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Subinterval Decomposition-Based Interval Importance Analysis Method

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Abstract: The importance analysis method represents a powerful tool for quantifying the impact of input uncertainty on the output uncertainty. When an input variable is described by a specific interval rather than a certain probability distribution, the interval importance measure of input interval variable can be calculated by the traditional non-probabilistic importance analysis methods. Generally, the non-probabilistic importance analysis methods involve the Monte Carlo simulation (MCS) and the optimization-based methods, which both have high computational cost. In order to overcome this problem, this study proposes an interval important analytical method avoids the time-consuming optimization process. First, the original performance function is decomposed into a combination of a series of one-dimensional subsystems. Next, the interval of each variable is divided into several subintervals, and the response value of each one-dimensional subsystem at a specific input point is calculated. Then, the obtained responses are taken as specific values of the new input variable, and the interval importance is calculated by the approximated performance function. Compared with the traditional non-probabilistic importance analysis method, the proposed method significantly reduces the computational cost caused by the MCS and optimization process. In the proposed method, the number of function evaluations is equal to one plus the sum of the subintervals of all of the variables. The efficiency and accuracy of the proposed method are verified by five examples. The results show that the proposed method is not only efficient but also accurate.

Keywords: Importance analysis method; interval variable; subinterval decomposition; performance function; MCS

1 Introduction

The input uncertainty in engineering structures leads to the uncertainty of its output response [1,2]. Quantifying the influence degree of input uncertainty on the output uncertainty is of great significance in the engineering model design [3]. Sensitivity analysis (SA) represents a powerful tool for quantifying the influence degree of input uncertainty [4–6]. At present, the sensitivity analysis methods can be broadly divided into two categories: local sensitivity analysis (LSA) methods [7–9] and global sensitivity analysis (GSA) methods [10–13]. The LSA essentially denotes a partial derivative of the performance function at the nominal point of the input variable, so it can quantify the sensitivity of the input variable only at a certain nominal point. In contrast, the GSA measures the degree over which the uncertainty of the input variable affects the uncertainty of the output response. The GSA is also known as the importance measure analysis, and the GSA methods include the variance-based methods [14–17], moment-independent methods [18,19], reliability sensitivity methods [20,21] and so on. However, the application of the GSA methods is based on the premise that input variable uncertainty can be modeled by particular distribution type and precise distribution parameters. Recently, it has shown that using incorrect distribution types or parameters can lead to incorrect results [22]. In fact, due to high experimental costs and/or inadequate design experience, it is difficult to obtain enough the amount of data needed to determine the distribution types and parameters of all of the variables in practical engineering problems correctly. In order to address the mentioned problems, a non-probabilistic importance analysis method has emerged.

Since the interval importance analysis needs to determine only the variation ranges of variables without high requirements for data amount, it has become one of the most commonly used non-probabilistic importance analysis methods. Guo et al. [23] proposed a non-probabilistic model of structural reliability on the basis of interval uncertainty for the first time. Based on this model, Li et al. [24] proposed a non-probabilistic reliability importance measure. This interval importance measure can be used to quantify the influence of the interval variable uncertainty on the non-probabilistic reliability index, and thus, is well suited to actual engineering problems with incomplete data amount. Wang et al. [3] used the non-probabilistic importance analysis method to analyze the importance of the clamp support in the aviation hydraulic pipeline system, and then combined this method with the traditional optimization method and proposed a dimension reduction optimization strategy based on sensitivity analysis. Han et al. [25] used the interval model to describe the uncertainty of cantilever parameters and used the interval sensitivity analysis method to analyze the influence of the design parameter fluctuations on the structural response. Although the non-probabilistic importance analysis has low requirements for the data of variables and good adaptability, its shortcomings severely limit its application in practice. Since the calculation process of non-probabilistic importance analysis represents a nested calculation process that combining the Monte Carlo simulation (MCS) and optimization method, the number of function evaluations required to estimate the interval importance measures is very large, which significantly limits the application of this analysis.

In view of the above problems, an efficient method for reducing the computational cost of non-probabilistic importance analysis is proposed. In the proposed method, the subinterval decomposition theory [26] is used to eliminate the optimization process of solving the upper and lower of response bounds, and the performance function is reconstructed by a series of decomposed univariate functions, where a new input variable represents the values of the univariate functions at the corresponding characteristic point. Accordingly, the computational cost required to estimate the importance is equivalent to one plus the sum of subintervals. Compared to the traditional non-probabilistic importance measure algorithms, the proposed method avoids the repeated optimization process and estimates the importance measure without a need for a large number of samples.

The rest of this study is organized as follows. The non-probabilistic importance measure based on interval uncertainty is presented in Section 2. A brief review of subinterval decomposition is given is Section 3. In Section 4, an efficient non-probabilistic interval importance analysis method based on subinterval decomposition is proposed. In Section 5, the efficiency and accuracy of the proposed method are verified by five examples. The results of the proposed method is compared to the results of the MCS, Sequential quadratic programming (SQP) and Genetic algorithm (GA). Finally, the conclusions are drawn in Section 5.

2 Interval Uncertainty Level-Based Importance Measure

2.1 Interval Uncertainty-Based Importance Measure Definition

Suppose $Y^I = g\left(\boldsymbol{X}^I\right)$ denotes the performance function of a structure with n-dimensional input vector expressed as $\boldsymbol{X}^I=\left[X_1^I, X_2^I, \cdots, X_n^I\right]$ and output denoted as Y^I. Since the uncertainty of the input variable is quantified by the interval model, the output Y^I is also an interval, that is, $Y^I = \left[Y^l, Y^u\right]$. The midpoint Y^c and radius Y^r of output Y^I are, respectively, expressed as follows:

$$Y^c = \frac{Y^u + Y^l}{2}$$

$$Y^r = \frac{Y^u - Y^l}{2} \tag{1}$$

where Y^u and Y^l denote the upper and lower bounds of Y^I, respectively. Thus, the uncertainty level of Y^I can be expressed as follows:

$$\delta = Y^r / Y^c \tag{2}$$

The interval uncertainty-based importance measure can be used to quantify the impact of interval uncertainty of input variable on the interval uncertainty of the output variable. When the interval variable X_i^I is fixed to any of its realization values x_i^*, the uncertainty level δ given by Eq. (2) can be expressed as $\delta_{X_i^I = x_i^*}$. When x_i^* changes over the entire interval of X_i^I, the upper and lower bounds of $\delta_{X_i^I}$ can be, respectively, expressed as follows:

$$\delta^u_{X_i^I} = \max\left(\delta_{X_i^I = x_i^*}, x_i^* \in \left[x_i^l, x_i^u\right]\right)$$

$$\delta^l_{X_i^I} = \min\left(\delta_{X_i^I = x_i^*}, x_i^* \in \left[x_i^l, x_i^u\right]\right) \tag{3}$$

Then, the average of $\delta_{X_i^I}$ can be expressed as follows:

$$\delta^{average}_{X_i^I} = \frac{\delta^u_{X_i^I} + \delta^l_{X_i^I}}{2} \tag{4}$$

It is worth noting that the two parameters describing the interval variables represent the midpoint and the radius that correspond to the mean and variance of the random variable obeying a certain distribution, respectively. Therefore, the midpoint can be used to represent the average value of $\delta_{X_i^I}$ that is given by Eq. (4). Then, the influence degree of the uncertainty of the entire interval variable X_i^I on the uncertainty level of output Y^I can be expressed as follows:

$$M_i = \frac{\left|\delta - \delta^{average}_{X_i^I}\right|}{\delta} \tag{5}$$

where M_i denotes the proposed interval uncertainty-based importance measure index, and $|*|$ represent the absolute operator.

In addition, it is worth noting that the conventional interval importance measure proposed by Li et al. [24] is based on the non-probabilistic reliability index proposed by Guo [23]. However, the interval importance measure proposed in this study is based on the uncertainty level of the output rather than the non-probabilistic reliability index, so the proposed measure differs from the one proposed in [24].

2.2 Interval Uncertainty Importance Measure Estimation

According to the above analysis, proper estimation of indicator M_i is curcial for obtaining the unconditional response uncertainty level δ and conditional response uncertainty level $\delta_{X_i^I = x_i^*}$, $x_i^* \in \left[x_i^l, x_i^u\right]$. Besides, the MCS or optimization algorithms required to calculate the upper and lower bounds of

Eq. (3), or Y^u and Y^l given by Eq. (1). In the following, the traditional MCS procedure for estimating M_i is presented:

1. Generate an $N \times n$ sample matrix of interval variables using the Latin hypercube sampling (LHS) and denote it as $\boldsymbol{\Omega}$ and each its element as x_{ji}, where j and i represent the row and column' indexes, respectively; and N is the number of simulated samples.
2. Calculate the corresponding output Y^I, and obtain its upper bound Y^u and lower bound Y^l.
3. Calculate the midpoint Y^c and radius Y^r by Eq. (1), and then calculate the uncertainty level δ by Eq. (2).
4. Fix the interval variable X_i^I at its realization x_{ji}, and obtain the changed sample matrix $\boldsymbol{\Omega}^{ji}$, where except the ith column whose elements are replaced with the element x_{ji}, the other columns are the same as the corresponding columns of $\boldsymbol{\Omega}$. Then, repeat Steps (2) and (3) using the change sample matrix $\boldsymbol{\Omega}^{ji}$, and obtain the conditional uncertainty level $\delta_{X_i^I=x_{ji}}$.
5. Change x_{ji} from x_{1i} to x_{Ni} and repeat Step (4) to obtain a total of N conditional uncertainty levels $\delta_{X_i^I=x_{ji}}$. Furthermore, obtain the upper bound $\delta^u_{X_i^I}$ and lower bound $\delta^l_{X_i^I}$ of $\delta_{X_i^I}$ by Eq. (3).
6. Calculate the average value of $\delta_{X_i^I}$ by Eq. (4), and finally, calculate the interval importance measure of X_i^I by Eq. (5).

Obviously, the MCS for solving the upper and lower bounds of the response can be replaced by some of the commonly used optimization algorithms, such as the SQP or the GA. Compared with the MCS, using the optimization algorithm to solve the upper and lower bounds can greatly reduce the computational cost. For instance, for a simple performance function with N = 10000 and n = 3 shown in Example 1, where N and n represent the number of simulated samples and the number of variables, the MCS requires $3 \times 10^8 + 10000$ function evaluations. However, the SQP and the GA need only about 15400 and 614040 function evaluations, respectively. In terms of computational cost, the SQP and the GA are obviously superior to the MCS. However, the computational cost of the SQP or the GA is still too large to be implemented in parctice. Therefore, highly efficient interval importance analysis method is urgently needed.

3 Review of Subinterval Decomposition

As explained in Section 2.1, the interval importance analysis requires many repeated calculations for estimating the upper and lower bounds of the output variable, which is the main reason for a large amount of calculation of the traditional calculation methods. Therefore, in order to reduce the computational cost, the subinterval decomposition method [26] is used for determing the upper and lower bounds of the output result.

First, a multiplicative dimensional reduction method (M-DRM) studied in [27] is adopted to approximate the original performance function as a combination of a series of univariate functions,which is given by the following equation:

$$Y = g(\boldsymbol{X}) \approx \sum_{i=1}^{n} g(X_i, \boldsymbol{c}_{-i}) - (n-1)g(\boldsymbol{c}) \tag{6}$$

where $g(X_i, \boldsymbol{c}_{-i}) = g(c_1, \cdots c_{i-1}, X_i, c_{i+1}, \cdots, c_n)$ denotes a univariate function in terms of X_i, $\boldsymbol{c} = [c_1, c_2, \cdots, c_n]^T$ is a specific reference point corresponding to $\boldsymbol{X} = [X_1, X_2, \cdots X_3]$, and $\boldsymbol{c}_{-i}$ is $\boldsymbol{c}$ where c_i is removed.

Generally, $\boldsymbol{c}$ represents the reference point used to bring the approximated function closer to the original performance function. For instance, for normally distributed variables, $\boldsymbol{c}$ is usually taken as their mean, so for interval variables, it represents the midpoint, that is, $\boldsymbol{c} = \left(X_1^c, X_2^c, \cdots, X_n^c\right) = \boldsymbol{X}^c$. Then, the original performance function can be approximated as follows:

$$Y^I \approx \sum_{i=1}^{n} g\left(X_i^I, \boldsymbol{X}_{-i}^c\right) - (n-1) g\left(\boldsymbol{X}_i^c\right) \tag{7}$$

Next, the upper and lower bounds of the two and three-dimensional problems are solved by subinterval decomposition.

For a two-dimensional problem, two interval variables are respectively divided into four subintervals, shown in Fig. 1a. According to Eq. (7), the response corresponding to the black triangle in Fig. 1a can be approximately expressed as follows [26]:

$$Y_{i_1 i_2}^I\left(X_1^l + i_1\Delta X_1, X_2^l + i_2\Delta X_2\right) \approx g\left(X_1^l + i_1\Delta X_1, X_2^c\right) + g\left(X_1^c, X_2^c + i_2\Delta X_2\right) - g\left(X_1^c, X_2^c\right), i_1, i_2 = 0,1,2,3,4 \tag{8}$$

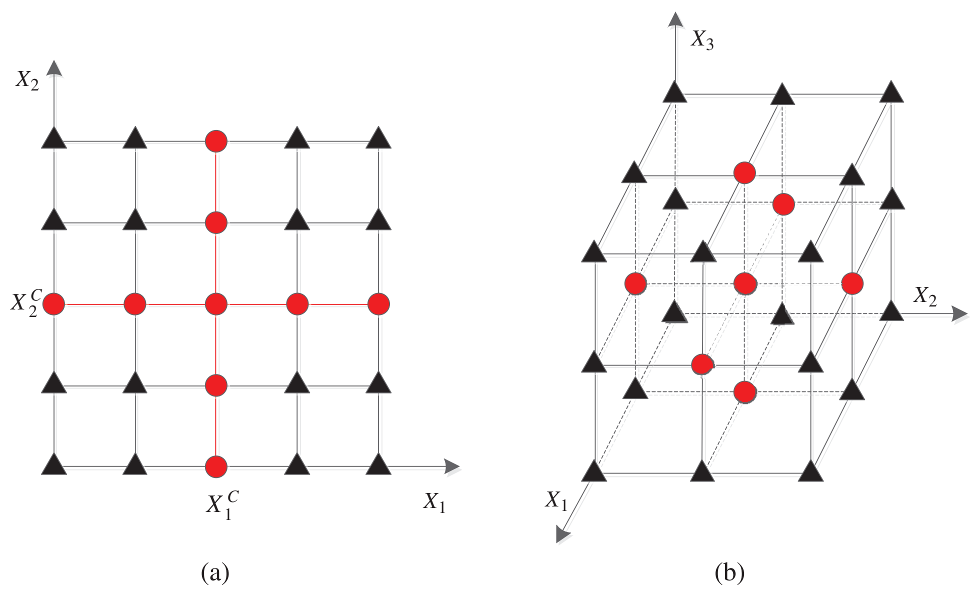

Figure 1: Illustrations of the interval decomposition at (a) n = 2 and (b) n = 3

where ΔX_i, X_i^l and X_i^c denote the length of the subinterval, lower bound and midpoint of X_i, respectively. Then, the structure response can be converted into the following form:

$$Y^I = \bigcup_{i_1,i_2=0,1,2,3,4} Y_{i_1 i_2}^I\left(X_1^l + i_1\Delta X_1, X_2^l + i_2\Delta X_2\right) \tag{9}$$

Finally, the upper and lower bounds of the two-dimensional function can be approximated as follows:

$$Y^u = \max_{i_1,i_2=0,1,2,3,4}\left\{Y_{i_1,i_2}\left(X_1^l + i_1\Delta X_1, X_2^l + i_2\Delta X_2\right)\right\} \tag{10}$$

$$Y^l = \min_{i_1,i_2=0,1,2,3,4}\left\{Y_{i_1,i_2}\left(X_1^l + i_1\Delta X_1, X_2^l + i_2\Delta X_2\right)\right\} \tag{11}$$

Consequently, if responses at the characteristic points $\left\{\left(X_1^l + i_1\Delta X_1, X_2^c\right), \left(X_1^c, X_2^l + i_2\Delta X_2\right), \left(X_1^c, X_2^c\right)\right\}_{i_1,i_2=0,1,2,3,4}$ are calculated, the upper and lower bounds that are given by Eqs. (10) and (11), respectively, can be directly obtained. In Fig. 1a, the characteristic points correspond to the red dots, thus indicating that the number of function evaluations required to calculate the upper and lower bounds in the two-dimensional problem with four subintervals represents the number of red dots, that is, 2 × 4 + 1 = 9.

For a three-dimensional problem, each interval variable is decomposed into two subintervals, as shown in Fig. 1b. According to Eq. (7), the upper and lower bounds of the three-dimensional problem can be, respectively, approximated as follows:

$$Y^u = \max_{i_1, i_2 = 0,1,2} \left\{ Y^I_{i_1,i_2,i_3}\left(X^l_1 + i_1\Delta X_1, X^l_2 + i_2\Delta X_2, X^l_3 + i_3\Delta X_3\right) \right\} \tag{12}$$

$$Y^l = \min_{i_1, i_2 = 0,1,2} \left\{ Y^I_{i_1,i_2,i_3}\left(X^l_1 + i_1\Delta X_1, X^l_2 + i_2\Delta X_2, X^l_3 + i_3\Delta X_3\right) \right\} \tag{13}$$

Similar to Eq. (8), according to Eq. (7), $Y_{i_1,i_2,i_3}\left(X^l_1 + i_1\Delta X_1, X^l_2 + i_2\Delta X_2, X^l_3 + i_3\Delta X_3\right)$ can be approximated as a combination of three univariate functions, which is given by the following equation:

$$\begin{aligned} &Y^I_{i_1,i_2,i_3}\left(X^l_1 + i_1\Delta X_1, X^l_2 + i_2\Delta X_2, X^l_3 + i_3\Delta X_3\right) \\ &\approx g\left(X^l_1 + i_1\Delta X_1, X^c_2, X^c_3\right) + g\left(X^c_1, X^l_2 + i_2\Delta X_2, X^c_3\right) \\ &\quad + g\left(X^c_1, X^c_2, X^l_3 + i_3\Delta X_3\right) - (3-1)g\left(X^c_1, X^c_2, X^c_3\right), i_1, i_2, i_3 = 0,1,2 \end{aligned} \tag{14}$$

According to Eq. (14), for the three-dimensional problem with two subintervals per interval variable, the number of function evaluations required to calculate the upper and lower bounds by Eqs. (12) and (13) is equal to 3 × 2 + 1 = 7, which represents the number of the red dots in Fig. 1b.

Following the above analysis, the upper and lower bounds of n-dimensional problem can be, respectively, expressed as follows:

$$Y^u = \max_{i_1=0,\cdots,m_1, i_2=0,\cdots,m_2,\cdots,i_n=0,\cdots,m_n} \left\{ Y^I_{i_1 i_2 \cdots i_n}\left(X^l_1 + i_1\Delta X_1, X^l_2 + i_2\Delta X_2, \cdots, X^l_n + i_n\Delta X_n\right) \right\} \tag{15}$$

$$Y^l = \min_{i_1=0,\cdots,m_1, i_2=0,\cdots,m_2,\cdots,i_n=0,\cdots,m_n} \left\{ Y^I_{i_1 i_2 \cdots i_n}\left(X^l_1 + i_1\Delta X_1, X^l_2 + i_2\Delta X_2, \cdots, X^l_n + i_n\Delta X_n\right) \right\} \tag{16}$$

where $Y^I_{i_1 i_2 \cdots i_n}\left(X^l_1 + i_1\Delta X_1, X^l_2 + i_2\Delta X_2, \cdots, X^l_n + i_n\Delta X_n\right)$ is given by the following equation:

$$\begin{aligned} &Y^I_{i_1 i_2 \cdots i_n}\left(X^l_1 + i_1\Delta X_1, X^l_2 + i_2\Delta X_2, \cdots, X^l_n + i_n\Delta X_n\right) \\ &\approx g\left(X^l_1 + i_1\Delta X_1, X^c_2, \cdots, X^c_n\right) + g\left(X^c_1, X^l_2 + i_2\Delta X_2, \cdots, X^c_n\right) \\ &\quad + \cdots + g\left(X^c_1, X^c_2, \cdots, X^l_n + i_n\Delta X_n\right) - (n-1)g\left(X^c_1, X^c_2, \cdots, X^c_n\right), \\ &\quad i_1 = 0,1,\cdots,m_1, i_2 = 0,1,\cdots,m_2,\cdots,i_n = 0,1,\cdots,m_n \end{aligned} \tag{17}$$

According to Eqs. (15)–(17), and assuming that the number of subintervals of the ith interval variable is m_i, the total number of function evaluations required to calculate the upper and lower bounds is equal to $\sum_{i=1}^{n} m_i + 1$. On this basis, an efficient interval importance measure analysis method based on the subinterval decomposition is proposed.

4 Subinterval Decomposition-Based Interval Importance Analysis Method

In this section, the detailed implementation process of the proposed method based on the subinterval decomposition is presented.

First, a new performance function for interval importance analysis is defined. According to Eq. (17), the performance function with an n-dimensional interval variable can be approximated as follow:

$$Y^I \approx \eta(\boldsymbol{\chi}) = \chi_1 + \chi_2 + \cdots + \chi_n - \gamma \tag{18}$$

where η denotes the approximate substitution of the original performance function Y^I, $\boldsymbol{\chi} = [\chi_1, \chi_2, \cdots \chi_n]$ denotes the new input vector of η, and γ is a constant. The elements of the vector and the constant are respectively expressed as follows:

$$\begin{aligned}
&\chi_1 = g\left(X_1^l + i_1\Delta X_1, X_2^c, \cdots, X_n^c\right) i_1 = 0, 1, \cdots, m_1 \\
&\chi_2 = g\left(X_1^c, X_2^l + i_2\Delta X_2, \cdots, X_n^c\right) i_2 = 0, 1, \cdots, m_2 \\
&\quad\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
&\chi_n = g\left(X_1^c, X_2^c, \cdots, X_n^l + i_n\Delta X_n\right) i_n = 0, 1, \cdots, m_n \\
&\gamma = (n-1) g\left(X_1^c, X_2^c, \cdots, X_n^c\right)
\end{aligned} \tag{19}$$

The detailed calculation procedure of the proposed interval importance analysis method is as follows:

1. Calculate the values of all univariate functions χ_k by Eq. (19), where $k = 1, 2, \cdots n$.

$$\begin{aligned}
&\chi_1 = \left[\chi_1^1, \chi_1^2, \cdots, \chi_1^{m_1}\right]^T \\
&\chi_2 = \left[\chi_2^1, \chi_2^2, \cdots, \chi_2^{m_1}\right]^T \\
&\quad\vdots \qquad\qquad \vdots \\
&\chi_n = \left[\chi_n^1, \chi_n^2, \cdots, \chi_n^{m_1}\right]^T
\end{aligned} \tag{20}$$

2. Calculate the new sample matrix of the approximated performance function given by Eq. (18). Arrange and combine the elements of all variables in Eq. (20) to generate a new sample matrix $\boldsymbol{\Phi}_{\varsigma\times n} = \left[\phi_1, \phi_2, \cdots, \phi_\varsigma\right]^T$, where $\varsigma = m_1 \times m_2 \times \cdots \times m_n$ and $\phi_i = \left[\chi_1^{*i}, \chi_2^{*i}, \cdots, \chi_n^{*i}\right]$ represent the ith group sample, which can be obtained by the following permutation and combination:

$$\left.\begin{aligned}
&\chi_1 = \left[\chi_1^1, \chi_1^2, \cdots, \chi_1^{m_1}\right]^T \\
&\chi_2 = \left[\chi_2^1, \chi_2^2, \cdots, \chi_2^{m_2}\right]^T \\
&\qquad\vdots \\
&\chi_n = \left[\chi_n^1, \chi_n^2, \cdots, \chi_n^{m_n}\right]^T
\end{aligned}\right\} \rightarrow \begin{bmatrix} \chi_1^{*i} \\ \chi_2^{*i} \\ \vdots \\ \chi_n^{*i} \end{bmatrix}^T = \phi_i \tag{21}$$

In Eq. (21), the ith element of ϕ_i is one of the elements of χ_i; thus, the total number of elements of ϕ_i is $\varsigma = m_1 \times m_2 \times \cdots \times m_n$.

3. Substitute $\boldsymbol{\Phi}_{\varsigma\times n}$ into Eq. (18), that is, calculate $\eta(\boldsymbol{\Phi}_{\varsigma\times n})$ and obtained ς responses $\boldsymbol{\eta} = \left[\eta_1, \eta_2, \cdots, \eta_\varsigma\right]^T$. Using Eqs. (15) and (16), the lower and upper bounds of $\boldsymbol{\eta}$ are directly obtained as follows:

$$\begin{aligned}
&\eta^u = \max(\boldsymbol{\eta}) \\
&\eta^l = \min(\boldsymbol{\eta})
\end{aligned} \tag{22}$$

   Then, the midpoint η^c and radius η^r are obtained by Eq. (1). The unconditional uncertainty level of η is further obtained by Eq. (2) and denoted as δ_η.

4. Fix χ_i at its realization χ_i^t, and obtain a new sample matrix $\boldsymbol{\Phi}^*_{\varsigma^*\times n}$ using the permutation and combination rules given in Step (2). Since χ_i is fixed, the value of ς^* is $m_1 \times m_2 \times m_{i-1} \times 1 \times m_{i+1} \cdots \times m_n$.
5. Repeat Step (3) with the new sample matrix $\boldsymbol{\Phi}^*_{\varsigma^*\times n}$ to obtain the conditional uncertainty level and denote it as $\delta_{\eta|\chi_i^t}$.
6. Change t from 1 to m_i, and repeat Steps (4) and (5) to obtain m_i conditional uncertainty level $\delta_{\eta|\chi_i} = \left[\delta_{\eta|\chi_i^1}, \delta_{\eta|\chi_i^2}, \cdots, \delta_{\eta|\chi_i^{m_i}}\right]$.

7. Using Eqs. (3) and (4), get the average value of $\delta_{\eta|\chi_i}$ and denote it as $\delta_{\eta|\chi_i}^{average}$.
8. Finally, calculate the interval importance index of χ_i by Eq. (5) and denote is as M_{χ_i}. Then, use M_{χ_i} to approximate the interval importance index of X_i.

Based on the presented steps, the computational cost of the proposed method depends only on the calculation burden required to calculate the values of all of the univariate functions. If the subinterval number of each interval variable denote as m_i is known, then the characteristic input points of all interval variables can be determined. The values of all of the univariate functions at the characteristic input points can be obtained using the original performance function, and this calculation process mainly difines the calculation burden of the proposed method because all of the other steps of the proposed method do not require additional computation. As mentioned previously, the computational cost, that is, the number of function evaluations, is equal to one plus the sum of subinterval numbers of all interval variables. Hence, the computational cost of the proposed method increases linearly rather than exponentially with the increase in input dimensionality. However, even though the computational cost fo the proposed method is affected by input dimensionality, this effect is relatively small.

Obviously, the accuracy and efficiency of the proposed method are determined by the number of subintervals of variables. Generally, the larger the number of the subintervals is, the higher the accuracy and the lower the efficiency will be, and vice versa. However, the error analysis can be used to balance accuracy and efficiency [26], but that is a very complex process, which is difficult to achieve. Based on the study of five examples presented in Section 5, in this study, the number of subintervals is suggested to be in the range of 2–5.

As mentioned above, the proposed method constructs the sample matrix by using the permutation and combination. If the number of interval variables or the number of subintervals of each interval variables is large. Even though this does not add any extra computation, too large number of samples will still affect calculation efficiency. For this reason, in this work, the number of samples is limited to 10^5. If the number of sample size exceeds 10^5, only the first 10^5 samples will be selected and used to form the sample matrix; otherwise, all of the smaples will be selected and used to form the sample matrix.

5 Case Studies

The accuracy and efficiency of the proposed interval importance analysis method were verified by five examples. In order to make the verification process more relevant, the MCS and two optimization-based algorithms were employed as reference algorithms, and their results were compared with the result obtained by the proposed algorithm. The key parameters of the GA optimization method, Generations, PopulationSize, CrossoverFraction and ParetoFraction were set to 50, 20, 0.8, and 0.5, respectively. In the MCS, the sample size N is 10^4, and in two optimization-based algorithms, the sample size was set to 10^2.

5.1 Example 1: Simple Example

A simple numerical example expressed by Eq. (23) was used to verify the correction of the proposed method:

$$Y = X_1 + X_2 + X_3 \tag{23}$$

In Eq. (23), X_1, X_2, and X_3 denote three interval variables with the upper bound of $\boldsymbol{X}^u = [4.5, 5, 5.5]$ and lower bound of $\boldsymbol{X}^l = [3.5, 3, 2.5]$. According to the definition of uncertainty level given by Eq. (2), the uncertainty levels of the three interval variables is 0.25, 0.5 and 0.75, respectively.

In this example, there were two subintervals per interval variable. According to the above-presented analysis, the numbers of function evaluations required by the four methods to estimate the interval importance measures were calculated, and they are listed in Tab. 1. According to the results presented in Tab. 1, the proposed method performed the best among all the methods, achieving the highest efficiency, and it was followed by the SQP, GA, and MCS method.

Table 1: Numbers of function evaluations required by the four methods in Example 1

Method	MCS	SQP	GA	Proposed method
Number of Function evaluations	3.0001×10^8	154	614040	$2 \times 3 + 1 =$ **7**

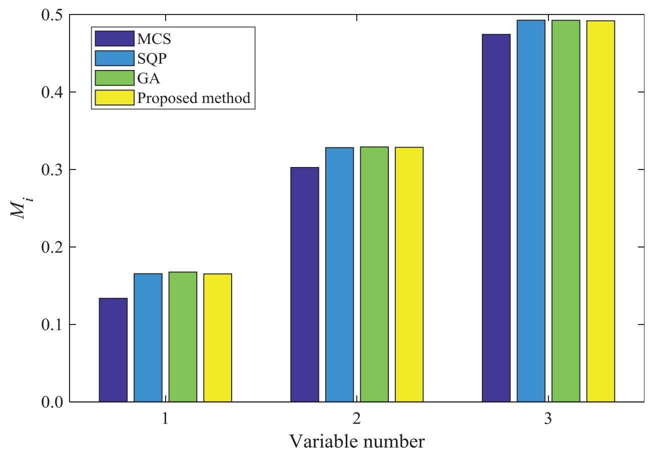

Figure 2: The interval importance measures of Example 1

The interval importance measures of Example 1 are shown in Fig. 2. As shown in Fig. 2, the four methods performed similarly, thus indicating that the proposed method had high accuracy.

According to the results presented in Fig. 2, the importance ranking of the three interval variables was $X_1 < X_2 < X_3$. In this example, the performance function was very simple and completely linear, which is why the influence degree of the uncertainty of the interval variables on the output result of the performance function depended only on the uncertainty level of each interval variable. The uncertainty levels of the three interval variables obtained by the proposed method were 0.25, 0.5, and 0.75 respectively, which indicated the correctness of the importance ranking obtained by the proposed method.

5.2 Example 2: A Probabilistic Risk Assessment Model

A probabilistic risk assessment model [4] was expressed as follows:

$$\begin{aligned} Y = {} & X_1X_3X_5 + X_1X_3X_6 + X_1X_4X_5 + X_1X_4X_6 + X_2X_3X_4 \\ & X_2X_3X_5 + X_2X_4X_5 + X_2X_5X_6 + X_2X_4X_7 + X_2X_6X_7 \end{aligned} \tag{24}$$

where, the midpoint of the seven interval variables was $\boldsymbol{X}^c = \left[2, 3, 1 \times 10^{-3}, 2 \times 10^{-3}, 4 \times 10^{-3}, 5 \times 10^{-3}, 3 \times 10^{-3}\right]$. The lower and upper bounds were calculated by $\boldsymbol{X}^l = \boldsymbol{X}^c - 0.1 \times \boldsymbol{X}^c$ and $\boldsymbol{X}^u = \boldsymbol{X}^c + 0.1 \times \boldsymbol{X}^c$, respectively. The importance indices of all interval variables obtained for two subintervals per interval variable are shown in Fig. 3.

As shown in Fig. 3, the proposed method matched well with the other three reference methods, especially with the SQP and GA method. The importance rankings of seven interval variables obtained by four different methods were completely consistent, that is, $X_3 < X_1 \approx X_7 < X_4 < X_5 < X_6 < X_2$.

The numbers of function evaluations required by the MCS, SQP, GA, and the proposed method to calculate the interval importance measures of the seven interval variables are listed in Tab. 2. Obviously, the proposed method was superior to the other three methods regarding efficiency.

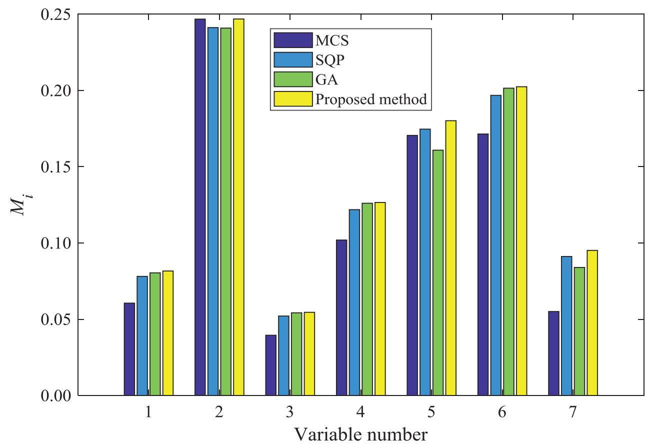

Figure 3: Interval importance measures of Example 2

Table 2: Number of function evaluations required by the four methods in Example 2

Method	MCS	SQP	GA	Proposed method
Number of function evaluations	3.0001×10^8	1386	1430040	2 × 7 + 1 = **15**

5.3 Example 3: Ishigami Function

A highly-nonlinear and non-monotone function studied in [28] was modified and adopted to verify the performance of the proposed method, and it was expressed as follows:

$$Y = \sin(X_1) + 0.3\sin^2(X_2) + 0.02X_3^4 \sin(X_1) \tag{25}$$

In this example, the lower and upper bounds of all three variables were given as 1 and 2, respectively. Therefore, they all have the same midpoint value of 1.5. The importance results obtained by four methods for three subintervals per variable are shown in Fig. 4.

As presented in Fig. 4, for this highly nonlinear and non-monotone function, although the indices of each variable obtained by the three reference methods were slightly different, their importance rankings were the same, that is, $X_2 < X_1 < X_3$, and they were the same as the importance ranking obtained by the proposed method, indicating that the proposed method performed well even for problems with the highly nonlinear and non-monotone performance function.

The numbers of function evaluations required by the MCS, GA, SQP, and proposed method to calculate the importance indices of all variables are listed in Tab. 3. According to the results given in Tab. 3 that, the computational efficiency of the proposed method was very high. Consequently, the proposed method could provide the correct importance ranking with extremely high efficiency even for highly nonlinear and non-monotone function.

5.4 Example 4: Composite Cantilever Beam Structure

A composite cantilever beam structure [29] shown in Fig. 5 was studied. The displacement Y of the free point under the load F_0 was calculated by using the mechanical analysis by:

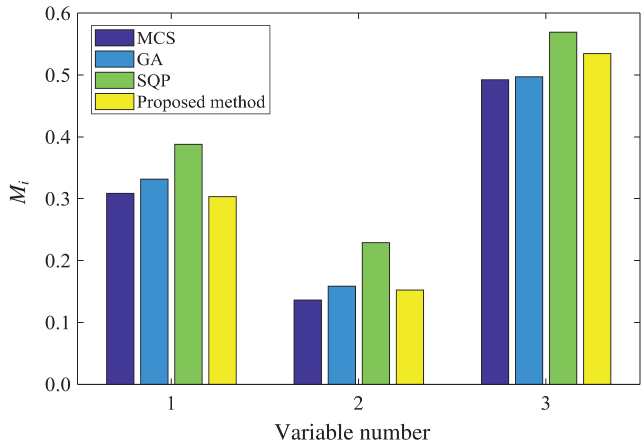

Figure 4: Interval importance measures of the Ishigami function

Table 3: Number of function evaluations required by the four methods in example 3

Method	MCS	SQP	GA	Proposed method
Number of function evaluations	3.0001×10^8	301	614040	$3 \times 3 + 1 =$ **10**

$$Y = \frac{F_0 L^3}{2h^3} \left(\frac{E_L^2 - 4G_{LT}E_T v_{LT}^2 + E_L(E_T + 4G_{LT} + 2E_T v_{LT})}{E_L G_{LT}(E_L + E_T + 2E_T v_{LT})} \right) \tag{26}$$

where F_0, L, h, E_L, E_T, G_{LT} and v_{LT} are denoted the applied load per width, length, height, longitudinal Young's modulus, transverse Young's modulus, shear modulus and Poisson's ratio, respectively, and the corresponding midpoint values were 3.81 cm, 50.8 cm, 530 KN/m, 9.38 Gpa, 0.036, 173, and 33.1, respectively. Their upper and lower bounds were respectively calculated as their midpoints plus and minus 0.1 times their corresponding midpoint. The interval importance results are shown in Fig. 6.

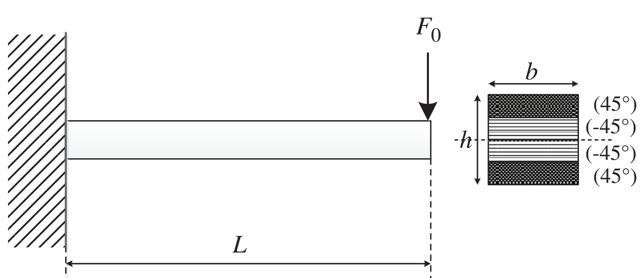

Figure 5: Composite cantilever beam structure model

In this example, there were two subintervals per interval variable. The numbers of function evaluations required by the four methods to estimate a set of interval importance results are listed in Tab. 4, where it can be seen that the proposed method achieved very high efficiency.

The repeated 10 importance results are represented by the box plots in Fig. 6. The short distribution range resulted a higher robustness; otherwise, higher variability was achieved. The results show that the GA had higher variability than the MCS and SQP methods. The results obtained by the proposed method were completely unchanged compared with other three methods and extremely stable. These results were caused by the

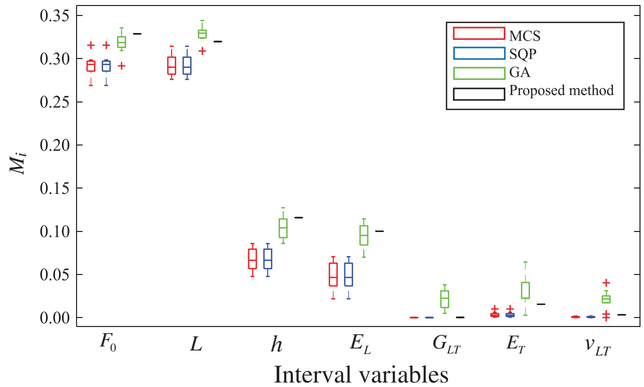

Figure 6: Interval importance measures of the composite cantilever beam structure, for 10 replicates

Table 4: Numbers of function evaluations required by the four methods in Example 4

Method	MCS	SQP	GA	Proposed method
Number of function evaluations	3.0001×10^8	330	1132040	2 × 7 + 1 = **15**

characteristic of the proposed method: namely, as long as the number of the subintervals per interval variable was determined, the feature points were determined, and the sample matrix generated by permutation and combination was also determined and invariant, so the calculation results were completely invariant.

In terms of interval importance ranking, the result of the proposed method was completely consistent with those of the other three reference methods. The importance of F_0 and L was similar and larger than those of the other variables; G_{LT}, E_T, and v_{LT} were of similar importance that was very small; h was slightly important than E_L, their results were large than those of G_{LT}, E_T, v_{LT} and small than those of F_0 and L.

5.5 Example 5: Roof Truss

A roof truss [4,30,31] shown in Fig. 7 was adopted, and the perpendicular displacement Y of the node C was calculated by the following equation:

$$Y = \frac{ql^2}{2}\left(\frac{3.81}{A_C E_C} + \frac{1.13}{A_S E_S}\right) \tag{27}$$

where q, l, A_C, A_S, E_C, and E_S represented the uniformly distributed load, the length of the concrete bars, and the sectional areas and elastic moduli of the steel bars and concrete bars, respectively. The midpoint values of the interval variables were set to 2×10^4 N/m, 12 m, 4×10^{-2} m^2, 9.83×10^{-4} m^2, 3×10^{10} MPa, and 2×10^{11} MPa, respectively. Their upper and lower bounds were, respectively, calculated as their midpoint plus and minus 0.1 times their corresponding midpoint.

Each interval variable was divided into two subintervals. The obtained interval importance measures of all interval variables are represented by box plots in Fig. 8. The numbers of function evaluations required by the MCS, SQP, GA, and the proposed method to calculate a set of interval importance measures is listed in Tab. 5. The proposed method was obviously very efficient.

As well know, the larger the sample size is, the closer the MCS result is to the true value will be. Therefore, under the condition of a sufficient number of samples, the result calculated by the MCS is of a great reference value. In this example, the MCS was used as a reference to evaluate the performance of the proposed method and the other two optimization-based methods. The result obtained by the GA

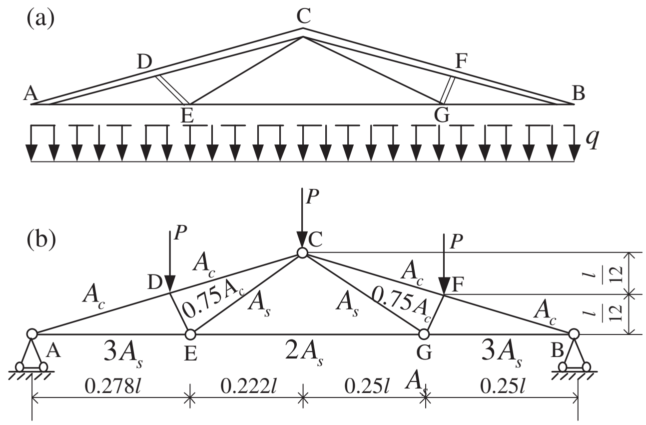

Figure 7: The schematic diagram of roof truss

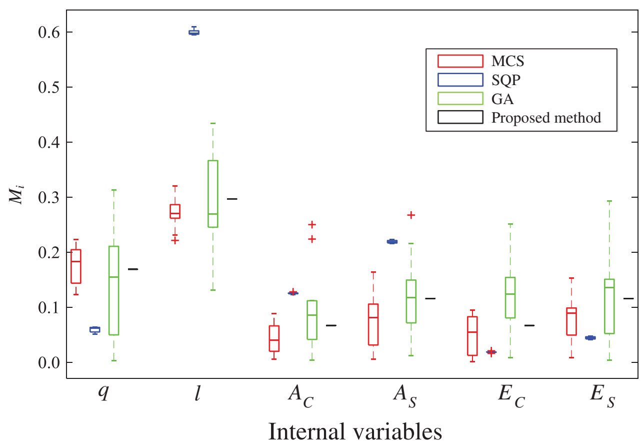

Figure 8: Interval importance measures of roof truss, 10 replicates

Table 5: Number of function evaluations required by the four methods in Example 5

Method	MCS	SQP	GA	Proposed method
Number of function evaluations	3.0001×10^8	546	1226040	$2 \times 6 + 1 =$ **13**

changes the most, indicating that its robustness was poor. Although the result obtained by the SQP had a small change, its importance ranking was slightly different from that obtained by the MCS. For instance, q was 2 in the MCS; however, it was about 4 in the SQP. At the same time, the result of the SQP on l was poor, and the error was relatively large, while the error of the proposed method was zero, which was due to the characteristic of the proposed method, which was mentioned in Example 3. Consequently, the proposed method obtained similar results as the MCS, and its importance ranking was consistent with that of the MCS, as shown in Fig. 8.

The above-presented five examples fully verify the efficiency, accuracy and robustness of the proposed method, demonstrating that the proposed method performs very well in both efficiency and accuracy.

6 Conclusions

The non-probabilistic importance measure is more applicable to practical engineering than the probabilistic distribution-based importance measure due to the low requirement for the prior knowledge on variable data. However, the high computational cost in conventional importance analysis methods limits their application in practical engineering. In view of this, a subinterval decomposition method is proposed in this study to approximate the interval importance measure. The main aims are to establish an approximate performance function by using the interval decomposition and generate the input sample matrix of the approximated performance function using the permutation and combination of univariate function values at the feature points. The proposed method has high accuracy, robustness, and efficiency, which was verified by five examples, where the proposed method was compared with three popular methods, namely, the MCS, SQP, and GA methods. For the highly nonlinear problem, the results of the two traditional optimization-based methods, the SQP and the GA, were quite different from those of the MCS method, which was used as a reference method; the efficiency of the SQP and the GA was low, the robustness was poor, especially that of the GA. However, the proposed method achieved excellent performance, especially in terms of computational cost.

Although the proposed method is highly efficient, and such that has important engineering application value, it is developed based on the assumption that variables are independent of each other. Therefore, there is room for further improvements, and the development of an efficient interval importance analysis method for correlation variables will be our next research objective.

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References

1. Radhika, B., Panda, S. S., Manohar, C. S. (2008). Time variant reliability analysis of nonlinear structural dynamical systems using combined Monte Carlo simulations and asymptotic extreme value theory. *Computer Modeling in Engineering and Sciences, 1(2),* 79.
2. Zhang, F., Deng, W., Nan, H., Zhang, L., Huang, Z. (2017). Reliability analysis of bleed air anti-icing system based on subset simulation method. *Applied Thermal Engineering, 115,* 17–21. DOI 10.1016/j.applthermaleng.2016.11.202.
3. Wang, W., Zhou, C., Gao, H., Zhang, Z. (2018). Application of non-probabilistic sensitivity analysis in the optimization of aeronautical hydraulic pipelines. *Structural and Multidisciplinary Optimization, 57(6),* 2177–2191. DOI 10.1007/s00158-017-1848-4.
4. Zheng, C. J., Chen, H. B., Matsumoto, T., Takahashi, T. (2011). Three dimensional acoustic shape sensitivity analysis by means of adjoint variable method and fast multipole boundary element approach. *Computer Modeling in Engineering and Sciences, 79(1),* 1.
5. Wang, W., Gao, H., Zhou, C. (2019). Extending the global reliability sensitivity analysis to the systems under double-stochastic uncertainty. *Advances in Structural Engineering, 22(3),* 626–640. DOI 10.1177/1369433218795570.
6. Zhang, F., Xu, X., Cheng, L., Wang, L., Liu, Z. et al. (2019). Global moment-independent sensitivity analysis of single-stage thermoelectric refrigeration system. *International Journal of Energy Research, 43(15),* 9055–9064. DOI 10.1002/er.4811.
7. Lu, Z., Song, S., Yue, Z., Wang, J. (2008). Reliability sensitivity method by line sampling. *Structural Safety, 30(6),* 517–532. DOI 10.1016/j.strusafe.2007.10.001.

8. Song, S., Lu, Z., Qiao, H. (2009). Subset simulation for structural reliability sensitivity analysis. *Reliability Engineering & System Safety, 94(2),* 658–665. DOI 10.1016/j.ress.2008.07.006.
9. Cattarin, G., Pagliano, L., Causone, F., Kindinis, A., Goia, F. et al. (2018). Empirical validation and local sensitivity analysis of a lumped-parameter thermal model of an outdoor test cell. *Building and Environment, 130,* 151–161. DOI 10.1016/j.buildenv.2017.12.029.
10. Tarantola, S., Gatelli, D., Kucherenko, S. S., Mauntz, W. (2007). Estimating the approximation error when fixing unessential factors in global sensitivity analysis. *Reliability Engineering & System Safety, 92(7),* 957–960. DOI 10.1016/j.ress.2006.07.001.
11. Yuan, Z., Liang, P., Silva, T., Yu, K., Mottershead, J. E. (2019). Parameter selection for model updating with global sensitivity analysis. *Mechanical Systems and Signal Processing, 115,* 483–496. DOI 10.1016/j.ymssp.2018.05.048.
12. Li, G. J., Xie, C. Y., Wei, F. Y., Wang, F. J. (2018). Moment-independence global sensitivity analysis for the system with fuzzy failure state and its Kriging method. *Journal of Systems Engineering and Electronics, 29(3),* 658–666. DOI 10.21629/JSEE.2018.03.23.
13. Buzzard, G. T., Xiu, D. (2011). Variance-based global sensitivity analysis via sparse-grid interpolation and cubature. *Communications in Computational Physics, 9(3),* 542–567. DOI 10.4208/cicp.230909.160310s.
14. Yun, W., Lu, Z., Jiang, X. (2018). An efficient sampling approach for variance-based sensitivity analysis based on the law of total variance in the successive intervals without overlapping. *Mechanical Systems and Signal Processing, 106,* 495–510. DOI 10.1016/j.ymssp.2018.01.015.
15. Zhou, C., Liu, F., Tang, C., Wang, W. (2017). Global sensitivity analysis of uncertain input variables in structural models. *Journal of Engineering Mechanics, 143(12),* 04017138. DOI 10.1061/(ASCE)EM.1943-7889.0001373.
16. Wei, P., Lu, Z., Hao, W., Feng, J., Wang, B. (2012). Efficient sampling methods for global reliability sensitivity analysis. *Computer Physics Communications, 183(8),* 1728–1743. DOI 10.1016/j.cpc.2012.03.014.
17. Saltelli, A., Annoni, P., Azzini, I., Campolongo, F., Ratto, M. et al. (2010). Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index. *Computer Physics Communications, 181(2),* 259–270. DOI 10.1016/j.cpc.2009.09.018.
18. Borgonovo, E., Castaings, W., Tarantola, S. (2010). Moment independent importance measures: new results and analytical test cases. *Risk Analysis an Official Publication of the Society for Risk Analysis, 31(3),* 404–428. DOI 10.1111/j.1539-6924.2010.01519.x.
19. Zadeh, F. K., Nossent, J., Sarrazin, F., Pianosi, F., van Griensven, A. et al. (2017). Comparison of variance-based and moment-independent global sensitivity analysis approaches by application to the SWAT model. *Environmental Modelling & Software, 91,* 210–222. DOI 10.1016/j.envsoft.2017.02.001.
20. Zhao, J., Zeng, S., Guo, J., Du, S. (2018). Global reliability sensitivity analysis based on maximum entropy and 2-layer polynomial chaos expansion. *Entropy, 20(3),* 202. DOI 10.3390/e20030202.
21. Papaioannou, I., Breitung, K., Straub, D. (2018). Reliability sensitivity estimation with sequential importance sampling. *Structural Safety, 75,* 24–34. DOI 10.1016/j.strusafe.2018.05.003.
22. Jiang, C., Zheng, J., Han, X. (2018). Probability-interval hybrid uncertainty analysis for structures with both aleatory and epistemic uncertainties: a review. *Structural and Multidisciplinary Optimization, 57(6),* 2485–2502. DOI 10.1007/s00158-017-1864-4.
23. Guo, S. (2001). A non-probabilistic model of structural reliability based on interval analysis. *Chinese Journal of Computational Mechanics, 18(1),* 56–60.
24. Li, G., Lu, Z., Tian, L., Xu, J. (2013). The importance measure on the non-probabilistic reliability index of uncertain structures. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability, 227(6),* 651–661. DOI 10.1177/1748006X13489069.
25. Han, Z. J., Wang, Z. Q. (2012). Non-probabilistic reliability research and sensitivity analysis of hanging beam based on interval finite element method. *Journal of Central South University, 43(5),* 1746–1752.
26. Fu, C. M., Cao, L. X., Tang, J. C., Long, X. Y. (2018). A subinterval decomposition analysis method for uncertain structures with large uncertainty parameters. *Computers & Structures, 197,* 58–69. DOI 10.1016/j.compstruc.2017.12.001.

27. Zhang, X., Pandey, M. D. (2013). Structural reliability analysis based on the concepts of entropy, fractional moment and dimensional reduction method. *Structural Safety, 43,* 28–40. DOI 10.1016/j.strusafe.2013.03.001.
28. Chun, M. H., Han, S. J., Tak, N. I. (2000). An uncertainty importance measure using a distance metric for the change in a cumulative distribution function. *Reliability Engineering & System Safety, 70(3),* 313–321. DOI 10.1016/S0951-8320(00)00068-5.
29. Yun, W., Lu, Z., Zhang, Y., Jiang, X. (2018). An efficient global reliability sensitivity analysis algorithm based on classification of model output and subset simulation. *Structural Safety, 74,* 49–57. DOI 10.1016/j.strusafe.2018.04.003.
30. Zhang, J., Xiao, M., Gao, L., Fu, J. (2018). A novel projection outline based active learning method and its combination with Kriging metamodel for hybrid reliability analysis with random and interval variables. *Computer Methods in Applied Mechanics and Engineering, 341,* 32–52. DOI 10.1016/j.cma.2018.06.032.
31. Shi, Y., Lu, Z., Chen, S., Xu, L. (2018). A reliability analysis method based on analytical expressions of the first four moments of the surrogate model of the performance function. *Mechanical Systems and Signal Processing, 111,* 47–67. DOI 10.1016/j.ymssp.2018.03.060.