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Article

Dynamic Simulation of Cracked Buildings for Damage Detection

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Abstract: A dynamic simulation method for cracked structures is implemented to determine their dynamic response with the purpose of evaluating their structural behavior. The procedure makes possible the simulation of three-dimensional cracked structures. The excitation force is randomly generated to simulate wind gusts. It is assumed the structure remains in the elastic range, which allows for each mode that contributes to its dynamic response to be decoupled. The results indicate that the presence of damage causes changes in the modals parameters of the structure as accurate as other similar methods proposed for simpler structures. Therefore, it is concluded that the proposed method is a reliable way to evaluate the dynamic behavior of three-dimensional cracked building structures.

Keywords: Dynamic simulation; cracked structures; damage detection; structural dynamic

1 Introduction

Over time, buildings can suffer damage caused by use, lack of maintenance and due to natural events of great magnitude, such as hurricanes, settlement, earthquakes and floods. During earthquakes of large magnitudes, damage induced to the structure may lead to collapse resulting in economic and/or human losses. To prevent such loss, structural evaluation is of great importance, as a tool for making decisions concerning maintenance and rehabilitation. As an engineering solution to this problem, a review process known as Structural Health Monitoring (SHM) has been developed. This process has been mainly used within the aerospace, civil and mechanical engineering infrastructure sectors, and makes it possible to distinguish between a damaged structure and an undamaged structure. It is well known that any damaged structural element presents changes in its dynamic parameters. This damage may cause material non-linearity at damage vicinities. It is in these vicinities that dynamic simulation plays an important role, since it is possible to simulate damage to structural elements and obtain their dynamic parameters. In this simulation, the parameters obtained can indicate whether a structure is damaged or not, allowing the development of accurate damage detection methods.

Regarding the field of civil engineering, damage detection techniques began to be used in the early 1980s, with the use of dynamic analysis of cracked beams in structures, which became a helpful tool for the development of analytical damage detection methods. In this context a first solution to determine the dynamic behavior of beams with open cracks was proposed by Christides et al. [1], who developed their theory based on the Euler-Bernoulli beam from the Hu-Washizu variational principle. The authors derived the differential equilibrium equation and associated it with the boundary conditions of the Euler-Bernoulli beam of uniform section with one or more pairs of symmetrical cracks. This theory was considered an important step in the development of more rigorous cracked beam vibration theories. In the mid-1990s, dynamic simulation of cracked beams began to be used as a tool to determine more reliable damage detection methods based on vibration analysis. For instance, Chondros et al. [2] proposed a more consistent method based on the theory developed by Christides et al. [1], in which they obtained a function that represents the modification of the stress field caused by cracks, which is based on the theory of fracture mechanics. Even though it is a recognized fact that cracks do not always remain open, the behavior of beams with cracks that open and close, referred to as breathing cracks, has not been studied well enough. Thus, if the static deformation is as large as the vibration amplitude, the cracks will remain open all the time and the problem is considered to be linear. However, if the static deformation is small compared to the vibration amplitude, the cracks will open and close over time and the problem is considered to be nonlinear. Chondros et al. [3] proposed a method to predict the behavior of a beam with cracks that open and close using a bilinear type model where there are only two states: completely open or completely closed. The transition from open to closed cracks is assumed to occur when the beam passes to its undeformed shape.

Another alternative to simulate damage in structures was developed by Zheng et al. [4]. They proposed a method based on Finite Elements (FE) to predict the dynamic behavior of structures with open cracks. An "overall additional flexibility matrix" is added to the flexibility matrix of the undamaged element to obtain the total flexibility matrix, and this matrix is inverted to obtain the stiffness matrix. This procedure is the first to incorporate the effect of the distance between the node of the right end and the crack location, which leads to more precise results. This effect had been neglected in previous methods based on the FE method. Mazaheri et al. [5] presented a simplified method to obtain the effect of cracks on beams. Cracks were modeled as rotational springs and solved using a FE method. The cracks were assumed to remain open using a linear approach. Eroglu et al. [6] developed a new FE formulation for straight beams with an edge crack, which they proposed for frame structures. The in-plane motion of the beam was composed of shear and axial deformation and rotational inertia. The crack was modeled by a rotational spring and two translational springs. Using a similar approach, Ozturk et al. [7] performed dynamic analysis of elastically supported cracked beams subjected to a concentrated moving load. They found that the reduction of natural frequencies caused by cracks is more pronounced for higher modes.

The dynamic simulation problem is more complicated when applied to frame structures. Ozturk et al. [8] investigated the dynamic stability of cracked multi-bay frame structures, in other words, the buckling behavior of cracked columns. A cracked element was developed using a FE method and the principles of fracture mechanics. They determined that the reduction of buckling load depends on crack location and size. Higher reductions of buckling loads were obtained in the supports and corners of the frame structure. Using a different procedure, Nikolakopoulos et al. [9] proposed a computational-graphical method to identify the crack depth and position in frame structures, by eigenfrequency measurements. To identify the location and depth of a crack in a frame structure, it is necessary to calculate the intersection point of the superposed contours that correspond to the measured eigenfrequency variations caused by presence of cracks. Caddemi et al. [10–11] presented an approach, aimed to evaluate frequencies and vibration modes for cracked frame structures. This method is based on the Euler-Bernoulli beam closed-form solution, to obtain the vibration modes of a beam with multiple cracks, regardless of the number of cracked cross sections. This exact solution of the dynamic stiffness matrix for a multi-cracked Euler-

Bernoulli beam represents the first robust backward solution to the problem of damage detection of frame structures. Later, Civera et al. [12] investigated the consequences of damage and presence of geometric nonlinearity due to large deflections. Breathing crack behavior and high driving force amplitude were assumed. A dynamic simulation using a FE model was used with the main purpose of learning when the nonlinear effects may not be safely neglected.

The main objective of this work is to propose a reliable method of dynamic simulation of cracked buildings which can be used to give precise evaluations of structural integrity by applying advanced damage detection techniques. The method used in this work is a modification of the procedure proposed by Zheng et al. [4], which was generalized for bar members with six Degrees Of Freedom (DOF) per node to analyze complex three-dimensional structures and validated with experimental data. The proposed method was implemented in a three-dimensional building. The modal analysis results without damage were compared with those determined using a structural analysis program. Next, one damage scenario was considered, and the different damage elements were identified. The proposed procedure makes possible the simulation of multiple open cracks in bar members with local loss of stiffness in the elastic range. Matrix analysis procedures can be used to obtain the global stiffness matrix (damaged or undamaged). Hence, the Rigid Body Transformation (RBT) can be used to consider the Rigid End Zones (REZ) in the beam-column joint in case a refined analysis is needed. Material and geometric nonlinearities can also be implemented as it is done for linear elastic procedures. The method can be applied to 3D structures based on bars members to simulate elastic-linear damaged or undamaged beam-column elements. The results of the dynamic simulation showed small changes in the dynamic parameters, such as vibration frequencies and mode shapes, of the same order of magnitude as those detected in dynamic tests in real-scale structures. The advantages of this proposed method compared with more robust ones, like using solid elements, are ease of implementation in computer programs and reduced computational time.

2 Cracked Building Simulation Proposal

The proposed dynamic simulation of cracked buildings is based on the procedure proposed by Zheng et al. [4] in which only three DOF per node and the effect of the distance between the right side end node of the element and the crack location were considered. The Zheng and Kessissoglou proposal was generalized, in this study, for bar members with up to six DOF per node to permit analysis of more complex structures, such as buildings.

Zheng and Kessissoglou proposed to add an overall flexibility matrix $\mathbf{C}_{ovl}$ to the undamaged flexibility matrix $\mathbf{C}_{in}$. The total flexibility matrix for six DOF per node $\mathbf{C}_{tot}$, in a bar with an open vertical crack can be determined using Eq. (1).

$$\mathbf{C}_{tot} = \mathbf{C}_{in} + \mathbf{C}_{ovl} \tag{1}$$

where

$$\mathbf{C}_{in} = \begin{bmatrix} \frac{L_e}{EA} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{L_e^3}{3EI} & 0 & 0 & 0 & \frac{L_e}{2EI} \\ 0 & 0 & \frac{L_e^3}{3EI} & 0 & \frac{-L_e^2}{2EI} & 0 \\ 0 & 0 & 0 & \frac{L_e}{GJ} & 0 & 0 \\ 0 & 0 & \frac{-L_e^2}{2EI} & 0 & \frac{L_e}{EI} & 0 \\ 0 & \frac{L_e^2}{2EI} & 0 & 0 & 0 & \frac{L_e}{EI} \end{bmatrix} \tag{1a}$$

$$\mathbf{C}_{ovl} = \begin{bmatrix} c_{11} & -c_{12} & -c_{13} & -c_{14} & -c_{15} & -c_{16} \\ -c_{21} & c_{22} & -c_{23} & -c_{24} & -c_{25} & c_{26} \\ -c_{31} & -c_{32} & c_{33} & -c_{34} & c_{35} & -c_{36} \\ -c_{41} & -c_{42} & -c_{43} & c_{44} & -c_{45} & -c_{46} \\ -c_{51} & -c_{52} & c_{53} & -c_{54} & c_{55} & -c_{56} \\ -c_{61} & c_{62} & -c_{63} & -c_{64} & -c_{65} & c_{66} \end{bmatrix} \tag{1b}$$

where L_e is the length of the element, E is the modulus of elasticity of the material, G is the shear modulus, J is the Saint Venant torsional constant, I is the second moment of inertia. Overall flexibility matrix elements can be calculated by:

$$c_{i,j} = \int_{A_c} \frac{\partial^2 \Gamma}{\partial P_i \partial P_j} dA \qquad i,j = 1,2,3 \tag{2}$$

$$\Gamma = \frac{1}{E}\left[(K_{I1} + K_{I2} + K_{I3} + K_{I5} + K_{I6})^2 + (K_{II2} + K_{II3})^2 + \frac{1+\upsilon}{E} K_{III4}^2\right] \tag{3}$$

here K is the Stress Intensity Factor (SIF) of the uniform cross section of a beam for the fundamental modes of fracture I, II and III caused by forces P_1, P_2, …, P_6. A_c and υ are the area of the cracked cross section and the Poisson's ratio of the material, respectively. Γ is the strain energy release rate function given by Eq. (3). The forces P_i and P_j of Eq. (2) are those that are acting on the right side of the element shown in Fig. 1.

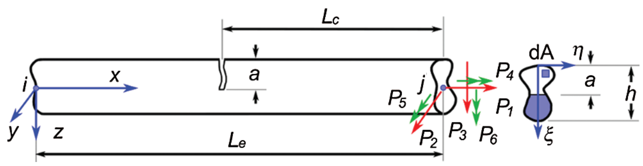


Figure 1: Proposed cracked element and forces. Adapted from [4]

The SIF factors were determined according to the procedure proposed by Ricci et al. [15] using Eqs. (4) to (8):

$$K_{I1} = P_1 \sqrt{\frac{\beta_{sP_1}}{bA}\left(\frac{A}{A_c} - 1\right)}\ ; \tag{4}$$

$$K_{I2,3} = P_{2,3} L_c \sqrt{\frac{\beta_{sP_{2,3}}}{bI_{z,y}}\left(\frac{I_{z,y}}{I_{cz,y}} - 1\right)}\ ; \tag{5}$$

$$K_{I5,6} = P_{5,6} \sqrt{\frac{\beta_{sP_{5,6}}}{bI_{z,y}}\left(\frac{I_{z,y}}{I_{cz,y}} - 1\right)}\ ; \tag{6}$$

$$K_{II2,3} = P_{2,3} \sqrt{\frac{\mathrm{X}\beta_{sP_{2,3}} 2(1+v)}{bA}\left(\frac{A}{A_c} - 1\right)}\ ; \tag{7}$$

$$K_{III4} = P_4 \sqrt{\frac{\beta_{sP_4}}{bJ}\left(\frac{J}{J_c} - 1\right)}\ ; \tag{8}$$

where β_s is the slope factor, I is the second moment of inertia, J is the Saint Venant torsional constant, b is the width of the cross section, χ is the shear factor and the subscript c indicates properties of the cracked section.

Slope factor β_s can be determined by means of a detailed FE model of the cracked area and by experimental cracked specimens. Several authors proposed slope factor values β_s. For instance, Kienzler et al. [13] achieved good agreement between analytical and experimental results in several applications using $\beta_s = 1$. Nobile [14] determined the SIF factors for a simply supported beam with rectangular cross-section, which were similar to those obtained by Ricci et al. [15] using $\beta_s = 1$. In addition, Dunn et al. [16] carried out a detailed FE model of a steel I-beam element using only bending moment to obtain the slope factor β_s. They found that β_s is a function of the crack depth ratio $\delta = a/h$ as indicated in Eq. (9).

$$\beta_s = 1.16\delta_1^{-0.374} \tag{9}$$

Once the SIF factors are calculated and the overall flexibility matrix is obtained, the stiffness matrix of a cracked element $\mathbf{K}_c$ with six DOF per node can be obtained using Eq. (10).

$$\mathbf{K}_c = \mathbf{L}\mathbf{C}_{tot}^{-1}\mathbf{L}^T \tag{10}$$

in which

$$\mathbf{L} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -L_e & 0 & -1 & 0 \\ 0 & -L_e & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{10a}$$

where the superscript T indicates the transpose of matrix $\mathbf{L}$.

3 Dynamic Simulation Algorithm of Cracked Buildings

The proposed simulation algorithm requires six fundamental parts for its implementation, which are shown in Fig. 2 and explained as follows:

- The required input data for the analysis is obtained (Step 1) as: geometric properties of the bar elements and their coordinates, damage and dynamic properties.
- If there is a cracked bar element, its geometrical properties are calculated.
- For the properties of the undamaged bars, the stiffness and mass matrices are calculated using traditional procedures (Step 2).
- For the properties of the damaged bars, the stiffness matrices are calculated using Eq. (10).
- The overall stiffness and mass matrices of the structure are obtained from the matrices of the individual bar elements.
- A random force is generated with a normal distribution and with a specified range of intensities (Step 3).
- A modal analysis of the structure is performed using eigenvalue method (Step 4).
- Using the modal parameters, the dynamic response is calculated: i.e., acceleration, velocity and displacement (Step 5).
- Finally, modes shape and dynamic response of the structure with and without damage can be compared (Step 6).

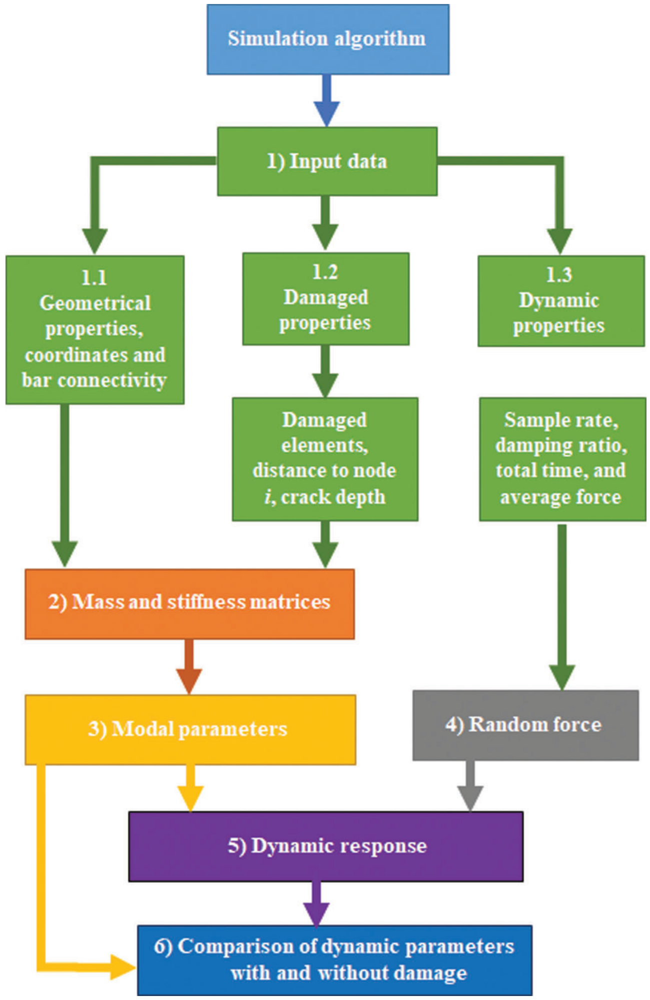


Figure 2: Flowchart of the proposed simulation method

4 Validation of the Proposed Dynamic Simulation Method

To validate the proposed simulation method, its dynamic parameters were compared with those determined by a simulation method for cracked beams proposed by Shifrin and Ruotolo [17]. Shifrin and Ruotolo tested a cracked beam experimentally to validate their procedure, which is well known and accepted in the literature. Hence, these experimental results [17] give parameters that can be used to determine the reliability of the proposed method.

The Shifrin and Ruotolo experiment consists of a cantilever beam with a length of 800 mm and a square cross section of 20 mm each side, in which the effect of a crack with a depth of 2 mm fixed at a distance of 120 mm from the clamped side and another crack with a depth of 2 mm that moved along the entire beam was analyzed, as shown in Fig. 3. The modulus of elasticity of the material was 210,000 MPa and the mass density was 7800 kg/m^3.

Shifrin and Ruotolo [17] determined the frequency relationships between the model without damage and with two cracks ω_i/ω_{oi}, where ω_i and ω_{oi} are the circular vibration frequencies of the beam with and without damage, respectively. In the model with the two cracks, the second crack moved from a distance of 150 mm to 800 mm from its clamped end. By using the proposed simulation method, the frequency relationship with and without damage was also determined, under the conditions mentioned earlier. The results of simulating the first 2 vibration frequencies with and without damage, using the calibrated model proposed by Shifrin et al. [17] and the method proposed here, are shown in Fig. 4.

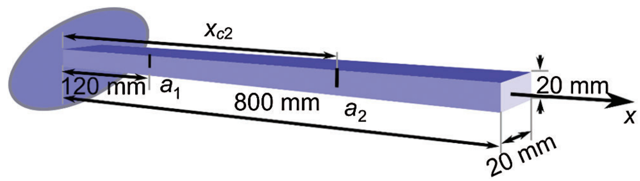


Figure 3: Experimental beam adopted for the calibration of the proposed method. Data taken from [17]

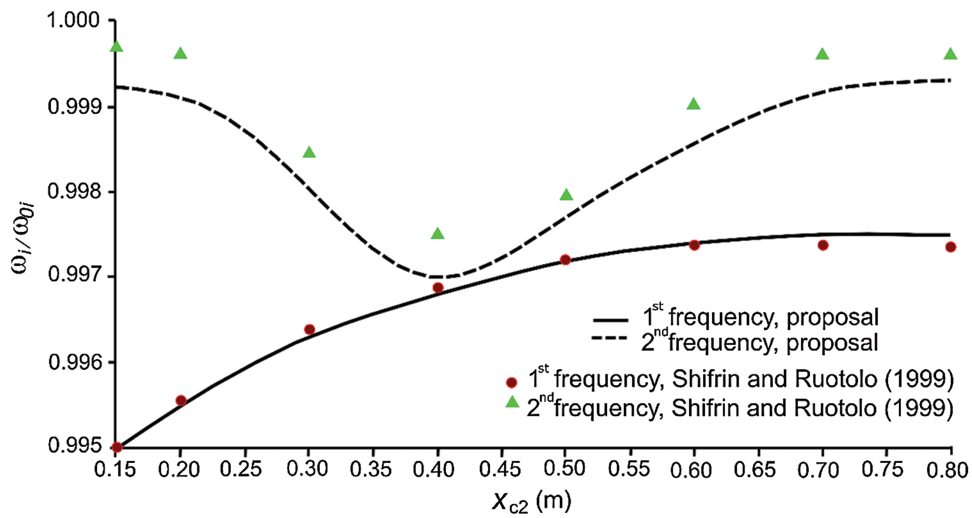


Figure 4: Comparison of frequency differences with and without damage of an experimental beam and result from 2 simulation methods. Adapted from [17]

Fig. 4 shows the difference in frequencies with and without damage between the experimental results determined by Shifrin et al. [17] and the method proposed here. The comparison of the first mode shapes between compared results indicates high correlation, with negligible differences. In the case of the second mode shape, it can be inferred that damaged circular frequency from proposed method get smaller values compared with the experimental ones. This indicates that proposed method is more sensitive to damage for the second mode shape. However, differences between compared results are still small and a good correlation was also obtained for the second mode shape. Considering these two findings, we can conclude that the proposed method has good accuracy for the dynamic simulation of cracked beams.

5 Implementation of the Proposed Method in a Three-Dimensional Building

The dynamic simulation method was implemented in a damaged Reinforced Concrete (RC) building. The model consists of a building with a Lateral Force Resisting System (LFRS) based on RC Moment Resisting Frames (MRFs), a floor system based on solid slabs that work as a rigid diaphragm, and a shallow foundation on rigid ground. Thus, a fixed-end condition was considered for the purpose of analysis. The cross sections of all the beam and column elements are 250 × 500 mm and 500 × 500 mm, respectively, with a concrete compressive strength of f’c = 25 MPa and a modulus of elasticity of the concrete of E = 22000 MPa. The geometry of the building for this study is shown in Fig. 5. Material and geometric nonlinearities were neglected because the excitation force used to analyze the building was not large enough to bring the structure to the nonlinear range.

The RC building model was chosen because this type of building is representative of the type of damage described earlier, and because the proposed dynamic simulation method could be easily implemented.

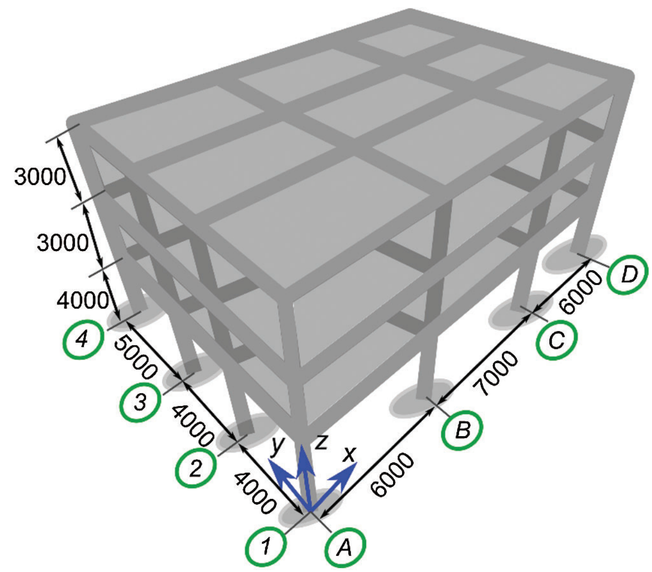


Figure 5: Overall geometry of the RC building to be evaluated (dimensions in mm)

5.1 Location of Cracks in Beams

The building was designed for office usage according to the Mexico City Design Code [18], taking into account strong column–weak beam theory, to avoid a mechanism of premature collapse and severe damage to the beams. The proposed crack pattern was obtained by increasing the live loads in different beams per story, simulating a scenario in which the building's use is changed to storage, different from what it was originally designed for. The additional live load for a storage building was distributed randomly in the selected beams per story (Fig. 6). Also, a simplified seismic Equivalent Static Force Procedure (ESFP) was applied to the building according to The Design Handbook of Civil Works for Seismic Design by the Mexican Federal Electricity Commission (CFE) [19] to observe the variation of bending moments in the beams. Most of the cracks are at the ends and center of the beams, locations where the bending moments reached maximum values due to the boundary conditions and external loads (Figs. 7, 8, 9, and Tab. 1). This model tries to represent realistic damage which is common in many parts of the world due to inadequate practices when changing the usage of existing buildings. The main objective was to create a crack pattern without a specific sequence in crack position and crack size, to test the effectiveness of the proposed procedure to simulate damage efficiently in several elements.

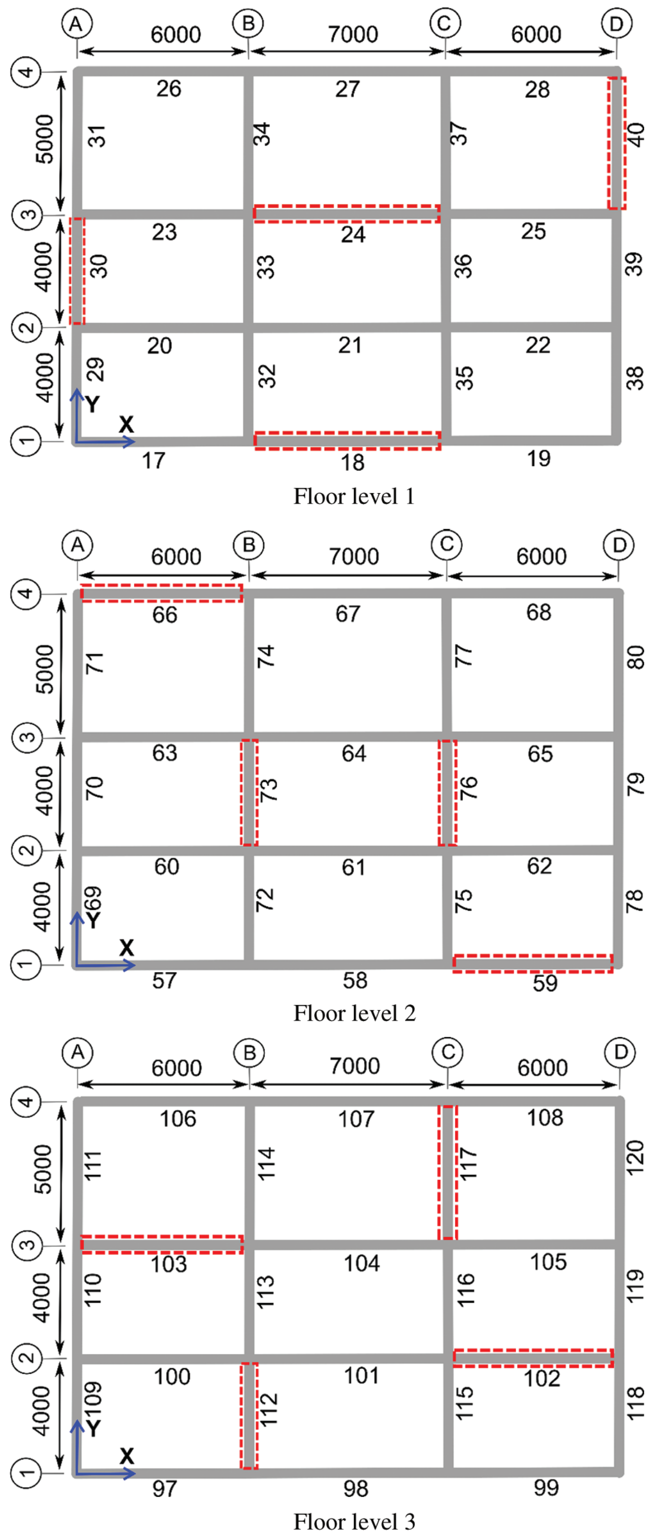


Figure 6: Location of elements with damage in levels 1, 2 and 3 (dimensions in mm). Damaged element and ID numbering

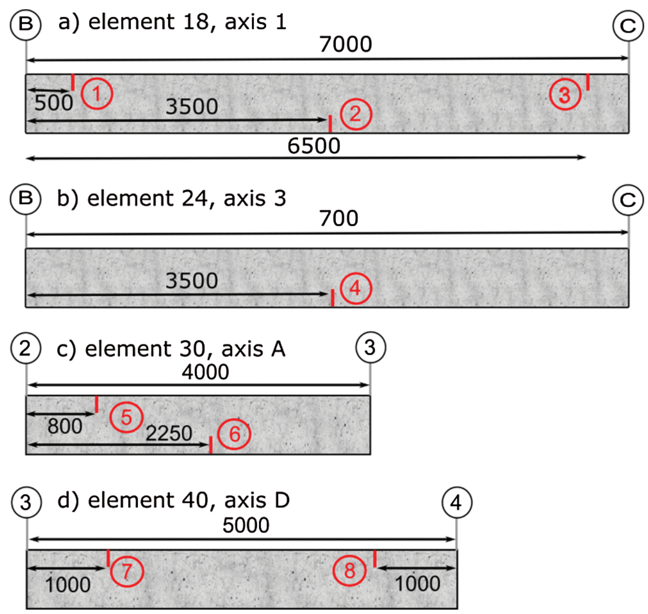


Figure 7: Damage location at first level beams (dimensions in mm). a) element 18, axis 1. b) element 24, axis 3. c) element 30, axis A. d) element 40, axis D

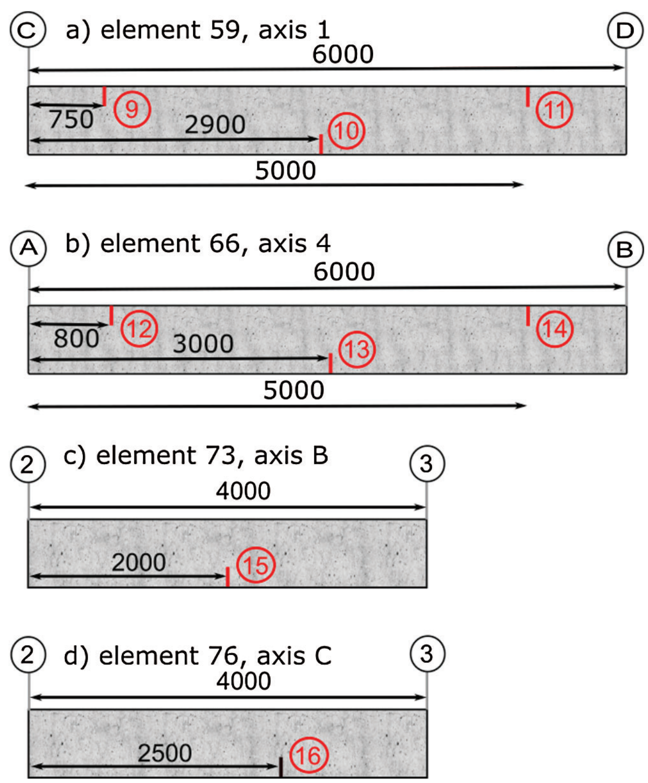


Figure 8: Damage location at second level beams (dimensions in mm). a) element 59, axis 1. b) element 66, axis 4. c) element 73, axis B. d) element 76, axis C

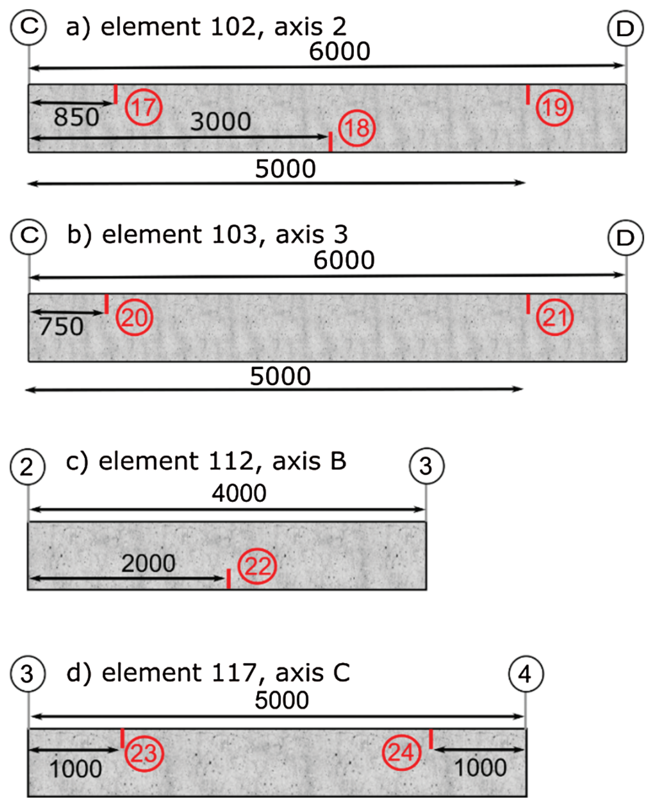


Figure 9: Damage location at third level beams (dimensions in mm). a) element 102, axis 2. b) element 103, axis 3. c) element 112, axis B. d) element 117, axis C

Table 1: Crack location and depth in damaged elements

Level	Bar number	Crack number	Distance from left node to crack (mm)	Crack depth (mm)
1	18	1	500	100
		2	3500	150
		3	6500	100
	24	4	3500	220
	30	5	800	100
		6	2250	250
	40	7	1000	120
		8	4000	120
2	59	9	750	80
		10	2900	220
		11	5000	100
	66	12	800	110
		13	3000	125
		14	5000	110

(Continued)

Table 1 (continued).

Level	Bar number	Crack number	Distance from left node to crack (mm)	Crack depth (mm)
	73	15	2000	250
	76	16	2500	100
3	102	17	850	80
		18	3000	120
		19	5000	100
	103	20	750	125
		21	5000	125
	112	22	2000	100
	117	23	1000	150
		24	4000	150

5.2 Dynamic Simulation of the Building with Damage Using the Proposed Method

The building was simulated using a three-dimensional frame numerical model with bar elements that represent the structural behavior of the columns and beams. The numerical model consisted of 64 nodes with a total of 288 DOF. The dynamic response of the cracked building was obtained using the proposed method, based on the Zheng and Kessissoglou method [4], and generalized for three-dimensional structures. The modal parameters needed for determining the solution of the dynamic response were determined by eigenvalue and eigenvector procedure using the damage stiffness matrix obtained after applying the generalized proposed procedure.

After obtaining the modal parameters (frequencies and mode shapes), the dynamic response of the cracked building was calculated using the recursive algorithm proposed by Wilson [20], which separates the response into its different modes, solves for each of them and couples them to obtain the final dynamic response. The simulated excitation was represented by a force which varied randomly in terms of both position and magnitude. This force is a representation of wind gusts and other environmental vibrations which are much smaller in magnitude, simulated along the nodes of the structural model. The magnitude of wind gust forces was calculated according to The Design Handbook of Civil Works for Wind Design by the CFE [21]. The dynamic response was determined with a sampling frequency of 60 Hz in a total time of 1000 s to obtain the modal parameters in the frequency range of interest.

6 Results and Discussion

To verify the dynamic simulation method proposed, the vibration frequencies and mode shapes of the adopted building were compared. First, this comparison was done assuming no damage in the building. Mode shapes and frequencies were determined using SAP2000® structural analysis program [22] and using the proposed procedure. Fig. 10 shows this comparison for the first 3 mode shapes and frequencies without damage using the 2 analysis methods mentioned earlier.

SAP2000® uses a more generalized procedure to determine the mass and stiffness matrix compared to the proposed procedure. Therefore, some differences between the procedures were observed. For instance, visual differences in mode shapes were observed due to different adopted perspectives. After aligning the perspectives to give the same view in both methods, the visual differences were reduced as shown in Fig. 10. Frequency differences of 0.05% were obtained, which indicates that for cases of undamaged structures, the dynamic response in free vibration is almost the same using both methods.

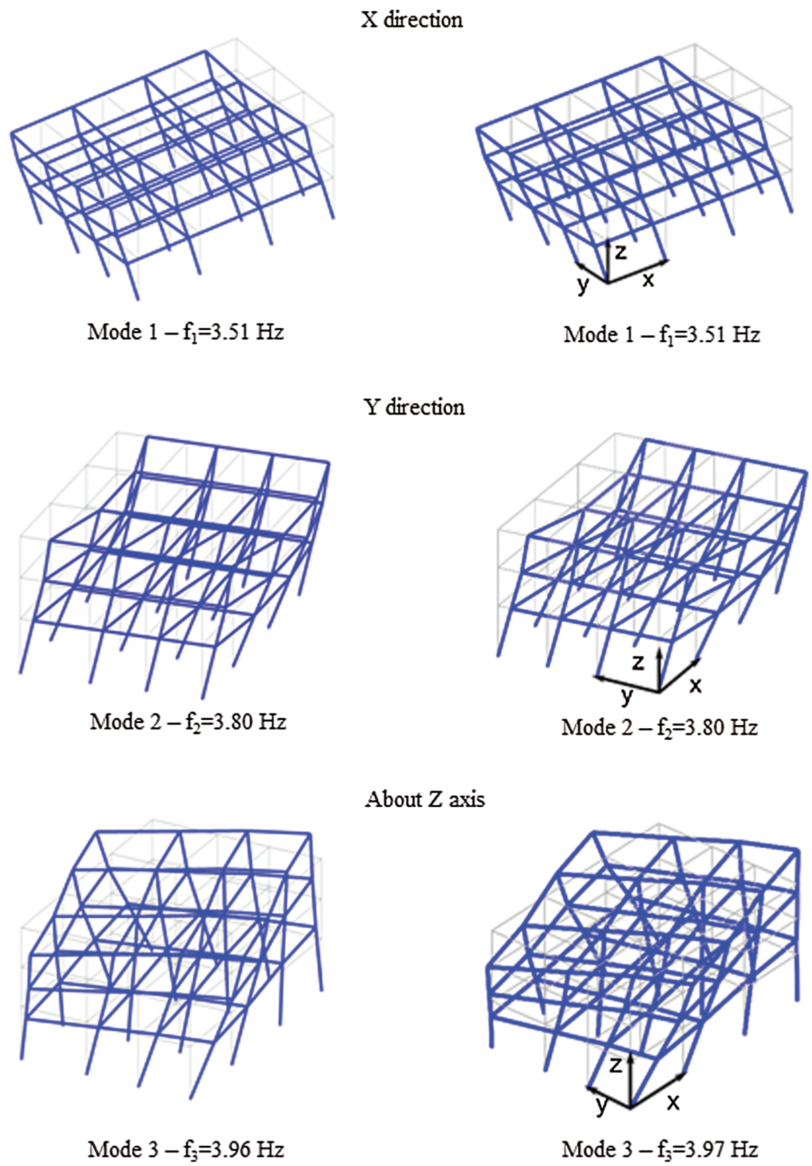


Figure 10: Comparison of the first 3 mode shapes of the building without damage calculated using SAP2000® (left column) and the proposed dynamic simulation method (right column)

To verify the accuracy of the obtained mode shapes, the Modal Assurance Criterion (MAC) method was used. This method calculates the projection of one vector over another. If both vectors are parallel, the MAC method obtains a value of one (no damage); if both vectors are not correlated at all (perpendicular to each other), MAC method gives a value of zero. The MAC method between 2 vectors is defined by Eq. (11) [23].

$$MAC\left(\varphi_i, \varphi_i^*\right) = \frac{\left|\varphi_i^T \varphi_i^*\right|^2}{\left(\varphi_i^T \varphi_i\right)\left(\varphi_i^{*T} \varphi_i^*\right)} \tag{11}$$

where φ_i represents the mode shape without damage, and φ_i^* is the mode shape with damage.

When using the MAC method to compare the modal forms obtained using the proposed method with those obtained using SAP2000®, values greater than 0.9999 were obtained for the 3 modes analyzed, which indicates that the modal forms are almost identical.

The MAC method gives values very close to 1 for very small variations of the compared vectors. Since the damage modifies modal forms only locally and subtly, it is to be expected that MAC values give values very close to 1. The Normalized Modal Difference (NMD) method was proposed with the objective to discern between several types of damage more precisely [24], and is a variant of the MAC method. It is defined in Eq. (12).

$$NMD\left(\varphi_i, \varphi_i^*\right) = \sqrt{\frac{1-MAC\left(\varphi_i, \varphi_i^*\right)}{MAC\left(\varphi_i, \varphi_i^*\right)}} \times 100 \tag{12}$$

When the compared vectors are equal, the NMD method gives 0 (no damage). The larger the difference between vectors, the larger the NMD value will be. When calculating the NMD value for a MAC of 0.9999, a value of 1% was obtained, indicating that the compared mode shapes are almost identical. It can be concluded that the modal analysis of the undamaged building using the proposed procedure was carried out correctly, since the three methods of comparison (visual mode shape, change of frequencies and change of mode shapes) show minimal differences.

Once it was concluded that the proposed method presented adequate results in the building without damage, the comparison of the first 3 mode shapes and frequencies without damage and with damage was obtained using the generalized dynamic simulation method. It is shown in Fig. 11.

To determine the damage level achieved by the damage simulation with the generalized dynamic simulation method, the NMD and MAC methods were used once again. The obtained frequency changes and NMD results are shown in Figs. 12 and 13, respectively.

As shown in Fig. 12, the frequency changes between the building with and without damage are 3.56% for the first mode, 3.14% for the second mode and 3.35% for the third mode. Although the simulated damage is intense, the frequency changes are less than 5%; a similar order of magnitude was reported in the literature (for example, [25] and [26]).

Comparing the mode shapes using the MAC and NMD method of Fig. 11, it can be seen that the NMD values increase and the MAC values decrease, with increasing mode shapes. This indicates that higher modes may display damage more readily than the first mode shape. However, higher mode shapes tend to be less accurate, since they have more complex mode shapes that require more points to obtain the dynamic response.

Finally, the accuracy of the dynamic response was determined using acceleration histories determined for a building with damage and without damage along the x and y directions (building characteristics are explained in Section 5.2). This was an attempt to represent a real modal identification. To do that, acceleration response was decimated by an order of 4 to get a final frequency of 15 Hz. This focuses modal identification on the frequency range of interest between 2 and 6 Hz. Next, acceleration response was separated into the x and y directions to facilitate detection of frequencies. The Power Spectra Density (PSD) Function was determined for all nodes in the x and y directions. Afterwards, the Singular Value Decomposition (SVD) of the PSD functions was carried out. The SVD were averaged for each direction. Resultant averaged SVD represents the PSD in each direction, as shown in Fig. 14. Next, peak frequencies were chosen for the fundamental frequency in both directions. For the undamaged building, peak frequencies were found to be 3.51 Hz and 3.80 Hz for the x and y directions, respectively. For the damaged building, peak frequencies were 3.37 Hz and 3.70 Hz for the x and y directions, respectively. These values match the numerical results of frequencies obtained using eigenvalue analysis, which were 3.51 Hz and 3.80 Hz for the x and y directions of the undamaged structure, respectively, and 3.39 Hz and 3.68 Hz for the x and y directions of the damaged structure, respectively. The differences between the numerical method using eigenvalues (given in Fig. 11) and the dynamic simulation of the experimental test using the PSD functions were caused by the random force, which introduced noise in the acceleration response.

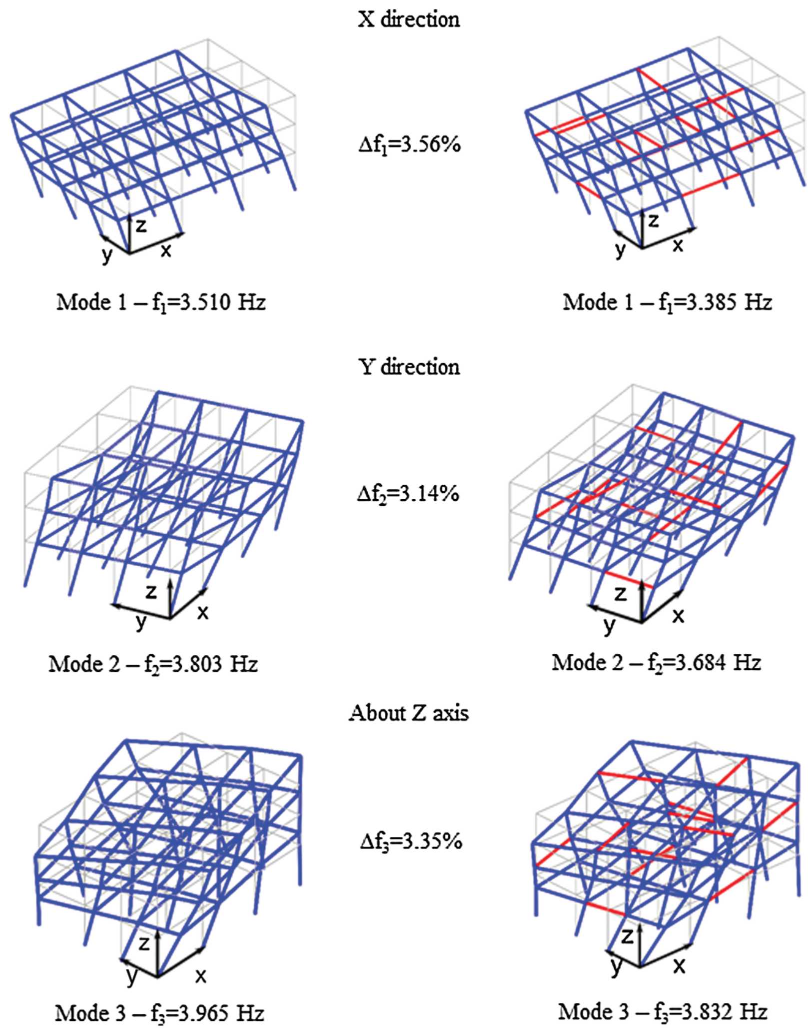


Figure 11: Comparison of the first 3 mode shapes of the building without damage (left column) and with damage (elements in red) calculated using the proposed simulation method (right column)

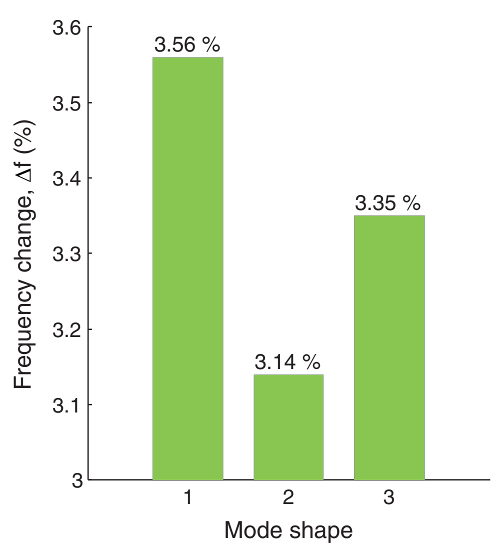


Figure 12: Frequency change method between the damaged and undamaged scenarios

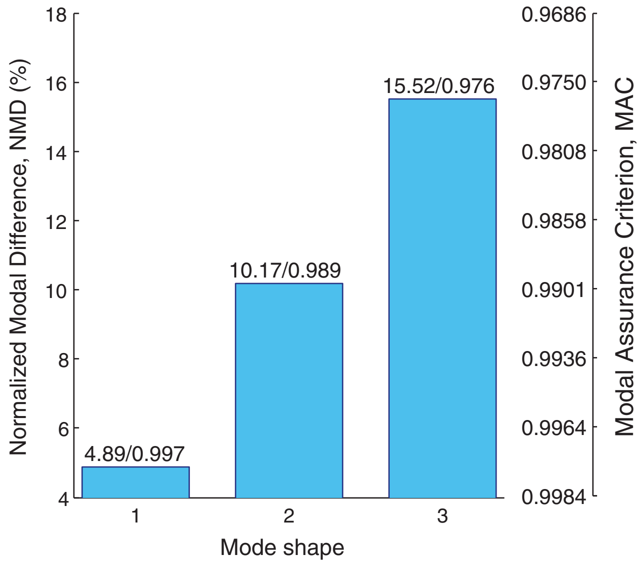
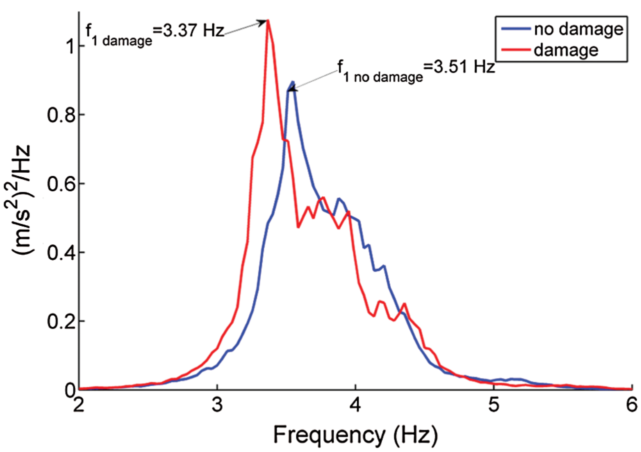
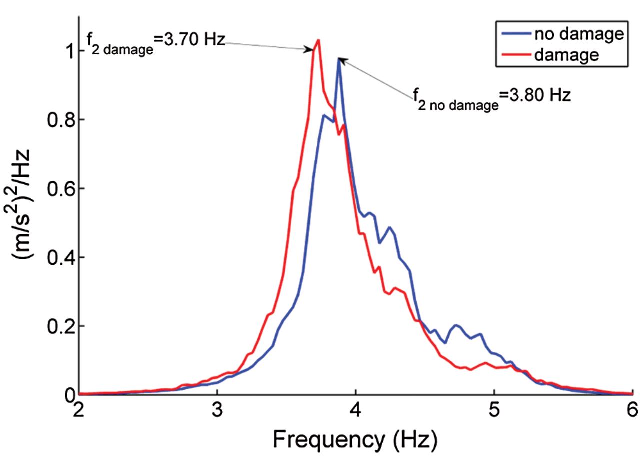


Figure 13: DMN and MAC results for the first 3 mode shapes of the damage scenario



(a)



(b)

Figure 14: Comparison of the power density spectrum, with and without damage in x (a) and y (b) directions

7 Conclusions

In this study, a method of dynamic simulation of cracked structures was implemented to evaluate behavior and structural integrity through methods of damage detection. This was carried out by generalizing the method of Zheng et al. [4] for three-dimensional structures, and validated by comparing resulting modal parameters with those obtained using the method of Shifrin et al. [17]. It was determined that the proposed method presented a behavior very similar to that of [17]. Subsequently, the proposed method was evaluated in a 3D undamaged RC building using the SAP2000® program [22], and differences in frequencies were 0.05%. Thus, the reliability of the proposed method to simulate three-dimensional frame structures was demonstrated.

From the application of the dynamic simulation method to the cracked building, it can be concluded that the differences between the building with and without damage do not change its frequencies more than 3.56%, despite the severe to moderate damage in several of the structural elements. This is consistent with the results from cracked structures in the literature, which reported a frequency change in buildings with severe damage of no more than 5%. These small changes in the vibration frequencies of cracked structures occur because the damage only affects the stiffness of the structure locally and the contribution is not significant to the overall stiffness of the system. The proposed method is a reliable way to assess the structural integrity of a damaged structure, with the advantage of being able to simulate damage to different elements and locations, and thus calibrate vibration-based damage detection methods.

Further research is required to validate this method with dynamic test results of a real building with damage. It is also recommended that the method should be expanded upon to consider geometric nonlinearities. Finally, it is recommended to simulate cracks that open and close due to the inversion of stress and/or deformation that occur in RC structures.

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