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Improved Teaching Learning Based Optimization and Its Application in Parameter Estimation of Solar Cell Models

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ABSTRACT
Weak global exploration capability is one of the primary drawbacks in teaching learning based optimization (TLBO). To enhance the search capability of TLBO, an improved TLBO (ITLBO) is introduced in this study. In ITLBO, a uniform random number is replaced by a normal random number, and a weighted average position of the current population is chosen as the other teacher. The performance of ITLBO is compared with that of five meta-heuristic algorithms on a well-known test suite. Results demonstrate that the average performance of ITLBO is superior to that of the compared algorithms. Finally, ITLBO is employed to estimate parameters of two solar cell models. Experiments verify that ITLBO can provide competitive results.

KEY WORDS: Evolutionary computation; parameter estimation; solar cell model; teaching learning based optimization

1 INTRODUCTION

OPTIMIZATION problems can be found in numerous fields. To solve them, a large number of optimization approaches have been proposed. Especially, the meta-heuristic algorithms, such as Genetic Algorithm (Holland, 1975) (GA), Artificial Bee Colony (Karaboga, 2005) (ABC), Particle Swarm Optimization (Kenndy & Eberhart, 1995) (PSO), Differential Evolution (Storn & Price, 1995) (DE), and Teaching Learning Based Optimization (TLBO) (Rao et al., 2011), have gained more and more attentions in the past decades. Until now, a lot of works (Cuevas et al., 2016; Fan et al., 2017; Fan & Yan, 2016; Hermosilla et al., 2017; Wang et al., 2015) have been reported as meta-heuristic algorithms. Among these methods, TLBO is a recently proposed population-based algorithm. In this algorithm, the teacher and learner phases are two main operators. TLBO has some attractive features, such as free from the algorithm specific parameters, ease in programming, high efficiency etc. (Satapathy & Naik, 2014). Therefore, it has become a popular tool for solving different optimization problems. However, the performance of TLBO needs to be further improved since it is directly influenced by the parameter settings and the operators in the teacher and learner phases.

To enhance the search efficiency of TLBO, a large number of works have been reported. For example, Rao and Patel (2012) introduced an elitist TLBO to solve constrained optimization problems wherein an elitism concept is used. Zhou et al. (2014) proposed an improved TLBO with dynamic group strategy, in which the mean of the class is replaced by the mean of the corresponding group. The results show that the proposed algorithm is effective. In Ref. (Ghasemi et al., 2015), an improved TLBO algorithm was proposed. In this algorithm, a Gaussian distribution function and a mutation strategy used in DE is employed to produce new individuals. Ghasemi et al. (2015) employed a levy mutation strategy to improve TLBO performance. Wang et al. (2016) proposed a novel TLBO variant with experience information and differential mutation (EI-TLBO), which are used to improve the global search capability. In Ref. (Li et al., 2016), an improved TLBO with group learning (GTLBO) was proposed to alleviate the population convergence problem. Sleesongsom and Burreerat (2017) introduced a self-adaptive population size TLBO (SAP-TLBO). In SAP-TLBO, the number of individuals may be different in the teacher and learner phases. In the literature (Tang et al., 2017), a hybrid TLBO (HTLBO) algorithm was

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proposed. In HTLBO, a variable neighborhood search is used to improve the local search capability. Farahani and Rashidi (2017) proposed a modified TLBO algorithm, in which a new teacher phase is used and a mutation strategy used in DE is employed to balance between the local and global search capabilities. The experimental results indicate that the performance of the proposed algorithm is better than that of the other compared algorithms. Zamli et al. (2017) utilized the mamdani fuzzy inference system to automatically select a suitable search operator in TLBO. The results present that the proposed algorithm exhibits better search capability when compared with the other competitors. Additionally, TLBO and its variants are usually used to solve numerous real-world optimization problems (Banerjee et al., 2015; Chatterjee & Mukherjee, 2016; Lin, et al., 2015; Patel, Savsani et al., 2017; Rao & Kalyankar, 2013).

Like PSO (Kenndy & Eberhart, 1995), although a uniform random number used in TLBO can help to enhance its global search capability, it may influence the search efficiency of TLBO. Moreover, the best individual selected as a teacher may result in trapping into local optimal. To alleviate the above mentioned problems, an improved teaching learning based optimization (ITLBO) is proposed in this study. In the ITLBO, a normal random number is used and a weighted average position of the current population is selected as the other teacher to guide the population evolution in the teacher phase. ITLBO can improve the global exploration capability of the original TLBO since the new teacher can increase random perturbation. It should be noted that the generalized opposition-based learning used by Chen et al. (2016) is also employed in our proposal. The main reason is that the generalized opposition-based learning can assist TLBO to improve the global search capability. The performance of ITLBO is compared with that of five meta-heuristic algorithms on 25 30-dimensional IEEE CEC2005 (Suganthan, et al., 2005) test functions. The experimental results verify that the overall performance of the proposed algorithm is the best among all the competitors. ITLBO is also utilized to estimate parameters of the single diode and double diode models. The results show that ITLBO is a competitive optimization tool for dealing with actual application problems.

The remainder of this paper is organized as follows. Sections 2 and 3 introduce the basic TLBO algorithm and ITLBO. In Section 4, the proposed algorithm is compared with five algorithms on a famous test suite. Moreover, the impact of a parameter (Sigma) and the effectiveness of the proposed teacher phase are investigated. The application of ITLBO in parameter estimation for two solar cell models is given in Section 5. Finally, Section 6 concludes this study.

2 TEACHING LEARNING BASED OPTIMIZATION

A minimization problem can be expressed as follows:

$$f(\boldsymbol{x}^*) = \min_{\boldsymbol{x}_i \in \Omega} f(\boldsymbol{x}_i),\ \boldsymbol{x}_i \in \boldsymbol{S} \subseteq \prod_{j=1}^{D}\left[L_j, U_j\right], \quad (1)$$

where f denotes an objective function; $\boldsymbol{x}_i = (x_{i,1}, \ldots, x_{i,D})$ is a D-dimensional vector; $\boldsymbol{x}^*$ is the global optimum solution, $\Omega \subseteq R^D$; L_j and U_j ($j = 1, \cdots, D$) are the lower and upper bounds of the jth variable of $\boldsymbol{x}_i$, respectively; $\boldsymbol{S}$ is the search space.

TLBO is an effective and efficient population-based optimization approach. Teacher and learner phases are two main operators in TLBO. $\boldsymbol{X}_{\text{old},i} = (x_{\text{old},i,1}, \cdots, x_{\text{old},i,D})$ and $\boldsymbol{X}_{\text{new},i} = (x_{\text{new},i,1}, \cdots, x_{\text{new},i,D})$ denote the old and the new positions of the ith solution (or the ith target vector) in the population, respectively. The population at the Gth generation is denoted as $\boldsymbol{P}^G = \{\boldsymbol{x}_1^G, \ldots, \boldsymbol{x}_{NP}^G\}$, which contains NP individuals. Two main operators of the original TLBO are described as follows.

2.1 Teacher phase

In the teacher phase, the best individual in the current population is selected as the teacher and is used to guide the mean position of the population. The teaching process can be described as follows:

$$\boldsymbol{X}_{new,i} = \boldsymbol{X}_{old,i} + rand \times (\boldsymbol{X}_{\text{Teacher}} - TF \times \boldsymbol{X}_{\text{Mean}}), \quad (2)$$

where $\boldsymbol{X}_{\text{Teacher}}$ and $\boldsymbol{X}_{\text{Mean}}$ are the teacher position and the mean position of the current population, respectively; $rand$ is a uniform random number within [0,1]; TF is a teaching factor, and its value can be either 1 or 2. If the fitness function value of $\boldsymbol{X}_{new,i}$ is better than that of $\boldsymbol{X}_{old,i}$, $\boldsymbol{X}_{new,i}$ will be accepted.

2.2 Learner phase

After the teaching operation, the new learner can be updated as follows:

$$\boldsymbol{X}_{new,i} = \begin{cases} \boldsymbol{X}_{old,i} + rand \times (\boldsymbol{X}_{old,i} - \boldsymbol{X}_{old,h}), & \text{if } f(\boldsymbol{X}_{old,i}) < f(\boldsymbol{X}_{old,h}), \\ \boldsymbol{X}_{old,i} + rand \times (\boldsymbol{X}_{old,h} - \boldsymbol{X}_{old,i}), & \text{otherwise} \end{cases} \quad (3)$$

where $\boldsymbol{X}_{old,h}$ ($i \neq h$) is randomly chosen from the current population; $\boldsymbol{X}_{new,i}$ will survive if a better fitness function value is given.

3 IMPROVED TLBO

ALTHOUGH the original TLBO is simple and efficient, its average performance is greatly impacted by the teacher and learner phases. To further enhance its performance, an improved TLBO (ITLBO) is proposed in the current study. In ITLBO, a normal random number and a weighted average position of the current population are used to balance the global and local search capabilities. Moreover, the generalized opposition-based learning is utilized to enhance the exploration capability of TLBO.

3.1 Improved teaching process

Generally, a uniform random number can provide good exploration capability, whereas a normal random number can speed up the convergence. Moreover, if only the best individual is used to guide the population evolution in the teacher phase; it may lead to premature convergence. Therefore, a normal random number and a weighted average position of the current population are utilized in the teacher phase. The main objective is to balance the global and local search capabilities of TLBO.

Based on the above introductions, a novel teaching process can be formulated as follows:

$$\boldsymbol{x}_{new,i} = \boldsymbol{x}_{old,i} + N(\mu,\sigma)\times(\boldsymbol{X}_{\text{Teacher}} - \boldsymbol{x}_{r1,i}) + N(\mu,\sigma)\times(\boldsymbol{X}_W - \boldsymbol{x}_{r2,i}) \quad , (4)$$

where N is the normal distribution function; $\boldsymbol{X}_W = \sum_{i=1}^{NP} w_i \times \boldsymbol{x}_i$ ($w_i = \frac{\left| f(\boldsymbol{x}_{old,i}) - \max(f(\boldsymbol{x}_{old,i}))\right|}{\sum_{i=1}^{NP}\left| f(\boldsymbol{x}_{old,i}) - \max(f(\boldsymbol{x}_{old,i}))\right|}$) denotes the weighted average position of the current population; μ and σ denote the mean and standard deviation values of the normal distribution function, respectively; r_1 and r_2 are integers randomly selected within the range [1, *NP*].

From Eq. (4), it can be seen that the advantages of the novel teaching process are twofold:

- The normal random number can provide steadier perturbation when compared with the uniform random number. It signifies that the search efficiency of the proposed teaching strategy may be better than that of the original teaching process. It should be noted that, the normal distribution function has been used in other studies (Ghasemi, Taghizadeh, et al., 2015; Zhao, 2015; Zou et al., 2014), but it is employed to generate new individuals. In the current work, the normal distribution function is only utilized to produce the scale factor.
- $(\boldsymbol{X}_W - \boldsymbol{x}_{r2,i})$ can increase a random perturbation towards $\boldsymbol{x}_{old,i}$, thus it is helpful for TLBO to improve the global exploration capability.

Additionally, $\mu = 0.5$ is chosen in the proposed algorithm. This is because the scale factor produced by the normal distribution function with the mean value 0.5 can balance the convergence speed and population diversity of TLBO in a certain extent. Moreover, the setting of σ is analyzed in Subsection 4.2.

3.2 Implementation of ITLBO

1) Initialization operation

Determine the population size *NP* and the maximum number of function evaluations *maxFEs*. The initial population $\boldsymbol{P}^0$ is generated through randomly and uniformly sampling from $\boldsymbol{S}$.

2) Teacher phase

In the teacher process, the new individual $\boldsymbol{x}_{new,i}$ can be generated by Eq. (4). If $\boldsymbol{x}_{new,i}$ is better than $\boldsymbol{x}_{old,i}$, it replaces $\boldsymbol{x}_{old,i}$.

3) Learner phase

In the learner phase, Eq. (3) is used to produce the new individual $\boldsymbol{x}_{new,i}$. If $\boldsymbol{x}_{new,i}$ is better than $\boldsymbol{x}_{old,i}$, it will survive in the next generation.

4) Generalized opposition-based generation

Based on the generalized opposition learning, the new individual $\boldsymbol{x}_{new,i}$ can be generated as follows:

$$\boldsymbol{x}_{new,i} = rand \times (L + U) - \boldsymbol{x}_{old,i} \quad . \quad (5)$$

If the fitness function value of $\boldsymbol{x}_{new,i}$ is better than that of $\boldsymbol{x}_{old,i}$, $\boldsymbol{x}_{new,i}$ will be accepted.

5) Stop until the number of function evaluations is equal to *maxFEs*; otherwise, repeat from Step 2.

4 EXPERIMENTAL RESULTS

IN this section, 25 30-dimensional IEEE CEC2005 test functions are used to evaluate the search capability of the proposed algorithm. The performance of ITLBO is compared with that of five meta-heuristic algorithms, which are CMA-ES (Hansen & Ostermeier, 2001), CLPSO (Liang et al., 2006), TLBO (Rao, et al., 2011), OTLBO (Roy et al., 2014), and GOTLBO (Chen, et al., 2016). All compared algorithms are implemented in Matlab (Matlab R2012a) and executed on Windows 7 operating system (64 bit). *maxFEs* is set to be 300,000 in all experiments. Moreover, Wilcoxon's rank sum test (Wilcoxon, 1945), Iman-Davenport test (García et al., 2009) and Friedman's test (Friedman, 1937) with a significant level of 0.05 are utilized to detect the differences between ITLBO and the other competitors. Because each algorithm has its own suitable *NP*, the settings of *NP* recommended in the literatures are used in the current study. Additionally, the population size of ITLBO is the same as that of the other TLBO variants, i.e., *NP* = 50.

4.1 Comparing ITLBO with five algorithms on 30-dimensional CEC2005 functions

In this part, ITLBO is compared with five selected algorithms on 25 30-dimensional test functions, which are introduced in IEEE CEC2005 and can be classified into four groups: (1) unimodal functions F1~F5; (2) basic multimodal functions F6~F12; (3) expanded multimodal functions F13 and F14; (4) hybrid composition functions F15~F20. For each test function, all experimental results shown in Table 1 are achieved by 30 independent runs and the best result is highlighted in bold. The statistical analysis results obtained by Wilcoxon's rank sum test are also presented in Table 1. According to the results shown in Table 1, the following conclusions can be given:

- For unimodal test functions F1~F5, from Table 1, it can be clearly seen that CMA-ES performs the best among all compared algorithms. This is because CMA-ES has good local exploitation capability. However, CLPSO, TLBO, and OTLBO perform better than ITLBO on only one test function. Moreover, GOTLBO cannot perform better than ITLBO on any unimodal functions. The main reason may be that, ITLBO uses the weighted average position of the current population and the normal random number to improve the search efficiency.
- For basic multimodal test functions F6~F12, Table 1 shows that the average performance of ITLBO is similar to that of CMA-ES. The overall performance of ITLBO is significantly superior to and inferior to that of CLPSO on four and two test functions, respectively. However, TLBO, OTLBO, and GOTLBO cannot perform better than our proposal on any basic multimodal functions. It can be found that the novel teacher process is helpful in finding a promising region and the generalized opposition-based learning can improve the global search capability of TLBO.
- For expanded multimodal functions F13 and F14, it can be seen from Table 1 that the performances of GOTLBO and ITLBO are similar. CMA-ES, TLBO, and OTLBO cannot outperform the proposed algorithm on any test functions. Compared with CLPSO, the performance of ITLBO is significantly inferior to that of CLPSO on function F13. The main reason may be that PSO can be regarded as a distributed model (DM), and TLBO can be regarded as the combination of a distributed model and a centralized model (CM) (Li, et al., 2015). In general, CM cannot perform well on multimodal test functions in most cases.
- For hybrid composition functions F15 and F25, they are difficult to solve and are more likely to verify the optimization performance of algorithms. It can be observed from Table 1 that the overall performance of ITLBO is better than that of TLBO, OTLBO, and GOTLBO. Compared with CLPSO, ITLBO can outperform CLPSO on six test functions and is inferior to CLPSO on three functions. Table 1 also indicates that CAM-ES provides significantly better results than ITLBO on three functions and is significantly surpassed by our proposal on five test functions. Based on the above comparisons, ITLBO can achieve the best average performance among all selected algorithms. It is because ITLBO not only uses the generalized opposition-based learning strategy and the weighted average position of the current population to improve the exploration capability, but also employs the normal random number to speed up the convergence. Therefore, ITLBO can achieve a good balance between the exploration and exploitation capabilities.

The statistical analysis results presented in Table 1 indicate that the average performance of ITLBO is significantly better than that of TLBO and its variants. ITLBO can outperform CLPSO on the majority of test functions. Note that, the searching capability of ITLBO is worse than that of CMA-ES on some basic test functions which include unimodal and multimodal functions. However, ITLBO can outperform CMA-ES on the majority of complex optimization problems. Based on the above analysis and introduction, we can find that the normal random number, the weighted average position of the current population, and the generalized opposition-based learning strategy used in TLBO can effectively improve the search efficiency.

4.2 Impact of sigma

In this experiment, 25 30-dimensional test functions are utilized to investigate the performance of ITLBO under different Sigma values, which are chosen from the set {0.1, 0.15, 0.2, 0.25, 0.3}. The times of independent runs for each function are set to be 30. The statistical analysis results achieved by Friedman and Iman-Davenport tests are presented in Table 2. As shown in Table 2, we can observe that the performance of ITLBO is not significantly impacted by the Sigma value. Therefore, it is convenient for users to set the Sigma value within our recommended range. Additionally, the rankings achieved by Friedman's test are plotted in the Appendix (Figure A1). It can be observed that the overall performance of ITLBO is the best when Sigma = 0.25. Therefore, Sigma = 0.25 is chosen to provide the best performance in the proposed algorithm. We can also conclude that a small or large Sigma value is not beneficial for the optimization performance of ITLBO, thus the Sigma value within the range of [0.15, 0.25] is suggested.

Table 1. **Results of all compared algorithms on 25 30-dimensional CEC2005 test functions**

Function	CLPSO Mean(Std)	CMA-ES Mean(Std)	TLBO Mean(Std)	OTLBO Mean(Std)	GOLBO Mean(Std)	ITLBO Mean(Std)
F1	**0.00E+00(0.00E+00)**−	1.78E-25(4.51E-26)+	1.85E-27(1.92E-27)+	2.88E-27(3.37E-27)+	1.16E-27(9.23E-28)+	5.77E-29(8.84E-29)
F2	8.28E+02(1.60E+02)+	**6.30E-25(2.19E-25)−**	5.05E-12(1.63E-11)+	4.70E-14(8.28E-14)+	1.86E-07(4.69E-07)+	5.48E-17(8.83E-17)
F3	1.55E+07(4.08E+06)+	**4.98E-21(1.51E-21)−**	4.86E+05(2.28E+05)−	3.76E+05(1.94E+05)−	7.96E+05(4.58E+05)≈	7.09E+05(3.53E+05)
F4	6.30E+03(1.23E+03)+	6.63E+05(2.27E+06)+	1.67E+02(4.86E+02)+	2.10E+01(2.99E+01)+	3.46E+02(4.79E+02)+	**7.47E-04(1.96E-03)**
F5	3.83E+03(3.56E+03)+	**3.33E-10(9.59E-11)−**	3.07E+03(8.63E+02)+	2.29E+03(6.64E+02)≈	3.37E+03(9.40E+02)+	2.02E+03(6.90E+02)
F6	4.62E+00(5.34E+00)+	**1.33E-01(7.28E-01)−**	2.18E+01(3.41E+01)+	2.86E+01(3.19E+01)+	3.78E+01(3.96E+01)+	1.00E+01(2.78E+01)
F7	1.24E+04(5.97E+02)+	**1.31E-03(4.23E-03)−**	2.24E-02(2.38E-02)≈	2.83E-02(3.52E-02)≈	2.75E-02(3.54E-02)≈	2.32E-02(1.86E-02)
F8	2.09E+01(5.81E-02)≈	**2.02E+01(4.81E-01)−**	2.10E+01(4.17E-02)≈	2.10E+01(5.52E-02)≈	2.10E+01(5.08E-02)≈	2.09E+01(5.67E-02)
F9	**0.00E+00(0.00E+00)**−	4.43E+02(1.38E+02)+	8.77E+01(1.71E+01)+	8.06E+01(1.23E+01)+	9.10E+01(1.62E+01)+	5.43E+01(1.64E+01)
F10	9.64E+01(1.76E+01)+	**4.80E+01(1.36E+01)**≈	1.11E+02(3.22E+01)+	1.02E+02(2.59E+01)+	9.69E+01(2.70E+01)+	5.39E+01(2.27E+01)
F11	**2.59E+01(1.44E+00)**−	6.37E+01(1.86E+00)+	3.10E+01(2.67E+00)+	2.98E+01(3.31E+00)≈	2.89E+01(4.23E+00)≈	2.83E+01(4.37E+00)
F12	1.72E+04(5.07E+03)+	1.15E+04(1.13E+04)+	1.00E+04(8.42E+03)+	1.08E+04(1.25E+04)+	5.56E+03(6.41E+03)≈	**3.28E+03(3.40E+03)**
F13	**2.10E+00(2.46E-00)**−	3.55E+00(7.87E-01)+	4.04E+00(9.72E-01)+	3.59E+00(6.74E-01)+	3.41E+00(9.85E-01)≈	3.00E+00(5.62E-01)
F14	1.27E+01(2.51E-01)≈	1.47E+01(1.85E-01)+	**1.25E+01(3.88E-01)≈**	1.29E+01(2.12E-01)+	1.28E+01(3.31E-01)≈	1.27E+01(3.56E-01)
F15	6.86E+01(6.17E+01)−	4.33E+02(2.61E+02)≈	4.49E+02(7.83E+01)+	4.29E+02(7.64E+01)+	4.45E+02(6.73E+01)+	**3.97E+02(8.92E+01)**
F16	1.68E+02(2.87E+01)≈	3.66E+02(2.58E+02)+	2.53E+02(1.59E+02)+	**1.53E+02(1.16E+02)**≈	2.17E+02(1.44E+02)+	1.93E+02(1.63E+02)
F17	2.47E+02(3.48E+01)+	4.10E+02(3.11E+02)+	2.16E+02(1.47E+02)+	2.14E+02(1.38E+02)+	1.96E+02(1.44E+02)≈	**1.70E+02(1.35E+02)**
F18	9.10E+02(1.90E+01)+	9.04E+02(1.85E-01)+	9.38E+02(2.40E+01)+	9.29E+02(3.83E+01)+	9.00E+02(0.00E+00)+	**8.86E+02(3.45E+01)**
F19	9.14E+02(1.44E+00)+	9.04E+02(3.01E-01)+	9.35E+02(3.45E+01)+	9.33E+02(1.95E+01)+	8.97E+02(1.83E+01)≈	**8.94E+02(2.56E+01)**
F20	9.14E+02(1.44E+01)	9.04E+02(2.23E-01)+	9.33E+02(3.21E+01)+	9.27E+02(3.12E+01)+	**9.00E+02(0.00E+00)≈**	**9.00E+02(3.40E+00)**
F21	**5.00E+02(2.61E-13)−**	**5.00E+02(2.50E-12)≈**	9.54E+02(3.33E+02)+	9.07E+02(3.46E+02)+	9.51E+02(3.32E+02)+	7.96E+02(3.29E+02)
F22	9.70E+02(1.71E+01)+	**8.32E+02(2.32E+01)−**	9.31E+02(2.64E+01)+	9.27E+02(2.43E+01)+	9.31E+02(2.77E+01)+	9.04E+02(2.00E+01)
F23	**5.34E+02(7.92E-05)−**	5.37E+02(3.54E+00)−	1.11E+03(1.65E+02)+	9.37E+02(2.72E+02)+	1.03E+03(2.00E+02)+	7.13E+02(2.30E+02)
F24	**2.00E+02(1.50E-12)≈**	**2.00E+02(7.90E-14)≈**	2.00E+02(1.31E-01)+	2.00E+02(5.78E-04)+	2.80E+02(2.72E+02)+	**2.00E+02(1.02E-12)**
F25	1.99E+03(3.88E+01)+	**2.06E+02(5.78E+00)−**	3.64E+02(3.66E+02)+	3.15E+02(3.05E+02)+	2.17E+02(2.34E+01)+	2.12E+02(2.58E+00)
+	14	12	21	19	15	
−	7	9	1	1	0	
≈	4	4	3	5	10	

("−", "+", and "≈" marks denote that the performance of ITLBO is significantly worse than, better than, and similar to that of the corresponding algorithm, respectively.)

Table 2. **Results of Friedman and Iman-Davenport tests with different Sigma values**

Friedman value	χ^2 value	p-value	Iman-Davenport value	F_F value	p-value
3.488	9.488	0.4797	0.8673	2.4665	0.4865

4.3 *Effectiveness of the proposed teaching process*

In this experiment, 25 30-dimensional test functions and two ITLBO variants are utilized to investigate the effectiveness of the proposed teaching process. For each test function, the run times are set as 30. Two different teaching processes are given as follows:

$$\boldsymbol{x}_{new,i} = \boldsymbol{x}_{old,i} + N(0.5, 0.25) \times (\boldsymbol{X}_{\text{Teacher}} - \boldsymbol{x}_{r1,i}), \quad (6)$$

$$\begin{aligned}\boldsymbol{x}_{new,i} = \boldsymbol{x}_{old,i} &+ N(0.5, 0.25) \times (\boldsymbol{X}_{\text{Teacher}} - \boldsymbol{x}_{r1,i}) \\ &+ N(0.5, 0.25) \times (X_{Mean} - \boldsymbol{x}_{r2,i})\end{aligned}. \quad (7)$$

According to the above introductions, two ITLBO variants using Eqs. (6) and (7) are called as ITLBO1 and ITLBO2, respectively. From Eq. (6), we can find that the perturbation is generated by the difference between the individual and the best position of the current population. It can be also observed from Eq. (7) that the perturbation towards $\boldsymbol{x}_{old,i}$ is produced by two differences, which are $(\boldsymbol{X}_{\text{Teacher}} - \boldsymbol{x}_{r1,i})$ and $(X_{Mean} - \boldsymbol{x}_{r2,i})$. The main targets of Eqs. (6) and (7) are to test the effectiveness of the $(X_{Mean} - \boldsymbol{x}_{r2,i})$ and the weighted average position $\boldsymbol{X}_W$, respectively.

All results are presented in Table A1 in the Appendix. From Table A1, the statistical analysis results present that ITLBO performs better than ITLBO1 and ITLBO2 on eight and nine test functions, respectively. However, the two variants outperform ITLBO on only one test function. Therefore, the proposed teaching process can provide the best overall performance for TLBO. The main reason is that the weighted average position can provide more useful evolution information to guide the evolution of each individual and increase random perturbation to improve the global search capability. However, Table A1 also indicates that ITLBO outperforms the two variants on the majority of basic test functions. For complex problems, the performance of ITLBO is slightly better than that of two algorithms. This is because these test functions are difficult to be solved by some other famous meta-heuristic algorithms. Additionally, the rankings obtained by Friedman's test are shown in Figure A2 in the Appendix. It is clearly that ITLBO achieves the best ranking among all the TLBO variants.

5 PARAMETER ESTIMATION OF SOLAR CELL MODELS USING ITLBO

RECENTLY, in order to improve the quality of the atmospheric environment, the traditional coal-fired power is transformed and reduced gradually. At the same time, the use of renewable and clean energy power is encouraged and supported. The PV (photovoltaic) system, due to its environment-friendly features, has drawn intense attention from academia and industry, see for examples (Askarzadeh & Rezazadeh, 2013; Easwarakhanthan, Bottin, Bouhouch, & Boutrit, 1986; Jiang, Maskell, & Patra, 2013; Oliva, Cuevas, & Pajares, 2014). For the PV system, it is a crucial problem to choose a suitable model which can describe the characteristics of PV cells appropriately. Normally, two main equivalent circuit models have been widely used in practical applications to describe the current-voltage (*I-V*) relationship in the PV cells, which are single diode model and double diode model. However, both models require the knowledge of all unknown parameters (Gong & Cai, 2013). Thus, parameter accuracy turns to be a significant problem as the parameters extracted from the *I-V* curves of the PV cell can be used in many applications (Jiang, et al., 2013).

5.1 Modelling of single and double diode

Since the single and the double diode models are two commonly used models among different types of solar cell models, we introduce the two models in the following parts.

5.1.1 Single diode model

The single diode model is commonly utilized to describe the *I-V* characteristics of solar cells because it is simple and accurate (Gow & Manning, 1999). Its output current can be calculated as follows:

$$I_L = I_{ph} - I_d - I_{sh}, \tag{8}$$

where I_L denotes the output current of the solar cell; I_{ph} and I_d denote the photo-induced current and diode current, respectively.

In Eq. (8), the diode current I_d based on the Shockley equation can be formulated as follows:

$$I_d = I_{sd}\left(exp\left(\frac{V_L + I_L \times R_s}{a \times V_t}\right) - 1\right), \tag{9}$$

where I_{sd} and V_L indicate the saturation current of the diode and the output voltage of the solar cell, respectively; a is the diode ideality constant; R_s is the series resistance. The junction thermal voltage V_t can be formulated as follows:

$$V_t = \frac{kT}{q}, \tag{10}$$

where k and q denote the Boltzmann constant (1.3806503E-23 J/K) and the electron charge (1.60217646E-19 C), respectively; T (K) indicates the temperature of the solar cell.

In Eq. (8), the shunt resistor current I_{sh} can be calculated as follows:

$$I_{sh} = \frac{V_L + I_L \times R_s}{R_{sh}}, \tag{11}$$

where R_{sh} denotes the shunt resistance.

In the single diode model, five parameters (i.e., I_{sd}, I_{ph}, R_s, R_{sh}, and a) are needed to be estimated. The objective function of the single diode model can be defined as follows:

$$\min RMSE = \sqrt{\frac{1}{m}\sum_{k=1}^{m}\left(I_{L,k} - \bar{I}_{L,k}\right)^2}, \tag{12}$$

where *RMSE* denotes the root mean square error; m denotes the number of experimental data; $\bar{I}_{L,k}$ denotes the experimental output current in the solar cell; $I_{L,k}$ is the calculated output current from Eq. (8).

5.1.2 Double diode model

The impact of recombination current loss is not considered in the single diode model, while it is considered in the double diode model. Therefore, the single diode model can be regarded as a simplified version of the double diode model.

The output current of the double diode model can be calculated as follows:

$$\begin{aligned} I_L &= I_{ph} - I_{d1} - I_{d2} - I_{sh} \\ &= I_{ph} - I_{sd1}\left(\exp\left(\frac{V_L + I_L \times R_s}{a_1 \times V_t}\right) - 1\right) \\ &\quad - I_{sd2}\left(\exp\left(\frac{V_L + I_L \times R_s}{a_2 \times V_t}\right) - 1\right) - \frac{V_L + I_L \times R_s}{R_{sh}}, \end{aligned} \tag{13}$$

where I_{d1} and I_{d2} denote the first and second currents, respectively; I_{sd1} and I_{sd2} indicate the diffusion and the saturation currents, respectively; a_1 denotes the diffusion diode ideality constant, and a_2 denotes the recombination diode ideality factor.

For the double diode model, I_{ph}, I_{sd1}, I_{sd2}, R_s, R_{sh}, a_1, and a_2 are seven optimized parameters. The objective function of the double diode model can be described as follows:

$$\min RMSE = \sqrt{\frac{1}{m}\sum_{k=1}^{m}\left(I_{L,k} - \bar{I}_{L,k}\right)^2}\,, \tag{14}$$

where $I_{L,k}$ is the calculated output current in the solar cell from Eq. (13).

5.2 Comparison with five meta-heuristic algorithms

In this experiment, ITLBO is utilized to estimate the parameters of two solar cell models, i.e., the single and the double diode models. The *I-V* characteristic of the two cell models is 57 mm diameter commercial (R.T.C. France) silicon solar cell at 33℃. The experimental data is directly taken from Ref. (Easwarakhanthan, et al., 1986). Moreover, the lower and upper boundaries of parameters of the two models (Askarzadeh & Rezazadeh, 2013; Chen et al., 2016; Gong & Cai, 2013) are presented in Table A2 in the Appendix. For all compared algorithms, the population size is set to be 20 except for CMA-ES. The maximum numbers of function evaluations are set to be 10000 and 20000 for the single diode model (five optimized parameters) and the double (seven optimized parameters) diode model, respectively. Moreover, all compared algorithms are executed for 30 independent runs to obtain the results, which are shown in Tables 3 and 4. The best results are highlighted in bold. Note that the results of four algorithms (i.e., CLPSO, TLBO, OTLBO, and GOTLBO) are directly taken from the literature (Chen, et al., 2016), and the results of CMA-ES are achieved through our own simulation experiment. Because the overall performance of GOTLBO is the best when the value of the jumping rate is equal to 1 on the benchmark test functions, the results of GOTLBO$_{Jr=1.0}$ are adopted in the current study. As shown in Table 3, the minimal RMSE value is achieved by OTLBO. ITLBO can obtain the second best result among all compared algorithms on the single diode model. Overall, the minimal RMSE value of ITLBO (9.86242E-04) is very close to that of OTLBO (9.86026E-04) on the single diode model. Table 3 indicates that the maximal RMSE value of ITLBO is the smallest among all compared algorithms. It means that the overall performance of the proposed algorithm is excellent. For the double diode model, it can be observed from Table 4 that ITLBO can achieve the best result (9.83263E-04) when compared with the other competitors. Moreover, the maximal RMSE value of ITLBO (1.58900E-03) is the smallest among all the maximal RMSE values. Additionally, the proposed algorithm can get the best mean value of RMSE among all the selected algorithms. Overall, ITLBO exhibits the best average performance on the parameter estimation of the double diode model when compared with the other algorithms.

Based on the above comparisons and analyses, we can find that ITLBO is a competitive optimization tool to deal with the actual application problems. Furthermore, the prediction results of the single and double diode models obtained by ITLBO are plotted in Figure 1 and 2, respectively.

Table 3. **Results of all compared algorithms for the single diode model**

	Min value	Max value	Mean
CLPSO	1.05672E-03	3.03829E-03	1.68858E-03
CMA-ES	2. 32757E-03	2.26969E-02	6.14180E-03
TLBO	9.87010E-04	1.78338E-03	1.29346E-03
OTLBO	**9.86026E-04**	1.86989E-03	**1.14239E-03**
GOTLBO$_{Jr=1.0}$	1.00684E-03	2.14982E-03	1.50696E-03
ITLBO	**9.86242E-04**	**1. 54205E-03**	1.20547E-03

Table 4. **Results of all compared algorithms for the double diode model**

	Min value	Max value	Mean
CLPSO	1.00720E-03	2.07598E-03	1.37731E-03
CMA-ES	1. 41367E-03	8.26330E-03	3.89138E-03
TLBO	9.96598E-04	2.31028E-03	1.35292E-03
OTLBO	9.84070E-04	1.78645E-03	1.23906E-03
GOTLBO$_{Jr=1.0}$	9.94041E-04	2.68617E-03	1.45316E-03
ITLBO	**9. 83263E-04**	**1. 58900E-03**	**1.17138E-03**

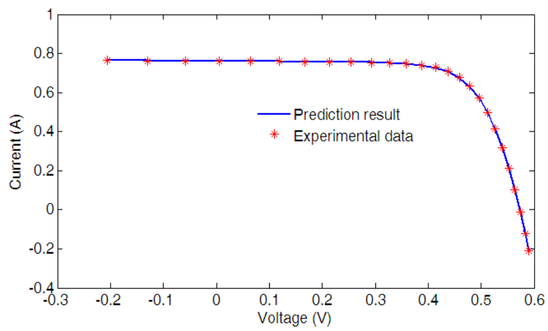

Figure 1. **Prediction results of the single diode model**

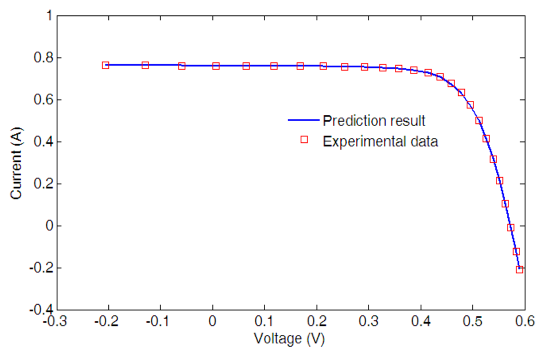

Figure 2. **Prediction results of the double diode model**

6 CONCLUSION

IN the current study, an improved teaching learning based optimization (ITLBO) is introduced. In ITLBO, the normal random number is used to enhance the search efficiency; the weighted average position of the current population is utilized to guide the evolution of each individual and enhance the global search capability. The main purpose of this study is to balance the convergence speed and population diversity during the evolutionary process. The performance of ITLBO is compared with that of the five algorithms on 25 30-dimensional IEEE CEC2005 test functions. The simulation and statistical analysis results indicate that ITLBO can perform better than the other algorithms, especially when compared with the original TLBO and its variants. The local search capability of ITLBO is slightly worse than that of CMA-ES on some unimodal test functions. However, ITLBO outperforms CMA-ES on some complex optimization problems. In addition, the Sigma and improved teaching operator are studied in Subsections 5.2 and 5.3, respectively. The results show that the parameter sensibility is low and the proposed teaching process is effective.

Besides the benchmark test functions, ITLBO is also utilized to estimate the parameters of two solar cell models. The results indicate that ITLBO performs better than, or at least comparably with, five meta-heuristic algorithm. Therefore, ITLBO is an efficient and effective tool for solving real-world optimization problems.

In the future work, more advanced machine learning methods would be used to improve the search efficiency of TLBO. Moreover, how to reduce useless computational cost in TLBO is also worth further study.

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8 DISCLOSURE STATEMENT

NO potential conflict of interest was reported by the authors.

9 REFERENCES

A. Askarzadeh & Rezazadeh, A. (2013). Artificial bee swarm optimization algorithm for parameters identification of solar cell models. *Applied Energy*, *102*, 943-949.

S. Banerjee, Maity, D., & Chanda, C. K. (2015). Teaching learning based optimization for economic load dispatch problem considering valve point loading effect. *International Journal of Electrical Power & Energy Systems*, *73*, 456-464.

S. Chatterjee & Mukherjee, V. (2016). PID controller for automatic voltage regulator using teaching–learning based optimization technique. *International Journal of Electrical Power & Energy Systems*, *77*, 418-429.

X. Chen, Yu, K., Du, W., Zhao, W., & Liu, G. (2016). Parameters identification of solar cell models using generalized oppositional teaching learning based optimization. *Energy*, *99*, 170-180.

E. Cuevas, Santuario, E., Zaldivar, D., & Perez-Cisneros, M. (2016). An improved evolutionary algorithm for reducing the number of function evaluations. *Intelligent Automation & Soft Computing*, *22*, 177-192.

T. Easwarakhanthan, Bottin, J., Bouhouch, I., & Boutrit, C. (1986). Nonlinear minimization algorithm for determining the solar cell parameters with microcomputers. *International Journal of Solar Energy*, *4*, 1-12.

Q.Q. Fan, Yan, X.F., & Zhang, Y.L. (2018). Auto-selection mechanism of differential evolution algorithm variants and its application. *European Journal of Operational Research*, 270, 636-653.

Q. Q. Fan & Yan, X. F. (2016). Self-Adaptive Differential Evolution Algorithm With Zoning Evolution of Control Parameters and Adaptive Mutation Strategies. *IEEE transactions on cybernetics*, *46*, 219-232.

H. Feshki Farahani & Rashidi, F. (2017). An improved teaching-learning-based optimization with differential evolution algorithm for optimal power flow considering HVDC system. *Journal of Renewable and Sustainable Energy, 9*, 035505.

M. Friedman. (1937). The use of ranks to avoid the assumption of normality implicit in the analysis of variance. *Journal of the American statistical association, 32*, 675-701.

S. García, Molina, D., Lozano, M., & Herrera, F. (2009). A study on the use of non-parametric tests for analyzing the evolutionary algorithms' behaviour: a case study on the CEC'2005 special session on real parameter optimization. *Journal of Heuristics, 15*, 617-644.

M. Ghasemi, Ghavidel, S., Gitizadeh, M., & Akbari, E. (2015). An improved teaching–learning-based optimization algorithm using Lévy mutation strategy for non-smooth optimal power flow. *International Journal of Electrical Power & Energy Systems, 65*, 375-384.

M. Ghasemi, Taghizadeh, M., Ghavidel, S., Aghaei, J., & Abbasian, A. (2015). Solving optimal reactive power dispatch problem using a novel teaching–learning-based optimization algorithm. *Engineering Applications of Artificial Intelligence, 39*, 100-108.

W. Gong & Cai, Z. (2013). Parameter extraction of solar cell models using repaired adaptive differential evolution. *Solar Energy, 94*, 209-220.

J. Gow & Manning, C. (1999). Development of a photovoltaic array model for use in power-electronics simulation studies. In *Electric Power Applications, IEE Proceedings-* (Vol. 146, pp. 193-200): IET.

N. Hansen & Ostermeier, A. (2001). Completely derandomized self-adaptation in evolution strategies. *Evolutionary computation, 9*, 159-195.

G. Hermosilla, Ruiz-del-Solar, J., & Verschae, R. (2017). An enhanced representation of thermal faces for improving local appearance-based face recognition. *Intelligent Automation & Soft Computing, 23*, 1-12.

J. H. Holland. (1975). Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence: U Michigan Press.

L. L. Jiang, Maskell, D. L., & Patra, J. C. (2013). Parameter estimation of solar cells and modules using an improved adaptive differential evolution algorithm. *Applied Energy, 112*, 185-193.

D. Karaboga. (2005). An idea based on honey bee swarm for numerical optimization. In: Technical report-tr06, Erciyes University, engineering faculty, computer engineering department.

J. Kenndy & Eberhart, R. (1995). Particle swarm optimization. In *Proceedings of IEEE International Conference on Neural Networks* (Vol. 4, pp. 1942-1948).

M. Li, Ma, H., & Gu, B. (2016). Improved teaching–learning-based optimization algorithm with group learning. *Journal of Intelligent & Fuzzy Systems, 31*, 2101-2108.

Y. L. Li, Zhan, Z. H., Gong, Y. J., Chen, W. N., Zhang, J., & Li, Y. (2015). Differential evolution with an evolution path: A DEEP evolutionary algorithm. *Cybernetics, IEEE Transactions on, 45*, 1798-1810.

J. J. Liang, Qin, A. K., Suganthan, P. N., & Baskar, S. (2006). Comprehensive learning particle swarm optimizer for global optimization of multimodal functions. *IEEE Transactions on Evolutionary Computation, 10*, 281-295.

W. Lin, Yu, D., Wang, S., Zhang, C., Zhang, S., Tian, H., Luo, M., & Liu, S. (2015). Multi-objective teaching–learning-based optimization algorithm for reducing carbon emissions and operation time in turning operations. *Engineering Optimization, 47*, 994-1007.

D. Oliva, Cuevas, E., & Pajares, G. (2014). Parameter identification of solar cells using artificial bee colony optimization. *Energy, 72*, 93-102.

J. Patel, Savsani, V., Patel, V., & Patel, R. (2017). Layout optimization of a wind farm to maximize the power output using enhanced teaching learning based optimization technique. *Journal of Cleaner Production, 158*, 81-94.

R. Rao & Patel, V. (2012). An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems. *International Journal of Industrial Engineering Computations, 3*, 535-560.

R. V. Rao & Kalyankar, V. (2013). Parameter optimization of modern machining processes using teaching–learning-based optimization algorithm. *Engineering Applications of Artificial Intelligence, 26*, 524-531.

R. V. Rao, Savsani, V. J., & Vakharia, D. (2011). Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems. *Computer-Aided Design, 43*, 303-315.

P. K. Roy, Paul, C., & Sultana, S. (2014). Oppositional teaching learning based optimization approach for combined heat and power dispatch. *International Journal of Electrical Power & Energy Systems, 57*, 392-403.

S. C. Satapathy & Naik, A. (2014). Modified Teaching–Learning-Based Optimization algorithm for global numerical optimization—A comparative study. *Swarm and Evolutionary Computation, 16*, 28-37.

S. Sleesongsom & Bureerat, S. (2017). Four-bar linkage path generation through self-adaptive population size teaching-learning based optimization. *Knowledge-Based Systems, 135*, 180-191.

R. Storn & Price, K. (1995). Differential evolution-a simple and efficient adaptive scheme for global optimization over continuous spaces (Vol. 3): ICSI Berkeley.

P. N. Suganthan, Hansen, N., Liang, J. J., Deb, K., Chen, Y.-P., Auger, A., & Tiwari, S. (2005). Problem definitions and evaluation criteria for the CEC 2005 special session on real-parameter optimization. *KanGAL report, 2005005*.

Q. Tang, Li, Z., Zhang, L., & Zhang, C. (2017). Balancing stochastic two-sided assembly line with multiple constraints using hybrid teaching-learning-based optimization algorithm. *Computers & Operations Research*, *82*, 102-113.

Z. Wang, Lu, R., Chen, D., & Zou, F. (2016). An Experience Information Teaching–Learning-Based Optimization for Global Optimization. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *46*, 1202-1214.

X. Wei, Wang, Y., Li, Z., Zou, T., & Yang, G. (2015). Mining Users interest navigation patterns using improved ant colony optimization. *Intelligent Automation & Soft Computing*, *21*, 445-454.

F. Wilcoxon. (1945). Individual comparisons by ranking methods. *Biometrics bulletin*, *1*, 80-83.

K. Z. Zamli, Din, F., Baharom, S., & Ahmed, B. S. (2017). Fuzzy adaptive teaching learning-based optimization strategy for the problem of generating mixed strength t-way test suites. *Engineering Applications of Artificial Intelligence*, *59*, 35-50.

X. H. Zhao. (2015). Improved teaching-learning based optimization for global optimization problems. In *Control Conference (CCC), 2015 34th Chinese* (pp. 2639-2644): IEEE.

F. Zou, Wang, L., Hei, X., Chen, D., Jiang, Q., & Li, H. (2014). Bare-bones teaching-learning-based optimization. *The Scientific World Journal*, *2014*.

F. Zou, Wang, L., Hei, X., Chen, D., & Yang, D. (2014). Teaching–learning-based optimization with dynamic group strategy for global optimization. *Information Sciences*, *273*, 112-131.

10 NOTES ON CONTRIBUTORS

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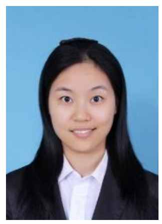

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Appendix

Table A1. **Result of ITBLO and ITBLO variants**

Function	ITLBO1 Mean(Std)	ITLBO2 Mean(Std)	ITLBO Mean(Std)
F1	2.35E-29(6.32E-29)−	1.40E-27(1.47E-27)+	5.77E-29(8.84E-29)
F2	6.08E-16(1.96E-15)+	2.00E-15(6.65E-15)+	5.48E-17(8.83E-17)
F3	6.42E+05(3.70E+05)≈	8.76E+05(4.89E+05)≈	7.09E+05(3.53E+05)
F4	4.05E-02(9.46E-02)+	2.75E-02(1.24E-01)+	7.47E-04(1.96E-03)
F5	1.99E+03(7.93E+02)≈	2.09E+03(6.83E+02)≈	2.02E+03(6.90E+02)
F6	2.26E+01(3.13E+01)+	1.66E+01(3.28E+01)+	1.00E+01(2.78E+01)
F7	2.05E-02(1.70E-02)≈	2.36E-02(2.02E-02)≈	**2.32E-02(1.86E-02)**
F8	2.09E+01(3.91E-02)≈	2.10E+01(3.39E-02)≈	2.09E+01(5.67E-02)
F9	7.55E+01(1.84E+01)+	6.30E+01(1.66E+01)+	5.43E+01(1.64E+01)
F10	7.32E+01(2.48E+01)+	6.58E+01(2.03E+01)+	5.39E+01(2.27E+01)
F11	2.71E+01(4.26E+00)≈	2.82E+01(3.34E+00)≈	2.83E+01(4.37E+00)
F12	7.89E+03(8.55E+03)+	2.88E+03(2.68E+03)≈	3.28E+03(3.40E+03)
F13	3.41E+00(9.05E-01)≈	3.27E+00(9.19E-01)≈	3.00E+00(5.62E-01)
F14	1.27E+01(3.04E-01)≈	1.29E+01(2.81E-01)≈	1.27E+01(3.56E-01)
F15	4.23E+02(7.83E+01)≈	3.97E+02(8.09E+01)≈	3.97E+02(8.92E+01)
F16	1.96E+02(1.75E+02)≈	2.02E+02(1.80E+02)≈	1.93E+02(1.63E+02)
F17	2.23E+02(1.34E+02)+	2.44E+02(1.44E+02)+	1.70E+02(1.35E+02)
F18	8.93E+02(2.55E+01)≈	8.80E+02(4.06E+01)≈	8.86E+02(3.45E+01)
F19	8.86E+02(3.45E+01)≈	8.87E+02(3.45E+01)≈	8.94E+02(2.56E+01)
F20	8.97E+02(1.82E+01)≈	8.83E+02(3.79E+01)−	9.00E+02(3.40E+00)
F21	7.98E+02(3.21E+02)≈	7.21E+02(2.93E+02)≈	7.96E+02(3.29E+02)
F22	9.12E+02(2.54E+01)≈	9.10E+02(1.42E+01)≈	9.04E+02(2.00E+01)
F23	7.71E+02(2.70E+02)≈	8.05E+03(2.99E+02)+	7.13E+02(2.30E+02)
F24	2.00E+02(1.75E-12)+	2.00 E+02(1.81E-12)+	2.00E+02(1.02E-12)
F25	2.13E+02(4.66E+00)≈	2.12E+02(2.60E+00)≈	**2.12E+02(2.58E+00)**
+	8	9	
−	1	1	
≈	16	15	

Table A2. **Lower and upper boundaries of parameters in two solar cell models**

Parameter	Lower bound	Upper bound
I_{ph}(A)	0	1.0
I_{sd}(μA)	0	1.0
R_s(Ω)	0	0.5
R_{sh}(Ω)	0	100
a	1.0	2.0

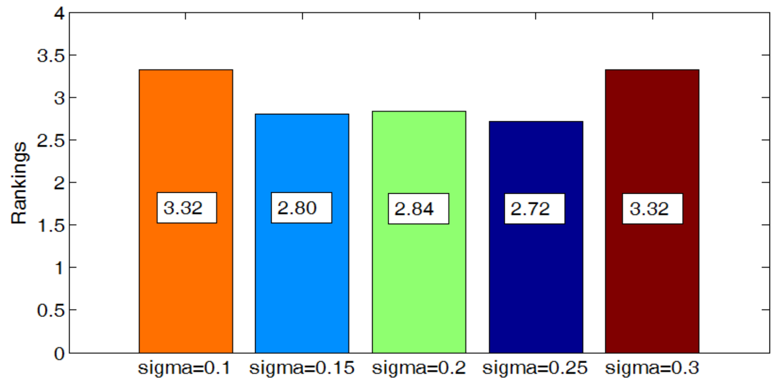

Figure A1. **Rankings of ITLBO under different values of Sigma**

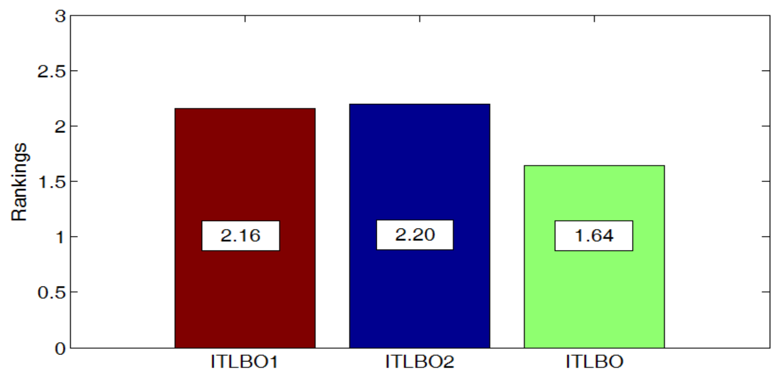

Figure A2. **Rankings of ITLBO with different teaching process**