if the observation process destroys the information in X, reconstruction is impossible. The observation process actually uses the *M* row vectors $\{\varphi_j\}_{j=1}^{M}$ of the $M \times N$ observation matrix Φ to project the sparse coefficient vector, that is, the inner product between Φ and each observation vector $\{\varphi_j\}_{j=1}^{M}$ is calculated, and M observation values $y_j = <\Theta, \varphi_j> (j = 1, 2, \cdots, \mathrm{M})$ are obtained, and the observation vector $Y = (y_1, y_2, \cdots, \mathrm{y}_M)$ is obtained. The formula is shown in Eq. (4).

$$Y = \Phi\Theta = \Phi\Psi^T X = A^{CS} X \tag{4}$$

Here, the sampling process is non-adaptive, that is, Φ does not have to change according to signal *X*, and the observation is no longer a point sampling of the signal but a more general *K*-linear functional of the signal.

Finding Θ from Eq. (4) for a given *Y* is a linear programming problem, but since M<<N, that is, the number of equations is less than the number of unknown quantity, this is an underdetermined problem, generally no Determine the solution. However, if Θ has K-term sparsely (K<<M), then the problem is expected to find a definite solution. At this point, as long as we try to determine the appropriate position of the K non-zero coefficients θ_i in Θ, since the observation vector *Y* is a linear combination of the *K* column vectors corresponding to these non-zero coefficients θ_i, a linear equation of M00K can be formed. Solve the specific values of these non-zero items. In this regard, $1 - \delta_k \le \frac{\|\Theta f\|_2^2}{\|f\|_2^2} \le 1 + \delta_k$, the finite equidistant property, gives the necessary and sufficient conditions for the existence of a deterministic solution. This necessary and sufficient condition is consistent with the geometrical properties that the sparse signals must maintain under the observation matrix. That is, to completely reconstruct the signal, it must be ensured that the observation matrix does not map two different *K*-term sparse signals into the same sample set, which requires each *M* column vector extracted from the

observation matrix. The matrix is non-singular. It can be seen that the key to the problem is how to determine the position of the non-zero coefficient to construct a solvable $M \times K$ linear equation system.

2.3 Signal Reconstruction

In the theory of compressed sensing, since the number of observations *M* is much smaller than the length *N* of the signal, it has to face the problem of solving the underdetermined equations $Y = A^{CS} X$. On the surface, solving the underdetermined equations seems to be hopeless. However, both [10] and [11] point out that because the signal is sparse or compressible, this premise fundamentally changes the problem, making the problem solvable. The observation matrix has RIP properties and provides a theoretical guarantee for accurate recovery of signals from M observations. To more clearly describe the signal reconstruction problem of compressed sensing theory, first define the $p-$ norm of vector $X = \{x_1, x_2, \cdots, \mathrm{x}_n\}$ as Eq. (5).

$$\|X\|_P = \left(\sum_{i=1}^{N} |x_i|^p \right)^{1/\mathrm{p}} \tag{5}$$

When $p = 0$ gets the $0-$ norm, it actually represents the number of non-zero entries in the middle.

$$\min \|\Psi^T X\|_0 \text{ s.t. } A^{CS} X = \Phi \Psi^T X = Y \tag{6}$$

However, it needs to list C_N^K possible linear combinations of all non-zero position positions in *M* in order to get the optimal solution. Therefore, the numerical calculation for solving, Eq. (6) is extremely unstable and is an NP-hard problem. Note that this and the sparse decomposition problem are mathematically the same problem. The existing algorithm of sparse decomposition can then be applied to CS reconstruction.

Solving a simpler l_1 optimization problem yields an equivalent solution Eq. (7) (requires Φ and Ψ not related)

$$\min \|\Psi^T X\|_1 \text{ s.t. } A^{CS} X = \Phi \Psi^T X = Y \tag{7}$$

The method does not consider the relationship of the sparse signal in each sub-band position when the BP, MP and OMP methods do not consider the multi-scale decomposition of the signal. The tree structure of the sparse coefficient is utilized to further improve the accuracy and speed of the reconstructed signal. The matching tracking algorithm is based on the greedy iterative algorithm, which replaces the number of samples required by the BP algorithm with the reduction of computational complexity. For example, the OMP algorithm requires $M \geq cK$ and $c \approx 2\ln(N)$ sampling points to recover the signal with a high probability. The computational complexity of signal reconstruction is $O(NK^2)$. It simplifies the OMP to a certain extent, and further increases the calculation speed (calculation complexity is *O(N)*) at the cost of approximating accuracy, and is more suitable for solving large-scale problems.

3 Orthogonal Matching Pursuit Reconstruction Algorithm

3.1 Principle of OMP Algorithm

The OMP algorithm essentially uses the atomic selection criterion in the matching pursuit algorithm, but orthogonally processes the selected atom using the Gram-Schmidt orthogonalization method, and then projects the signal on the space formed by these orthogonal atoms to obtain a signal. The components and margins on each selected atom are then decomposed in the same way. In each step of decomposition, the selected atoms satisfy certain conditions, so the margin decreases rapidly with the decomposition process.

Iteratively optimizes the iterative optimality by recursively orthogonalizing the selected set of atoms, thus effectively overcoming the problem that the matching pursuit algorithm often needs more iterations to obtain a better convergence effect.

The OMP reconstruction algorithm is reconstructed for a given number of iterations. This forced iterative process stops so that OMP requires a lot of linear measurements to ensure accurate reconstruction. In short, it selects the columns in a greedy iterative way so that the selected column in each iteration is most correlated with the current redundant vector, subtracting the relevant part from the measurement vector and iterating iteratively until the number of iterations is sparse Degree K, forced iteration to stop.

3.2 OMP Algorithm Implementation Steps

Algorithm 1: CS recovery using OMP

Input: The CS observation y, and a measurement matrix $\Omega=\Phi\Psi=\{\omega_i,\text{i}=1,2,\cdots,\text{m}\}$ where $\Phi \in R^{n\times m}$ and $\Psi \in R^{m\times m}$.

Initialization: Index $I=\varnothing$, residual $r=y$, sparse representation $\theta=0 \in R^m$.

Iteration:

While (stopping criterion false)

$i=\arg\max_j |<r,\omega_j>|$;

$I=I\cup\{i\}$;

$r=y-\Omega(:,I)[\Omega(:,I)]^{\dagger}y$

end while

$\theta(I)=[\Omega(:,I)]^{\dagger}y$

Output: Sparse representation θ, and the original signal $x=\Psi\theta$.

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4 Compressed Sensing Image Reconstruction Simulation

When processing images, we need to transform the image, such as FFT, DCT, wavelet transform, etc., transform the image into sparse coefficients under the corresponding basis, then process the coefficient matrix in columns, and finally reverse the processed coefficients. After transforming back, you can get a sparsely reconstructed image.

In order to make the image more sparse, firstly, the original image is wavelet transformed, and then an N × N orthogonal matrix is used. Why use an orthogonal matrix? To be honest, I have been thinking about this problem for a long time, because the orthogonal basis matrix is not related to the original image, and the number of iterations can be reduced during reconstruction. After the original signal is wavelet transformed, it becomes more sparse, and the information in the high frequency band is basically gone, so that the image is reduced a lot, and it is convenient to save and transmit. Next, the wavelet transform signal is measured and reconstructed. When they reconstruct the signal, the reconstructed image is obtained after inverse wavelet transform (the number of measurements in this process is M = 190). It can be concluded from the experimental results that the reconstruction error between the original image and the reconstructed image is 30.6127. The orthogonal matching pursuit algorithm (OMP) is quite effective in reconstructing two-dimensional signals.

Next, I simulated the Lena 256×256 image with the sampling rate (M/N) of 0.9, 0.7, 0.5, 0.3, and 0.1, respectively. The restored image is shown in Fig. 2.

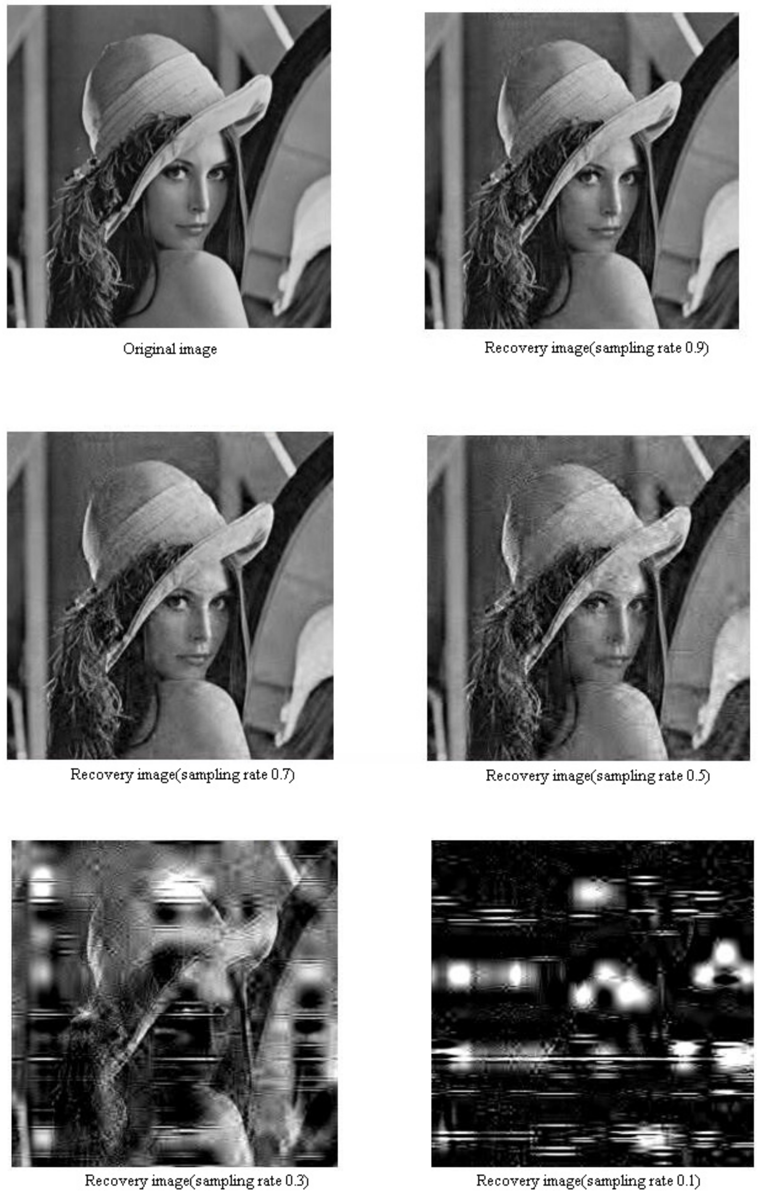

Figure 2: OMP recovery image comparison chart at different sampling rates

In order to compare the recovery effect of OMP algorithm at different sampling rates more intuitively, the PSNR value of OMP algorithm after reconstruction at 0.1, 0.3, 0.5, 0.7, 0.9 is given, as shown in Tab. 1.

Table 1: Comparison of experimental results between different M/N and PSNR

M/N	PSNR
0.1	6.13
0.3	12.62
0.5	26.80
0.7	30.13
0.9	30.61

As can be seen from the above table, as the sampling rate M/N value gradually increases, the PSNR value also gradually increases.

5 Conclusions

From the above one-dimensional signal to the two-dimensional image compression simulation reconstruction simulation can draw the following conclusions. The orthogonal matching pursuit algorithm has excellent restoration and recovery for one-dimensional signals. For two-dimensional image signals, the reconstruction of the orthogonal matching pursuit algorithm (OMP) is better, and its reconstruction time is shorter. Although the restored image of the base tracking is the clearest, its reconstruction time is far higher than the other algorithms. The OMP algorithm can take into account the reconstruction time and reconstruction quality, and is a practical reconstruction algorithm. Therefore, the orthogonal matching pursuit algorithm is more general for applications where image reconstruction requirements are not particularly high.

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