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A New Idea of Fractal-Fractional Derivative with Power Law Kernel for Free Convection Heat Transfer in a Channel Flow between Two Static Upright Parallel Plates

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Abstract: Nowadays some new ideas of fractional derivatives have been used successfully in the present research community to study different types of mathematical models. Amongst them, the significant models of fluids and heat or mass transfer are on priority. Most recently a new idea of fractal-fractional derivative is introduced; however, it is not used for heat transfer in channel flow. In this article, we have studied this new idea of fractal fractional operators with power-law kernel for heat transfer in a fluid flow problem. More exactly, we have considered the free convection heat transfer for a Newtonian fluid. The flow is bounded between two parallel static plates. One of the plates is heated constantly. The proposed problem is modeled with a fractal fractional derivative operator with a power-law kernel and solved via the Laplace transform method to find out the exact solution. The results are graphically analyzed via MathCad-15 software to study the behavior of fractal parameters and fractional parameter. For the influence of temperature and velocity profile, it is observed that the fractional parameter raised the velocity and temperature as compared to the fractal operator. Therefore, a combined approach of fractal fractional explains the memory of the function better than fractional only.

Keywords: Fractal-fractional derivative, power law kernel, convection heat transfer, upright parallel plates.

1 Introduction

The phenomenon of heat transfer is studied by different researchers in many fields of study. Convection heat transfer plays a significant role in fluids dynamics. There is a lot

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of work done by scientists in heat transfer in fluids dynamics such as [Sooppy, Khan, Khan et al. (2019)] investigated the phenomenon of heat transfer in a drilling nanoliquid to study the effect of clay nanoparticles. Shah et al. [Shah, Zafar and Fetecau (2018)] studied the analysis of free convection flow for a fractional viscous fluid. They used to apply shear stress to the fluid and the geometry is taken as a vertical plate. Furthermore, Rout et al. [Rout, Parida, and Panda (2013)] investigated the heat and mass transfer in a moving geometry. The heat source is considered for convective surface conditions. Shah et al. [Shah and Khan (2016)] discussed the heat transfer for a second-grade fluid, where the geometry is an oscillating plate. The solution is obtained in exact form via the Laplace transform method. Some recent applications via different researchers are reported as Sheikholeslami et al. [Sheikholeslami, Hayat and Alsaedi (2017)], [Ghara, Das, Maji et al. (2012)], [Raju, Sandeep, Sugunamma et al. (2016)] and [Vieru, Fetecau and Fetecau (2015)]. Imran et al. [Imran, Shah and Khan et al. (2018)] investigated the natural convection flow as an application of non-integer Caputo time-fractional derivatives.

Recently, fractional calculus took the attention of scientists due to its significant role in daily life applications. Especially in fluid dynamics, it plays a significant role to generalize the models. Khan et al. [Khan, Abro, Tassaddiq et al. (2017)] using a fractional operator to study the starting solution of second-grade fluid. The operator is based on the exponential function. Atangana et al. [Atangana and Nieto (2015)] claimed that this fractional operator is suitable for some physical problems not for all. Ali et al. [Ali, Saqib, Khan et al. (2016)] used this fractional operator to find out the exact solution for free convection flow with the influence of MHD. The Walters-B fluid model is studied for the applications of this operator. However, the existing operators have some issues of non-singularity and non-locality. To solve this problem, Atangana et al. [Atangana and Baleanu (2016a)], [Atangana and Baleanu (2016b)] investigated the new fractional operator based on Mittag-Leffler function. Alkahtani [Alkahtani (2016)] used Atangana-Baleanu fractional operator to find out the solution or Chua's circuit model. Furthermore, Sheikh et al. [Sheikh, Ali, Saqib et al. (2017)] reported a comparison of Atangana and Baleanu and Caputo-Fabrizio fractional operators. Casson fluid model is studied with the chemical reaction and heat generation. The enchantment of heat transfer is reported by Abro et al. [Abro, Memon, Abro et al. (2019)] using a fractional operator to generalize the study. The Jeffrey nanofluid is reported as an application in solar energy. Recently Arif et al. [Arif, Ali, Sheikh et al. (2019)] reported a comparative study for Couette flow with couple stress fluids. A time-fractional model with a non-local kernel is taken. Atangana and Baleanu and Caputo-Fabrizio operators are used for the comparison. Furthermore, Li et al. [Li, Feng and Chen (2017)] investigate improved kernel results for some FPT problems based on simple observations. The same author Li et al. [Li, Liu, Wang et al. (2019)] use the linear kernel for complementary maximal strip recovery. Recently, Abro et al. [Abro, Memon and Uqaili (2018)] used the same fractional operator for the comparative analysis of RL and RC electrical circuits. The Laplace technique is used to elaborate on the solution. Fan et al. [Fan and He (2012)] investigated the fractal derivative model to study air permeability as a porous medium. Chen [Chen (2006)] and Chen et al. [Chen, Sun, Zhang et al. (2010)] investigated the diffusion modeling for the diffusion equation by incorporate the fractal derivative and fractal time-space. Recently, Atangana [Atangana (2017)] investigated the connectivity between fractal and fractional derivatives to calculate the problems in daily life problems. Furthermore, in the previous year Atangana et al. [Atangana and Khan (2019)], [Atangana and Shafiq (2019)] and Atangana et al. [Atangana and Qureshi (2019)] reported a detailed analysis on newly introduced operators to solve fractal fractional differential equations. Recently in 2020, Imran [Imran (2020)] investigated the fractal fractional derivative to find out the solution or the viscose fluid flow. The flow is in between two parallel infinite plates with the influence of MHD. Up to date, no one has applied the new idea of the fractal-fractional derivative to study the heat transfer problem with fluid flow via fractal-fractional derivative approach. Therefore, this new approach of the fractal-fractional derivative is applied here in this work for the first time to heat transfer problem combined with fluid motion.

More specifically, this article is focused on the application of the fractal-fractional model of Newtonian viscose fluid between parallel infinite plates. Free convection flow with heat transfer is considered with physical boundary conditions. The fluid motion is between an upright channel such that the parallel plates are in a static position and the motion is induced due to convection heating of the plates. Such a study using the new idea of fractal-fractional derivatives with the power-law kernel is not reported yet. The problem solution is obtained by using the Laplace technique. The effects of both fractal, as well as fractional parameters on temperature and velocity, are plotted via Mathcad-15 and discussed.

2 Mathematical formulation

Here we supposed that an incompressible Newtonian viscous fluid in two infinite parallel plates. The distance between plates is L in a coordinate system. It is assumed that both plate are at reset and the temperature of the lower pate is Θ_0, while the upper plate temperature is Θ_w. The flow of the fluid is due to free convection as show in Fig. 1. The Reynolds number neglected and no pressure gradient is assumed in the direction of flow, the governing equation with assumed boundary conditions are:

$$\rho \frac{\partial u(y,t)}{\partial t} = \mu \frac{\partial^2 u(y,t)}{\partial y^2} + g\rho\beta_\Theta \left(\Theta(y,t) - \Theta_\infty\right). \tag{1}$$

$$\rho C_p \frac{\partial \Theta(y,t)}{\partial t} = k \frac{\partial^2 \Theta(y,t)}{\partial y^2}, \tag{2}$$

The physical conditions at initial and boundary are:

$$u(y,0) = 0,\ \Theta(y,0) = \Theta_0 \text{ for all } y \geq 0, \tag{3}$$

$$u(0,t) = 0,\ \Theta(0,t) = \Theta_0,\ t > 0, \tag{4}$$

$$u(L,t) = 0,\ \Theta(L,t) = \Theta_w,\ t > 0, \tag{5}$$

the dimensionless variables are elaborate for non-dimensionalization:

$$t^* = \frac{U^2}{\nu} t,\ y^* = \frac{U}{\nu} y,\ \ u^* = \frac{u}{U},\ \ \theta = \frac{\Theta - \Theta_0}{\Theta_w - \Theta_0}, \tag{6}$$

By using (6) the dimensionless form of 1-5 is, for simplicity * sign is dropped.

$$\frac{\partial u(y,t)}{\partial t}=\frac{\partial^2 u(y,t)}{\partial y^2}+Gr\theta(y,t). \tag{7}$$

$$\Pr\frac{\partial \theta(y,t)}{\partial t}=\frac{\partial^2 \theta(y,t)}{\partial y^2}, \tag{8}$$

$$u(y,0)=0,\ \theta(y,0)=0 \text{ for all } y\geq 0, \tag{9}$$

$$u(0,t)=0,\ \theta(0,t)=0,\ t>0, \tag{10}$$

$$u(1,t)=0,\ \theta(1,t)=1,\ t>0, \tag{11}$$

3 Solution with fractal fractional model

The fractal fractional model of Eqs. (7) and (8) as follows:

$$^{C}D_t^{\alpha}u(y,t)=\beta t^{\beta-1}\left\{\frac{\partial^2 u(y,t)}{\partial y^2}+Gr\theta(y,t)\right\}-\frac{u(y,0)}{\Gamma(1-\alpha)}t^{-\alpha}. \tag{12}$$

$$^{C}D_t^{\alpha}\theta(y,t)=\beta t^{\beta-1}\left\{\frac{1}{\Pr}\frac{\partial^2 \theta(y,t)}{\partial y^2},\right\}-\frac{\theta(y,0)}{\Gamma(1-\alpha)}t^{-\alpha} \tag{13}$$

By applying Laplace transform of Eqs. (9)-(13) we get

$$\begin{aligned} q^{\alpha}u(y,q)-u(y,0)=\beta\Gamma(\beta)q^{-\beta}&\left\{\frac{\partial^2 u(y,q)}{\partial y^2}+Gr\theta(y,q)\right\}\\ &-\frac{u(y,0)}{\Gamma(1-\alpha)}q^{-\alpha}\Gamma(1-\alpha). \end{aligned} \tag{14}$$

$$q^{\alpha}\theta(y,q)-\theta(y,0)=\beta\Gamma(\beta)q^{-\beta}\left\{\frac{1}{\Pr}\frac{\partial^2 \theta(y,q)}{\partial y^2}\right\}-\frac{\theta(y,0)}{\Gamma(1-\alpha)}q^{-\alpha}\Gamma(1-\alpha). \tag{15}$$

$$u(y,0)=0,\ \theta(y,0)=0 \text{ for all } y\geq 0, \tag{16}$$

$$u(0,q)=0,\ \theta(0,q)=0,\ q>0, \tag{17}$$

$$u(1,q)=0,\ \theta(1,q)=\frac{1}{q},\ q>0, \tag{18}$$

Solution of Eqs. (14) and (15) subjected to Eqs. (16)-(18) is

$$\theta(y,q)=\frac{1}{q}\frac{\sinh\left(y\sqrt{a\,q^{\gamma}}\right)}{\sinh\left(\sqrt{a\,q^{\gamma}}\right)}, \tag{19}$$

$$u(y,q)=\frac{\mathrm{c}\,Gr}{q^{\gamma+1}}\left(\frac{\sinh\left(y\sqrt{b\,q^{\gamma}}\right)}{\sinh\left(\sqrt{b\,q^{\gamma}}\right)}-\frac{\sinh\left(y\sqrt{a\,q^{\gamma}}\right)}{\sinh\left(\sqrt{a\,q^{\gamma}}\right)}\right), \tag{20}$$

where $\gamma=\alpha+\beta,\ \frac{1}{a}=\beta\Gamma(\beta)$

Eqs. (19) and (20) can be written as in the form of:

$$\theta(y,q)=\frac{1}{q}\sum_{n=0}^{\infty}\left\{\sum_{p_1=0}^{\infty}\frac{(1+2n-y)^{p_1}}{p_1!}\left(aq^{\gamma}\right)^{\frac{p_1}{2}}-\sum_{p_2=0}^{\infty}\frac{\left(-(1+2n-y)^{p_2}\right)}{p_2!}\left(aq^{\gamma}\right)^{\frac{p_2}{2}}\right\} \tag{21}$$

$$u(y,q)=\mathrm{c}\,Gr\left\{\begin{array}{l}\frac{1}{q^{\gamma+1}}\sum_{n=0}^{\infty}\left\{\sum_{p_1=0}^{\infty}\frac{(1+2n-y)^{p_1}}{p_1!}\left(bq^{\gamma}\right)^{\frac{p_1}{2}}-\sum_{p_2=0}^{\infty}\frac{\left(-(1+2n-y)^{p_2}\right)}{p_2!}\left(bq^{\gamma}\right)^{\frac{p_2}{2}}\right\}\\ -\frac{1}{q^{\gamma+1}}\sum_{n=0}^{\infty}\left\{\sum_{p_1=0}^{\infty}\frac{(1+2n-y)^{p_1}}{p_1!}\left(aq^{\gamma}\right)^{\frac{p_1}{2}}-\sum_{p_2=0}^{\infty}\frac{\left(-(1+2n-y)^{p_2}\right)}{p_2!}\left(aq^{\gamma}\right)^{\frac{p_2}{2}}\right\}\end{array}\right\} \tag{22}$$

After Laplace inverse of Eqs. (21) and (22) we get:

$$\begin{aligned}\theta(y,t)=&\sum_{n=0}^{\infty}\sum_{p_1=0}^{\infty}\sum_{q_1=0}^{\infty}\frac{(y-1-2n)^{p_1}}{p_1!}\frac{a^{\frac{p_1}{2}}}{q_1!}\times\frac{t^{q_1-\frac{\gamma p_1}{2}}}{\Gamma\left(1+q_1-\frac{\gamma p_1}{2}\right)}\frac{\Gamma\left(q_1+\frac{p_1}{2}\right)}{\Gamma\left(\frac{p_1}{2}\right)}\\ &-\sum_{n=0}^{\infty}\sum_{p_2=0}^{\infty}\sum_{q_2=0}^{\infty}\frac{\left(-(y+1+2n)\right)^{p_2}}{p_2!}\frac{a^{\frac{p_2}{2}}}{q_2!}\times\frac{t^{q_2-\frac{\gamma p_2}{2}}}{\Gamma\left(1+q_2-\frac{\gamma p_2}{2}\right)}\frac{\Gamma\left(q_2+\frac{p_2}{2}\right)}{\Gamma\left(\frac{p_2}{2}\right)}\end{aligned} \tag{23}$$

$$u(y,t)=\mathrm{c\ Gr}\left\{\int_0^t\left[\begin{array}{l}\dfrac{(t-s)^{\gamma-2}}{\Gamma(\gamma-1)}\sum_{n=0}^{\infty}\sum_{p_1=0}^{\infty}\sum_{q_1=0}^{\infty}\dfrac{(y-1-2n)^{p_1}}{p_1!}\dfrac{a^{\frac{p_1}{2}}}{q_1!}\times\dfrac{s^{q_1-\frac{\gamma p_1}{2}}}{\Gamma\left(1+q_1-\frac{\gamma p_1}{2}\right)}\dfrac{\Gamma\left(q_1+\frac{p_1}{2}\right)}{\Gamma\left(\frac{p_1}{2}\right)}\\ -\sum_{n=0}^{\infty}\sum_{p_2=0}^{\infty}\sum_{q_2=0}^{\infty}\dfrac{\left(-(y+1+2n)\right)^{p_2}}{p_2!}\dfrac{a^{\frac{p_2}{2}}}{q_2!}\times\dfrac{s^{q_2-\frac{\gamma p_2}{2}}}{\Gamma\left(1+q_2-\frac{\gamma p_2}{2}\right)}\dfrac{\Gamma\left(q_2+\frac{p_2}{2}\right)}{\Gamma\left(\frac{p_2}{2}\right)}\end{array}\right]ds\right\}$$
$$-\mathrm{c\ Gr}\left\{\int_0^t\left[\begin{array}{l}\dfrac{(t-s)^{\gamma-2}}{\Gamma(\gamma-1)}\sum_{n=0}^{\infty}\sum_{p_1=0}^{\infty}\sum_{q_1=0}^{\infty}\dfrac{(y-1-2n)^{p_1}}{p_1!}\dfrac{a^{\frac{p_1}{2}}}{q_1!}\times\dfrac{s^{q_1-\frac{\gamma p_1}{2}}}{\Gamma\left(1+q_1-\frac{\gamma p_1}{2}\right)}\dfrac{\Gamma\left(q_1+\frac{p_1}{2}\right)}{\Gamma\left(\frac{p_1}{2}\right)}\\ -\sum_{n=0}^{\infty}\sum_{p_2=0}^{\infty}\sum_{q_2=0}^{\infty}\dfrac{\left(-(y+1+2n)\right)^{p_2}}{p_2!}\dfrac{a^{\frac{p_2}{2}}}{q_2!}\times\dfrac{s^{q_2-\frac{\gamma p_2}{2}}}{\Gamma\left(1+q_2-\frac{\gamma p_2}{2}\right)}\dfrac{\Gamma\left(q_2+\frac{p_2}{2}\right)}{\Gamma\left(\frac{p_2}{2}\right)}s\end{array}\right]ds\right\} \tag{24}$$

4 Results and discussion

A new approach of a fractal-fractional derivative is used in this problem to study the heat transfer due to free convection when the flow is between two parallel upright plates. The fractal-fractional model is developed with given physical boundary conditions. After the dimensionless analysis, the problem is solved via the Laplace technique to find the exact solution to the problem. The influence of fractal, as well as fractional operators on different profiles, is plotted graphically and discussed. Figs. 2 and 3 show the influence of fractal fraction parameters on temperature and velocity respectively. It is observed that influence of fractal fractional parameter decrease the temperature and as a result, the velocity decreases. The behavior of the fractional parameter without a fractal operator on temperature and velocity is highlighted in Figs. 3 and 8 respectively. It is observed that increasing fractional parameter, decrease both the temperature and velocity. It is because of the power-law kernel. This shows the memory effect of the velocity and temperature at a certain time. The variation of $\mathbf{Pr}$ on temperature is plotted in Fig. 4, the lower temperature is found out at the greeter value $\mathbf{Pr}$. Fig. 5 shows the critical point for fractal fractional parameters on temperature while Fig. 6 shows the critical point for fractional parameter on temperature. It is observed that critical pints are different for fractal and fractional parameters. The thermal Grashof number for the fractal fractional derivative is highlighted in Fig. 9. It is noticed that the greater value of Gr results the greater velocity. The effect of $\mathbf{Pr}$ drop the temperature, as a result, decreases the velocity which is shown in Fig. 10. Figs. 11 and 12 highlights the critical point for Figs. 7 and 8 respectively.

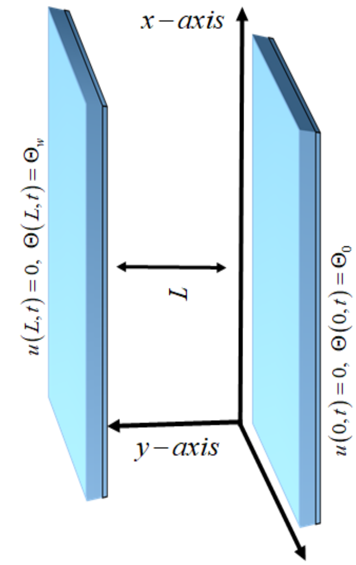

Figure 1: Physical sketch

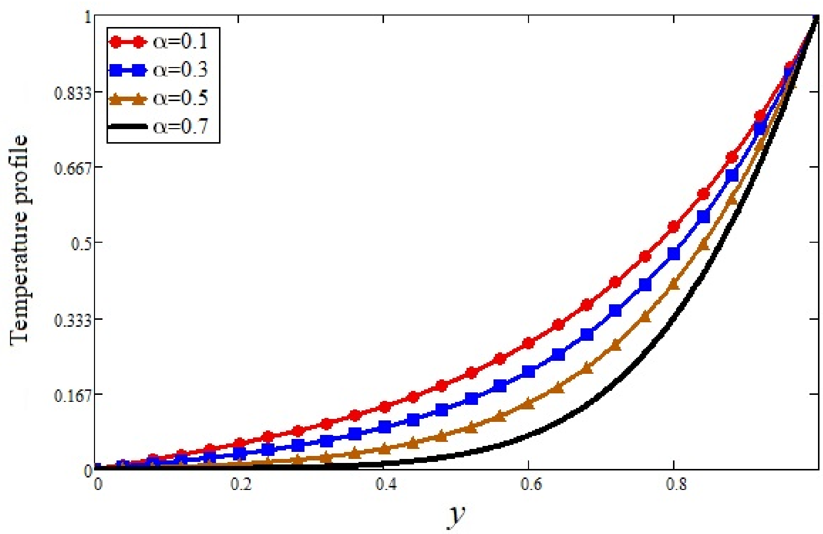

Figure 2: Variation of fractal parameter on temperature

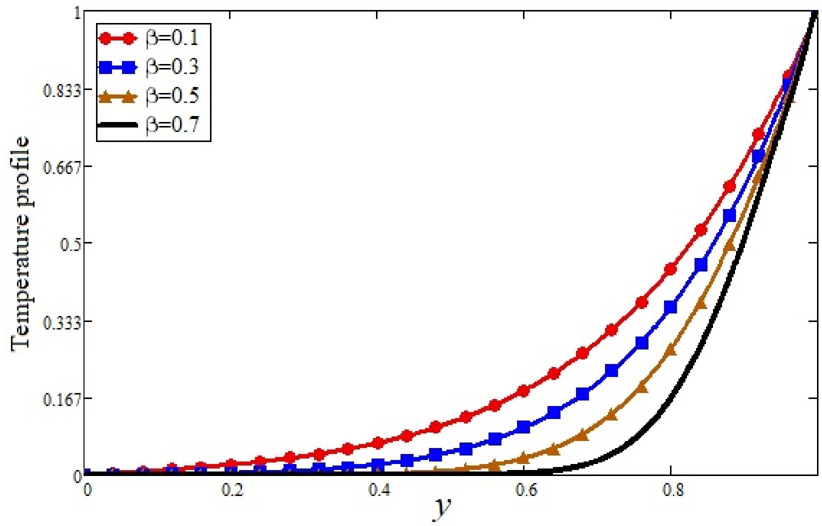

Figure 3: Variation of fractional parameter on temperature

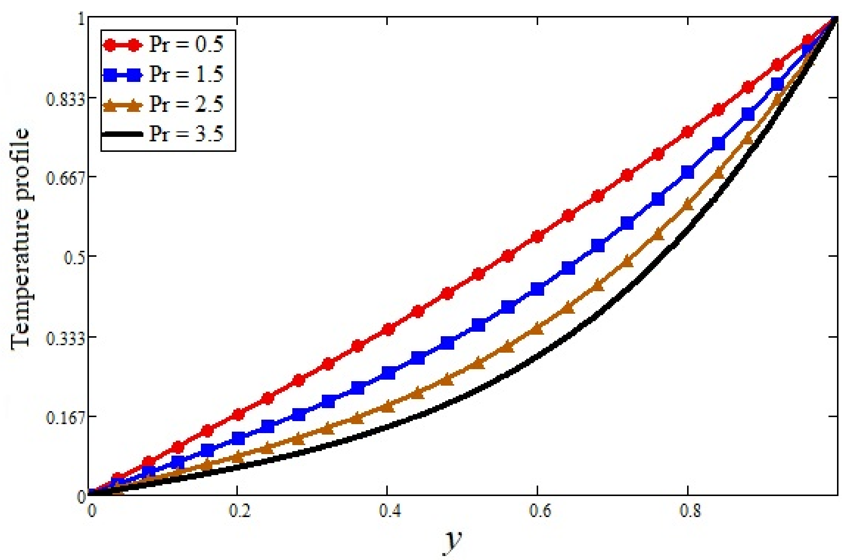

Figure 4: Variation of Pr on temperature

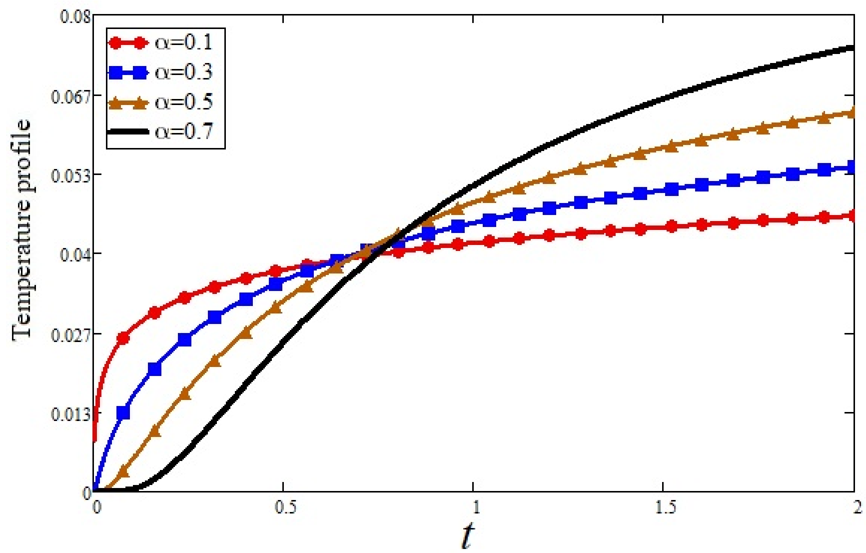

Figure 5: Variation of t for fractal parameter on temperature

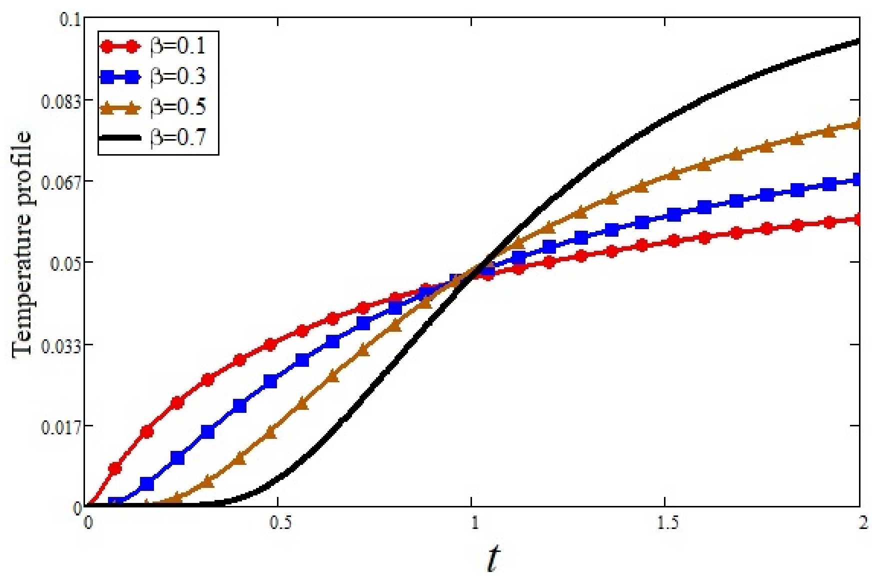

Figure 6: Variation of t for fractional parameter on temperature

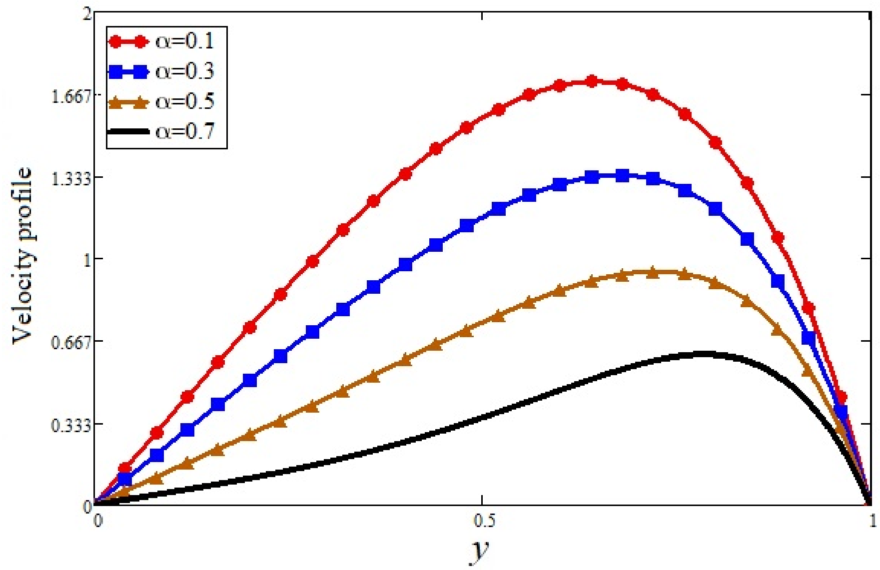

Figure 7: Variation of fractal parameter on velocity

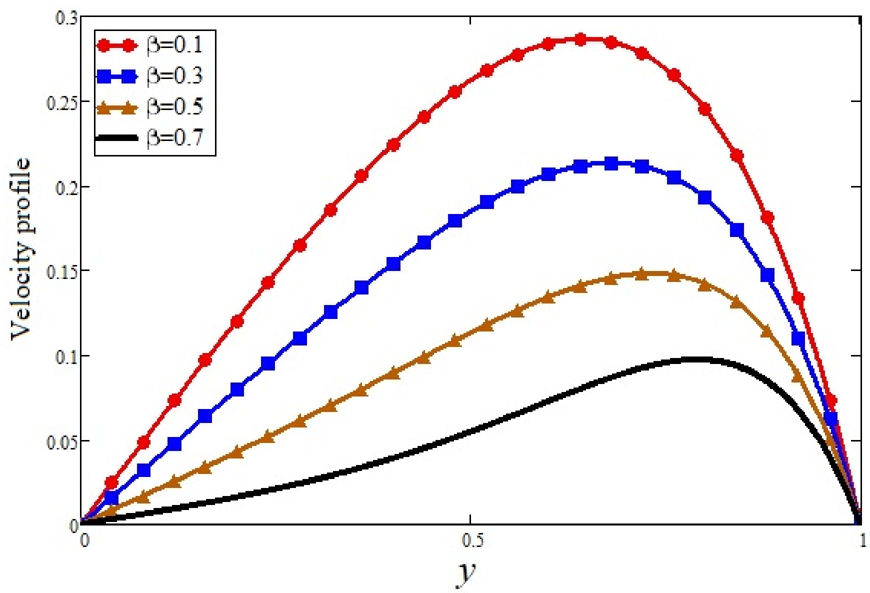

Figure 8: Variation of fractional parameter on velocity

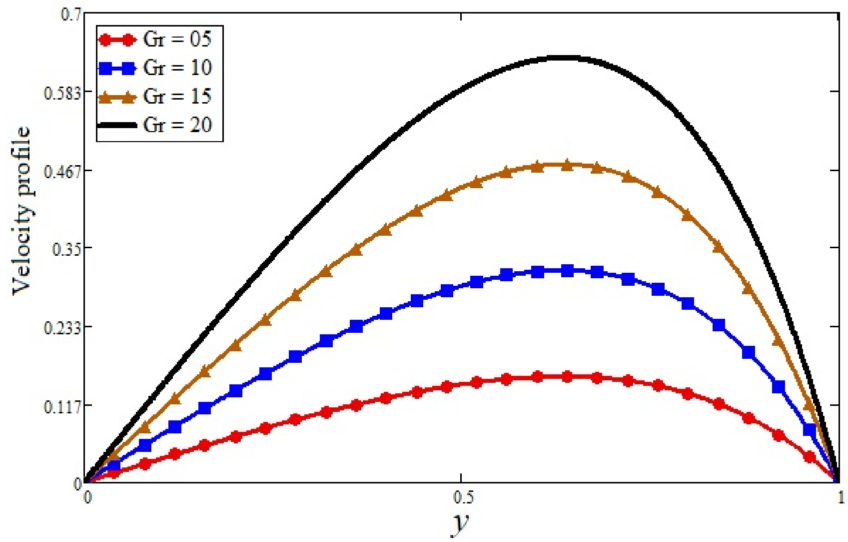

Figure 9: Variation of Gr on velocity

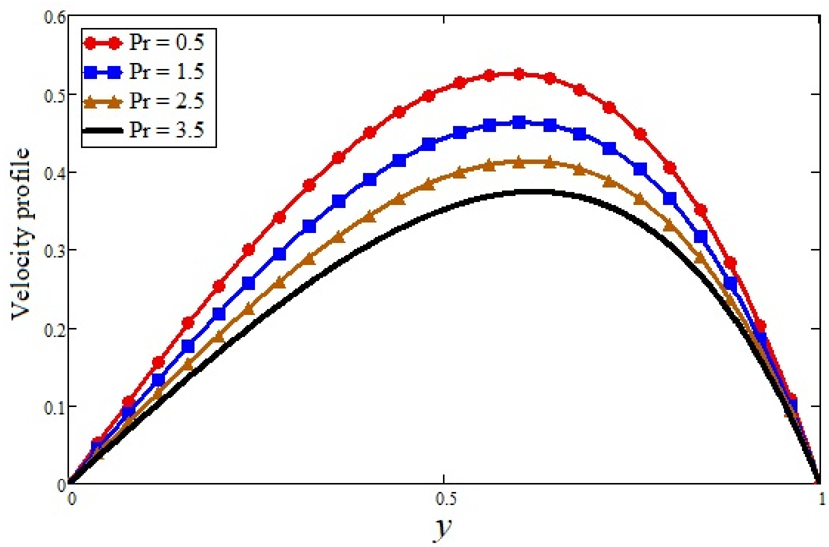

Figure 10: Variation of $\Pr$ on velocity

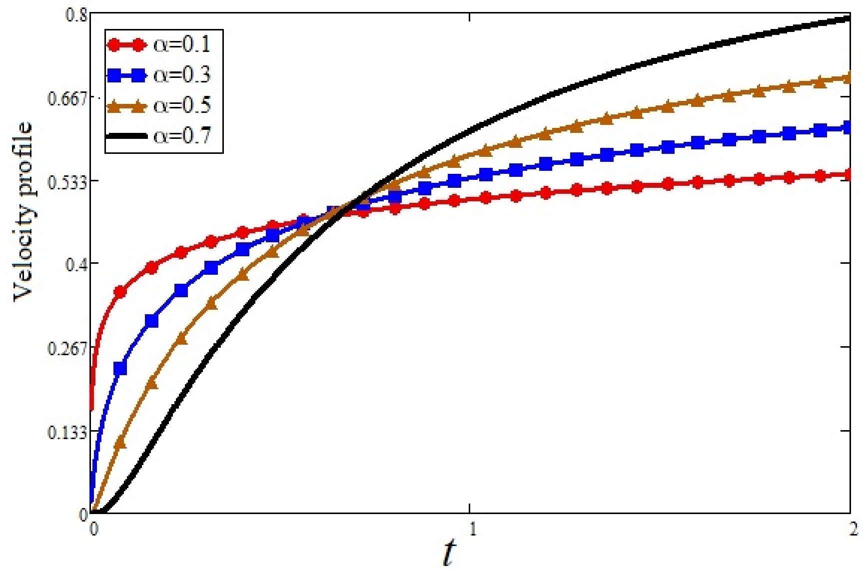

Figure 11: Variation of t for fractal parameter on velocity

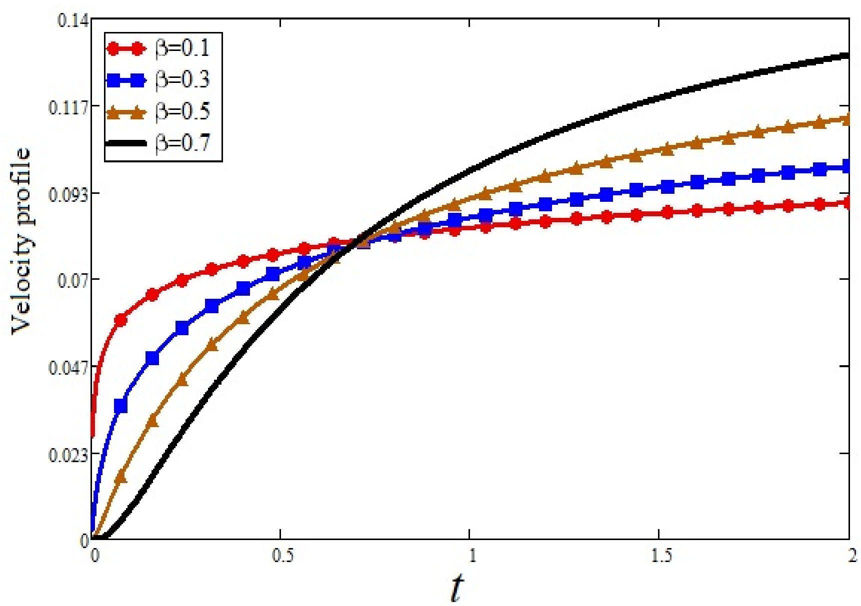

Figure 12: Variation of t for fractional parameter on velocity

5 Conclusion

This article is dealt with the new idea in fractional calculus that is fractal fractional derivative in fluid dynamic flow to the study of free convection flow of viscous Newtonian fluid in an upright infinite parallel plates channel. The main outcomes of this study are given below.

- Fractal fractional and fractional parameter highlight the decreasing effect for both velocity as well as temperature for the large value of fractal-fractional and fractional parameter.
- For the greater fractal-fractional parameter greater value and for the fractional parameter for temperature and velocity.
- The behaviour of temperature for the lower time is decreased and after critical point it is revised for both fractal-fractional and fractional parameters and the same behaviour is notified for velocity.
- For velocity the fractal-fractional parameter gives better results for the memory effect compare to the fractional parameter.

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Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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