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Article

Image Reconstruction Based on Compressed Sensing Measurement Matrix Optimization Method

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Abstract: In this paper, the observation matrix and reconstruction algorithm of compressed sensing sampling theorem are studied. The advantages and disadvantages of greedy reconstruction algorithm are analyzed. The disadvantages of signal sparsely are preset in this algorithm. The sparsely adaptive estimation algorithm is proposed. The compressed sampling matching tracking algorithm supports the set selection and culling atomic standards to improve. The sparse step size adaptive compressed sampling matching tracking algorithm is proposed. The improved algorithm selects the sparsely as the step size to select the support set atom, and the maximum correlation value. Half of the threshold culling algorithm supports the concentration of excess atoms. The experimental results show that the improved algorithm has better power and lower image reconstruction error under the same sparsely criterion, and has higher image reconstruction quality and visual effects.

Keywords: Block compressed sensing; sparse representation; reconstruction algorithm

1 Introduction

Since the traditional Nyquist sampling theorem is discarded in the data transmission process, some scholars have proposed that since some data is to be discarded, whether the compressed data information of the signal can be directly obtained when the sampled data is compressible, this not only avoids the waste of storage resources, but also improves the sampling efficiency. After many scholars continue to explore, a new sampling theorem based on data sparsely is proposed, namely Compressed Sampling or compressed sensing. The theorem theory states that as long as the sampled signal is sparse the signal itself is sparse or sparse in a certain transform domain; the signal can be sampled by a standard far below the Nyquist sampling theorem. The signal is mapped to a measurement matrix that is uncorrelated with the sparse transformation matrix, and the measured value can be recovered from the original signal under the under sampling frame. The compressed sensing method directly combines sampling and data compression in the signal sampling stage, avoiding the redundant information acquisition of the Nyquist theorem sampling, and then using the optimization method to process the compressed samples in the digital signal processing. The theorem is an underdetermined linear inverse problem for signal reconstruction. The sampled data is usually recovered by an optimal iterative algorithm. The theorem has attracted the interest and research of many scholars, and is widely used in face recognition, speech recognition; military imaging radar imaging, remote sensing imaging; nuclear magnetic resonance in the medical field; deep space exploration in the field of astronomy. The compressed sensing theorem was named the top ten technology progress award in the United States in 2007.

2 Compressed Sensing Theory Mathematical Model

Compressed sensing also known as compression sensing or compressed sampling, is a technique for signal reconstruction using sparse or compressible signals [1]. Or it can be said that the signal is compressed while sampling, which greatly reduces the sampling rate. Compressed sensing skips the step of acquiring samples and directly obtains a representation of the compressed signal. The CS theory utilizes many natural signals with a compact representation on a particular basis. That is, these signals are sparse or compressible [2–3]. Due to this characteristic, the signal encoding and decoding framework of the compressed sensing theory is quite different from the traditional compression process, which mainly includes three aspects: signal sparse representation, coding measurement and reconstruction algorithm.

Signal sparsely is the premise of the CS theorem, and signal sparsely is the core step of CS sampling [4]. It is often necessary to sparsely convert non-sparse signals. Suppose the original signal is the vector in the N-dimensional space. If the vector X has only a few non-zero coefficients, it can be seen that the vector itself is sparse [5]. For non-sparse signals, if X maps a transform coefficient within a transform domain can be expressed as Eq. (1).

$$X = \psi\theta = \sum_{N} \psi_i \theta_i \tag{1}$$

where $\psi = \{\psi_1, \psi_2, \cdots \psi_N\}$ is also called sparse basis, $\theta = \{\theta_1, \theta_2, \cdots \theta_N\}$ is called the sparse representation vector of X, the sparse representation vector θ contains K (N>>K) non-zero coefficients, then θ is K sparse or vector X has K sparsely, as shown in Fig. 1 as a 3-sparse signal [6].

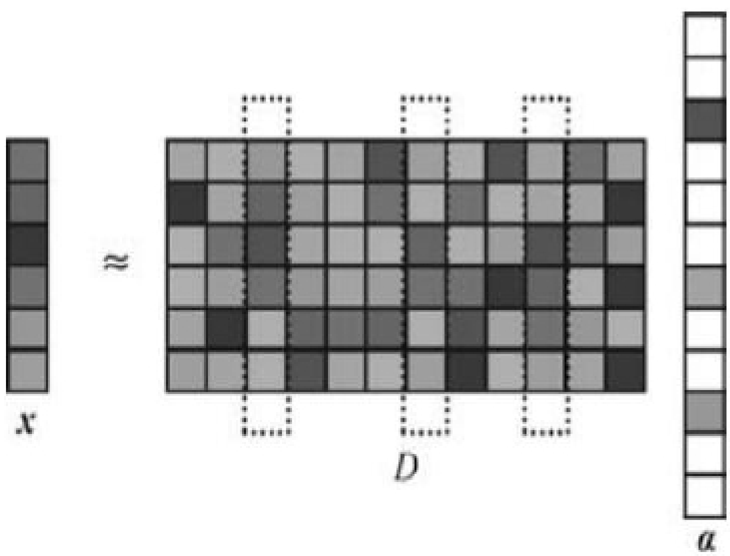

Figure 1: Signal sparse representation

From the process of compressed sensing, it can be seen that its process includes two main aspects. One is the measurement matrix, whose main function is to compress the N dimensional signal X to the M dimensional signal Y, the amount of data collected is far less than the traditional sampling method, but the measurement matrix must meet certain constraints to ensure that most of the information of the signal is not lost [7-8]. The second is the reconstruction algorithm. The main function is to obtain the original N dimensional signal X from the M dimensional signal Y through nonlinear projection. In the reconstruction process, the signal sparsely is used to obtain a reconstructed signal by solving a nonlinear optimization problem [9]. The choice of measurement matrix and reconstruction algorithm affects the error between the reconstructed signal and the original signal, and the measurement matrix in turn affects the acquisition of measured values and the performance of the reconstruction algorithm. Therefore, the core problem in compressed sensing is the design and construction of the measurement matrix [10]. The measurement matrix must meet the compression characteristics to break through the Nyquist sampling rate and also meet the requirements of the reconstruction algorithm. This paper mainly studies the content of measurement matrix in compressed sensing, and focuses on the optimization method and construction method of measurement matrix [11]. Due to the close relationship between the measurement matrix and the reconstruction algorithm, we first introduce the reconstruction algorithm and analyze which properties of the measurement matrix affect the performance of the reconstruction algorithm.

2.1 Reconstruction Algorithm

The compressed sensing reconstruction algorithm is based on the measurement matrix Φ to obtain $\hat{x}$ from the M dimensional measurement signal *Y* through a nonlinear optimization method. Since *N*>*M*, there is no array solution of $y = \Phi x$, which is ill-conditioned equation [12]. When $\Theta = \Phi\psi$ satisfies certain conditions, the solution with the most sparse characteristics is the desired solution. The mathematical model of the compressed sensing reconstruction algorithm can be described as Eq. (2).

$$\min\|s\|_0 \, st.y = \Phi\psi s = \Theta s \tag{2}$$

Where $\|s\|_0$ represents the number of non-zero elements in the vector. Due to *N*>*M* and *K* sparsely, solving all possible sparse situations can find the most sparse form of solution, but this is an NP-hard problem, so it cannot be directly Solution [13]. Therefore, how to use the conversion algorithm to solve the problem becomes the main research work of the reconstruction algorithm. The following are the main types of commonly used reconstruction algorithms.

(1) Minimum 1-norm method

At present, it has been theoretically proved that the solution of the norm l_0 in compressed sensing is equivalent to the solution of the norm l_1, and the same solution can be obtained, and the norm l_1 is a convex function to facilitate the solution. Equation (1-4) can be transformed into the following Eq. (3).

$$\min\|s\|_1 \, st \cdot y = \Phi\Psi s = \Theta s \tag{3}$$

The Eq. (3) is mathematically a convex programming problem, which can be transformed into a linear programming optimization problem and solved by the optimization method [14]. The basis pursuit method often adopts the interior point method and the gradient projection method. The reconstruction algorithm based on the minimum norm l_1 has high reconstruction accuracy and requires few measurements, but the speed is slow and the algorithm complexity is high [15]. The incoherence of the column vectors of the sensing matrix Θ directly affects the search direction, the local optimal solution, and the accuracy and reconstruction time of the reconstruction algorithm.

(2) Matching pursuit algorithm

Based on the l_1 norm minimum problem, the speed of the algorithm is slow, and an approximately minimized l_0 norm model is proposed. With a certain reconstruction error, the solution model is as shown in Eq. (4).

$$\min\|s\|_0 \, st \cdot \|y - \Phi\Psi s\| < \varepsilon \tag{4}$$

This kind of algorithm is a greedy iterative algorithm. According to the sparsity of the signal, the signal can be represented linearly by the column vector of the matrix by selecting the most matching atom from the measurement matrix and approximating the signal with the least square method. Its main feature is that the reconstruction speed is fast, but it can not guarantee the convergence to the global optimal solution, so the reconstruction accuracy is low, that is to say, when the reconstruction accuracy is high, it needs more measurement values [16]. Its rapidity makes it one of the most commonly used reconstruction algorithms in experiments and engineering. Therefore, there are more researches on these algorithms, including matching pursuit algorithm, orthogonal matching pursuit algorithm, regularized orthogonal matching pursuit algorithm, subspace tracking algorithm, adaptive matching pursuit algorithm in case of unknown sparsity and regularized adaptive matching pursuit algorithm. The performance of various matching pursuit algorithms has been improved in turn. This paper only studies the performance of different measurement matrices. The reconstruction algorithms used in the experiment are all algorithms. Although the reconstruction accuracy is general, the reconstruction speed is fast, which facilitates the simulation and observation of measurement matrix in the laboratory.

The common feature of greedy algorithms is that they are approximately expressed by the column vectors of the sensing matrix. The orthogonality of column vectors is related to the selectivity of atoms. Therefore, the non correlation of the sensor matrix column vector has an important impact on the accuracy and speed of the reconstruction algorithm [17]. The stronger the nonlinear correlation is, the less the number of iterations and the shorter the reconstruction time, the better the reconstruction effect. Through the above analysis, it can be seen that the properties of the measurement matrix not only have an important impact on the signal compression and sampling process, but also indirectly affect the reconstruction effect and the speed of reconstruction. Therefore, it is very meaningful to study the constraint conditions of measurement matrix and construct a measurement matrix with good performance. The research of measurement matrix has always been the focus and difficulty in the theoretical research and application research of compressed sensing, and is the core part and application foundation of the whole compressed sensing.

(3) Optimization of measurement matrix

Measurement matrix is the core part of compressed sensing, but its structure is not arbitrary and must meet certain constraints. Restricted isometry property, for any vector $s \in R^{[T]}$ and constant $\delta \in (0,1)$ with K sparse properties, the sensor matrix $\Theta = \Phi\psi$ has a scale constraint isometric, If the formula conditions are met.

$$(1-\delta)\|s\|_2^2 \le \|\Theta_T s\|_2^2 \le (1+\delta)\|s\|_2^2 \tag{5}$$

The measurement matrix must meet certain conditions, which can be achieved by designing a measurement matrix with good properties. Although the principle gives the condition that the measurement matrix should satisfy in theory, but in practice, the condition is difficult to be used to guide the design of measurement matrix [18]. At the same time, the principle is a sufficient condition rather than a necessary condition, which has limitations in the design of measurement matrix. In order to guide the design of measurement matrix, some scholars give the correlation discriminant method, which uses the correlation number to measure the condition of compression reconstruction. The theoretical measurement matrix can not be implemented in hardware very smoothly in practice, and the measurement matrix that can be applied in practice must meet more constraints. The fast and effective measurement matrix constructed in practical applications has the following characteristic properties. The measurement matrix is irrelevant to the sparse matrix, which guarantees a high reconstruction accuracy the number of measured values should be as close as possible to the theoretical value, reducing the acquisition of measured values The cost of the matrix has a certain structure, can be quickly sampled and quickly reconstructed less storage space, simple elements, easy to implement in hardware. These characteristics become the basis for constructing a measurement matrix that is easy to implement in hardware in this paper. According to the above measurement matrix in theory and application Some of the measurement matrices have been studied in succession. The measurement matrices can be divided into two categories from the randomness and certainty of matrix elements.

3 An optimization Method of Measurement Matrix Based on Eigenvalue Decomposition

Some optimization methods of measurement matrix have their own shortcomings, such as too many iterations and high computational complexity. This paper presents an optimization method of measurement matrix based on matrix eigenvalue decomposition. The matrix is constructed by measuring matrix and sparse transformation matrix, and a global mutual coherence coefficient based on non-diagonal elements of matrix is defined. On the basis of studying the mathematical relationship between the eigenvalues of the matrix and the mutual interference coefficient, the measurement matrix is gradually optimized by averaging the eigenvalues greater than zero in the matrix. This method has fast approximation speed and little iteration, which greatly reduces the computational complexity.

3.1 Global Mutual Interference Coefficient Based on Matrix

The mutual coherence coefficient is defined as the maximum value of the non-diagonal elements of a matrix. The disadvantage of this definition is that it can only describe local coherence. It is possible that the inner product of two columns in a matrix is relatively large, but the inner product between other columns is very small, This leads to the result that the local mutual correlation coefficient is relatively large, but the performance of the matrix is not too bad; it is also possible that the inner product of any two columns in the matrix is relatively close and not very large, but their overall correlation coefficient is relatively large, and the performance of the measurement matrix is not very good.Due to the shortcomings of the mutual coherence coefficients defined, this paper proposes a global mutual coherence coefficient based on all non-diagonal elements of the matrix. The definition formula is as Eq. (5).

$$\mu_{all} = \sum_{i \neq j} (g_{ij})^2 \tag{5}$$

This definition method describes the global coherence of a matrix. The criterion requires that any particular column of the matrix is approximately orthogonal. In this paper, the global coherence is more stable with the criterion. In addition, there is a close relationship between the global mutual coherence coefficient defined in this paper and the eigenvalue of the matrix, and the related mathematical formula can be deduced. This method can reduce the overall mutual coherence of matrix, and achieve the purpose of optimizing the measurement matrix.

3.2 Eigenvalue Decomposition Method for Reducing Mutual Interference Coefficient

Set the measurement matrix $\Phi \in R^{M \times N1}$, rank of matrix is M, $\Psi \in \mathrm{R}^{\mathrm{M \times N}}$ is a sparse transformation matrix. $D = \Phi\Psi$, Gram matrix $G = \tilde{D}^T\tilde{D}$, $\tilde{D}$ represents the matrix D after column unitization of pair. If the semi definite matrix G has M eigenvalues $\lambda_k > 0$, it is shown in Eq. (6) and (7).

$$\sum_{k=1}^{M} \lambda_k = N \tag{6}$$

$$\sum_{k=1}^{\mathrm{M}} (\lambda_k)^2 = \sum_{i,j=1}^{N} (< \tilde{d}_i, \tilde{d}_j >)^2 \tag{7}$$

After averaging the eigenvalues of each matrix, the matrix $\hat{G}$ with changed eigenvalues is obtained. Then $\hat{G}$ is decomposed into $G = \tilde{D}^T\tilde{D}$, then $\hat{D}$ is unitized to get $\tilde{\hat{D}}$, and then the eigenvalue of $G_{new} = \tilde{D}^T\tilde{D}$ is calculated. Every time the eigenvalue is modified, the eigenvalue of the new matrix will approach. The sum of squares of all the non-diagonal elements in the new matrix gradually decreases, so the eigenvalues of the optimized matrix are fast approaching to the limit value. Because the real symmetric matrix is orthogonal after eigenvalue decomposition, the orthogonally will become stronger and the non-diagonal elements of the matrix will be reduced after averaging the eigenvalues in each optimization iteration. Finally, after several iterations and optimizations, the measurement moment is obtained. Firstly, the global mutual coherence coefficient between the measurement matrix and the sparse transformation matrix is defined. Then, the global mutual interference coefficient is gradually reduced by the eigenvalue decomposition of the matrix and averaging the eigenvalues greater than zero, and then the optimized measurement matrix is obtained. The experimental results show that the performance and reconstruction quality of compressed sensing can be improved by using the optimized measurement matrix based on reducing the overall cross-correlation coefficient.

4 Experiment Simulations and Result Analysis

When processing images, we need to transform the image, such as FFT, DCT, wavelet transform, etc., transform the image into sparse coefficients under the corresponding basis, then process the coefficient matrix in columns, and finally reverse the processed coefficients. After transforming back, you can get a sparsely reconstructed image. Next, it simulated the Lena 256 × 256 image, respectively. The restored image is shown in Figs. 2 and 3.

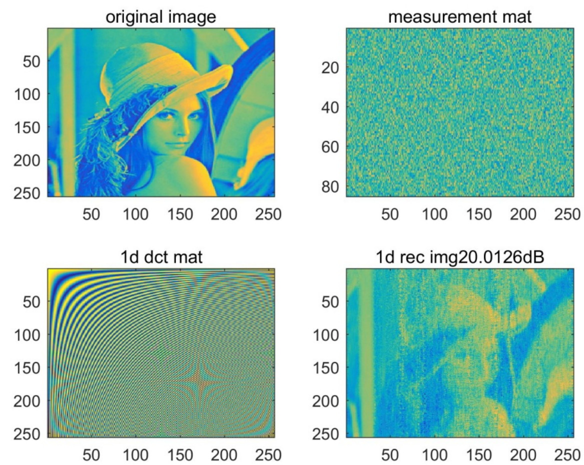

Figure 2: CoSaMP recovery image

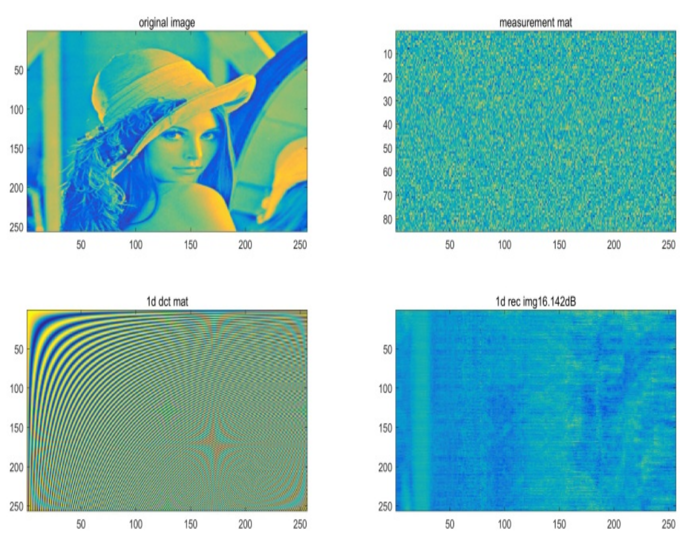

Figure 3: CS_IHT recovery image

As can be seen from the above Figs. 2 and Figs. 3, as the same sampling rate, the CaSaMP'S value is much larger than CS_IHT.

5 Conclusions

In this paper, we focus on the reconstruction algorithm used for image reconstruction based on CS. Considering that the inner product of two vectors cannot reflect the degree of their similarity to the full extent, we replace the inner product of two vectors with their correlation coefficients to propose an improved CoSaMP algorithm based on correlation coefficient. Experimental results show that this improved algorithm can improve the quality of the reconstructed image. Since the computational complexity of the correlation coefficient of two vectors is greater than the inner product of two vectors, the improved CoSaMP algorithm reduces the reconstruction efficiency. This will be the next problem to research.

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