INTELLIGENT AUTOMATION & SOFT COMPUTING, 2016
http://dx.doi.org/10.1080/10798587.2016.1261955

Fault Diagnoses of Hydraulic Turbine Using the Dimension Root Similarity Measure of Single-valued Neutrosophic Sets

Jun Ye

Department of Electrical and Information Engineering, Shaoxing University, 508 Huancheng West Road, Shaoxing, Zhejiang Province 312000, P.R. China

ABSTRACT
This paper proposes a dimension root distance and its similarity measure of single-valued neutrosophic sets (SVNSs), and then develops the fault diagnosis method of hydraulic turbine by using the dimension root similarity measure of SVNSs. By the similarity measures between the fault diagnosis patterns and a testing sample with single-valued neutrosophic information and the relation indices, we can determine the main fault type and the ranking order of various vibration faults for predicting some possible fault trend. Then, the comparison of the fault diagnoses of hydraulic turbine based of the proposed dimension root similarity measure and the existing cotangent similarity measure of SVNSs is provided to demonstrate the effectiveness and rationality of the proposed fault diagnosis method. The fault diagnosis results of hydraulic turbine show that the proposed fault diagnosis method not only gives the main fault types of hydraulic turbine, but also provides useful information for multi-fault analyses and future possible fault trends. The developed fault diagnosis method is effective and reasonable in the fault diagnosis of hydraulic turbine under single-valued neutrosophic environment.

KEYWORDS
Single-valued neutrosophic set; Dimension root distance; Similarity measure; Hydraulic turbine; Fault diagnosis

1. Introduction

Due to the complex structure of turbine-generator sets, if there is a fault of the equipment, it will produce a chain reaction and cause the fault of other parts or equipment, and then will seriously impact the reliability of power generation. Therefore, one is convinced of the importance of fault diagnoses. Various diagnosis methods have been developed and applied in hydraulic turbine-generator sets. For example, the vibration fault diagnosis of hydraulic turbine generating unit was presented based on the wavelet packet analysis and support vector machine (Peng & Luo, 2006; Zhang, Chen, & Zhang, 2013). The neural network was applied to the vibration fault diagnosis of hydraulic turbine by the particle swarm optimization (Jia et al., 2009). The fault diagnosis expert system was built by the fuzzy synthesized evaluation combined with the practical experience of experts (Xiong, Zhang, & Zhang, 2009). The data stream mining method of the associative rule classification was applied to the fault diagnosis of hydraulic turbine generator unit (Su, Nan, Yu, et al., 2012). Furthermore, fault diagnosis methods were investigated in wind turbines (Dey, Pisu, & Ayalew, 2015; Liu et al., 2015). The fault diagnosis of steam turbine generator unit was introduced based on the support vector machine (Sang & Zhang, 2013).

However, the above mentioned diagnosis methods imply some disadvantages. For instance, the fault diagnoses based on the neural network and the support vector machine are only suitable for unique fault diagnosis and require learning process for updating fault knowledge. Then, the fault diagnosis expert systems indicate the diagnosis complexity and need the practical experience of experts. Therefore, the above motioned diagnosis methods are difficult to handle multiple fault diagnoses and fault prediction. In many real situations, the diagnosis data cannot provide deterministic values, because the fault testing data obtained by experts are usually imprecise or uncertain due to a lack of data, time pressure, or experts' limited attention and knowledge. Therefore, for expressing imprecise and incomplete information in real problems, Zadeh firstly proposed fuzzy sets (Zadeh, 1965). Then, fuzzy sets have been extended to intuitionistic fuzzy sets (IFSs) (Atanassov, 1986), vague sets (Gau & Buehrer, 1993), and interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov & Gargov, 1989) and so on. Therefore, the intelligent fault diagnosis frameworks have been developed based on the fuzzy integral (Karaköse, Aydın, & Akın, 2010). Some researchers have developed the fault diagnosis methods of steam turbine based on the similarity measures of vague sets (Lu & Ye, 2013; Shi & Ye, 2013; Ye, Qiao, Wei, & Li, 2005) and the cross entropy of vague sets (Ye, 2009). However, existing diagnosis methods cannot deal with fault diagnosis problems with incomplete, indeterminate, and inconsistent information comprehensively, which exists in real world. Then, a neutrosophic set proposed by Smarandache (1998) is a powerful tool to deal with incomplete, indeterminate, and inconsistent information in the real world and extends the theory of fuzzy sets, vague sets, IFSs, and IVIFSs. The neutrosophic set is characterized by a truth-membership degree, an indeterminacy-membership degree, and a falsity-membership degree independently, which lie within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. Specially, the indeterminacy presented in the neutrosophic set is independent on the truth and falsity values and can include inconsistent information; while vague sets, IFSs, and IVIFSs are only characterized by a truth-membership degree and a falsity-membership degree, and then they can include the incomplete and uncertain information (hesitant degree), which is dependent on the truth and

CONTACT Jun Ye yehjun@aliyun.com

falsity values, but not include indeterminate and inconsistent information. Hence, the neutrosophic set is more suitable for expressing incomplete, indeterminate, and inconsistent information comprehensively. However, the neutrosophic set is difficult to apply directly in engineering fields, because the range of the truth-membership, indeterminacy-membership and falsity-membership degrees is within the nonstandard unit interval $]^-0, 1^+[$. Therefore, their range can be restrained within the real standard unit interval [0, 1] to apply easily in engineering problems. For this purpose, the concepts of a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS) were introduced as the subclasses of the neutrosophic set (Wang, Smarandache, Zhang, & Sunderraman, 2005, 2010). In a fault diagnosis problem, various symptoms usually imply a lot of incomplete, uncertainty, and inconsistent information for a fault, which characterizes a relation between symptoms and a fault. Thus, we work with the uncertainties and inconsistencies to lead us to some proper fault diagnosis. Since SVNSs are the generalization of IFSs (vague sets) and fuzzy sets, SVNSs can express the indeterminate and inconsistent information, which the intuitionistic fuzzy sets (vague sets) and the fuzzy sets cannot represent. Therefore, SVNSs have been applied to decision-making (Biswas, Pramanik, & Giri, 2016; Liu & Wang, 2014; Şahin & Küçük, 2015; Ye, 2014), clustering analysis (Ye, 2016), and medical diagnosis (Ye & Fu, 2016). As for fault diagnosis problems with single-valued neutrosophic information, Ye (2015) proposed cotangent similarity measures between SVNSs based on the cotangent function and successfully applied them to the fault diagnosis of steam turbine under single-valued neutrosophic environment. Then, Wang, Wei, and Ye (2016) presented a misfire fault diagnosis method of gasoline engines based on the cosine measure of SVNSs. Till now neutrosophic theory has not been applied to the fault diagnoses of hydraulic turbine. However, existing fault diagnosis methods for hydraulic turbine cannot handle the fault diagnosis problems with neutrosophic information. To extend existing fault diagnosis methods with neutrosophic information to hydraulic turbine in single-valued neutrosophic setting, the main purposes of this paper are to propose a new similarity measure based on a dimension root distance of SVNSs and its fault diagnosis method for the vibration fault diagnosis of hydraulic turbine with single-valued neutrosophic information.

The remainder of this paper is organized as follows: Section 2 briefly describes some basic concepts of SVNSs. Section 3 presents a dimension root distance of SVNSs and its similarity measure of SVNSs (called the dimension root similarity measure of SVNSs) and investigates the properties. Based on the dimension root similarity measure of SVNSs, we establish a fault diagnosis method for the vibration fault diagnosis of hydraulic turbine under single-valued neutrosophic environment, and then its diagnosis results demonstrate the effectiveness and nationality of the developed fault diagnosis method in Section 4. Section 5 gives conclusions and future research.

2. Some Basic Concepts of SVNSs

Smarandache (1998) firstly introduced the concept of the neutrosophic set from philosophical point of view. As mentioned above, it is difficult to apply the neutrosophic set to real problems. Therefore, Wang et al. (2010) introduced the concept of SVNS, which is a subclass of the neutrosophic set, and gave the following definition.

Definition 1. (Wang et al., 2010). Let X be a universal set. A SVNS N in X is characterized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$ and a falsity-membership function $F_N(x)$. Then, a SVNS N can be denoted by the following form:

$$N = \left\{ \left\langle x, T_N(x), I_N(x), F_N(x) \right\rangle | x \in X \right\},$$

where $T_N(x)$, $I_N(x)$, $F_N(x) \in [0, 1]$ for each point x in X. Obviously, the sum of $T_N(x)$, $I_N(x)$ and $F_N(x)$ satisfies the condition $0 \le T_N(x) + I_N(x) + F_N(x) \le 3$.

Let $N = \left\{ \left\langle x, T_N(x), I_N(x), F_N(x) \right\rangle | x \in X \right\}$ and $M = \left\{ \left\langle x, T_M(x), I_M(x), F_M(x) \right\rangle | x \in X \right\}$ be two SVNSs. Then there are the following relations (Wang et al., 2010):

(1) Complement: $N^c = \left\{ \left\langle x, F_N(x), 1 - I_N(x), T_N(x) \right\rangle | x \in X \right\}$;
(2) Inclusion: $N \subseteq M$ if and only if $T_N(x) \le T_M(x)$, $I_N(x) \ge I_M(x)$, and $F_N(x) \ge F_M(x)$ for any x in X;
(3) Equality: $N = M$ if and only if $N \subseteq M$ and $M \subseteq N$.

3. Dimension Root Distance and its Similarity Measure of SVNSs

In this section, we propose a dimension root distance and its similarity measure between SVNSs.

Definition 2. Let two SVNSs N and M in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ be $N = \left\{ \left\langle x_j, T_N(x_j), I_N(x_j), F_N(x_j) \right\rangle | x_j \in X \right\}$ and $M = \left\{ \left\langle x_j, T_M(x_j), I_M(x_j), F_M(x_j) \right\rangle | x_j \in X \right\}$. Then, a dimension root distance between SVNSs N and M is defined as

$$D_R(N, M) = \frac{1}{n} \sum_{j=1}^{n} \left\{ \frac{1}{3} \left(\left| T_N(x_j) - T_M(x_j) \right|^2 + \left| I_N(x_j) - I_M(x_j) \right|^2 + \left| F_N(x_j) - F_M(x_j) \right|^2 \right) \right\}^{1/3}, \tag{1}$$

Proposition 1. The dimension root distance measure $D_R(N, M)$ satisfies the following properties (D1)-(D4):

(D1) $0 \le D_R(N, M) \le 1$;
(D2) $D_R(N, M) = 0$ if and only if $N = M$;
(D3) $D_R(N, M) = D_R(M, N)$
(D4) If P is a SVNS in X and $N \subseteq M \subseteq P$, then $D_R(N, P) \ge D_R(N, M)$ and $D_R(N, P) \ge D_R(M, P)$.

Proof:

It is obvious that $D_R(N, M)$ satisfies the properties (D1)-(D3). Hence, we only prove the property (D4). Since $N \subseteq M \subseteq P$, this implies $T_N(x_j) \le T_M(x_j) \le T_P(x_j)$, $I_N(x_j) \ge I_M(x_j) \ge I_P(x_j)$, and $F_N(x_j) \ge F_M(x_j) \ge F_P(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Then, we have the following inequalities:

$$\left| T_N(x_j) - T_M(x_j) \right| \le \left| T_N(x_j) - T_P(x_j) \right|,$$
$$\left| T_M(x_j) - T_P(x_j) \right| \le \left| T_N(x_j) - T_P(x_j) \right|,$$
$$\left| I_N(x_j) - I_M(x_j) \right| \le \left| I_N(x_j) - I_P(x_j) \right|,$$
$$\left| I_M(x_j) - I_P(x_j) \right| \le \left| I_N(x_j) - I_P(x_j) \right|,$$
$$\left| F_N(x_j) - F_M(x_j) \right| \le \left| F_N(x_j) - F_P(x_j) \right|, \text{ and}$$
$$\left| F_M(x_j) - F_P(x_j) \right| \le \left| F_N(x_j) - F_P(x_j) \right|$$

Hence, there are $\left|T_N(x_j) - T_M(x_j)\right|^2 + \left|I_N(x_j) - I_M(x_j)\right|^2 + \left|F_N(x_j) - F_M(x_j)\right|^2 \leq \left|T_N(x_j) - T_P(x_j)\right|^2 + \left|I_N(x_j) - I_P(x_j)\right|^2 + \left|F_N(x_j) - F_P(x_j)\right|^2$ and $\left|T_M(x_j) - T_P(x_j)\right|^2 + \left|I_M(x_j) - I_P(x_j)\right|^2 + \left|F_M(x_j) - F_P(x_j)\right|^2 \leq \left|T_N(x_j) - T_P(x_j)\right|^2 + \left|I_N(x_j) - I_P(x_j)\right|^2 + \left|F_N(x_j) - F_P(x_j)\right|^2$.

Combining the above inequalities with the defined distance formula (1), we can obtain that $D_R(N, P) \geq D_R(N, M)$ and $D_R(N, P) \geq D_R(M, P)$.

Thus, we complete the proof of the properties.

Considering the importance of elements in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, one needs to give the weight w_j of the element x_j $(j = 1, 2, \ldots, n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Then, the weighted dimension root distance measure between SVNSs N and M can be defined as:

$$W_R(N, M) = \sum_{j=1}^{n} w_j \left\{ \frac{1}{3} \left(\left|T_N(x_j) - T_M(x_j)\right|^2 + \left|I_N(x_j) - I_M(x_j)\right|^2 + \left|F_N(x_j) - F_M(x_j)\right|^2 \right) \right\}^{1/3}, \tag{2}$$

Especially, when $w_j = 1/n$ $(j = 1, 2, \ldots, n)$, Equation (2) reduces to Equation (1). Obviously, the weighted dimension root distance measure also satisfies the above properties (D1)-(D4).

Based on the complementary relationship between the distance measure and the similarity measure, we can define the similarity measure based on the weighted dimension root distance.

Definition 3. Let two SVNSs N and M in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ be $N = \left\{ \left\langle x_j, T_N(x_j), I_N(x_j), F_N(x_j) \right\rangle | x_j \in X \right\}$ and $M = \left\{ \left\langle x_j, T_M(x_j), I_M(x_j), F_M(x_j) \right\rangle | x_j \in X \right\}$Then, the similarity measure based on the weighted dimension root distance between SVNSs N and M is defined as follows:

$$S_R(N, M) = 1 - \sum_{j=1}^{n} w_j \left\{ \frac{1}{3} \left(\left|T_N(x_j) - T_M(x_j)\right|^2 + \left|I_N(x_j) - I_M(x_j)\right|^2 + \left|F_N(x_j) - F_M(x_j)\right|^2 \right) \right\}^{1/3}, \tag{3}$$

which is called the dimension root similarity measure of SVNSs.

Proposition 2. The dimension root similarity measure $S_R(N, M)$ satisfies the following properties (S1)–(S4):

(S1) $0 \leq S_R(\mathrm{N, M}) \leq 1$

(S2) $S_R(N, M) = 1$ if and only if $N = M$;

(S3) $S_R(\mathrm{N, M}) = S_R(\mathrm{M, N})$

(S4) If P is a SVNS in X and $N \subseteq M \subseteq P$, then $S_R(N, P) \leq S_R(N, M)$ and $S_R(N, P) \leq S_R(M, P)$.

According to the above distance properties and the complementary relationship between the distance and the similarity measure, we can easily prove that the dimension root similarity measure should satisfy the above properties (S1)-(S4).

4. Fault Diagnosis Method of Hydraulic Turbine based on the Dimension Root Similarity Measure

For a volume of fault feature information obtained from modem measurement technologies, the fault information may contain a lot of incomplete, uncertain, and inconsistent information. In some practical situations, some fault features may include truth and falsity information and indeterminacy information, which are expressed by SVNSs. Hence, the dimension root similarity measure of SVNSs is a suitable tool to deal with fault diagnosis problems with single-valued neutrosophic information. This section proposes a fault diagnosis method for hydraulic turbine by using the proposed similarity measure of SVNSs.

4.1. Fault Diagnosis Method

For a fault diagnosis problem of hydraulic turbine, the fault diagnosis of hydraulic turbine realized by the frequency features extracted from the vibration signals of hydraulic turbine is a simple and effective method (Su et al., 2012; Zhang et al., 2013). The spectrum analysis of the vibration signals measured by the sensors is carried out, and the different frequency components in the spectrum are composed of different fault characteristic frequencies, which reflect the different fault reasons. The frequency components in the spectrum of the fault vibration data are used as the fault feature vectors. The type of faults can be determined by these fault feature vectors (Su et al., 2012; Zhang et al., 2013).

Assume that a set of m fault diagnosis patterns (fault diagnosis knowledge) is $D = \{D_1, D_2, \ldots, D_m\}$ and a set of n frequency features is $S = \{s_1, s_2, \ldots, s_n\}$. Then, the information of a fault diagnosis pattern D_k $(k = 1, 2, \ldots, m)$ with respect to a frequency feature s_i $(i = 1, 2, \ldots, n)$ is represented by a SVNS D_k $(k = 1, 2, \ldots, m)$:

$$D_k = \left\{ \left\langle s_j, T_{D_k}(s_j), I_{D_k}(s_j), F_{D_k}(s_j) \right\rangle | s_j \in S \right\}.$$

Then, the information of testing samples is represented by a SVNS R_t $(t = 1, 2, \ldots, q)$:

$$R_t = \left\{ \left\langle s_j, T_{R_t}(s_j), I_{R_t}(s_j), F_{R_t}(s_j) \right\rangle | s_j \in S \right\}.$$

The measure value δ_k $(k = 1, 2, \ldots, m)$ can be obtained by the following similarity measure between D_K and R_t:

$$\begin{aligned} \delta_k &= S_R(D_k, R_t) \\ &= 1 - \sum_{j=1}^{n} w_j \left\{ \frac{1}{3} \left(\left|T_{D_k}(s_j) - T_{R_t}(s_j)\right|^2 + \left|I_{D_k}(s_j) - I_{R_t}(s_j)\right|^2 + \left|F_{D_k}(s_j) - F_{R_t}(s_j)\right|^2 \right) \right\}^{1/3}. \end{aligned} \tag{4}$$

For easy fault diagnosis analyses, the measure values of δ_k $(k = 1, 2, \ldots, m)$ are normalized into the values of relation indices by the following formula:

$$\rho_k = \frac{2\delta_k - \delta_{\min} - \delta_{\max}}{\delta_{\max} - \delta_{\min}}, k = 1, 2, \ldots, m, \tag{5}$$

where ρ_k lies within the interval $[-1, 1]$, $\delta_{\min} = \min_{1 \leq k \leq m} \{\delta_k\}$ and $\delta_{\max} = \max_{1 \leq k \leq m} \{\delta_k\}$.

According to the relation indices, we can rank faults and determine the main fault type or predict possible fault trend for the tested equipment.

In this fault diagnosis process, if the maximum value of the relation indices is $\rho_{\max} = \max_{1 \leq k \leq m} \{\rho_k\} = 1$ in the kth relation

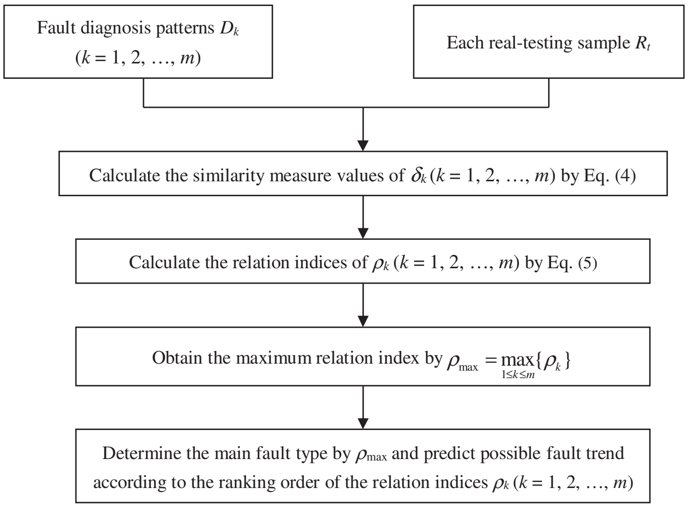

Figure 1. Overall Block Diagram of the Fault Diagnosis Approach of Hydraulic Turbine.

index, then we can determine that the testing sample R_t should belong to the main fault diagnosis pattern D_k. Then, we can also predict possible fault trend according to the ranking order of the relation indices ρ_k ($k = 1, 2, \ldots, m$). Thus, the overall block diagram of the fault diagnosis approach of hydraulic turbine based on the dimension root similarity measure of SVNSs is shown in Figure 1.

4.2. *Vibration Fault Diagnosis of Hydraulic Turbine*

In this subsection, the proposed fault diagnosis method is applied to the vibration fault diagnosis of hydraulic turbine to illustrate its effectiveness.

In hydraulic turbine-generator sets, interaction effects in the factors such as the unbalance and offset center of rotor, the bearing clearance show the vibration of the turbine-generator sets. In the vibration fault diagnosis of hydraulic turbine, the relation between the cause and the fault symptoms of the hydraulic turbine has been established by means of the analyses of frequency features in the fault frequency spectrum (Su et al., 2012; Zhang et al., 2013). Now, we investigate the vibration fault diagnosis of hydraulic turbine by use of the proposed similarity measure of SVNSs to demonstrate the effectiveness and rationality of the fault diagnosis method in this study.

Let us consider a set of four fault diagnosis patterns D = {D_1(unbalance of rotor), D_2(offset center of rotor), D_3(bigger bearing clearance), D_4(vortex band of tail water pipe)} as the fault diagnosis knowledge and a set of five frequency features in the fault frequency spectrum $S = \{s_1(0.5f), s_2(f), s_3(2f), s_4(3f), s_5(>3f)\}$ under operating frequency f as a frequency feature set (Su et al., 2012; Zhang et al., 2013). Then, the information of the fault diagnosis pattern D_k ($k = 1, 2, 3, 4$) with respect to the frequency feature s_i (i = 1, 2, 3, 4, 5) can be expressed by the form of SVNSs, as shown in Table 1. Assume that the weight of each feature s_j is $w_j = 1/5$ for j = 1, 2, 3, 4, 5.

From Table 1, the fault diagnosis patterns can be expressed as the form of SVNSs:

$$D_1 = \{<s_1, 0.01, 0.04, 0.95>, <s_2, 0.9, 0.1, 0>, <s_3, 0.08, 0.03, 0.89>, <s_4, 0.02, 0.03, 0.95>, <s_5, 0.15, 0.06, 0.79>\},$$

$$D_2 = \{<s_1, 0.01, 0.08, 0.91>, <s_2, 0.69, 0.02, 0.29>, <s_3, 0.88, 0.1, 0.02>, <s_4, 0.65, 0.1, 0.25>, <s_5, 0.1, 0.11, 0.79>\},$$

$$D_3 = \{<s_1, 0.04, 0.03, 0.93>, <s_2, 0.13, 0.03, 0.84>, <s_3, 0.08, 0.07, 0.85>, <s_4, 0.02, 0.04, 0.94>, <s_5, 0.93, 0.06, 0.01>\},$$

$$D_4 = \{<s_1, 0.88, 0.04, 0.08>, <s_2, 0.2, 0.01, 0.79>, <s_3, 0.03, 0.09, 0.88>, <s_4, 0.02, 0.21, 0.77>, <s_5, 0.1, 0.18, 0.72>\},$$

For the vibration fault diagnosis of hydraulic turbine, eight real-testing samples are given as actual examples, which are expressed by the following SVNSs:

$$R_1 = \{<s_1, 0.03, 0, 0.97>, <s_2, 0.95, 0, 0.05>, <s_3, 0.1, 0, 0.9>, <s_4, 0.04, 0, 0.96>, <s_5, 0.2, 0, 0.8>\},$$

$$R_2 = \{<s_1, 0.07, 0, 0.93>, <s_2, 0.4, 0, 0.6>, <s_3, 0.12, 0, 0.88>, <s_4, 0.03, 0, 0.97>, <s_5, 0.96, 0, 0.04>\},$$

$$R_3 = \{<s_1, 0.04, 0, 0.96>, <s_2, 0.16, 0, 0.84>, <s_3, 0.08, 0, 0.92>, <s_4, 0.06, 0, 0.94>, <s_5, 0.93, 0, 0.07>\},$$

$$R_4 = \{<s_1, 0.01, 0, 0.99>, <s_2, 0.85, 0, 0.15>, <s_3, 0.98, 0, 0.02>, <s_4, 0.71, 0, 0.29>, <s_5, 0.07, 0, 0.93>\},$$

$$R_5 = \{<s_1, 0.9, 0, 0.1>, <s_2, 0.2, 0, 0.8>, <s_3, 0.05, 0, 0.95>, <s_4, 0.02, 0, 0.98>, <s_5, 0.18, 0, 0.82>\},$$

Table 1. Vibration Fault Diagnosis Knowledge.

Fault diagnosis pattern	Frequency feature $S_1(0.5f)$	$S_2(f)$	$S_3(2f)$	$S_4(3f)$	$S_5(>3f)$
D_1 (Unbalance of rotor)	[0.01, 0.05]	[0.9, 1]	[0.08, 0.11]	[0.02, 0.05]	[0.15, 0.21]
D_2 (Offset center of rotor)	[0.01, 0.09]	[0.69, 0.71]	[0.88, 0.98]	[0.65, 0.75]	[0.1, 0.21]
D_3 (Bigger bearing clearance)	[0.04, 0.07]	[0.13, 0.16]	[0.08, 0.15]	[0.02, 0.06]	[0.93, 0.99]
D_4 (Vortex band of tail water pipe)	[0.88, 0.92]	[0.2, 0.21]	[0.03, 0.12]	[0.02, 0.23]	[0.1, 0.28]

Table 2. Results of the Relation Indices and Fault Diagnoses Based on the Dimension Root Similarity Measure.

No.	R_t	Relation indices (ρ_k) D_1	D_2	D_3	D_4	Fault diagnosis results	Actual faults
1	R_1	1.0000	−0.7579	−0.7824	−1.0000	D_1 (Unbalance of rotor)	D_1
2	R_2	0.0529	−1.0000	1.0000	−0.4695	D_3 (Bigger bearing clearance)	D_3
3	R_3	−0.1641	−1.0000	1.0000	−0.3188	D_3 (Bigger bearing clearance)	D_3
4	R_4	0.0050	1.0000	−1.0000	−0.9600	D_2 (Offset center of rotor)	D_2
5	R_5	−0.1633	−1.0000	−0.1676	1.0000	D_4 (Vortex band of tail water pipe)	D_4
6	R_6	1.0000	−0.7430	−0.8233	−1.0000	D_1 (Unbalance of rotor)	D_1
7	R_7	−0.0425	1.0000	−1.0000	−0.8805	D_2 (Offset center of rotor)	D_2
8	R_8	−0.2255	−1.0000	−0.3712	1.0000	D_4 (Vortex band of tail water pipe)	D_4

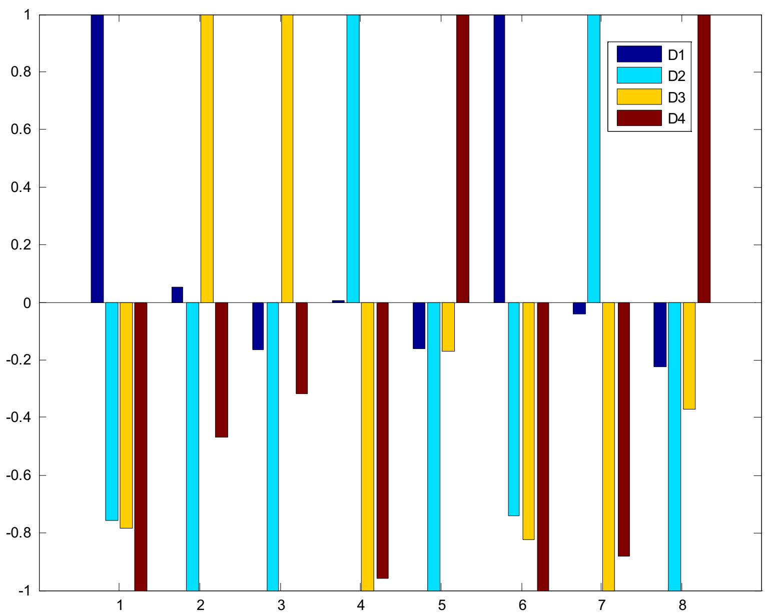

Figure 2. Fault Diagnosis Results Based on the Dimension Root Similarity Measure of SVNSs.

$$R_6 = \{< s_1, 0.02, 0, 0.98 >, < s_2, 1, 0, 0 >, < s_3, 0.08, 0, 0.92 >, < s_4, 0.03, 0, 0.97 >, < s_5, 0.18, 0, 0.82 >\},$$

$$R_7 = \{< s_1, 0.09, 0, 0.91 >, < s_2, 0.71, 0, 0.29 >, < s_3, 0.88, 0, 0.12 >, < s_4, 0.65, 0, 0.35 >, < s_5, 0.1, 0, 0.9 >\},$$

$$R_8 = \{< s_1, 0.88, 0, 0.12 >, < s_2, 0.21, 0, 0.79 >, < s_3, 0.12, 0, 0.88 >, < s_4, 0.04, 0, 0.96 >, < s_5, 0.1, 0, 0.9 >\}.$$

For the fault diagnosis of each real-testing sample R_t for $t = 1, 2, \ldots, 8$, the similarity measures and relation indices between $D_k (k = 1, 2, 3, 4)$ and $R_t (t = 1, 2, \ldots, 8)$ are calculated by Eqs. (4) and (5), then the calculating results are shown in Table 2 and Figure 2.

To show the diagnosis process in detail, we give the fault diagnosis analyses of the two testing samples R_1 and R_2 as the illustration.

As for No. 1 (the real-testing sample R_1) in Table 2 and Figure 2, we can see that the fault type of the hydraulic turbine is D_1 (unbalance of rotor) according to the maximum relation index (1.0000), which is in agreement with the actual fault. Obviously, the fault types of D_2, D_3, D_4 have very low possibility due to the negative relation indices. Therefore, the unbalance of rotor causes the violent vibration of the hydraulic turbine. Hereby, the ranking order of all faults is $D_1 \to D_2 \to D_3 \to D_4$.

As for No. 2 (the real-testing sample R_2) in Table 2 and Figure 2, we can see that the fault type of the hydraulic turbine is D_3 (bigger bearing clearance) according to the maximum relation index (1.0000), which is in agreement with the actual fault. Then, the unbalance of rotor (D_1) may exit

Table 3. Results of the Relation Indices and Fault Diagnoses Based on the Cotangent Similarity Measure.

No.	R_t	Relation index (ρ_k) D_1	D_2	D_3	D_4	Fault diagnosis result	Actual fault
1	R_1	1.0000	−0.9150	−0.5518	−1.0000	D_1(Unbalance of rotor)	D_1
2	R_2	0.1093	−1.0000	1.0000	−0.4877	D_3(Bigger bearing clearance)	D_3
3	R_3	−0.0391	−1.0000	1.0000	−0.2987	D_3(Bigger bearing clearance)	D_3
4	R_4	0.0632	1.0000	−0.8881	−1.0000	D_2(Offset center of rotor)	D_2
5	R_5	0.0563	−1.0000	0.0083	1.0000	D_4(Vortex band of tail water pipe)	D_4
6	R_6	1.0000	−0.9468	−0.5833	−1.0000	D_1(Unbalance of rotor)	D_1
7	R_7	−0.1818	1.0000	−0.9308	−1.0000	D_2(Offset center of rotor)	D_2
8	R_8	0.0094	−1.0000	−0.0465	1.0000	D_4(Vortex band of tail water pipe)	D_4

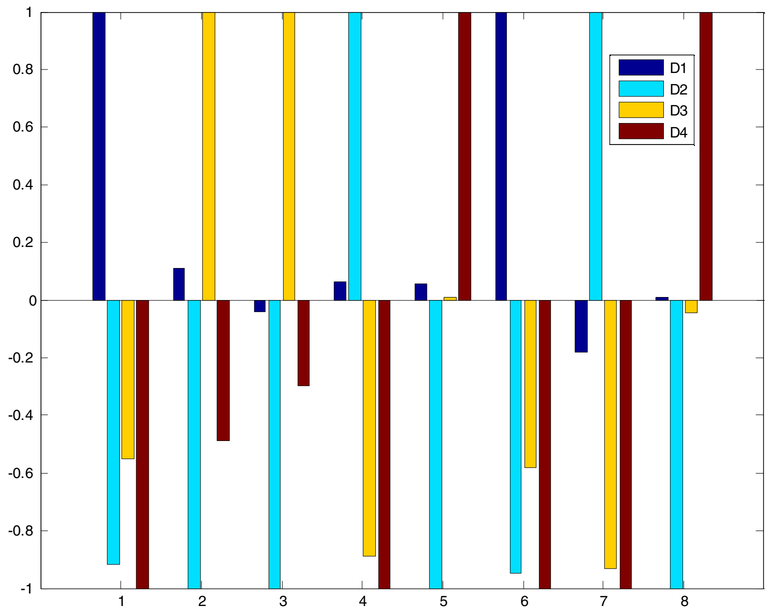

Figure 3. Fault Diagnosis Results Based on the Cotangent Similarity Measure of SVNSs.

low possibility, because its relation index is 0.0529, while the fault types D_2 and D_4 have very low possibility due to the negative relation indices. Therefore, the bigger bearing clearance is the main cause of producing the violent vibration of the hydraulic turbine. Hereby, the ranking order of all faults is $D_3 \to D_1 \to D_4 \to D_2$.

As for the rest of the real-testing samples (R_3-R_8), by the similar diagnosis analyses we can give the main faults and the ranking orders of all faults according to the relation indices. Obviously, we can see from Table 2 that all diagnosis results based on the dimension root similarity measure are in agreement with the actual faults of the hydraulic turbine.

The fault diagnosis results of the hydraulic turbine show that the diagnosis method proposed in this study not only indicates the main fault type of the hydraulic turbine, but also provides useful information for multi-fault analyses and future possible fault trend. Therefore, the fault diagnosis method developed in this paper is effective and reasonable in the vibration fault diagnoses of hydraulic turbine.

4.3. Comparative Analyses of Related Fault Diagnosis Methods

In existing literature, there is not any diagnosis method of hydraulic turbine in a neutrosophic environment until now. In fact, the fault diagnosis method of steam turbine proposed by Ye (2015) is similar to the one of hydraulic turbine except the difference of their similarity measures of SVNSs in the flow diagram of the fault diagnoses. Because a similarity measure is a key mathematical tool in the fault diagnosis, we use the cotangent similarity measure of SVNSs proposed by Ye (2015) for the comparison of the fault diagnosis problem of hydraulic turbine to demonstrate the effectiveness of the fault diagnosis method of hydraulic turbine based on the proposed dimension root similarity measure.

To realize the comparison of the fault diagnosis problem of hydraulic turbine, the dimension root similarity measure of Equation (4) can be replaced by the cotangent similarity measure of SVNSs proposed in (Ye, 2015) as the following form:

$$\delta_k = C_w(D_k, R_t) = \sum_{j=1}^{n} w_j \cot\left[\frac{\pi}{4} + \frac{\pi}{12}\left(\begin{array}{l}\left|T_{D_k}(s_j) - T_{R_t}(s_j)\right| + \\ \left|I_{D_k}(s_j) - I_{R_t}(s_j)\right| + \\ \left|F_{D_k}(s_j) - F_{R_t}(s_j)\right|\end{array}\right)\right]. \tag{6}$$

Then by using Eqs. (6) and (5), we can calculate the cotangent similarity measures and relation indices between D_k ($k = 1, 2, 3, 4$) and R_t ($t = 1, 2, \ldots, 8$) respectively, and then the results are shown in Table 3 and Figure 3.

From Tables 2 and 3, Figures 2 and 3 we can see that all the main faults given by the dimension root similarity measure and the cotangent similarity measure are identical in the fault diagnoses of the hydraulic turbine and in agreement with the actual faults. Hence, the fault diagnosis method of hydraulic turbine based on the proposed dimension root similarity measure of SVNSs is effective.

Furthermore, the fault diagnosis method of hydraulic turbine proposed in this study is simpler and more effective than the diagnosis methods of hydraulic turbines based on the neural networks (Jia et al., 2009) and support vector machine (Peng & Luo, 2006; Zhang et al., 2013), where their diagnostic results are within the accurate range of 86.32% and 92.68% (Peng & Luo, 2006; Zhang et al., 2013). It is obvious that the fault diagnosis method of hydraulic turbine proposed in this study can overcome the disadvantages of diagnosis methods based on the neural networks and support vector machine, which are only suitable for unique fault diagnosis result and require the learning process for updating fault knowledge; while the fault diagnosis method in this study does not require the learning process for updating fault knowledge and can provide multiple fault diagnosis information and future possible fault trends. Therefore, the proposed diagnosis method in this paper is easily implemented by some personal computer software, and then it is superior to the existing fault diagnosis methods of hydraulic turbine.

5. Conclusion

This paper proposed the dimension root distance and its similarity measure between SVNSs. Then, the dimension root similarity measure was applied to the vibration fault diagnosis of hydraulic turbine under single-valued neutrosophic environment. The fault diagnosis results demonstrated the effectiveness and rationality of the fault diagnosis method proposed in this study, and then it can not only indicate the main fault type of hydraulic turbine, but also predict future possible fault trend according to the relation indices. Furthermore, the proposed fault diagnosis method can deal with the fault diagnosis problems with incomplete, uncertain, and inconsistent information, which are not handled by existing fault diagnosis methods of hydraulic turbine. Therefore, the fault diagnosis method proposed in this study extends existing fault diagnosis methods and provides a new effective and simple way for the multiple fault diagnosis and fault predictions in real world applications.

In the future, the fault diagnosis method developed in this study will be further extended to fault diagnoses of other mechanical equipment, such as aircraft engines and wind turbines.

Disclosure statement

No potential conflict of interest was reported by the author.

Notes on contributor

Jun Ye received an M.S. degree in Automation and Robotics from the Technical University of Koszalin, Poland in 1997. In 2012, he was a visiting scholar in the School of Engineering of Southern Polytechnic State University in USA. Now, he is a professor in the Department of Electrical and Information Engineering, Shaoxing University, P.R. China. He has more than 30 years of experience in teaching and research. His research interests include soft computing, decision-making, intelligent control, robotics, pattern recognitions, and fault diagnosis. He has published more than 150 papers in journals.

References

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*, 87–96.

Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 31*, 343–349.

Biswas, P., Pramanik, S., & Giri, B.C. (2016). TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. *Neural Computing and Applications, 27*, 727–737.

Dey, S., Pisu, P., & Ayalew, B. (2015). A comparative study of three fault diagnosis schemes for wind turbines. *IEEE Transactions on Control Systems Technology, 23*, 1853–1868.

Gau, W.L., & Buehrer, D.J. (1993). Vague sets. *IEEE Transactions on Systems, Man, and Cybernetics, 23*, 610–614.

Jia, R., Zhang, X.W., Chen, X.Y., Li, H., Liu, J., & Song, X.F. (2009). Hydraulic turbines vibration fault diagnosis by RBF neural network based on particle swarm optimization. *Proceedings on Asia-Pacific Power and Energy Engineering Conference*, 1–4. China: Wuhan.

Karaköse, M., Aydın, İ, & Akın,E. (2010). The intelligent fault diagnosis frameworks based on fuzzy integral. IEEE Power Electronics Electrical Drives Automation and Motion Conference, Pisa, Italy. 1634-1639.

Liu, P.D., & Wang, Y.M. (2014). Multiple attribute decision making method based on single-valued neutrosophic normalized weighted Bonferroni mean. *Neural Computing and Applications, 25*, 2001–2010.

Liu, W.Y., Tang, B.P., Han, J.G., Lu, X.N., Hu, N.N., & He, Z.Z. (2015). The structure healthy condition monitoring and fault diagnosis methods in wind turbines: A review. *Renewable and Sustainable Energy Reviews, 44*, 466–472.

Lu, Z.K., & Ye, J. (2013). Cosine similarity measure between vague sets and its application of fault diagnosis. *Research Journal of Applied Sciences, Engineering and Technology, 6*, 2625–2629.

Peng, W.J., & Luo, X.Q. (2006). Research on vibrant fault diagnosis of hydro-turbine generating unit based on wavelet packet analysis and support vector machine. *Proceedings of the CSEE., 26*, 164–168.

Şahin, R., & Küçük, A. (2015). Subsethood measure for single valued neutrosophic sets. *Journal of Intelligent and Fuzzy Systems, 29*, 525–530.

Sang, L., & Zhang, T.S. (2013). Fault diagnosis of steam turbine generator unit based on support vector machine. *International Journal of Hybrid Information Technology, 6*, 101–108.

Shi, L.L., & Ye, J. (2013). Study on fault diagnosis of turbine using an improved cosine similarity measure for vague sets. *Journal of Applied Sciences, 13*, 1781–1786.

Smarandache, F. (1998). *Neutrosophy. neutrosophic probability, set, and logic*. Rehoboth, USA: Amer. Res. Press.

Su, L., Nan, H.P., Yu, X.Y., Wu, L.C., & Wang, J. (2012). The application of data stream mining method of associative rule classification in fault diagnosis of hydro-turbine generating unit. *Journal of Xi'an University of Technology, 28*, 106–110.

Wang, H., Smarandache, F., Zhang, Y.Q., & Sunderraman, R. (2005). *Interval neutrosophic sets and logic: Theory and applications in computing*. Phoenix, AZ: Hexis.

Wang, H., Smarandache, F., Zhang, Y.Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure, 4*, 410–413.

Wang, X.Q., Wei, J.L., & Ye, J. (2016). Misfire fault diagnosis of gasoline engines using the cosine measure of single-valued neutrosophic sets. *Journal of New Theory, 10*, 39–44.

Xiong, C.K., Zhang, L.D., & Zhang, Y.N. (2009). Fault diagnosis study of hydraulic turbine based on expert system. *Water Resources and Power, 27*, 161–163.

Ye, J. (2009). Fault diagnosis of turbine based on fuzzy cross entropy of vague sets. *Expert Systems with Applications, 36*, 8103–8106.

Ye, J. (2014). Single valued neutrosophic cross-entropy for multicriteria decision making problems. *Applied Mathematical Modelling., 38*, 1170–1175.

Ye, J. (2015). Single valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine. *Soft Computing*. DOI:10.1007/s00500-015-1818-y.

Ye, J. (2016). Single-valued neutrosophic clustering algorithms based on similarity measures. *Journal of Classification*. DOI:10.1007/s00357.

Ye, J., & Fu, J. (2016). Multi-period medical diagnosis method using a single valued neutrosophic similarity measure based on tangent function. *Computer Methods and Programs in Biomedicine, 123*, 142–149.

Ye, J., Qiao, X.L., Wei, H.L., & Li, W.B. (2005). Fault diagnosis of turbine based on similarity measures between vague sets. In S.C. Li, Y.J. Wang, & P. Huang (Eds.), *Proceedings in Safety Science and Technology, Science Press* (pp. 1358-1362). Beijing, P.R. China. Vol.V.

Zadeh, L.A. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.

Zhang, X.W., Chen, W.X., & Zhang, F.L. (2013). Hydraulic generating vibration faults diagnosis by support vector machine based on particle swarm optimization. *Journal of Xi'an University of Technology, 29*, 172–175.