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Failure Mode and Effects Analysis based on Z-numbers

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ABSTRACT
The main objective of this paper is to propose a new method for failure mode and effects analysis (FMEA) based on Z-numbers. In the proposed method, firstly, Z-numbers are used to perform the valuations (Z-valuation) of the risk factors like occurrence (O), severity (S) and detection (D). Secondly, the Z-valuations of the risk factors are integrated by fuzzy weighted mean method. A new risk priority number named as ZRPN is calculated to prioritize failure modes based on a modified method of ranking fuzzy numbers. Finally, a case study for the rotor blades of an aircraft turbine is performed to demonstrate the feasibility of the proposed method.

KEYWORDS
Z-numbers; Failure mode and effects analysis; ZRPN; Z-valuation; Fuzzy weighted mean method; Risk assessment

1. Introduction

Failure mode and effects analysis (FMEA), which was developed in the late 1950s, was originally used to study the problems that might arise from malfunctions of military systems. Nowadays, it has become one of the most efficient reliability analysis techniques for identifying and prioritizing potential failures in systems, designs, process and/or services. This methodology was intended to gather information for making risk management decisions, has been widely applied to several industry fields such as aerospace, engineering design and manufacturing (Braglia, Frosolini, & Montanari, 2003; Kim, Yun, Jeon, Lee, & Cho, 2010; Liu, You, Fan, & Lin, 2014; Silveira, Atxaga, & Irisarri, 2010). FMEA cannot only help analysts to identify known and potential failure modes and their causes and effects, but also help the designers to identify the key design that requires special controls for manufacturing, and to highlight areas for improvement in characteristic control or performance (Du, Lu, Su, Hu, & Deng, 2016; Yang, Huang, He, Zhu, & Wen, 2011).

Traditional FMEA uses the risk priority number (RPN) to prioritize failure modes. A RPN is obtained by multiplying the ratings of occurrence (O), severity (S) and detection (D) of a failure mode. The three factors O, S and D are all evaluated using ratings (also called rankings or scores) from 1 to 10. The failures with higher RPNs are assumed to be more important and should be given higher priorities. Owing to its visibility and simplicity, the traditional RPN method has been widely used in a number of industries as a solution to various reliability problems. However, some setbacks are still exposed to the traditional FMEA, which have been criticized by many researches (Chin, Wang, Poon, & Yang, 2009; Gargama & Chaturvedi, 2011; Pillay & Wang, 2003; Ravi Sankar & Prabhu, 2001; Sawant, Dieterich, Svatos, & Keall, 2010; Wang, Chin, Poon, & Yang, 2009).

Uncertainty is a key concept in risk conceptualisation and risk assessment (Aven, 2016). In order to measure and process uncertain information more adequately, evidence theory (Dempster, 1967; Shafer, 1976), fuzzy set theory (Zadeh, 1965) and some other new concepts like Deng entropy (Deng, 2016), evidential reasoning (Fu, Yang, & Yang, 2015), Power Average Operator (Jiang, Wei, Tang, & Zhou, 2017; Jiang, Wei, Zhan, Xie, & Zhou, 2016; Song, Jiang, Xie, & Zhou, 2017; Yager, 2001) D numbers (Mo & Deng, 2016; Zhou, Deng, Deng, & Mahadevan, 2017) and so on (Du et al., 2016; Hong, Zhang, Cao, & Du, 2016; Ning, Yuan, & Yue, 2016; Ning, Yuan, Yue, & Ramirez-Serrano, 2014; Ning et al., 2016;) were used to model and process uncertain information.

Evidence theory has been employed to quantify the imprecision and uncertainty in reliability and risk analysis (Deng, Han, Dezert, Deng, & Shyr, 2016; Deng, Jiang, & Zhang, 2017; Deng, Xiao, & Deng, 2017; Jiang, Xie, Zhuang, & Tang, 2017; Jiang & Zhan, 2016; Jiang, Zhuang, Xie, & Wu, 2017). For example, Yang et al. (2011) analyzed the risks of failure modes based on evidence theory and applied it to the application of rotor blades of an aircraft engine. Song and Jiang (Song & Jiang, 2016) applied evidence theory to engine fault diagnosis based on senor data fusion.

Fuzzy sets theory (Zadeh, 1965) is widely used in many applications due to its efficiency to model fuzzy information (Chou, 2016; Liu, 2014; Noori, 2015; Song, Wang, Lei, & Xue, 2015); such as environmental impact assessment (Rikhtegar et al., 2014), risk analysis (Akyar, 2016) and decision-making (Kahraman, Onar, & Oztaysi, 2015). It is also applied in FMEA under uncertain environments. For example, Liu, Liu, and Lin (2013) proposed a risk priority model for FMEA based on fuzzy evidential reasoning (FER) and belief-rule based (BRB) methodology to overcome the shortcomings of the traditional fuzzy FMEA. The FER method was used to aggregate the valuations of the experts and the BRB approach was used to model the uncertainty and nonlinear relationships between risks factors and corresponding risk levels.

Researchers tried to find a method, which can describe the uncertainty of information more reasonably and adequately, a model, which can simulate the process of cognitive and

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decisions of humans more properly. However, information in real life is characterized not only by fuzziness restrictions on values of variables, but also by partial reliability. Indeed the risk assessments, be it precise or not, are subject to the confidence in sources of knowledge background and experience. Therefore, fuzzy valuations or other uncertainty treatment approaches cannot adequately figure out the real-word problems. Therefore, reliability should be taken into account in the procedure of risk assessment. The concept of a Z-number is developed by L.A. Zadeh (2011) in 2011, which expresses both the restriction and the reliability of an evaluation, and is suggested as a more adequate formal construct for description of real-life information. Now, we can see more and more researchers employ Z-numbers to uncertainty modeling and decision-making. For example, Jiang et al. (2011) introduced the Z-number to model fuzziness and reliability of the sensor data and applied it to fault diagnosis. Kang et al. (2016) proposed a methodology for supplier selection with Z-numbers. In Aliev & Memmedova (2015), Z-number is applied to psychological research to increase precision and reliability of data processing results.

In this paper, we develop a model for FMEA based on Z-numbers. Firstly, we construct a Z-valuation structure, where the assessments of the risk factors are expressed by Z-numbers. Then, we use fuzzy weighted mean method (FWM) to integrate the Z-valuations of risk factors and synthesize the integrated valuations of the experts in FMEA team to a fuzzy number by taking into consideration the weights of them. Finally, ZRPNs are calculated by a modified method of ranking fuzzy numbers to rank failure modes. The advantages of this approach are that; the risk assessments are based on Z-numbers, which contain the reliability of the uncertain valuations; besides, in comparison with the traditional Fuzzy FMEA methodologies, this method overcomes the setbacks of defining a large number of membership functions and if-then rules, which are time-consuming and difficult. Last but not least, the proposed method is flexible and simple, and is much applicable for real-word applications.

The rest of the article is organized as follows: Section 2 recalls the basic theoretical background of failure risk analysis and the basic concepts of the fuzzy number and Z-number. Section 3 describes the new method developed in this study. Section 4 the validity of the proposed approach is tested in a real problem. The study is briefly summarized in Section 5.

2. Theoretical Background

In this section, we will briefly introduce some basic concepts about the FMEA, Z-number and fuzzy numbers used in this paper.

2.1. FMEA

FMEA is a technique used for defining, identifying and eliminating known and/or potential failures, problems, errors and so forth from the system, design, process, and/or service before they occur (Stamatis, 1995). Decision makers usually take the RPN as a criterion for prioritizing failures. The RPN is obtained by multiplying the ratings of O, S and D, which represents occurrence, severity and detection, respectively. Each of the three risk factors is evaluated with a rating from 1 to 10. The failure mode with a higher RPN should be given more concerns than that with a lower one. Due to the merits of simplicity and practicality, this method is widely used in many fields such as aerospace, engineering design and manufacturing, etc. However, the risk factors like O, S and D are difficult to be determined precisely in real-word applications. Some other setbacks and irrationalities of the traditional FMEA methods are listed as follows (Chin et al., 2009; Gargama & Chaturvedi, 2011; Pillay & Wang, 2003; Ravi Sankar & Prabhu, 2001; Sawant et al., 2010; Wang et al., 2009):

- The relative importance among risk factors is not taken into consideration in determining the priority of the failures. The three factors are assumed to be of equal importance, but this may not be the case in practical applications.
- The RPN considers only three factors mainly in terms of safety. Other possible influencing factors such as economical aspects are ignored.
- Different sets of O, S and D ratings may produce exactly the same value of RPN, although their hidden risk implications may be totally different. For example, two different failures with the O, S and D values of 1, 4, 9 and 1, 6, 6, respectively, have the same RPN value of 36.
- The RPN elements have many duplicate numbers. Although 1000 numbers are assumed to be produced from the product of O, S and D, only 120 of them are unique.

2.2. Fuzzy Numbers

The theory of fuzzy numbers (Dubois & Prade, 1978) is based on the theory of fuzzy sets. It can well express information that is vague and imprecise, thus, is widely used in statistics, computer programming, engineering, experimental science and so on. Corresponding definitions along with some basic notions on fuzzy sets are given as follows:

Definition 2.1. A fuzzy set A is defined on an universe X may be given as:

$$A = \{\mu_A(x) | x \in X\} \tag{1}$$

Where $\mu_A \to [0, 1]$ is the membership function A. The membership value $\mu_A(x)$ describes the degree of $x \in X$ in A.

Definition 2.2. A fuzzy number A is a fuzzy subset of the real line X with the membership function A. The triangular fuzzy number and trapezoidal fuzzy number are the two most widely used fuzzy numbers, which are defined as follows:

$$\mu_A = \begin{cases} 0, & x \le a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & a_3 \le x \end{cases} \tag{2}$$

$$\mu_A = \begin{cases} 0, & x \le a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0, & x \le a_4 \end{cases} \tag{3}$$

Definition 2.3. The α-cut, $\alpha \in (0,1]$, of a fuzzy number A is a crisp set defined as:

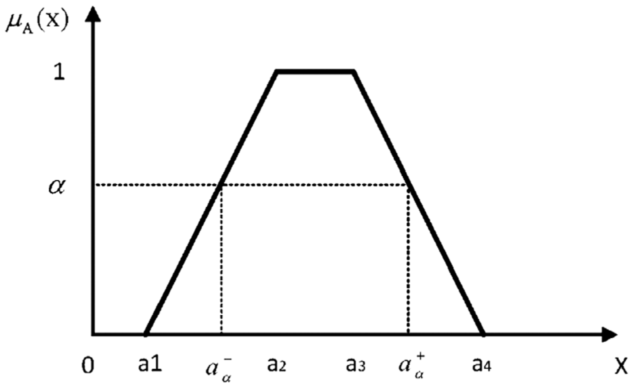

Figure 1. A α-cut Set of the Fuzzy Number A.

$$A_\alpha = \{x|x \in X, A(x) > \alpha\} \quad (4)$$

A α – cut of a fuzzy number A can be noted as a closed interval, which can be shown in Figure 1:

$$A_\alpha = [a_\alpha^-, a_\alpha^+] = [infA_\alpha, supA_\alpha] \quad (5)$$

2.3. Z-number

The real word information is characterized not only by fuzziness; the other essential property of the information is its partial reliability. In view of this, the concept of a Z-number is proposed by L.A. Zadeh to describe the real-world information more adequately. According to Zadeh (2011), a Z-number is an ordered pair fuzzy numbers denoted as $Z = (A, B)$. The first component A is a restriction on the values of an uncertain variable X. The second component B is a measure of reliability of the component A.

In real life, much of everyday reasoning and decision-making is based on a collection of Z-valuations.

For a random variable X, the ordered triple (X, A, B) refers to a Z-valuation and it is equal to the statement X is (A, B) (Yager, 2012; Zadeh, 2011). This can be interpreted as Prob(X *is* A), and

$$\text{Prob}(X \textit{ is } A) = P(A) = \int_R \mu_A(u)p_X(u)\mathrm{d}u \quad (6)$$

Where $p_X(u)$ is the underlying probability distribution of X. So the fuzzy number B plays a role of a soft constraint on a value of probability measure $P(A)$ of A. As $P(A)$ is determined on $p_X(u)$, fuzziness of $P(A)$ implies that actual probability distribution is not known (Aliev, Alizadeh, & Huseynov, 2015).

Some examples of the Z-valuations are as follows:

(Oil price will be significantly higher than 100 dollars/barrel, very likely) and

(Anticipated budget deficit, close to 3 million dollars, likely) and

(Population of Spain, about 47 million, quite sure).

3. Description of the New Method for FMEA

The main idea of the proposed approach in this paper is to develop a new priority model for FMEA that can better model and process uncertain information. The flowchart in Figure 2 shows the overall procedure of the proposed approach. In the Z-valuation process, a Z-valuation triplet is developed to express the uncertain valuations of the three risks factors by using Z-numbers. And the ranking Z-numbers process is a procedure used to prioritize risks by processing and ordering the information of the Z-valuations. The details of the arithmetic's are described in the following subsections.

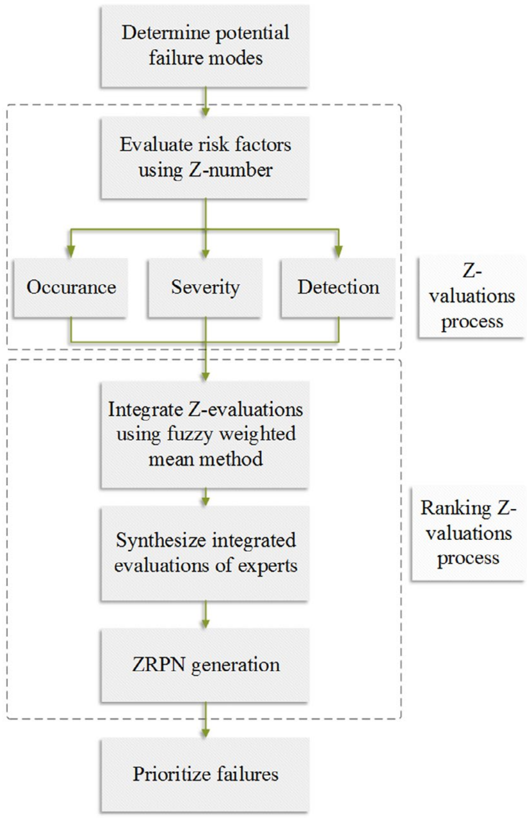

Figure 2. The Flowchart of the Proposed Method.

3.1. Z-valuation for Risks

In this section, a Z-valuation triplet for risk assessment for FMEA is defined, where the assessments of the risk factors are expressed by Z-numbers. To reduce the analytical and computational complexity, component A and B in a Z-number are assumed to be trapezoidal (triangle) fuzzy numbers.

Definition 3.1. Suppose there are K experts conducting risk assessments of L risk factors for M potential failure modes. Then the kth expert's Z-valuation of the lth risk factor for failure mode i can be defined as an ordered triplet:

$$\tilde{Z}_l^k = \left(FM_{il}, A_l^k, B_l^k\right) \quad (7)$$

Where, $i = 1\cdots M$, $l = 1\cdots L$, $k = 1\cdots K$. FM_{il} represents the risk degree of risk factors l with respect to the ith failure mode.

$A_l^k = \left(a_{l1}^k, a_{l2}^k, a_{l3}^k, a_{l4}^k\right)$ and $B_l^k = \left(b_{l1}^k, b_{l2}^k, b_{l3}^k\right)$

Represents the restraint of risk and the related reliability, respectively, where $a_{lj}^k(j = 1, 2, 3, 4) \in [0, 10]$,

$a_{lr}^k(r = 1, 2, 3) \in [0, 1]$. Then the defined Z-valuation can be interpreted as; the fuzzy probability of (FM_{il} is A_l^k) is B_l^k.

The Z-valuation model can be shown in Figure 3. The first component A is defined within the interval [0, 10]. The second component B is within the interval [0, 1], the more the confidence or reliability of valuation, the larger the value B represented. Some simple examples about the triplets of Z-valuation are listed as follows:

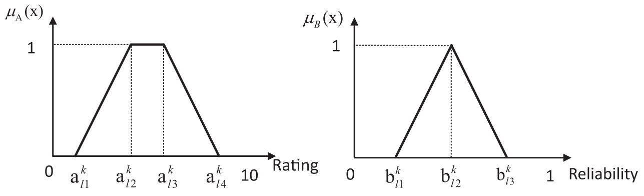

Figure 3. The Proposed Z-valuation Model.

$$\tilde{Z}_O^1 = (FM_{1O}, (4,5,5,6), (0.5,0.7,0.9))$$
$$\tilde{Z}_S^2 = (FM_{3S}, (0,1,2,4), (0.6,0.7,0.8))$$
$$\tilde{Z}_D^3 = (FM_{7D}, (6,7,8,9), (0.7,0.7,0.7))$$

3.2. Fuzzy Weighted Mean for Integrating Z-valuations

Ranking fuzzy numbers is a very important issue in a fuzzy set theory. It can give decision makers the best alternative and the ranking of the other alternatives. Therefore, similarly, the method of ranking Z-numbers can be a solution to prioritize the risks in FMEA. The existing methods of ranking Z-numbers, such as Bakar & Gegov (2015) and Kang, Wei, Li, & Deng (2012), usually transform the Z-numbers to fuzzy numbers by converting the component B to a crisp number, then use the methods of ranking fuzzy numbers to solve the problem. However, by reducing the fuzzy number B to a single number, we will lose the information we have purposely been keeping throughout the risk assessment. Therefore, in the process of integrating Z-valuations, we try to keep the fuzzy information of the B. Meanwhile, take the information of A as the main effect on the risk assessment (Jiang, Xie, Luo, & Tang, 2017). Taking into account these concerns, we employ the FWM to address this issue. This method takes the reliability as the fuzzy weight of the uncertain valuations and avoids transforming the fuzzy information of component B into a crisp number. The FWM has been studied in depth in recent years (Dong & Wong, 1987; Kao & Liu, 2001). For example, in Chen & Chen (2003) and Chen, Munif, Chen, Liu, & Kuo (2012), Chen et.al used the FWM to aggregate the valuation items of the components of the manufactory for risk analysis. Wang et al. (2009) proposed a fuzzy weighted geometric mean method (FWGM) and applied it to FMEA. In this paper, we employ FWM to integrate Z-valuations of the risk factors for each of the experts, the arithmetic is as follows:

$$\tilde{V} = \left(\sum_{l=1}^{L} A_l^k \times B_l^k \right) / \sum_{l=1}^{L} B_l^k = (a_1^k, a_2^k, a_3^k, a_4^k) \tag{8}$$

Where $\tilde{V}$ is an integrated fuzzy number; $l = 1 \cdots L$ represents the risk factors O, S, D, etc.; $k = 1 \cdots K$ represents the K experts.

Considering that the weights of the experts are different, the fuzzy numbers$\tilde{V}$ of the K experts are synthesized with the following formula:

$$\bar{V} = \left(\sum_{k=1}^{k} w_k a_1^k, \sum_{k=1}^{k} w_k a_2^k, \sum_{k=1}^{k} w_k a_3^k, \sum_{k=1}^{k} w_k a_4^k \right) = \left(\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4\right) \tag{9}$$

Where $\bar{V}$ is a synthesized fuzzy number, is denoted as $\left(\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4\right)$; w_k is the weight of the kth expert.

Take into consideration that the value of $\bar{a}_4$ may exceed the maximum of the rating (i.e. 10). The following method, which is based on α-cut set is used to standardize the fuzzy number.

For a synthesized valuation $\bar{V} = \left(\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4\right)$, α is defined as;

$$\alpha = \begin{cases} \frac{\bar{a}_4 - 10}{\bar{a}_4 - \bar{a}_3}, & \bar{a}_4 > 10 \\ 0, & \bar{a}_4 \leq 10 \end{cases} \tag{10}$$

Then,

$$\begin{aligned} a_\alpha^- &= \alpha\left(\bar{a}_2 - \bar{a}_1\right) + \bar{a}_1, \\ a_\alpha^+ &= \alpha\left(\bar{a}_4 - \bar{a}_3\right) + \bar{a}_4 \end{aligned} \tag{11}$$

The normalized fuzzy number can be noted as $\bar{V}_\alpha$, the definition of which is defined as follows:

$$\bar{V}_\alpha = \left(a_\alpha^-, \bar{a}_2, \bar{a}_3, a_\alpha^+\right) \tag{12}$$

3.3. The ZRPN Generation Method

In order to prioritize the risks of the failure modes, a RPN is usually employed as a ranking reference standard. The idea and method of producing a RPN is various. For example, a classical RPN is obtained by multiplying the scores of O, S and D, directly. This method is simple and practical, and is widely used in many applications. However, many shortcomings are exposed to this method, which has been discussed in previous. In Yang et al. (2011), evidence theory is used to express more uncertain information in the process of evaluating risks and integrating information. These methods can overcome some setbacks of the traditional FMEA; however, the final RPN generation methods in these papers back to produce the classical RPN, which may confront with the same problems with the traditional FMEA in some cases. Take into account this fact; we proposed a novel RPN generation method. Firstly, we employ the FWM to integrate the Z-valuations of the O, S, and D. Then, the integrated valuations are synthesized by considering the weights of the experts. Finally, a modified method of Chen et al. (2012) is presented for defuzzification to produce the ZRPN.

In (2012) Chen et. al proposed a ranking method for generalized fuzzy numbers with different left heights and right heights. This method takes into consideration of the position information of a fuzzy number and can well address the issue of ranking fuzzy numbers. Based on this method, we present a new method to produce the ZRPN.

$$ZRPN_i = \frac{M_i - N_i}{M_i + N_i + (10 - COG(A_i))} \tag{13}$$

Where, $M_i = LN_i + RN_i$, $N_i = LP_i + RP_i$; LN_i (Left Negative area), RN_i (Right Negative area), LP_i (Left Positive area) and RP_i (Right Positive area) are the four shaded areas in Figure 4, respectively. $COG\left(A_i\right)$ is the center-of-gravity (COG) of the fuzzy number A_i, which is defined as:

$$COG(A_i) = \frac{\int_{x_1}^{x_2} x g_1(x)\mathrm{d}x + \int_{x_2}^{x_3} x\mathrm{d}x + \int_{x_3}^{x_4} x g_3(x)\mathrm{d}x}{\int_{x_1}^{x_2} g_1(x)\mathrm{d}x + \int_{x_2}^{x_3} \mathrm{d}x + \int_{x_3}^{x_4} g_3(x)\mathrm{d}x} \tag{14}$$

4. Test Problems and Discussion of Results

To demonstrate the feasibility of the proposed risk assessment and priority model, an illustrative example about the

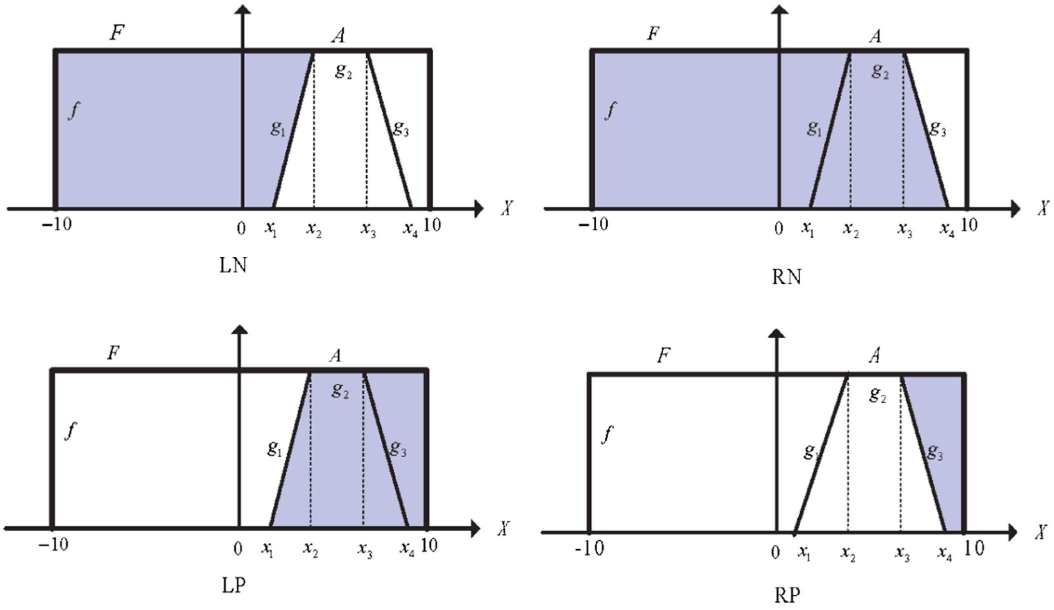

Figure 4. The Areas on the Negative Side and the Positive Side of the Fuzzy Number $\underline{A}_i$.

rotor blades of an aircraft turbine is presented. The rotor blade includes two different subsystems; the compressor rotor blades and the turbo rotor blades. The effects of each failure mode on the system are studied. For each of the failure modes, the system is investigated for any alarms of conditions monitoring arrangement (Yang et al., 2011). There are nine potential failure modes and seven of them with their effects and alarms on the system are adopted as a FMEA case performed in this paper. The following steps lead to the risk priority of the seven failure modes according to the arithmetic's defined above.

Step 1 Construct Z-valuations for the failure modes.

The risk assessments for the three risk factors are performed by 3 experts. The weights of the three experts are 0.4, 0.3, 0.3, respectively. Suppose the Z-valuations have the same components *B* (reliability), *B* = (0.7, 0.8, 0.9), and the components *A* (restriction) of the Z valuations are shown in Table 1.

Step 2 Calculate the integrated valuations $\tilde{V}$ and the synthesized valuations $\bar{V}(\bar{V}_\alpha)$

Integrate each experts' Z-valuations of O, S, D with respect to a certain failure mode with Equation (8). Then synthesize and standardize the integrated information of three experts with Equation (9) and Equation (12). The integrated valuations of the risk factors $\tilde{V}$ and the synthesized valuations $\bar{V}(\bar{V}_\alpha)$ are obtained in Tables 2 and 3.

Step 3 Calculate the ZRPNs of the seven failure modes then prioritize the risks of the failure modes.

The Z-valuations are converted to a series of fuzzy numbers, finally. According to Equation (13) and Equation (14), the ZRPNs belonging to the seven failure modes are calculated and listed in Table 3.

It can be seen from Table 3 that failure mode 1 has the largest ZRPN in the seven failure modes, followed by failure modes 5, 2, 7, 4, 6, 3. The larger the ZRPN is the more attention should be paid to the corresponding failure mode. Therefore, the failure modes 1, 5, 2 have higher priority than failure modes 7, 3, 6 and 4, and the risk of the failure mode 1 has the top priority to be concerned. So the final risk priority of the seven failure modes is: FM1 > FM5 > FM2 > FM7 > FM4 > FM6 > FM3.

To further illustrate the effectiveness of the proposed method, we make comparisons with the traditional RPN method and the FWGM method proposed by Wang et al. (2009) based on the above experimental data. Table 4 gives the RPNs of the three methods with their related rankings.

From Table 4, we can find that the result of the proposed ZFMEA is in accordance with the FWGM method (Wang et al., 2009). That is FM1 > FM5 > FM2 > FM7 > FM4 > FM6 > FM3. Further, from Figure 5, we can clearly find the trend of RPN with respect to the two methods goes similarly. As

Table 1. The Components A of the Z-valuations for the Three Risk Factors with Respect to Seven Failure Modes.

		Component *A* of Z-valuations		
Expert	Failure mode	O	S	D
Expert 1	FM_1	(6,7,8,9)	(5,6,7,8)	(6,8,8,10)
	FM_2	(2,3,4,5)	(7,8,9,10)	(3,4,5,7)
	FM_3	(0,2,3,5)	(1,3,3,5)	(0,1,2,5)
	FM_4	(1,2,3,5)	(5,6,7,9)	(2,4,5,7)
	FM_5	(6,8,9,10)	(3,4,5,6)	(6,8,9,10)
	FM_6	(0,1,1,3)	(0,1,2,3)	(5,7,7,9)
	FM_7	(5,7,8,10)	(3,4,4,5)	(2,4,4,6)
Expert 2	FM_1	(4,6,6,8)	(6,7,7,8)	(6,8,8,10)
	FM_2	(2,4,4,5)	(6,7,7,8)	(3,4,5,7)
	FM_3	(0,2,3,4)	(0,1,1,3)	(0,1,2,5)
	FM_4	(3,4,4,5)	(5,6,7,8)	(2,4,5,7)
	FM_5	(6,8,8,10)	(2,4,4,6)	(6,8,8,10)
	FM_6	(2,3,4,5)	(1,3,3,5)	(5,7,7,9)
	FM_7	(5,6,6,7)	(3,4,5,6)	(2,4,4,6)
Expert 3	FM_1	(6,7,7,8)	(4,6,7,9)	(5,8,8,10)
	FM_2	(1,3,4,6)	(6,7,8,9)	(2,4,5,7)
	FM_3	(1,2,3,5)	(0,2,3,5)	(0,1,1,5)
	FM_4	(2,4,4,5)	(2,6,6,10)	(2,3,4,5)
	FM_5	(5,7,8,10)	(2,5,5,7)	(7,8,8,10)
	FM_6	(0,1,1,3)	(0,1,1,4)	(7,8,8,9)
	FM_7	(6,7,8,9)	(2,4,5,7)	(1,3,3,5)

Table 2. The Integrated Valuation $\tilde{V}$.

Failure mode	Expert 1 (w_1 = 0.4)	Expert 2 (w_2 = 0.3)	Expert 3 (w_3 = 0.3)
FM_1	(4.4, 7, 8, 11.6)	(4.1, 7, 7, 11.1)	(3.9, 7, 7, 11.6)
FM_2	(3.1, 5.3, 6, 10.3)	(2.9, 5, 5, 8.6)	(2.3, 4.7, 6, 9.4)
FM_3	(0.3, 2, 3, 6.4)	(0, 1.3, 2, 5.14)	(0.3, 1.7, 2, 6.4)
FM_4	(2.1, 4, 5, 9)	(2.6, 4.7, 5, 8.6)	(1.6, 4.3, 5, 8.57)
FM_5	(3.9, 6.7, 8, 11.1)	(3.6, 6.7, 7, 11.1)	(3.6, 6.7, 7, 11.1)
FM_6	(1.3, 3, 3, 6.4)	(2.1, 4.3, 5, 8.1)	(1.3, 3, 3, 7.3)
FM_7	(2.6, 5, 5, 9)	(2.6, 4.7, 5, 8.2)	(2.3, 4.7, 5, 9)

Table 3. The Synthesized Valuations and Risk Priority Ranking of the Seven Failure Modes.

Failure mode	$\bar{V}$	α	$\bar{V}_\alpha$	ZRPN	Priority ranking
FM_1	(4.2,7,7.4,11.4)	0.47	(5.4, 7, 7.4, 10)	0.7	**1**
FM_2	(2.8,5,5.8,9.5)	0	(2.8, 5, 5.8, 9.5)	0.52	**3**
FM_3	(0.2,1.7,2.4,6)	0	(0.2, 1.7, 2.4, 6)	0.22	**7**
FM_4	(2.1,4.3,5,8.7)	0	(2.1, 4.3, 5, 8.7)	0.45	**5**
FM_5	(3.7,6.7,7.2,11.1)	0.28	(4.5, 6.7, 7.1, 10)	0.66	**2**
FM_6	(1.5,3.4,3.7,7.2)	0	(1.5,3.4,3.7,7.2)	0.34	**6**
FM_7	(2.5,4.8,5.2,8.7)	0	(2.5,4.8,5.2,8.7)	0.48	**4**

Table 4. The Comparisons of the Proposed Method with Two Available Methods.

Failure mode	Traditional RPN		FWGM method		Proposed ZFMEA	
	RPN	Ranking	FRPN	Ranking	ZRPN	Ranking
1	337.68	1	7.16	1	0.70	1
2	97.68	4	5.03	3	0.52	3
3	4.2	7	1.96	7	0.22	7
4	71.04	5	4.49	5	0.45	5
5	264.88	2	6.67	2	0.66	2
6	17.92	6	2.82	6	0.34	6
7	99.16	3	4.82	4	0.48	4

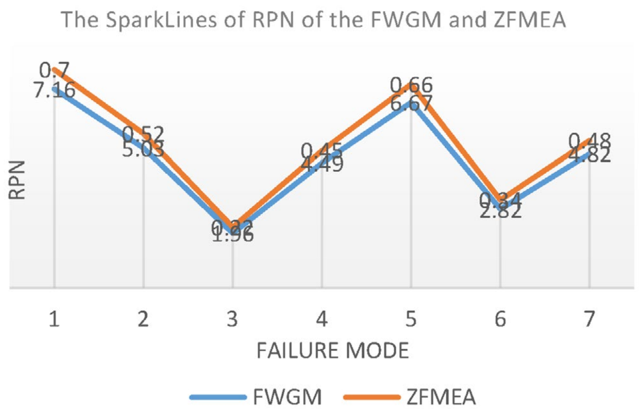

Figure 5. The Sparklines of RPN of the FWGM and ZFMEA.

to the traditional FMEA, the ratings of failure mode 2 and 7 are opposite to the other methods. Then we can find the advantage of FWGM method and ZFMEA, which are more sensitive to capture and distinguish the uncertainty and similar information. As to the FWGM methods, Wang analyzed the related importance of the risk factors and model them with fuzzy numbers, the weights of the expert are also considered; but the reliability of the fuzzy valuation is overlooked. While in the proposed ZFMEA, we can express the uncertainty and reliability simultaneously with Z-numbers. Besides, the proposed method has the merit of low computational complexity and simpleness when compared with the FWGM method. So from the above analysis, the proposed ZFMEA are more reasonable and effective for risk analysis.

5. Conclusion

Reliability of information is a very important issue in decision-making, management of information and risk valuation and analysis. This study presented a novel method for properly evaluating the level of risk. The main novelty introduced in the paper is taking into account the reliability part of the uncertain valuation with Z-numbers. Results obtained in the case study of the rotor blades of an aircraft turbine demonstrate the validity of the proposed approach.

The proposed approach has been proved to be useful and practical, but can still be improved in some aspects. For example, and the weights of the risk factors were not accounted for in this study.

Disclosure statement

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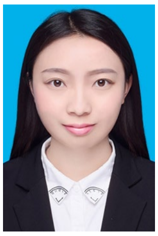

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