

Proportional Fairness-Based Power Allocation Algorithm for Downlink NOMA 5G Wireless Networks

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Abstract: Non-orthogonal multiple access (NOMA) is one of the key 5G technology which can improve spectrum efficiency and increase the number of user connections by utilizing the resources in a non-orthogonal manner. NOMA allows multiple terminals to share the same resource unit at the same time. The receiver usually needs to configure successive interference cancellation (SIC). The receiver eliminates co-channel interference (CCI) between users and it can significantly improve the system throughput. In order to meet the demands of users and improve fairness among them, this paper proposes a new power allocation scheme. The objective is to maximize user fairness by deploying the least fairness in multiplexed users. However, the objective function obtained is non-convex which is converted into convex form by utilizing the optimal Karush-Kuhn-Tucker (KKT) constraints. Simulation results show that the proposed power allocation scheme gives better performance than the existing schemes which indicates the effectiveness of the proposed scheme.

Keywords: 5G, NOMA, user fairness, resource optimization, multiple access scheme.

1 Introduction

The popularity of multimedia applications places higher demands on future wireless networks. In addition, due to the rapid development of the Internet of Things, the number of terminal devices has also increased rapidly, and will soon exceed the capacity of the current system [Dai, Wang, Ding et al. (2018); Saraereh, Alsaraira, Khan et al. (2019); Lee, Patil, Hunt et al. (2019); Jameel, Risaniemi, Khan et al. (2019); Saraereh, Alsaraira, Khan et al. (2020)]. Therefore, the new generation of mobile communication systems (5G) needs to further expand the system capacity to meet the emerging new business [Alemaishat, Saraereh, Khan et al. (2019); Ding, Lei, George et al. (2017)].

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In the face of increasingly tight spectrum resources, traditional multiple access technology has got difficulties to meet the demand, and it is urgent to introduce a new type of multiple access technology. Non-orthogonal multiple access (NOMA) technology not only supports large-scale user access but also greatly improves spectrum efficiency, so it is considered as one of the key technologies of 5G [Alemaishat, Saraereh, Khan et al. (2019); Kim, Park and Hong (2019); Liu, Qin, Elkashlan et al. (2017)]. In the face of the needs of the new generation of wireless networks, traditional multiple access technologies have got difficulties to meet, especially in terms of system throughput and user rate experience [Bakht, Jameel, Ali et al. (2019); Jabeen, Ali, Khan et al. (2019); Islam, Ayazov, Dobre et al. (2017)]. Therefore, the industry proposes to adopt a new type of multiple access technology in 5G, namely nonorthogonal multiple access (NOMA). NOMA can improve system throughput and spectrum efficiency, and can also increase the access of system equipment by multiple times. In some 5G scenarios, such as uplink dense scenes and wide coverage multi-node access, power multiplexing is adopted. The NOMA has obvious performance advantages over traditional orthogonal multiple access and is more suitable for future system deployment.

Unlike traditional orthogonal multiple access transmission, NOMA introduces the power domain multiplexing concept with the help of continuous interference cancellation technology. Multiple users share the same resource block (such as the same frequency resource), and at the transmitting end, due to the nonorthogonal multiple access mode, the interference information is actively introduced [He, Xie, Xie et al. (2019)]. The user with good channel conditions allocates less power, and the user with poor channel conditions allocates larger power. Correct demodulation is achieved at the receiving end by successive interference cancellation (SIC) technology. Receiver complexity is increased compared to orthogonal transmission, but higher spectral efficiency can be achieved. The basic idea of nonorthogonal transmission is to use complex receiver design in exchange for higher spectral efficiency. With the enhancement of chip processing capability, the application of nonorthogonal transmission technology in practical systems will become possible.

Many scholars have studied the problem of NOMA power allocation and ensuring the fairness of cell edge users [Choi (2016); Cui, Ding and Fan (2016); Jiang, Tang, Gu et al. (2020)]. Otao et al. [Otao, Kishiyama and Higuchi (2012)] introduces the performance analysis of NOMA using the proportional fair method in resource allocation and introduces three power allocation methods: Iterative water injection method, fixed power distribution method, and fractional power distribution method. Seyama et al. [Seyama and Seki (2015)] introduce the user set selection method based on the proportional fair scheduling strategy, which can effectively reduce the computational complexity of user scheduling. Liu et al. [Liu, Onen and Petrova (2015, 2016)] introduce a power allocation method based on proportional fairness. The objective function makes the sum of the fairness factors of multiple users the largest. Wang et al. [Wang and Chen (2016)] conducted a method study to maximize the sum rate of two users under the condition that the total power is limited and the minimum rate of each user is satisfied. However, Otao et al. [Otao, Kishiyama and Higuchi (2012)] only considers the maximization of the proportional fairness factor and does not involve the user's Quality-of-Service (QoS) constraints; while Liu et al. [Liu, Onen and

Petrova (2015, 2016)] considers the maximization of the proportional fairness factor and the lowest data rate of the user when the channel condition of the cell edge user is very poor, the overall performance and rate are not as good as the traditional orthogonal access technology under the same conditions. Similarly, although Wang et al. [Wang and Chen (2016)] consider the user's QoS and rate maximization, the fairness of the edge users is guaranteed, but the system performance and rate are still lower than the traditional orthogonal access technology under the same conditions. Zhu et al. [Zhu, Wang, Huang et al. (2017)] proposed an optimal power allocation that has been characterized in closed or semi-closed forms and explicitly considered the power order constraints in power allocation problems and introduced the concept of the SIC-stability to avoid an equal power allocation on each channel. It jointly optimizes the channel assignment and power allocation by exploiting the matching algorithm along with the optimal power allocation. The algorithm has an impact and improving the system performance. However, it requires more power consumption and a large number of iterations to reach the optimal system performance. Timotheou et al. [Timotheou and Krikidis (2015)] proposed an algorithm to maximize the fairness among users in terms of data-rate under full CSI and outage probability under average CSI. Although the resulting problems are nonconvex, simple low-complexity algorithms are developed that provide the optimal solution. The results of fairness performance are approximately an order of magnitude better than TDMA in the considered configurations. However, it lacks to address the performance under SIC and unable to compare the performance with other state-of-the-art schemes and also the performance is lower than conventional OMA and OFDMA.

Therefore, the above research still cannot balance the relationship between user fairness and system data rate. To address this problem, this paper proposes a new downlink NOMA power allocation algorithm. NOMA adopts different power policies of different users, which may lead to unbalanced user communication quality, unfair problems, and affect service quality. This paper mainly studies the power allocation problem of nonorthogonal multiple access based on QoS and proposes a power allocation algorithm. The main goal is to multiplex multiple users in the same resource block on the downlink to ensure the fairness of users, that is, quality of service (QoS). The idea of this paper is to optimize the users with the worst proportional fairness while maximizing the overall rate of all users. The data rate of the users multiplexed in the sub-band is not less than the data rate of the orthogonal multiple access under the same constraint condition (the lowest fair rate is traded off). The simulation shows that the performance of the proposed method is better than the performance of Liu et al. [Liu, Onen and Petrova (2015)] and other existing schemes.

The rest of the paper is organized as follows. In Section 2, the system model is described. In Section 3, the proposed algorithms and their principle are analyzed. In Section 4, the solution of the optimal distribution factor is described. Section 5 provides the simulation results, while Section 6 concludes the paper.

2 System model

In the downlink cellular system, as shown in Fig. 1, there is a base station with M antennas and single-antenna users. The system is divided into N_{SC} sub-bands, the total bandwidth is W_T , and the bandwidth of each sub-band is $W_{SC} = W_T/N_{SC}$. Assume that the maximum number of multiplexed users is m_{max} in each sub-band and the base station transmits information $S_{m,n}$ to the m -th user ($m \in \{1, 2, 3, \dots, M\}$) of the n -th sub-band ($n \in \{1, 2, 3, \dots, N_{SC}\}$). The term $P_{m,n}$ is the power of the m -th user ($m \in \{1, 2, 3, \dots, M\}$) in the n -th sub-band ($n \in \{1, 2, 3, \dots, N_{SC}\}$), where $\sum_j^{N_{SC}} \sum_{i=1}^M P_{i,j} = P_T$, and P_T is the total transmitted power. Then in the n -th sub-band, the signal received by the m -th user is expressed as

$$\begin{aligned} \mathbf{y}_{m,n} &= \sum_{i=1}^{m_{max}} \sqrt{P_{i,n}} \mathbf{h}_{m,n} \mathbf{S}_{i,n} \\ &= \sqrt{P_{m,n}} \mathbf{h}_{m,n} \mathbf{S}_{m,n} + \sum_{i=1, i \neq m}^{m_{max}} \sqrt{P_{i,n}} \mathbf{h}_{m,n} \mathbf{S}_{i,n} \end{aligned} \quad (1)$$

where $\mathbf{h}_{m,n} = \mathbf{g}_{L,n} PL^{-1}(d)$ represents the channel parameter of the m -th user from the base station to the n -th sub-band, assuming $\mathbf{g}_{L,n}$ is the Rayleigh fading channel gain, $PL^{-1}(d)$ is the path loss, and d is the distance between the base station to the user, N_i represents the additive white Gaussian noise. The symbol $\mathbf{y}_{m,n}$ represents the reception information of the m -th user, $S_{m,n}$ represents the transmitted signal of the m -th user of the n -th sub-band whereas $S_{i,n}$ is the interfering signal from the i -th user of the n -th sub-band.

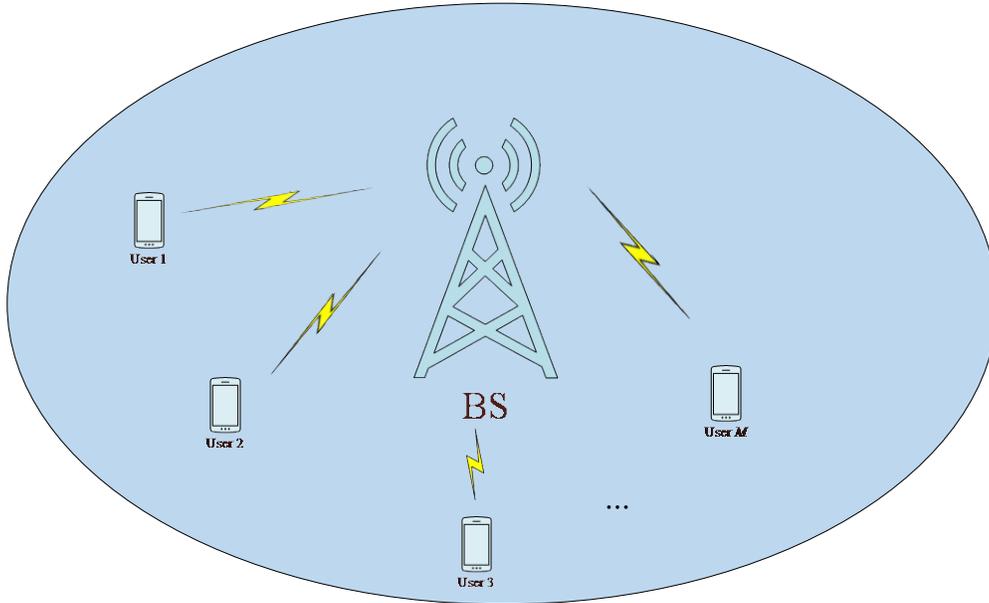


Figure 1: The system model of downlink NOMA

At the receiver, when the SIC technique is not used, the signal-to-interference-and-noise ratio (SINR) at the receiver is as shown in Eq. (2).

$$\begin{aligned} \gamma_{k,n}^{\text{SINR}} &= \frac{P_{k,n} |h_{k,n}|^2}{|h_{k,n}|^2 \sum_{i=1, i \neq k}^{m_{\max}} P_{i,n} + W_{\text{SC}} N_0} \\ &= \frac{P_{k,n} \varphi_{k,n}}{\varphi_{k,n} \sum_{i=1, i \neq k}^{m_{\max}} P_{i,n} + 1} \end{aligned} \quad (2)$$

where N_0 represents the power spectral density of the noise, $\varphi_{k,n} = \frac{|h_{k,n}|^2}{W_{\text{SC}} N_0}$ represents the channel response normalized by noise (CRNN). According to Shannon's equation, the throughput in the n -th sub-band can be calculated as

$$\begin{aligned} R_n &= W_{\text{SC}} \sum_{i=1}^{m_{\max}} \log_2(1 + \gamma_{k,n}^{\text{SINR}}) \\ &= W_{\text{SC}} \sum_{i=1}^{m_{\max}} \log_2 \left(1 + \frac{P_{k,n} \varphi_{k,n}}{I_{k,n} + 1} \right) \end{aligned} \quad (3)$$

where $I_{k,n} = \varphi_{k,n} \sum_{i=1, i \neq k}^{m_{\max}} P_{i,n}$ indicates that the user $m_{k,n}$ interferes with other users in the n -th sub-band.

At the receiving end, SIC technology [Wang and Chen (2016); Zhu, Wang, Huang et al. (2017); Timotheou and Krikidis (2015); Xia, Jiang and Wang (2019); Xia, Hu and Luo (2017)] is used. As shown in Fig. 2, this paper assumes that there are two users in the cell, m_1 and m_2 , of which m_1 has better channel conditions (internal cell users) and m_2 channel conditions are poor (edge cell users). That is, the user with the worst channel condition m_2 directly demodulates its own signal, and filters the interference of the user m_1 as noise. The user with good channel condition m_1 performs SIC, first demodulates the user m_2 , subtracts the received signal from the received signal m_2 , and then demodulates the signal. It is worth mentioning that, the same method is used for M number of users in the proposed study. That is if the user m_{M-1} needs to be demodulated, then user m_M and the maximum number of multiplexed users is m_{\max} .

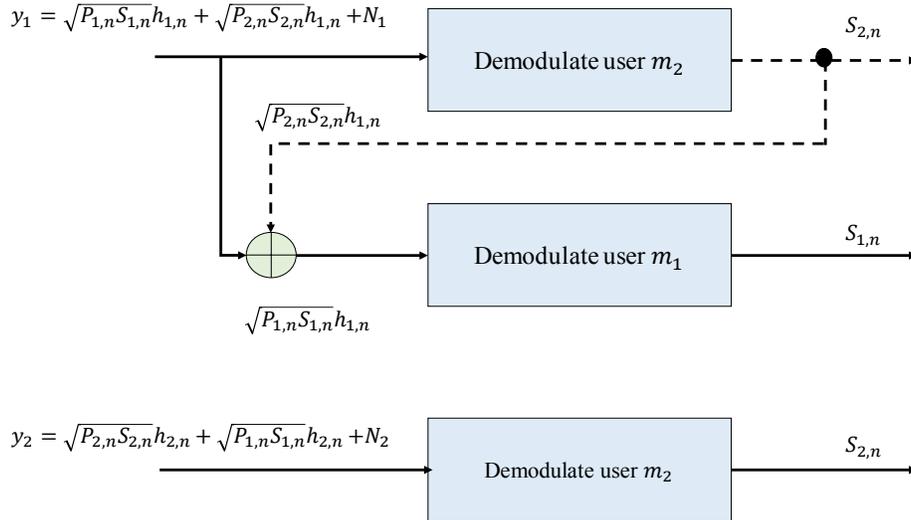


Figure 2: Illustration of the demodulation process using the SIC technique for two users

If more users are considered, assume that in the n -th channel, m_{max} users ($m_{max} \in \{1,2,3,\dots, M\}$) are multiplexed, and these users are arranged in descending order of CRNN, i.e., $\varphi_{1,n} > \varphi_{2,n} > \dots > \varphi_{m_{max}}$, after the SIC demodulation, the k -th user SINR is expressed as

$$\begin{aligned} \tilde{\gamma}_{k,n}^{\text{SINR}} &= \frac{P_{k,n} |\mathbf{h}_{k,n}|^2}{|\mathbf{h}_{k,n}|^2 \sum_{i=1}^{k-1} P_{i,n} + W_{\text{SC}} N_0} \\ &= \frac{P_{k,n} \varphi_{k,n}}{\varphi_{k,n} \sum_{i=1}^{k-1} P_{i,n} + 1} \end{aligned} \tag{4}$$

Therefore, the throughput of the user $m_{k,n}$ is

$$\begin{aligned} R_{k,n} &= W_{\text{SC}} \log_2(1 + \tilde{\gamma}_{k,n}^{\text{SINR}}) \\ &= W_{\text{SC}} \log_2 \left(1 + \frac{P_{k,n} \varphi_{k,n}}{\varphi_{k,n} \sum_{i=1}^{k-1} P_{i,n} + 1} \right) \end{aligned} \tag{5}$$

The overall throughput of the system is

$$R = \sum_{j=1}^{N_{\text{SC}}} \sum_{i=1}^{m_{max}} R_{i,j} \tag{6}$$

3 Proportional fair power allocation

3.1 Generalized user minimum data rate constraints

Suppose there are two users in the n -th sub-band, $m_{1,n}$ (user 1) has better channel conditions, $m_{2,n}$ (user 2) has poor channel conditions (cell edge users), i.e., $\varphi_{1,n} > \varphi_{2,n}$. $P_{\text{SC}} = \frac{P_T}{N_{\text{SC}}}$ denotes the transmit power of each sub-band, assuming a power distribution factor of α is assigned to $m_{1,n}$, i.e., $P_{1,m} = \alpha P_{\text{SC}}$, then, $P_{2,m} = (1 - \alpha) P_{\text{SC}}$. Considering the fact that the NOMA system should have a larger improvement in spectrum efficiency and data rate than the OMA system, so it is assumed that the data rates of $m_{1,n}$ and $m_{2,n}$ are not less than the data rate of OMA under the same condition [Fang, Zhang, Chaeng (2016)], as shown in Fig. 3, that is, $R_{1,n}^{\text{NOMA}} \geq R_{1,n}^{\text{OMA}}$; $R_{2,n}^{\text{NOMA}} \geq R_{2,n}^{\text{OMA}}$. It is also clear from the depiction of Fig. 3a that the NOMA scheme uses the same sub-band but the users are allocated a different power whereas, in the OMA scheme in Fig. 3b, the users are allocated a half-the-bandwidth but with full power which makes it ineffective than the NOMA scheme.

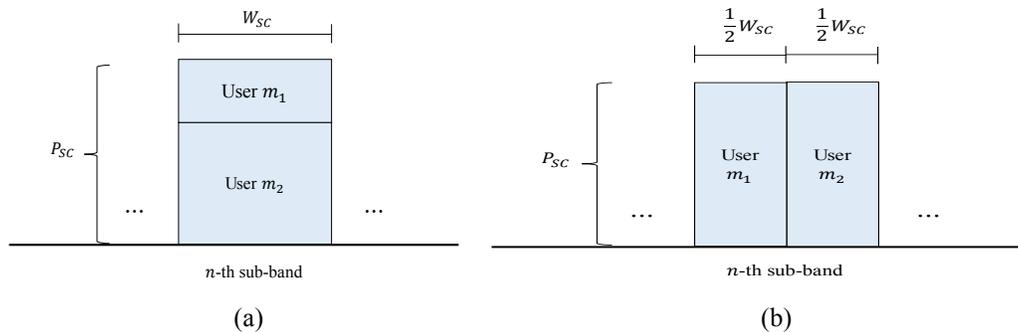


Figure 3: Relationships under the same resource conditions of NOMA and OMA schemes. (a) NOMA; (b) OMA

Mathematically, the relationship between the NOMA and OMA from Figs. 3(a) and 3(b) can be expressed as

$$\begin{cases} W_{SC} \log_2 \left(1 + \frac{\alpha P_{SC} |h_1|^2}{N_0 W_{SC}} \right) \geq \frac{1}{2} W_{SC} \log_2 \left(1 + \frac{P_{SC} |h_1|^2}{\frac{1}{2} W_{SC} N_0} \right) \\ W_{SC} \log_2 \left(1 + \frac{(1-\alpha) P_{SC} |h_2|^2}{N_0 W_{SC} + \alpha P_{SC} |h_2|^2} \right) \geq \frac{1}{2} W_{SC} \log_2 \left(1 + \frac{P_{SC} |h_2|^2}{\frac{1}{2} W_{SC} N_0} \right) \end{cases} \quad (7)$$

To obtain the range of the power distribution factor α , we solve Eq. (1) in terms of Eq. (7) and get the value range of α as follows

$$\frac{\sqrt{1+2\theta_1}-1}{\theta_1} \leq \alpha \leq \frac{\sqrt{1+2\theta_2}(1+\theta_2-\sqrt{1+2\theta_2})}{\theta_2(1+2\theta_2)} \quad (8)$$

where, $\theta_1 = \frac{P_{SC} |h_1|^2}{N_0 W_{SC}}$, $\theta_2 = \frac{P_{SC} |h_2|^2}{N_0 W_{SC}}$, assume $\alpha_{\max} = \frac{\sqrt{1+2\theta_2}(1+\theta_2-\sqrt{1+2\theta_2})}{\theta_2(1+2\theta_2)}$, $\alpha_{\min} = \frac{\sqrt{1+2\theta_1}-1}{\theta_1}$, that is, $\alpha_{\min} \leq \alpha \leq \alpha_{\max}$. According to the concept of NOMA, users with good channel conditions allocate less power, and users with poor channel conditions allocate larger power. Therefore, two $\alpha_{\max} < \frac{1}{2}$ and $\alpha_{\min} \leq \alpha_{\max}$ conditions are met simultaneously.

The above is a constraint in the case of multiplexing two users in a sub-band. It is assumed that $\alpha_1, \alpha_2, \dots, \alpha_k, \dots, \alpha_m$ represent the power allocation factors of these m users, respectively. The constraint is

$$r_1^{\text{NOMA}} \geq r_1^{\text{OMA}}, r_2^{\text{NOMA}} \geq r_2^{\text{OMA}}, \dots, r_k^{\text{NOMA}} \geq r_k^{\text{OMA}}, \dots, r_m^{\text{NOMA}} \geq r_m^{\text{OMA}} \quad (9)$$

The following constraints can be obtained by calculation using the methods of Alemaishat et al. [Alemaishat, Saraereh, Khan et al. (2019); Ding, Lei, George et al. (2017); Alemaishat, Saraereh, Khan et al. (2019); Kim, Park, Hong et al. (2019); Liu, Qin, Elkashlan et al. (2017)].

$$\begin{cases} \alpha_1 \geq \frac{(1+m\theta_1)^{\frac{1}{m}}-1}{\theta_1} \\ \alpha_2 \geq \frac{\left((1+m\theta_2)^{\frac{1}{m}}-1 \right) (1+\alpha_1\theta_2)}{\theta_2} \\ \vdots \\ \alpha_k \geq \frac{\left((1+m\theta_k)^{\frac{1}{m}}-1 \right) (1+(\alpha_1+\alpha_2+\dots+\alpha_{k-1})\theta_k)}{\theta_k} \\ \vdots \\ \alpha_{m-1} \geq \frac{\left((1+m\theta_{m-1})^{\frac{1}{m}}-1 \right) (1+(\alpha_1+\alpha_2+\dots+\alpha_{m-1})\theta_{m-1})}{\theta_{m-1}} \\ \alpha_1 + \alpha_2 + \dots + \alpha_{m-1} \leq \frac{\theta_m - \left((1+m\theta_m)^{\frac{1}{m}}-1 \right)}{\theta_m (1+m\theta_m)^{\frac{1}{m}}} \end{cases} \quad (10)$$

where $\theta_1, \theta_2, \dots, \theta_m$ represents $\frac{P_{SC} |h_1|^2}{N_0 W_{SC}}, \frac{P_{SC} |h_2|^2}{N_0 W_{SC}}, \dots, \frac{P_{SC} |h_m|^2}{N_0 W_{SC}}$, $\alpha_m = 1 - (\alpha_1 + \alpha_2 + \dots + \alpha_{m-1})$. According to the NOMA theory, it is also necessary to satisfy $\alpha_1 <$

$\alpha_2 < \dots < \alpha_m$. The solution of the power distribution coefficient that satisfies this condition can be obtained according to the actual situation.

3.2 Proportional fairness power allocation algorithm

Proportional fairness [Choi (2016); Cui, Ding and Fan (2016); Otao, Kishiyama and Higuchi (2012); Seyama and Seki (2015)] has been proven to maximize the logarithm of user throughput and, therefore, to ensure user spectrum efficiency and user fairness. A lot of references have adopted this method to allocate user power and user scheduling. Proportional fairness takes into account instantaneous user data and average user data rate, which is defined as follows

$$T_m(t+1) = \left(1 - \frac{1}{t_c}\right)T_m(t) + \frac{1}{t_c} \sum_{n=1}^{N_{SC}} x_{m,n}(t)R_{m,n}(t) \quad (11)$$

$n = 1, 2, \dots, N_{SC}; m = 1, 2, \dots, M$

where $T_m(t)$ represent the duration of time slot of the m -th user at t -th time slot, t_c refers to the average window length; $R_{m,n}(t)$ means at the t -th frame time, instantaneous data rate of the m -th user in the n -th sub-band; $x_{m,n}(t)$ represents the 0-1 sequence scheduled by the user, if the user m is at the t -th frame time, the n -th sub-band is scheduled, then, $x_{m,n}(t) = 1$; otherwise, $x_{m,n}(t) = 0$.

The proportional fair scheduling strategy is to maximize the logarithm of the average data rate over a period of time, which is equivalent to the geometric mean rate. To achieve this goal, user scheduling and power allocation need to maximize the criteria of (12) as follows.

$$\prod_{m=1}^M \left[1 + \sum_{n=1}^{N_{SC}} \frac{x_{m,n}(t)R_{m,n}(t)}{(t_c-1)T_m(t)}\right] \quad (12)$$

Eq. (12) clearly indicates that user fairness is increased by taking the geometric mean of the average data rate of the users over a certain time frame. This obviously maximizes fairness among users, where $t_c - 1$ in Eq. (12) can be approximated as

$$\left[1 + \frac{1}{t_c-1} \sum_{n=1}^{N_{SC}} \sum_{m=1}^M x_{m,n}(t) \left[\frac{R_{m,n}(t)}{T_m(t)}\right]\right] \quad (13)$$

It can be seen from Eq. (13) that $\frac{R_{m,n}(t)}{T_m(t)}$ is a factor of fairness selection and user fairness is independent of the average window length t_c , which results in improved user fairness. Although references [Choi (2016); Cui, Ding and Fan (2016); Otao, Kishiyama and Higuchi (2012); Seyama and Seki (2015)] maximizes the sum of sub-band multiplexed user fairness factors, it does not mostly consider the fairness of poor users. Therefore, this paper proposes a new power allocation algorithm for this problem, that is, selecting the user with poor fairness factor in the sub-band as the optimization target, so that the user has the best fairness. Due to the guarantee of fairness, it is necessary to sacrifice the data rate, which will inevitably reduce the fairness. In this paper, the OMA data rate under the same conditions is introduced as the trade-off rate of the guaranteed NOMA system, that is, the best user is guaranteed. Under the fairness condition, the user's data rate is also not lower than the OMA data rate under the same conditions, achieving a trade-off between fairness and data rate. In other words, the optimum user fairness is obtained by selecting

the maximum value of the average data rate over a certain time period of the users. The analytical relation is expressed in Eq. (14).

$$\operatorname{argmax}_{\alpha} \left\{ \frac{R_{1,n}(t)}{T_1(t)}, \frac{R_{2,n}(t)}{T_2(t)}, \dots, \frac{R(t)_{Mmax}}{T(t)_{Mmax}} \right\} \quad (14)$$

The constraint in Eq. (14) gives a detailed explanation in Eq. (10).

4 The solution of optimal distribution factor

Suppose there are two users multiplexed in a sub-band by NOMA. For the sake of analysis, the time t and sub-band n in Eq. (14) is omitted, and the optimization expression is

$$\operatorname{argmax}_{\alpha} \left\{ \frac{R_1}{T_1}, \frac{R_2}{T_2} \right\} \quad (15)$$

Subject to: $\alpha \max_{min}$

Since Eq. (15) is nonconvex, since it requires supplementary elements such as the power distribution factor value of each user, average data rate of each user and exact time duration, so it is difficult to solve. Therefore, this paper considers an approximate method. For a larger number X , satisfying $X > 0$, the following approximation can be made.

$$X^{-\min_m \left\{ \frac{R_1}{T_1}, \frac{R_2}{T_2}, \dots, \frac{R_m}{T_m} \right\}} \approx \sum_{i=1}^m X^{T_i} \quad (16)$$

Since X^{-k} is monotonically decreasing for $k \geq 0$, the optimization method in Eq. (15) can be equivalent to

$$\operatorname{argmin}_{\alpha} \left\{ X^{-\frac{R_1}{T_1}} + X^{-\frac{R_2}{T_2}} \right\} \quad (17)$$

Subject to: $\alpha \max_{min}$

The objective function of Eq. (17) can be further simplified as

$$\begin{aligned} f(\alpha) &= \left\{ X^{-\frac{R_1}{T_1}} + X^{-\frac{R_2}{T_2}} \right\} \\ &= X^{-\frac{W_{SC}}{T_1} \ln(1+\alpha\theta_1)} + X^{-\frac{W_{SC}}{T_2} \ln\left(\frac{1+\theta_2}{1+\alpha\theta_2}\right)} \\ &= (1 + \alpha\theta_1)^{-n_1} + m(1 + \alpha\theta_2)^{n_2} \end{aligned} \quad (18)$$

where, $X > 0$, $n_1 = \frac{W_{SC}}{T_1} \ln X$, $n_2 = \frac{W_{SC}}{T_2} \ln X$, $m = X^{-\frac{W_{SC}}{T_2} \ln(1+\theta_2)}$.

Now take the second-order of the objective function $f(\alpha)$, we get

$$f''(\alpha) = n_1(n_1 + 1)(1 + \alpha\theta_1)^{-n_1-2} + m(n_2 - 1)n_2(1 + \alpha\theta_2)^{-n_2-2} \quad (19)$$

In solving the problem of Eq. (17), this paper uses the optimal Karush-Kuhn-Tucker (KKT) condition [Fang, Zhang and Chaeng (2016); Benjebbour, Li and Saito (2013); Saito, Benjebbour and Kishiyama (2013); Al-Abbasi and Daniel (2015)] for solving the inequality constraint. First, define the Lagrangian expression as

$$\begin{aligned} L(\alpha, \lambda) &= f(\alpha) + \lambda_1(\alpha_{min} - \alpha) + \lambda_2(\alpha - \alpha_{max}) \\ &= X^{-\frac{W_{SC}}{T_1} \ln(1+\alpha\theta_1)} + X^{-\frac{W_{SC}}{T_2} \ln\left(\frac{1+\theta_2}{1+\alpha\theta_2}\right)} \end{aligned}$$

$$+\lambda_1 \left(\frac{\sqrt{(1+2\theta_1)}-1}{\theta_1} - \alpha \right) + \lambda_2 \left(\alpha - \frac{\sqrt{(1+2\theta_2)}(1+\theta_2-\sqrt{(1+2\theta_2)})}{\theta_2(1+2\theta_2)} \right) \quad (20)$$

The established KKT conditions are

$$\begin{aligned} \frac{\partial L(\alpha, \lambda)}{\partial \alpha} &= \left(\frac{W_{SC}}{T} \ln X \right) \theta_1 (1 + \alpha \theta_1)^{-\frac{W_{SC}}{T_1} \ln X - 1} \\ &+ \left(\frac{W_{SC}}{T_2} \ln X \right) X^{-\frac{W_{SC}}{T_2} \ln(1+\theta_2)} \theta_2 (1 + \alpha \theta_2)^{\frac{W_{SC}}{T_2} \ln X - 1} - \lambda_1 + \lambda_2 = 0 \end{aligned} \quad (21)$$

$$\lambda_1 \geq 0, \lambda_2 \geq 0 \quad (22)$$

$$\lambda_1 \left(\frac{\sqrt{(1+2\theta_1)}-1}{\theta_1} - \alpha \right) = 0 \quad (23)$$

$$\lambda_2 \left(\alpha - \frac{\sqrt{(1+2\theta_2)}(1+\theta_2-\sqrt{(1+2\theta_2)})}{\theta_2(1+2\theta_2)} \right) = 0 \quad (24)$$

$$\frac{\sqrt{(1+2\theta_1)}-1}{\theta_1} \leq \alpha \leq \frac{\sqrt{(1+2\theta_2)}(1+\theta_2-\sqrt{(1+2\theta_2)})}{\theta_2(1+2\theta_2)} \quad (25)$$

For the above KKT constraints, the following four conditions are discussed.

1) When $\lambda_1 > 0, \lambda_2 > 0$, Eqs. (22) and (23) hold simultaneously, that is, α is equal to the lower bound and the upper bound at the same time. This situation does not hold in reality; therefore, this condition is excluded.

2) When $\lambda_1 > 0, \lambda_2 = 0$, at this time, according to Eqs. (21) and (23), $\alpha^* = \alpha_{min}$, since $\lambda_1 > 0$, is satisfied, therefore, in Eq. (21) when $f'(\alpha^*) > 0$, i.e., $f'(\alpha_{min}) > 0$, the optimal value $\alpha^* = \alpha_{min}$ can be obtained. It can also be explained by a one-dimensional continuous function derivative: Since the second-order derivative $f''(\alpha) > 0$ of the objective function $f(\alpha)$ is constant, then $f'(\alpha)$ is a monotonically increasing function. According to the analysis condition, when $\lambda_1 > 0, f'(\alpha_{min}) > 0$, indicating that $f''(\alpha) > 0$, then $f(\alpha)$ is a monotonically increasing function, f (the minimum value of α) is obtained at α_{min} , so the optimal value of the objective function at $f'(\alpha_{min}) > 0$ is at $\alpha^* = \alpha_{min}$.

3) When $\lambda_1 = 0, \lambda_2 > 0$, according to Eq. (24), $\alpha^* = \alpha_{max}$, when the α_{max} is substituted into Eq. (21), the solution is $\lambda_2 < 0$, which contradicts the hypothesis, therefore, ignore this optimal value.

4) When $\lambda_1 = 0, \lambda_2 = 0$, solving Eq. (21), $\left(\frac{W_{SC}}{T} \ln X \right) \theta_1 (1 + \alpha \theta_1)^{-\frac{W_{SC}}{T_1} \ln X - 1} + \left(\frac{W_{SC}}{T_2} \ln X \right) X^{-\frac{W_{SC}}{T_2} \ln(1+\theta_2)} \theta_2 (1 + \alpha \theta_2)^{\frac{W_{SC}}{T_2} \ln X - 1} = 0$, the solution of the equation can be solved by MATLAB software, assuming the solution at this time for α_{opt} , since it is true by calculating $f'(\alpha_{max}) > 0$, only the relationship between α_{opt} and α_{min} needs to be considered. If $\alpha_{opt} > \alpha_{min}$, then $\alpha^* = \alpha_{opt}$; if $\alpha_{opt} \leq \alpha_{min}$, then $\alpha^* = \alpha_{min}$.

5 Simulation results and performance analysis

In this paper, the proposed power allocation algorithm is tested and simulated by MATLAB software. The channel condition is selected by the Rayleigh fading channel. The base station coverage is within 500 m, and the user is 50 m away from the base station. Considering the performance conditions of the receiver, between the two users

with a distance of at least 40 m, this simulation compares the two user data rates in one of the sub-bands of the base station. $N_0 = -174$ dBm, $W_{SC}=12$ Mbit/s, in order to compare the results of the simulation, this paper defines the fairness criterion of the user with the worst channel condition, that is, the data rate of the edge user accounts for the percentage of the system rate.

Fairness of users with the worst channel conditions

$$= \frac{\text{The data rate of the user with the worst channel conditions}}{\text{System rate}}$$

In Condition 2 of Section 4, KKT conditional analysis, i.e., when $\alpha^* = \alpha_{min}$ (both users are within 200 m from the base station), the NOMA and OMA data rates as shown in Fig. 4 and Fig. 5 are obtained. Comparing from Fig. 4, it can be seen that the rate of NOMA User 1 is just the rate of OMA under the same conditions. At this time, the rate of NOMA User 2 is greatly improved compared with OMA, indicating that NOMA can well overcome the cell problem of poor quality of edge users. When User 2 is not at the cell edge, the proposed algorithm is also compared with Liu et al. [Liu, Onen and Petrova (2015)] algorithm (the method for m_2 fairness of users with poor channel conditions: User 1 with better channel conditions). Since the rate of User 1 in Liu et al. [Liu, Onen and Petrova (2015)] is set by itself, the proposed study sets this value to be not less than the rate of OMA under the same conditions. In Fig. 5, the proposed algorithm achieves a better sum rate as compared with OMA and Liu et al. [Liu, Onen and Petrova (2015)]. Moreover, the rate of the edge user is much larger obtained by the proposed algorithm than the result in Liu et al. [Liu, Onen and Petrova (2015)]. At the same time, this paper averages the fairness of the users with the worst channel conditions at different transmit powers. The fairness of users with the worst channel conditions using the proposed scheme is 45.39%, and the fairness of users in Choi [Choi (2016)] is 37.11%. Therefore, it is proved that the proposed algorithm is superior to Choi [Choi (2016)] algorithm in fairness.

In Condition 4 of the analysis under KKT conditions in Section 4, when the two user channel conditions are very different (such as one close to the base station and the other at the cell edge). At this time, for each transmits power P , α_{opt} solved by MATLAB is always in the range of $[\alpha_{min} \leq \alpha \leq \alpha_{max}]$, that is, $\alpha^* = \alpha_{opt}$. As can be seen from Fig. 6, the rate increase of NOMA User 1 is relatively slow, and the growth rate of NOMA User 2 is relatively large. This is because the value of α_{opt} decreases as the power increases, resulting in User 1 has a slower growth rate, while User 2 has a power allocation factor of $1 - \alpha_{opt}$, so User 2 is growing faster than User 1. Compared with the data obtained in Liu et al. [Liu, Onen and Petrova (2015)], it is found that when User 2 is at the cell edge, the sum rate of the system is lower than that in condition 1, because the User m_2 is closer to the cell edge. The channel conditions are poor, so the rate of NOMA User 2, the rate of OMA User 2, and the rate of Liu et al. [Liu, Onen and Petrova (2015)] User 2 are both reduced. Compared with Fig. 4, the proposed NOMA power allocation algorithm increases the User 1 data rate, but the User m_2 (cell edge user) and the system's sum-rate decreases, that is, the fairness is relatively reduced, but the OMA user rate of m_2 (cell edge users) is also greatly reduced, so that the constraints of this paper (the rate of

NOMA User 2 is not less than the rate of OMA User 2) are still satisfied, thus verifying the correctness of the results. In Liu et al. [Liu, Onen and Petrova (2015)] algorithm, due to the limitation of the User m_1 rate, although the edge user has a large transmission power, the data rate is not improved due to poor channel conditions and large loss. It can be seen visually from Fig. 6 that the data rate of edge users in using the proposed scheme is always higher than the data rate in Liu et al. [Liu, Onen and Petrova (2015)].

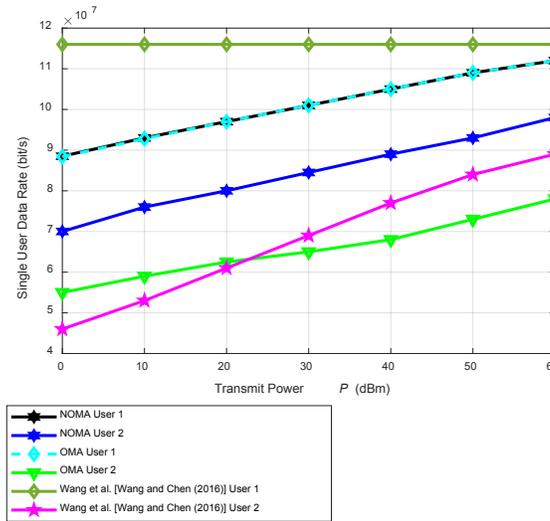


Figure 4: Comparison of the single user data rates of the algorithms at different transmit powers when $\lambda_1 > 0, \lambda_2 = 0$

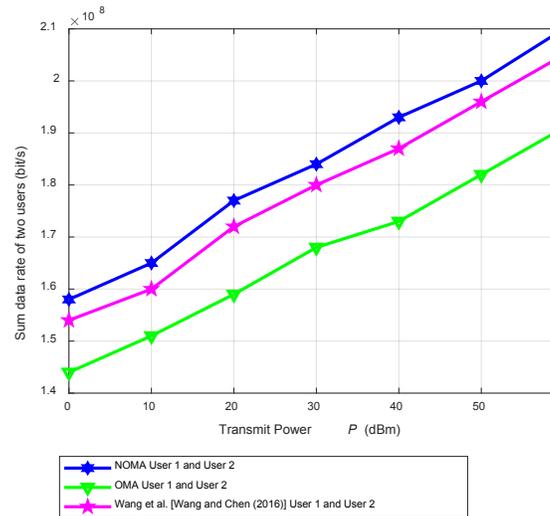


Figure 5: Comparison of the sum data rates of the two users of the algorithms at different transmit powers when $\lambda_1 > 0, \lambda_2 = 0$

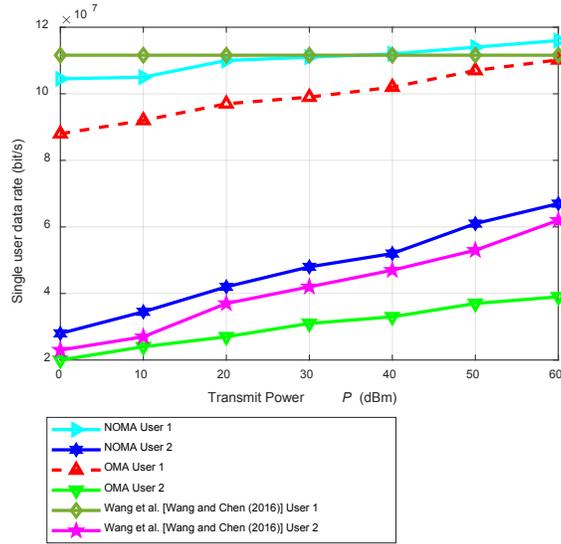


Figure 6: Comparison of the single user data rates of algorithms at different transmit powers when $\lambda_1 = 0, \lambda_2 = 0, \alpha^* = \alpha_{opt}$

In Fig. 7, the difference between the sum rates of the two is not large. At the same time, the fairness of the user with the worst channel conditions at different transmit powers is averaged. The fairness of the user with the worst channel conditions is 26.76% in Liu et al. [Liu, Onen and Petrova (2015)] and the worst user fairness of channel conditions is 29.27% by the proposed algorithm, which confirms that the proposed algorithm is superior to the method of Liu et al. [Liu, Onen and Petrova (2015)] in fairness.

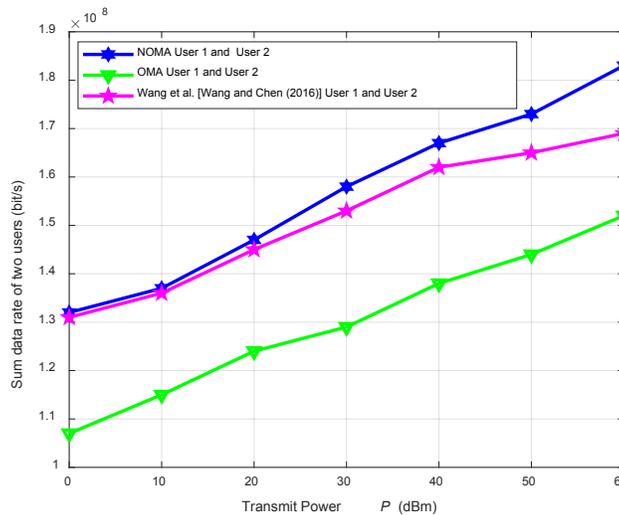


Figure 7: Relationship between the sum data rate of two users of algorithms under different transmit powers when $\lambda_1 = 0, \lambda_2 = 0, \alpha^* = \alpha_{opt}$

In Condition 4 of KKT analysis, there is another case. When the difference between channel conditions of the two users is small, the α_{opt} value may decrease $\alpha_{\text{opt}} < \alpha_{\text{min}}$ as the transmission power increases. In this case, the optimal value is α_{min} . The simulation results are shown in Fig. 8 when the transmit power is greater than 44 dBm, it is the same as in Condition 2 in the KKT condition analysis. Compared with Liu et al. [Liu, Onen and Petrova (2015)], it can be seen from Fig. 8 that the data rate of User 2 of the proposed algorithm is greater than the rate of User 2 in Liu et al. [Liu, Onen and Petrova (2015)]. At the same time, the fairness of the user with the worst channel conditions at different transmit powers is averaged. The fairness of the users with the worst channel conditions of the proposed algorithm is 35.86%, and the fairness of users with the worst channel conditions in Liu et al. [Liu, Onen and Petrova (2015)] is 27.59%, which confirms that the proposed algorithm is superior to Liu et al. [Liu, Onen and Petrova (2015)] in fairness. Fig. 9 compares the data rate of two users against different values of transmit power. As can be seen from Fig. 9, the data rate performance of the proposed algorithm is better than OMA and Liu et al. [Liu, Onen and Petrova (2015)] algorithm for each value of transmit power which makes it clear that the proposed algorithm has better performance.

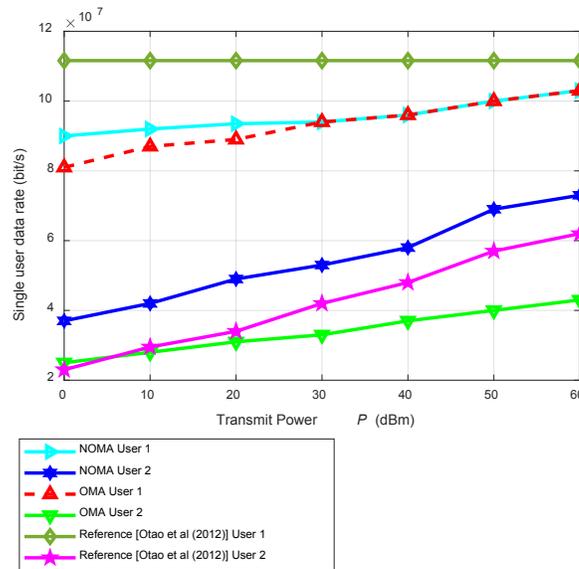


Figure 8: Relationship between single-user data rates of algorithms under different transmit powers when $\lambda_1 = 0, \lambda_2 = 0$, and $\alpha^* = \alpha_{\text{min}}$

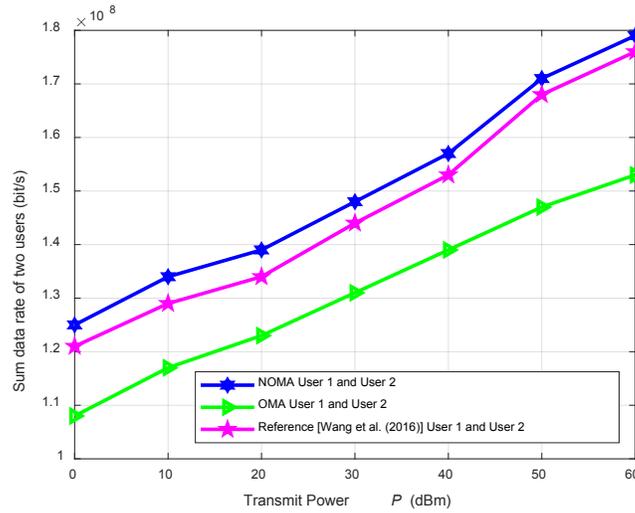


Figure 9: Comparison of the sum data rate of algorithms for two users against different values of transmit power when $\lambda_1 = 0, \lambda_2 = 0$, and $\alpha^* = \alpha_{min}$

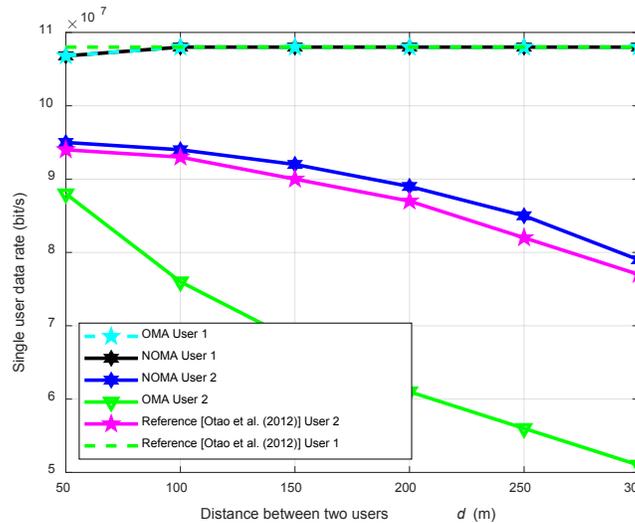


Figure 10: Comparison of the single user data rates of algorithms under the distance between the two users

For the relationship between the distance between the two users and the data rate, it is assumed that the position of User 1 is unchanged, and the position of User 2 is changed. As the distance increases, User 2 is getting closer to the edge of the cell, so as to ensure the value of User 1 in the reference is set to be the same as the data in this paper. As can be seen from Fig. 10 and Fig. 11, the rate is declining, and the decline of the data rate of NOMA User 2 is slower than that of OMA User 2. It can be seen from Fig. 10 that the rate of User 2 calculated by the proposed algorithm is larger than the rate of User 2 in Liu

et al. [Liu, Onen and Petrova (2015)], which proves once again that the proposed algorithm is superior to the reference [Liu, Onen and Petrova (2015)] algorithm in the fairness of users.

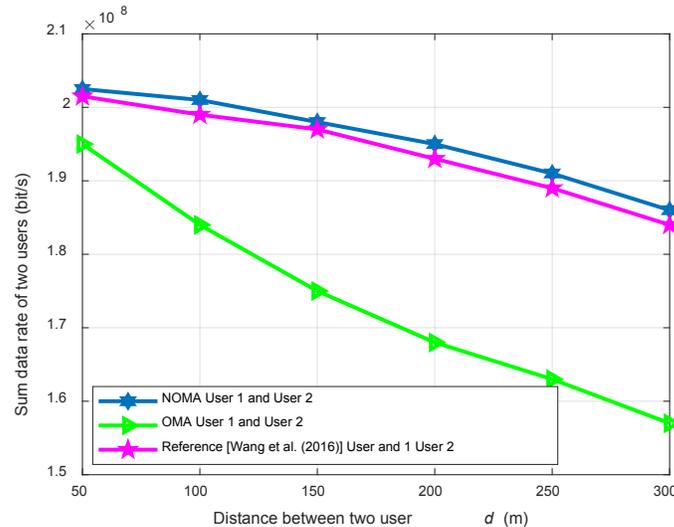


Figure 11: Comparison of the sum data rates of two users for different algorithms under the distance between the two users

To further elaborate on the effectiveness of the proposed study, Fig. 12 compares the spectral efficiency of the proposed algorithm and Zhu et al. [Zhu, Wang, Huang et al. (2017); Timotheou and Krikidis (2015)] algorithms with an increasing number of users. As can be seen from the results of Fig. 12, the spectral efficiency of the proposed algorithm is better than Zhu et al. [Zhu, Wang, Huang et al. (2017); Timotheou and Krikidis (2015)] algorithms, whereas, the spectral efficiency performance of Zhu et al. [Zhu, Wang, Huang et al. (2017)] is better than Timotheou et al. [Timotheou and Krikidis (2015)] algorithm. The results clearly indicate the superiority of the proposed scheme over the existing schemes, especially, in large-number of user scenarios.

To evaluate the relative effectiveness of the proposed study from the perspective of energy efficiency, Fig. 13 compares the energy efficiency versus the number of users. It can be seen from Fig. 13 that; the energy efficiency of the proposed algorithm is better than Zhu et al. [Zhu, Wang, Huang et al. (2017); Timotheou and Krikidis (2015)] for every number of users. It is also revealed from these results that the energy efficiency gap of the proposed and Zhu et al. [Zhu, Wang, Huang et al. (2017); Timotheou and Krikidis (2015)] algorithms is large, which means that the proposed algorithm is more energy-efficient than the existing schemes and requires less amount of energy for data transmission.

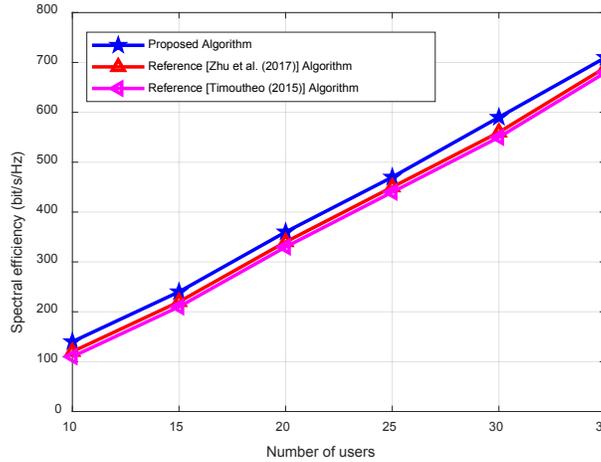


Figure 12: Comparison of the spectral efficiency of the proposed and existing algorithms with an increasing number of users

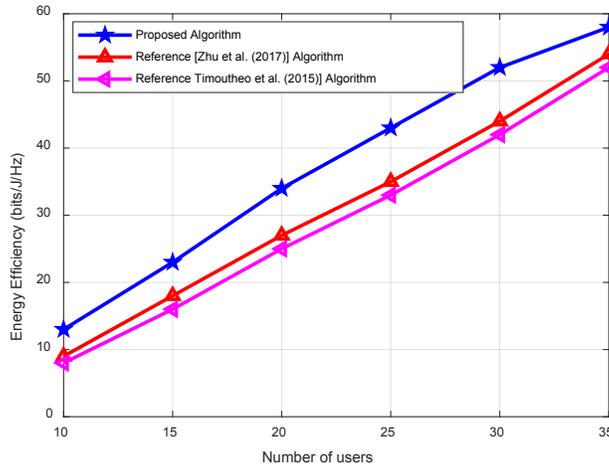


Figure 13: Comparison of the energy efficiency of the proposed and existing algorithms with an increasing number of users

6 Conclusions and future recommendations

This paper mainly studies the power allocation problem of proportionally fair NOMA. The objective is to ensure the fairness of the users at the edge of the cell. The objective function is to maximize the fairness of multiplexed users. In the NOMA scenario, the sum data rate of the two users is not less than the OMA scenario under the same conditions. The data rate also satisfies the basic idea of NOMA, that is, users with poor channel conditions allocate larger power, and users with better channel conditions allocate less power. This paper subtly transforms the nonconvex optimization problem in the objective function into a convex optimization problem and obtains the optimal value

through KKT constrained optimization conditions. The simulation results show that compared to OMA, the proposed NOMA algorithm has a better data rate and spectrum utilization. Moreover, it has a larger improvement and compared with Liu et al. [Liu, Onen and Petrova (2015)], it is superior in terms of user fairness. The above description in the KKT condition of Section 4 is based on the case where two users are multiplexed in the sub-band. Of course, the solution of m users is also theoretically satisfied. This part is the focus of the next step of this paper which is the future work.

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