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Output Consensus of Heterogeneous Multi-agent Systems under Directed Topologies via Dynamic Feedback

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ABSTRACT
This paper discusses the problem of dynamic output consensus for heterogeneous multi-agent systems (MAS) with fixed topologies. All the agents possess unique linear dynamics, and only the output information of each agent is delivered throughout the communication digraphs. A series of conditions and protocols are set for reaching the consensus. With the proper feedback controllers, the output consensus of the overall system is guaranteed. An application illustrates the theorems.

KEYWORDS
Multi-agent system; Heterogeneous; Output consensus; Dynamic feedback; Directed topology

Introduction

In recent years, the distributed control of MAS is becoming more and more important in the control research field, which is widely applied in the researching fields of transportation and power grid (Karfopoulos & Hatziargyriou, 2013), and is also highly appreciated by emerging industries (Logenthiran, Srinivasan, & Khambadkone, 2011; Zeng, Liu, Wu, & Ngan, 2011).

A critical problem in distributed MAS coordinated control is to find a control protocol so that all agents can reach an agreement regarding a certain quantity of interest that depends on the states or outputs of all agents. This problem is usually called the consensus problem. The consensus ability of MAS is a fundamental problem, since the starting of the consensus protocol design (Wang, Duan, Wen, & Chen, 2015). Ma and Zhang discussed the necessary condition of consensus ability in the fixed topology of linear time-invariant systems (Ma & Zhang, 2010). Olfati-Saber and Murray established the frame of dynamic MAS, which focused on the consensus problems on directed and undirected fixed or switching topology (Olfati-Saber & Murray, 2004). Ren and Beard had improved the theory in the reference above and pointed that the system could reach a consensus only if the topology contains a spanning tree (Ren & Beard, 2005). Afterward Ren himself brought a double-integrator dynamics and proposed a linear protocol with states of position and velocity (Ren, 2008). Meanwhile, the consensus focused on the linear dynamics, which would get much closer to the field of control. Tuna discussed the feasibility of consensus based on the state (2009), where he analysed sufficient certain conditions for the state-consensus, as well found an algorithm with a feedback law. Seo et al. had placed emphasis on the study of output consensus (Seo, Shim, & Back, 2009), and Scardovi et al. established the exploration on the time-varying structure (Scardovi & Sepulchre, 2009). Li et al. proposed a concept of consensus area, which based their research on the output observer (Z. K. Li, Duan, Chen, & Huang, 2010). As for the question of nonlinear models, Shi et al. had studied a single-integrator nonlinear dynamics in a switching topology, targeting the state consensus in a convex set (Shi & Hong, 2009). And Wen et al. proposed a control protocol based on a time-delay input. Focusing on the double-integrator nonlinear dynamics (Wen, Duan, Yu, & Chen, 2013), Yu et al. enhanced the definition of algebraic connectivity by using the information of position and velocity (Yu, Chen, & Cao, 2011), thus discussed the sufficient condition of consensus ability in the communication network.

Great progress has been made on consensus of homogeneous multi-agents over the past decade. Most consensus protocols in existing works are designed based on state feedback (Du, Li, & Ding, 2013; Xin & Cheng, 2014) and output feedback (Z. Li, Duan, & Huang, 2009; Xu, Xie, Li, & Lum, 2013). One advantage of the consensus protocols based on output feedback as compared to those based on state feedback is that the system states are not required to be measurable. It is noted that most of these works only consider consensus of homogeneous MAS. However, many practical MAS usually consist of agents with different dynamics, and it thus desirable to investigate the consensus problem of heterogeneous MAS.

Some approaches based on output regulation theory have reported to solve the consensus problem of heterogeneous MAS. Wieland et al. presented a necessary and sufficient condition for the output consensus of MAS without considering disturbances for agents (Wieland, Sepulchre, & Allgöwer, 2011). Kim et al. presented an approach based on internal reference models for output consensus of heterogeneous MAS, where the state matrices of internal reference models are assumed to have only eigenvalues with zero real parts (Hongkeun, Hyungbo, & Jin Heon, 2011).

In this paper, the output consensus in the system consisting of different linear dynamics is discussed. Through some assumptions and conditions, the output feedback consensus of heterogeneous MAS can be guaranteed on condition that the overall connection is satisfied, and the outputs are stabilized. Three sections will be expanded in the following part of this paper. The heterogeneous MAS description and a low gain feedback controller are also provided in this section. By the proof of a feedback compensator used in the stabilizing, the

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output-based consensus among agents with linear dynamics, which is stabilized and detectable, is studied. An application is provided to prove the results in the Third section. Finally, conclusions can be seen in the last section.

Output consensus with fixed topology

System description and basic knowledge of graph theory

In this paper, the heterogeneous MAS with N agents is described as

$$\begin{cases} \dot{x}_i(t) = A_{g_i} x_i(t) + B_{g_i} u_i \\ y_i(t) = C_{g_i} x_i(t) \end{cases} \quad i = 1, 2, \ldots, N \tag{1}$$

where the subscript i represents the subsystem agent $x_i \in \mathbb{R}^n$, $y_i \in \mathbb{R}^p$ and $u_i \in \mathbb{R}^m$ are state, output and control input vectors of any individual agent respectively. Each agent is supposed to own unique linear dynamics, which may differ from the others numerically, and $(C_{g_i}, A_{g_i}, B_{g_i})$ is assumed to be stable and detectable.

Definition 1. *For linear MAS (1), if there exists $u_i(t)$ such that for any initial value $x_i(0)$*

$$\lim_{t\to\infty} \left\| y_j(t) - y_i(t) \right\| = 0 \tag{2}$$

Then the system (1) is said to be output consensus able with respect to u_i.

A weighted digraph $G = (V, E, A)$ consists a vertex set $V = \{v_1, \ldots, v_n\}$, an edge set $E \subseteq V \times V$ and a weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ satisfying $a_{ij} > 0$ if $(v_i, v_j) \in E$, while $a_{ij} = 0$, otherwise. The in-degree of vertex v_i is defined as $deg_{in} = \sum_{j=1}^{n} a_{ij} i \in \mathcal{I}_n$. Then degree matrix of a weighted digraph G is defined as $D = diag\{deg_{in}(v_1), deg_{in}(v_2), \ldots, deg_{in}(v_n)\}$. The Laplacian matrix $L = (l_{ij})_{n\times n}$ of a weighted digraph G is defined as $L = D\ A$. The Laplacian matrix L has exact one zero eigenvalue if and only if the digraph G has a spanning tree.

A low gain feedback stable controller

The consensus problem by way of a stable controller $\mathcal{K}_i(s) = C_{\mathcal{K}_i}(sI - A_{\mathcal{K}_i})^{-1} B_{\mathcal{K}_i}$, which is represented in the state-space form

$$\begin{cases} \dot{\chi}_i = A_{\mathcal{K}_i} \chi_i + B_{\mathcal{K}_i} z_i \\ u_i = C_{\mathcal{K}_i} \chi_i \end{cases} \tag{3}$$

with the controller state $\chi_i \in \mathcal{R}^{p^i}$ and the set of neighbours of the i agent $N_i = \{ j \in N : a_{ij} \neq 0 \}$. $A_{\mathcal{K}_i}$, $B_{\mathcal{K}_i}$ and $C_{\mathcal{K}_i}$ are the corresponding matrices with the corresponding dimensions. z_i is defined as the output information of its neighbours' agents by

$$z_i(t) = -\sum_{j\in N_i} l_{ij} y_j(t) \tag{4}$$

The closed-loop system of ith agent is

$$\begin{aligned} \dot{\bar{x}}_i &= \bar{A}_i \bar{x}_i + \bar{B}_i z_i \\ y_i &= \bar{C}_i \bar{x}_i \\ z_i &= -\sum_{j=1}^{N} l_{ij} y_j \end{aligned} \tag{5}$$

where

$$\bar{x}_i = \begin{bmatrix} x_i \\ \chi_i \end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_{g_i} & B_{g_i} C_{\mathcal{K}_i} \\ 0 & A_{\mathcal{K}_i} \end{bmatrix}, \bar{B}_i = \begin{bmatrix} 0 \\ B_{\mathcal{K}_i} \end{bmatrix}, \bar{C}_i = \begin{bmatrix} C_{g_i} & 0 \end{bmatrix}.$$

Lemma 1 (Horn & Johnson, 2012). *According to the partitioned matrix $P = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, if matrices A and D are invertible, then*

$$|P| = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A| \cdot |D - CA^{-1}B| \tag{6}$$

Lemma 2 (Horn & Johnson, 2012). *According to the partitioned matrix $P = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$, if matrices A and D are inverse, then*

$$P^{-1} = \begin{bmatrix} A^{-1} & 0 \\ -DCA^{-1} & D^{-1} \end{bmatrix} \tag{7}$$

Lemma 3 (Horn & Johnson, 2012). *The necessary and sufficient conditions of a negative semi-definite matrix P with $n \times n$ orders are the leading principal minors D_k, $k = 1, 2, \cdots, n$ satisfy*

$$|D_1| \geq 0, \quad |D_2| \leq 0, \ldots, (-1)^n |D_n| \leq 0 \tag{8}$$

Theorem 1. *There exists a positive constant $\bar{\epsilon}$, such that, for each $\epsilon \in (0, \bar{\epsilon}]$, the controller given by*

$$\begin{cases} \dot{\chi}_i &= A_{g_i}\chi_i - B_{g_i} B_{g_i}^T P_i(\epsilon)\chi_i - H_i(y_i - C_{g_i}\chi_i) \\ &= (A_{g_i} + H_i C_{g_i} - B_{g_i} B_{g_i}^T P_i(\epsilon))\chi_i - H_i y_i \\ u_i &= B_{g_i}^T P_i(\epsilon)\chi_i \end{cases} \tag{9}$$

where H_i is a matrix such that $(A_{g_i} + H_i C_{g_i}) \leq 0$, and $P_i(\epsilon) = P_i^T(\epsilon) > 0$ is the unique solution of

$$A_{g_i}^T P_i(\epsilon) + P_i(\epsilon) A_{g_i} - P_i(\epsilon) B_{g_i} R^{-1} B_{g_i}^T P_i(\epsilon) + \epsilon I = 0 \tag{10}$$

in which, $R^{-1} = \tau I$, and $\tau = \min_{i=2,\ldots,N} Re\{\lambda_i(L)\} > 0$, is the stable dynamic output feedback controller (3) for (5).

According to the **Theorem 1**, the close-loop system matrices of (5) can be rewritten as

$$\bar{A}_i = \begin{bmatrix} Ag_i & B_{g_i} C_{\mathcal{K}_i} \\ 0 & A_{\mathcal{K}_i} \end{bmatrix} = \begin{bmatrix} A_{g_i} & B_{g_i} B_{g_i}^T P_i() \\ 0 & A_{g_i} + H_i C_{g_i} \end{bmatrix} = \begin{bmatrix} A_{g_i} & 0 \\ 0 & A_{g_i} + H_i C_{g_i} \end{bmatrix},$$

$$\bar{B}_i = \begin{bmatrix} 0 \\ B_{\mathcal{K}_i} \end{bmatrix}, \bar{C}_i = [\, C_{g_i} \quad 0 \,].$$

Where $\lim_{\epsilon\to 0} P_i(\epsilon) = \lim_{\epsilon\to 0} P_i^T(\epsilon) = 0$. Then with N agents, the overall closed-loop system dynamics can be described by

$$\dot{\tilde{x}}_i = \tilde{A}\tilde{x} \tag{11}$$

Where $\tilde{x} = \begin{bmatrix} \bar{x}_1 \\ \vdots \\ \bar{x}_N \end{bmatrix}$, $\tilde{A} = \begin{bmatrix} \bar{A}_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \bar{A}_N \end{bmatrix} - \begin{bmatrix} l_{11}\bar{B}_1\bar{C}_1 & \dots & l_{1N}\bar{B}_1\bar{C}_N \\ \vdots & \ddots & \vdots \\ l_{N1}\bar{B}_N\bar{C}_1 & \dots & l_{NN}\bar{B}_N\bar{C}_N \end{bmatrix}$.

Supposed that $V_{N\times N} = \tilde{A}$, and

$$V_{N\times N} = \tilde{A} = \begin{bmatrix} \begin{bmatrix} A_{g_1} & 0 \\ l_{11}H_1C_{g_1} & A_{g_1} + H_1C_{g_1} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ l_{12}H_1C_{g_2} & 0 \end{bmatrix} & \dots & \begin{bmatrix} 0 & 0 \\ l_{1N}H_1C_{g_N} & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ l_{21}H_2C_{g_1} & 0 \end{bmatrix} & \begin{bmatrix} A_{g_2} & 0 \\ l_{22}H_2C_{g_2} & A_{g_2} + H_2C_{g_2} \end{bmatrix} & \dots & \begin{bmatrix} 0 & 0 \\ l_{2N}H_2C_{g_N} & 0 \end{bmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{bmatrix} 0 & 0 \\ l_{N1}H_NC_{g_1} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ l_{N2}H_NC_{g_2} & 0 \end{bmatrix} & \dots & \begin{bmatrix} A_{g_N} & 0 \\ l_{NN}H_NC_{g_N} & A_{g_N} + H_NC_{g_N} \end{bmatrix} \end{bmatrix}.$$

The negative semi-definite matrix $V_{N\times N}$ can be obtained by using Mathematical Induction (Gunderson, 2010).

Proof:

(1) For $A_{g_1} \le 0$ and $(A_{g_1} + H_1C_{g_1}) \le 0$,

$$|V_{1\times 1}| = |\bar{A}_1| = \begin{bmatrix} A_{g_1} & 0 \\ l_{11}H_1C_{g_1} & A_{g_1} + H_1C_{g_1} \end{bmatrix} \le 0.$$

(2) Based on **Lemma 1** and **2**,

$$\begin{aligned} |V_{2\times 2}| &= \begin{vmatrix} \bar{A}_1 & \begin{bmatrix} 0 & 0 \\ l_{12}H_1C_{g_2} & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ l_{21}H_2C_{g_2} & 0 \end{bmatrix} & \bar{A}_2 \end{vmatrix} \\ &= |\bar{A}_1| \cdot \left| \bar{A}_2 - \begin{bmatrix} 0 & 0 \\ l_{21}H_2C_{g1} & \end{bmatrix} \cdot \begin{bmatrix} A_{g_1}^{-1} & 0 \\ M_1 & (A_{g1} + H_1C_{g1})^{-1} \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ l_{12}H_1C_{g_2} & 0 \end{bmatrix} \right| \\ &= |\bar{A}_1| \cdot \left| \bar{A}_2 - \begin{bmatrix} 0 & 0 \\ l_{21}H_2Cg_1A_{g_1}^{-1} & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ l_{12}H_1C_{g_2} & 0 \end{bmatrix} \right| \\ &= |\bar{A}_1| \cdot |\bar{A}_2| \ge 0 \end{aligned}$$

where $M_1 = -l_{11}(A_{g_1} + H_1C_{g_1})^{-1}H_1C_{g_1}A_{g_1}^{-1}$.

(3) Supposed that, $|V_{(N-1)\times(N-1)}| = \prod_{i=1}^{N-1}|\bar{A}_1|$, and $(-1)^{\left[\frac{N+1}{2}\right]}|V_{(N-1)\times(N-1)}| \ge 0$. According to the definition of similar matrix (Horn & Johnson, 2012), there would be

$$V_{(N-1)\times(N-1)}^{-1} \sim \begin{bmatrix} \bar{A}_1^{-1} & 0 & \dots & 0 \\ 0 & \bar{A}_2^{-1} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & \bar{A}_{N-1}^{-1} \end{bmatrix} =$$

$$\begin{bmatrix} \begin{bmatrix} A_{g_1}^{-1} & 0 \\ M_1 & (A_{g_1} + H_1C_{g_1})^{-1} \end{bmatrix} & 0 & \dots & 0 \\ 0 & \begin{bmatrix} A_{g_2}^{-1} & 0 \\ M_2 & (A_{g_2} + H_2C_{g_2})^{-1} \end{bmatrix} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & \begin{bmatrix} A_{g_N}^{-1} & 0 \\ M_N & (A_{g_N} + H_NC_{g_N})^{-1} \end{bmatrix} \end{bmatrix}$$

where $M_i = -l_{ii}(A_{g_i} + H_iC_{g_i})^{-1}H_iC_{g_i}A_{g_i}^{-1}$. Then,

$$
|\tilde{A}| = |V_{N\times N}| = |V_{(N-1)\times(N-1)}| \cdot \left| \bar{A}_N - \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ l_{N1}H_N C_{g_1} & 0 & \dots & l_{N(N-1)H_N C_{g_{N-1}}} \end{bmatrix} \cdot \begin{bmatrix} \begin{bmatrix} A_{g_1}^{-1} & 0 \\ M_1 & (A_{g_1}+H_1C_{g_1})^{-1} \end{bmatrix} & 0 & \dots & 0 \\ 0 & \begin{bmatrix} A_{g_2}^{-1} & 0 \\ M_2 & (A_{g_2}+H_2C_{g_2})^{-1} \end{bmatrix} & \dots & 0 \\ \vdots ; & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \begin{bmatrix} A_{g_N}^{-1} & 0 \\ M_N & (A_{g_N}+H_NC_{g_N})^{-1} \end{bmatrix} \end{bmatrix} \right|
$$

$$
= \left|V_{(N-1)\times(N-1)}\right| \cdot \left| \bar{A}_N - \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ l_{N1}H_N C_{g_1} A_{g_1}^{-1} & 0 & \dots & l_{N\times(N-1)}H_N C_{g_{N-1}} A_{g_{N-1}}^{-1} & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ l_{1N}H_1C_{g_N} & 0 \\ \vdots & \vdots \\ 0 & 0 \\ l_{(N-1)\times N}H_{N-1}C_{g_N} & 0 \end{bmatrix} \right|
$$

$$
= \left|V_{(N-1)\times(N-1)}\right| \cdot |\bar{A}_N| = \prod_{i=1}^{N} |\bar{A}_1|
$$

and $(-1)^{\left[\frac{N+1}{2}\right]} \prod_{i=1}^{N} |\bar{A}_i| \geq 0$ Where $[\cdot]$ represents the rounding function. Therefore $\tilde{A} \leq 0$.

Simulations and applications

In this section, a verification platform with 4 motors is provided to prove the consensus algorithm. Especially, one motor (Agent 4) is different from the others (shown in Figure 1). Every motor controlled by an Arduino© UNO board consist an agent, the communication between the agents with a Router and cables.

The transfer function models of 4 motors are

$$
f_1(s) = \frac{1.64}{0.1024s+1}, \quad f_2(s) = \frac{1.64}{0.1024s+1},
$$
$$
f_3(s) = \frac{1.64}{0.1024s+1}, \quad f_4(s) = \frac{1.61}{0.0796s+1}.
$$

Suppose that a communication network is given by Figure 2(a), which is represented by the Laplacian

$$
L = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}
$$

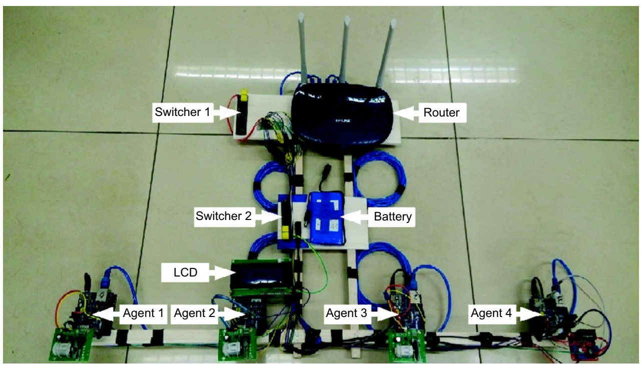
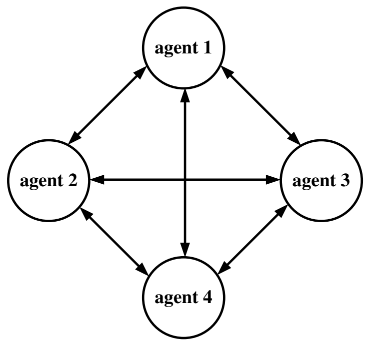


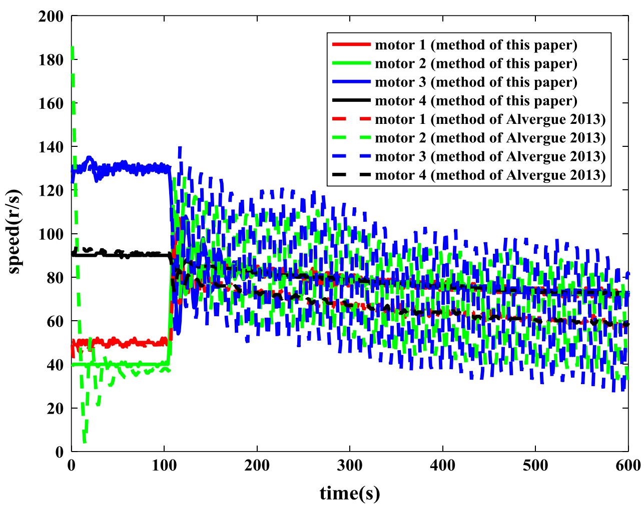
Figure 1. Structure of 4 motors.

We take $\epsilon = 0.001$ so that $(A_{g_i} + H_i C_{g_i})$ is Hurwitz for all $i = 1, 2, 3, 4$, from which $\mathcal{K}_i(s)$ becomes

$$
\mathcal{K}_1(s) = 0.71 + \frac{0.003}{s}, \quad \mathcal{K}_2(s) = 0.71 + \frac{0.003}{s},
$$



(a) Full-linked topology.



(b) Comparison of theoretical simulation results with testbed simulation results.

Figure 2. Consensus of 4 Motors under Full-linked Digraph. (a) Full-linked Topology. (b) Comparison of Theoretical Simulation Results with Testbed Simulation Results.

$$\mathcal{K}_3(s) = 0.71 + \frac{0.003}{s},\ \mathcal{K}_1(s) = 1.28 + \frac{0.05}{s}.$$

Under fixed topology (shown in Figure 2(a)), the consensus result can be seen in Figure 2(b). In the earlier 100s of this experiment, the initial speeds of 4 motors are 49r/min, 40r/min, 131r/min and 90r/min respectively. Under the fixed topology, the synchronization speed is 68r/min.

Figure 2(b) shows the comparison of results with the results based on the literature (Alvergue, Pandey, Gu, & Chen, 2013). As the figure shows, the dash lines denote the traditional method, and the solid lines denote the consensus algorithm introduced in this paper. Obviously, the performance of the algorithm introduced in this paper is better than the traditional one.

Conclusions

In this article, an output dynamic feedback controller design method is provided for analysis of heterogeneous MAS output consensus with fixed or dynamic topologies. With the proper feedback controllers, the MAS was proven to be stable, thus the stabilized consensus of the overall system is guaranteed. The MAS stabilized by the proper controllers will trace the convergent trajectory as well if one of the topologies has a spanning tree. An equivalent topology structure, which is formed by all switching candidate topologies and an Equivalent Laplacian Matrix (ELM) were designed to prove that if the equivalent structure has a spanning tree, then the overall system can reach the consensus.

Disclosure statement

No potential conflict of interest was reported by the authors.

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